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Existence and uniqueness of positive solutions for hybrid nonlinear fractional differential equations

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Abstract

In this paper, we study the existence, uniqueness, monotonicity and positivity of the solution of hybrid nonlinear fractional differential equations by the method of upper and lower control functions and using Dhage and Banach fixed point theorem. Two examples of the obtained results is given

Keywords: Fractional differential equation, Positive solutions, Fixed point theorem, Existence and uniqueness.

Introduction

Fractional differential equations have been of great interest recently. It is caused both by the intensive development of the theory of fractional calculus itself and by the applications. Particularly, the existence of positive solution of fractional differential equations are considered indepth in the last years . Although the tools of fractional calculus have been available and applicable to various fields of study (science, engineering, physics, chemistry, biology, medicine, atomic...).

In recent years, theory of hybrid differential equations have attracted much attention.

In [1] Zhao and Sun are develop the theory of fractional hybrid differential equations involving Riemann-Liouville differential operators of order $0<\alpha<1$. Are consider fractional hybrid differential equations

$$\begin{cases} D^{\alpha}\left(\dfrac{x(t)}{g(t,x(t))}\right) = f(t,x(t)), \ t \in J, \\ x(0) = 0, \end{cases}$$

where $f:J\times\mathbb{R}\to\mathbb{R}$,and $g:J\times\mathbb{R}\to\mathbb{R}\setminus\{0\}$, are given continuous functions. They established existence of solution by a fixed point theorem in Banach algebra due to Dhage.

Matar in [2] investigated the existence of positive solutions for the hybrid fractional differential equation

$$\begin{cases} {}^{C}D_{t_0}^{\alpha}\left(\dfrac{x(t)}{g(t,x(t))}\right) = f(t,x(t)), \ t \in J = [t_0,T], \\ x(t_0) = \theta \geq 0, \end{cases}$$

where $0<\alpha\leq 1$, $f:J\times\mathbb{R}\to\mathbb{R}$,and $g:J\times\mathbb{R}\to\mathbb{R}\setminus\{0\}$, are given continuous functions such that $g(t,x(t))=\lambda>0$. By using the method of the upper and lower solution and Dhage and Banach fixed point theorems, the author obtained the existence and uniqueness of a positive solution.

In this paper, we are interested in the analysis of qualitative theory of the problems of the positive solutions of the following fractional differential equations

$$\begin{cases} {}^{C}D_{t_0}^{\alpha}\left(\dfrac{x(t)}{p(t)+\frac{1}{\Gamma(\beta)}\int_{t_0}^{t}(t-s)^{\beta-1}g(s,x(s))\,ds}\right)=f(t,x(t)),\ t\in J=[t_0,T], \\ x(t_0)=p(t_0)\,\theta\geq 0, \end{cases}$$

where $0<\alpha,\beta\leq 1$, $f,g:J\times\mathbb{R}\to\mathbb{R}$,and $p:[t_0,T]\to\mathbb{R}$,are given continuous functions. To show the existence and uniqueness and monotonicity property of the positive solution, we transform (<ref>1</ref>) into an integral equation and then by the method of upper and lower solutions and use Dhage and Banach fixed point theorems.

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