Overall Reliability Optimization of a Production System

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***Abstract*— At the present time, the competitiveness in the industrial world became more and more harsh, which requires that the system must be as reliable as possible. In most of the optimization problems, hard fitness functions are considered. These functions cannot be solved by the traditional mathematical methods. An alternative solution to the conventional approaches is the use of meta-heuristic optimization techniques, due to their ability to obtain global or near-global optimum solutions. In the present paper, we address the overall system reliability-redundancy allocation optimization problem of a production system (pharmaceutical production line), using a powerful algorithm called the Stochastic Fractal Search (SFS). The constraints of the problem are handled by resorting to the penalty function method.**

Keywords—Reliability, Production system, Optimization, Stochastic fractal search.

I. INTRODUCTION

Nowadays, high performance in the industrial field became more important than ever. The competitive level can be reached by improving the overall system reliability. The designer may practice three main methods to reach the optimal: the use of redundant components (redundancy allocation), the increase of components reliabilities (reliability allocation), or both of them (reliability-redundancy allocation) [1-3].

These methods are applied to maximize the reliability without violating the considered design constraints, such as the cost, weight and volume. The aim is to determine the optimal number of redundant components, the reliability of each one, or both [4, 5].

The meta-heuristic optimization techniques have been used as an alternative to the classical mathematical approaches to obtain global or near-global optimum solutions. Owing to their high aptness of detecting promising regions in the search space and exploring it at an accurate time. It has been proved through the last two decades that nature-inspired algorithms are attractive, because they do not apply mathematical assumption to the optimization problems and have better global search abilities over conventional optimization algorithms [6].

The goal of this paper is to increase the overall reliability of a pharmaceutical plant by applying a powerful algorithm, called the Stochastic Fractal Search (SFS).

The reminder of the paper is organized as follow: Section 2 presents the different types of the reliability optimization problems. Section 3 gives some definitions, the basic principles and the pseudo-code of the applied algorithm. Section 4 presents the numerical case study. The results with a discussion are presented in Section 5. Finally, conclusions with suggestions for further works are given in the last section.

II. PROBLEM DESCRIPTION

The reliability optimization problems mainly exist in three ways: reliability allocation (finding the reliability of the components), redundancy allocation (finding the optimal number of components to add in parallel), and the reliability-redundancy-allocation (both ways) [7]–[11].

A. Reliability allocation problem

The general mathematical formulation of the reliability allocation problem is given as follows:

$$\text{Maximize } R_S(r) = R_S(r_1, r_2, \ldots, r_m) \tag{1}$$

subject to

$$g_j(r_1, r_2, ..., r_m) \le b \tag{2}$$
$$0 \le r_i \le 1; i = 1, 2, \ldots, m$$
$$r_i \in [0,1] \subset R^+$$

where R_s=(.) is the objective function of the problem (overall system reliability), g(.) is the set of constraints, r_i is the reliability of the ith subsystem, m is the number of subsystems in the system, and b is the vector of the resource limitations.

B. *Redundancy allocation problem*

The general mathematical formulation of the redundancy allocation problem is given as follows:

$$\text{Maximize } R_S(n) = R_S(n_1, n_2, \ldots, n_m) \tag{3}$$

subject to

$$g_j(n_1, n_2, ..., n_m) \le b \tag{4}$$

$$0 \le n_i \le, n_{i\max} = 1, 2, \ldots, m$$

$$n_i \in Z^+$$

where R_s=(.) is the objective function of the problem (overall system reliability), g(.) is the set of constraints, n_i is the number of redundant components in the ith subsystem, m is the number of subsystems in the system, and b is the vector of the resource limitations.

C. *Reliability-redundancy allocation problem*

The general mathematical formulation of the redundancy allocation problem is given as follows:

$$\text{Maximize } R_S(r, n) = R_S(r_1, r_2, \ldots, r_m; n_1, n_2, \ldots, n_m) \tag{5}$$

subject to

$$g_j(r_1, r_2, ..., r_m; n_1, n_2, ..., n_m) \le b \tag{6}$$

$$0 \le r_i \le 1;\ 0 \le n_i \le n_{i\max} = 1, 2, \ldots, m$$

$$r_i \in [0,1] \subset R^+; n_i \in Z^+$$

where R_s=(.) is the objective function of the problem (overall system reliability), g(.) is the set of constraints, n_i is the number of redundant components in the ith subsystem, r_i is the reliability of the ith system, m is the number of subsystems in the system, and b is the vector of the resource limitations. The problem involves mixed variables (real-integer).

III. Solution Approach

A. *Stochastic fractal search*

The stochastic fractal search (SFS) has been proposed by Salimi in 2013 [12]. It is a new meta-heuristic algorithm inspired by the natural phenomenon of growth and uses a mathematic concept, called the fractal, which is a theory that defines the mathematical model of diffusing a particle in similar way of the patterns in nature, such as snowflakes, rivers, network, and blood vessels [13-[15].

The basic principles of the SFS are as follows [16]:

- Initialize a random sample of points cloud.
- Move the points using a diffusion process (random walk).
- Update the position of each point according to the quality of the solution and identify the best one.
- Repeat the procedures until the stopping criterion is reached. Figure 1 illustrates its flowchart [15].

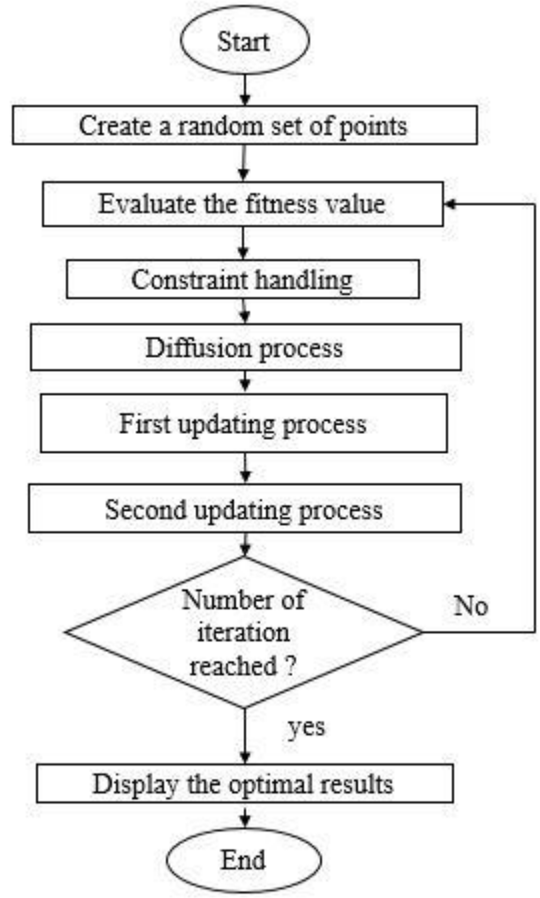

Fig.1 Flowchart of the implemented SFS.

Since the three reliability optimization problems described in the previous section are constrained, a constraint handling method should be used. In this paper, a penalty function is implemented to keeping the solution inside the search space.

The overall baseline of the implemented SFS is [12]:

- Input the parameters.
- Generate the initial vector of λ solution points: each solution point is a combination of reliabilities $r_1 = 1, 2, ...m.$ and /or number of components n_i .
- Evaluate the objective function.
- Constraint handling.
- Diffusion process (search for new solution): the new solution are created by diffusing new solution points according to Gaussian random walks.
- Two updating process: this step is performed in order to undertake the intensification process of the

evolutionary algorithm and preserve good solutions for the next iteration.

- Stopping criterion: once the number of iteration is reached, stop and display results; otherwise go to step 3.

The pseudo-code of the stochastic fractal search is described in Algorithm 1 [12].

Algorithm 1: Pseudo-code of the implementation SFS.

1-Input the parameters : λ, N_{iter} , ϕ_j

2-Generate an initial vector of solution points

3-While z < N_{iter}

4-Evaluate the fitness function according to equation

$$Objective\ function = -R_s(r_1 r_2,...,r_m, n_1, n_2...,n_m)$$

5-Constraint handling (according to the equations in section 2.4)

6-Difussion process performed by equations

$$GW_1 = Gaussian(\mu_{BP}, \sigma) + (\varepsilon \times BP - \varepsilon \times P_i')$$
$$GW_2 = Gaussian(\mu_{BP}, \sigma)$$

7-First updating process performed by equations

$$P_{ai} = \frac{rank(P_i)}{\lambda}$$

8-Second updating process performed by equations.

$$P_i'' = P_i' - \varepsilon' \times (P_t' - BP) \quad If\ \varepsilon' \le 0.5$$
$$P_i'' = P_i' - \varepsilon' \times (P_t' - P_r') \quad If\ \varepsilon' > 0.5$$

9-End while

10-Display the results.

B. Constraint handling technique

A handling method is applied since the reliability optimization problems are constrained and hold in nonlinear constraint functions, such as limits on volume, weight and cost. The constraint handling methods can be listed into five categories, according to their fundament [15,16]: (i) penalty function, (ii) pre-serving feasibility of solutions, (iii) decoders, (iv) making a distinction between feasible and infeasible solutions, and (v) hybrid methods.

In the present work, a penalty function is implemented in the algorithm so as to make sure that the search is guided towards the best solutions into the feasible space. The trick is to add penalty terms to the objective function value [17, 18], in order to decrease the fitness of the infeasible solutions. It is applied as follows:

$$Fitness\ value = -R_S\left(r_1, r_2, \ldots, r_m, n_1, n_2, \ldots, n_m\right) + \psi\left(r_1, r_2, \ldots, r_m, n_1, n_2, \ldots, n_m\right) \quad (7)$$

where $\psi\left(r_1, r_2, \ldots, r_m, n_1, n_2, \ldots, n_m\right)$ is the penalty term, calculated as follows:

$$\psi\left(r_1, r_2, \ldots, r_m, n_1, n_2, \ldots, n_m\right) = \sum_{j=1}^{M} \phi_j . \max\left(0, g_j\left(r_1, r_2, \ldots, r_m, n_1, n_2, \ldots, n_m\right)\right)^2 \quad (8)$$

where ϕj is a positive constant, called the penalty parameter or penalty factor.

IV. Numerical Case Study

The meta-heuristic mentioned in the previous section is illustrated through a production system (a pharmaceutical plant) in order to maximize its overall reliability under a set of design constraints. Figure 2 shows the configuration of this plant.

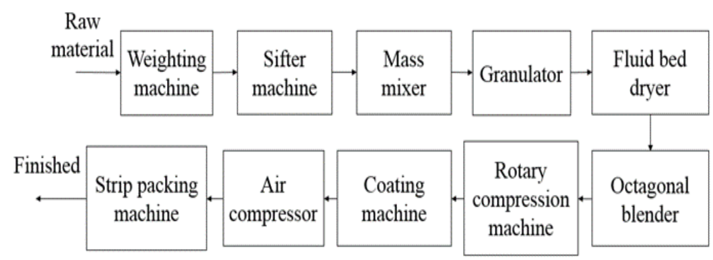

Fig. 2 Pharmaceutical plant.

The pharmaceutical plant (see figure 1) consists of ten subsystems connected in series, weighing machine, shifter machine, mass mixer, granulator, fluid bed dryer, octagonal blender, rotary compression machine, coating machine, air compressor, and strip packing machine [19], the raw material is transferred from a subsystem to another chronologically till the end of the production line.

The nonlinear mathematical model of the above system is formulated as follows [19]:

$$\text{Maximize } R_S = \prod_{i=1}^{4}\left[1-\left(1-r_i\right)^{n_i}\right] \quad (9)$$

subject to

$$g_1(r,n) = \sum_{i=1}^{4} C(r_i)(n_i + exp(\frac{n_i}{4}) \le C \quad (10)$$

$$g_2\left(r,n\right) = \sum_{i=1}^{4} v_i n_i^2 \le V$$

$$g_3\left(r,n\right) = \sum_{i=1}^{4} w_i (n_i * exp\left(\frac{n_i}{4}\right)) \le W$$

$$0.5 \le r_i \le 1-10^{-6}, r_i \in \left[0,1\right]$$

$$1 \le n_i \le 10, n_i \in \mathbb{Z}^+$$

Table 1 reports the data of the system.

Table 1. Data of the system.

Subsystem i	$10^5 \alpha_i$	β_i	v_i	w_i	C	v	w	$T(h)$
1	0.611360	1.5	4	9	553	289	483	1000
2	4.032464	1.5	5	7				
3	3.578225	1.5	3	5				
4	3.654303	1.5	2	9				
5	1.163718	1.5	3	9				
6	2.966955	1.5	4	10				
7	2.045865	1.5	1	6				
8	2.649522	1.5	1	5				
9	1.982908	1.5	4	8				
10	3.516724	1.5	4	6				

V. Results and Discussion

The implemented algorithm has been programmed using MATLAB R2014a and run on an Intel Core I7 with 6 GB of RAM and (2.20 GHz, windows 7, 64 bits).

Knowing that the specific parameters of the algorithm (as shown in Table 2) where chosen based on trial-and-error method in order that the algorithm can work in the most optimal way.

Table 2. SFS Parameters.

Parameters	values
Number of points	40
Number of decision variables	20
Maximum diffusion number	2
Diffusion walk	0.25

The numerical results of the case study are reported in Table 3, where 10 independent runs have been considered. The overall system reliability of each run has been reported.

From Table 4, it can be observed that the maximum overall reliability obtained is 0.958862840935210 and the minimum is 0.957977502246962, with an average reliability of the ten runs of 0.958772542523011.

Table 3. Results of ten runs.

Run #	R_S
1	0.958862128800166
2	0.957977502246962
3	0.958856112842219
4	0.958862840935210
5	0.958856566211402
6	0.958860104634159
7	0.958862404094019
8	0.958862655370792
9	0.958862348385879
10	0.958862761709307

Moreover, taking into account the criterions of small standard deviation (see Table 4), we can clearly observe that the robustness of the implemented approach for solving the system reliability optimization problem due to its small standard deviation.

From Table 5, we can observe that the redundancy and the reliability of each subsystem of the highest overall reliability.

Table 5. Numerical results of the best run.

i	n_i	r_i
1	3	0.881778125377022
2	3	0.821335699420238
3	3	0.825623648413994
4	3	0.825337848700228
5	3	0.864129695422615
6	3	0.832814443484730
7	3	0.846128972255311
8	3	0.837001933011588
9	3	0.847454216112727
10	3	0.826528292262859
$\boldsymbol{R_S}$	0.958862840935210	

Table 4. Statistical results.

	Worst	Mean	Best
Rs	0.957977502246962	0.958772542523011	0.958862840935210
Standard deviation		2.650244646203211E−04	

VI. CONCLUSIONS

The main goal of this paper was to present and implement a powerful meta-heuristic algorithm (namely the SFS) in order to maximize the overall reliability of a production system under a set of design constrains. A pharmaceutical plant containing ten subsystems connected in series has been considered for this purpose. A penalty function has been implemented in order to eliminate the infeasible solutions.

Future works will be devoted to the development of a multi-objective meta-heuristic in order to get better performances for this kind of optimization problems and allows the decision maker to have more information about both objectives, unlike the single objective approach.

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