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Improving the Voltage THD of the Inverter Outputs using Selective Harmonic Elimination Technique

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Abstract

This dissertation focuses on analyzing and comparing various control techniques for voltage source inverters (VSIs), with the goal to improve the quality of the output waveform by minimizing Total Harmonic Distortion (THD). Three conventional methods are explored: 180° conduction switching, Sinusoidal PWM (SPWM), and Space Vector Modulation (SVM). The study then highlights the Selective Harmonic Elimination Technique (SHE) control method, which uses mathematically computed switching angles to eliminate selected low-order harmonics.

MATLAB/Simulink simulations and mathematical modeling are used to evaluate each method's performance in terms of THD, waveform accuracy, and implementation complexity. The results show that SHE controls offers significantly better harmonic suppression, making it a suitable solution for high-performance power electronics applications.

Keys works : Selective Harmonic Elimination, THD, MATLAB

الملخص:

تهدف هذه المذكرة إلى دراسة وتحليل طرق التحكم المختلفة في العواكس الكهربائية، مع التركيز على تحسين جودة التوتر الخارج من خلال تقليل التشوه التوافقي الكلي .(THD) تم تناول ثلاث تقنيات تقليدية: طريقة التشغيل والإيقاف بـ 180 درجة، وتقنية تعديل عرض النبضة الجيبية(SPWM) ، وتقنية التعديل الشعاعي الفضائي .(SVM) بعد ذلك، تم التركيز على تقنية التحكم بالحساب المسبق(SHE) ، والتي تعتمد على حساب زوايا التبديل مسبقًا لإلغاء التوافقيات ذات الرتب المنخفضة.

تم استخدام النمذجة الرياضية والمحاكاة بواسطة برنامج MATLAB/Simulink لمقارنة الأداء من حيث THD ، الدقة، والتعقيد في التنفيذ. أظهرت النتائج أن طريقة التحكم بالحساب المسبق تحقق تشوهًا توافقيًا أقل بكثير مقارنة بالطرق الأخرى، مما يجعلها خيارًا فعالًا للتطبيقات التي تتطلب طاقة نظيفة وفعالة.

الكلمات المفتاحية: إزالة التوافقيات الانتقائية، معامل التشوه، برنامج ماتلاب.

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DEDICATION

All thanks and praise are due to God alone, with no partner after Him

He who does not thank people does not thank God.

We extend our thanks first to our mothers,

without whom we would not have reached this point.

Perhaps one of the reasons for our gratitude is to thank our esteemed families

who have been patient with us and who,

by the grace of God and then their grace,

have enabled us to achieve this work.

May God reward them on our behalf.

And to our colleagues at the college and everyone who

has extended a helping hand, whether near or far, even with a sincere prayer.

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List of abbreviations

AC	Alternatif current
DC	Direct current
UPS	Uninterruptible power supply
VSI	Voltage source inverters
V.F.I	Voltage fed inverters
IGBT	Insulated gate bipolar transistore
MOSFET	Metal oxide semiconductor fieled effeet
PWM	Puler width modulation
EEE	Electrical and electronics engineering
IM	Induction motor
RMS	Root mean square
DSPs	Digital signal processorcs
FPGA	Field programmable gate array
CSI	Current Source Inverters
CFI	Current-fed inverter
m-MC	The modular multilevel converter
SM	Sub-modules
VSC	Voltage source converter
THD	Total harmonic distortion
SPWM	Sinusoidal Pulse Width Modulation
SVM	Space Vector Modulation
pwm	Pulse Width Modulation
svpwm	Space Vector Pulse Width Modulation
PMSM	permanent magnet synchronous motors
UPS	Uninterruptible power supply systems
SHE	Selective Harmonic Elimination

General Introduction

General Introduction

In the context of today's rapidly evolving technological landscape, the demand for efficient and high-quality electrical power conversion has become more critical than ever. A wide array of modern applications ranging from industrial motor drives and renewable energy systems such as photovoltaic installations, to electric vehicles and smart grids rely heavily on voltage source inverters (VSIs) to perform the essential function of converting direct current (DC) into alternating current (AC). This conversion process must not only be efficient, but also capable of producing output waveforms with high fidelity to minimize energy losses and ensure the reliable operation of downstream equipment.

One of the most persistent and challenging issues in inverter operation is the mitigation of total harmonic distortion (THD). Harmonic distortion occurs when the output waveform deviates from a pure sine wave due to the presence of unwanted frequency components typically introduced by switching operations. Elevated levels of THD can lead to significant energy losses, cause electromagnetic interference (EMI) with nearby devices, accelerate thermal stress, and reduce the lifespan of sensitive electrical equipment [1]. Therefore, reducing THD while maintaining the desired voltage amplitude and frequency has become a primary objective in power electronics research.

To address this issue, a variety of inverter control strategies and modulation techniques have been developed. Traditional methods, such as six-step conduction, offer simplicity and ease of implementation but generally suffer from high harmonic content and limited control flexibility. On the other hand, pulse width modulation (PWM) techniques such as sinusoidal PWM and Space vector modulation provide improved waveform quality and finer control over output voltages. However, even these advanced methods often struggle to fully eliminate low-order harmonics without the aid of bulky and expensive filters [2].

In recent years, pre-calculation control also known as Selective Harmonic Elimination (SHE) has gained significant attention as a promising alternative. This technique involves calculating optimal switching angles in advance to cancel out selected low-order harmonics while preserving the fundamental component. By precisely timing the inverter's switching events, this method enables the generation of quasi-sinusoidal waveforms with greatly reduced harmonic content. Moreover, it shifts the remaining harmonics to higher frequencies where they are easier to filter, allowing for smaller and more cost-effective filter components [3].

This dissertation presents a comprehensive and comparative study of the pre-calculation control approach in contrast with conventional methods, including six-step switching and various PWM techniques. The investigation is grounded in both analytical modeling and practical implementation through MATLAB/Simulink simulations. Key performance indicators such as waveform accuracy, harmonic distortion, and control complexity are evaluated in detail.

The ultimate aim of this work is to demonstrate how pre-calculation control can contribute to the advancement of inverter design, offering a compelling balance between harmonic reduction and implementation feasibility. By proving its effectiveness through quantitative results and simulation-based validation, this research contributes to the broader goal of developing more reliable, compact, and energy-efficient power electronic converters suited for the demands of next-generation electrical systems.

This dissertation has been divided into three chapters. The first chapter talked about the electrical inverter, its operating principle and some of its types. The second chapter talked about the three traditional control techniques: the 180-degree method, the vertical method and the Sines Triangle method. The third chapter focused only on the Selective Harmonic Elimination, which is called SHE , and we presented the results and comparison.

Chapter I:

General information about the inverter

I.1. Introduction

The development of electrical power conversion systems has become a fundamental pillar of the modern era, thanks to rapid advances in the field of power electronics. Among the most important components of these systems is the inverter, which plays a pivotal role in converting power from direct current (DC) to alternating current (AC). This is critical in a vast number of industrial and domestic applications, particularly in renewable energy systems, motor drives, and backup power grids [4].

Inverters come in a variety of shapes and designs depending on their application, including single-phase inverters, three-phase inverters, and multi-level inverters. These types differ in their structure, load handling capacity, output signal quality, and control complexity [5].

Understanding the operating principle of an inverter is a fundamental step in any study aimed at improving its performance or developing new control strategies. The primary task of an inverter is to produce alternating current at a specific frequency and voltage from a DC source, while minimizing harmonics and achieving maximum efficiency. Achieving this goal requires a detailed study of the inverter's structure, classifications, and operation, as well as the characteristics of the voltage and current generated [6].

This chapter aims to provide a basic and comprehensive overview of electrical inverters, including their definition, types, characteristics, and areas of use. It also paves the way for the following chapters, which address the various control techniques used in these devices.

I.2. Inverter (Onduleur) :

Inverse, or what is called in French as (onduleur) it is an electronic device that converts direct current(DC) into alternating current(AC) and is widely used in energy extracted from solar panels Uninterruptible power supply systems (UPS).

Figure I.1: Inverter

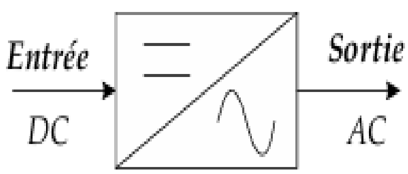

Figure I.2 : Inverter symbol

I.3. Types of inverters:

There are many types of electrical inverters, including:

I.3.1 Voltage Source Inverters – VSI:

Voltage Source Inverters is a type of converter that converts DC voltage to AC voltage. It is also known as voltage-fed inverter (VFI). A VSI consists of a DC power source, transistors (thyristors, IGBT, MOSFET, etc.) for switching, and a DC link capacitor (to provide filtering and minimize fluctuations). An ideal VSI keeps the voltage constant throughout the process.

The DC input source can be batteries stacked in series or parallel, photovoltaic cells, or rectified output from another AC power source. It can be used in both single phase and three phase topologies.

Voltage Source Inverter can operate in two modes:

- PWM (Pulse width modulation) mode.
- Square wave mode

Voltage source inverter is available in three configurations - single-phase half-bridge, single-phase full-bridge, and three phase [2]

I.3.2.Single-Phase

In a single phase there are two types:

I.3.2.1. Half-Bridge Inverter(Onduleur en demi-ponts):

In Figure 1.3 we have a half-bridge inverter circuit consisting of two capacitors and two switches T1 and T2. We cannot open them at the same time because it will produce an electrostatic discharge.

if T1= on and T2=off >>

$$V0 = Vs/2$$

$$T2 = Vs \quad . \quad Ig2\ (IT2) = Is$$

if T1= off and T2=on >>

$$V0 = -Vs/2$$

$$T1 = Vs \ . \quad Ig1\ (IT1) = Is$$

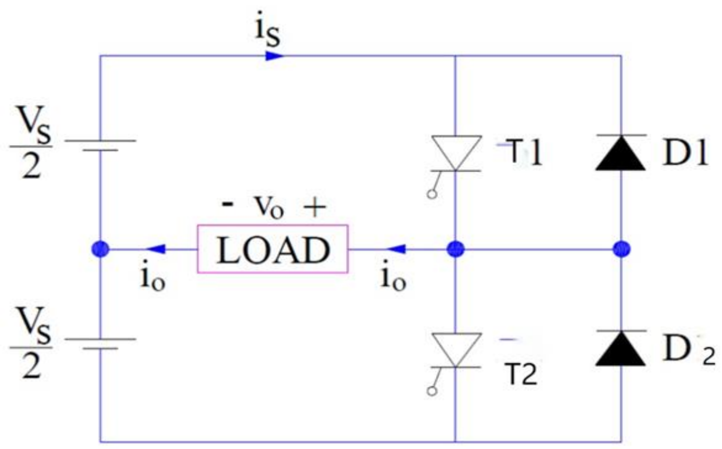

Figure I.3: Half-bridge inverter

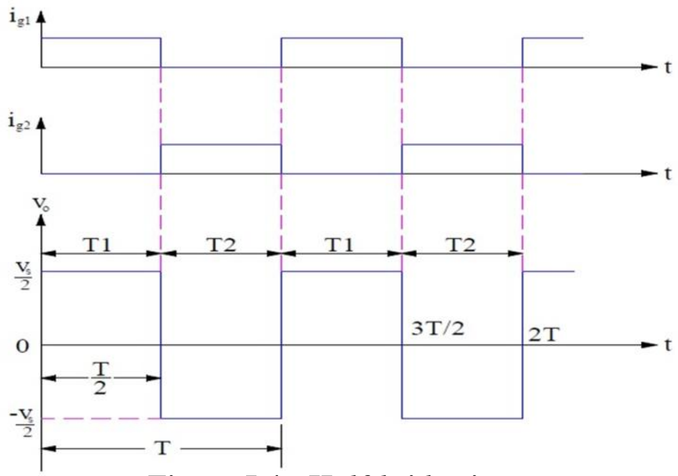

Figure I.4 : Half-bridge inverter

I.3.2.2. Full-Bridge Inverter (Onduleur en ponts):

The full bridge inverter is controlled by switches that operate periodically with time t in two periods. In the first period, the time period is (0<t<T/2), in which switches K1, K2 are closed and K3, K4 are open. In the second period, it is (T>t>T/2), in which switches K1, K2 are open and K3, K4 are open.

"T/2>t>0"

V0=E . T1=T2=-D1=-D2=0

is=so

"T> t> T/2"

V0= -E . T3=T4=-D3=-D4 =0

Is=i0

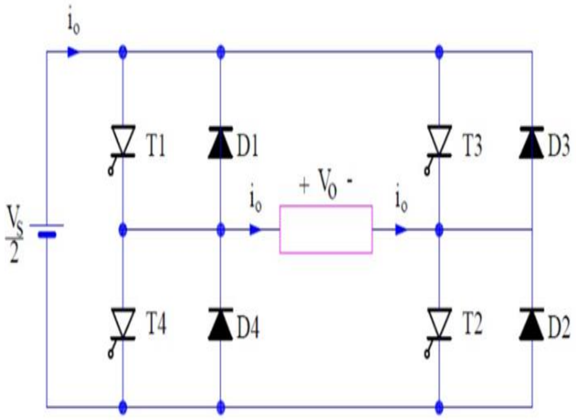

Figure I.5 : Half bridge inverter working diagram

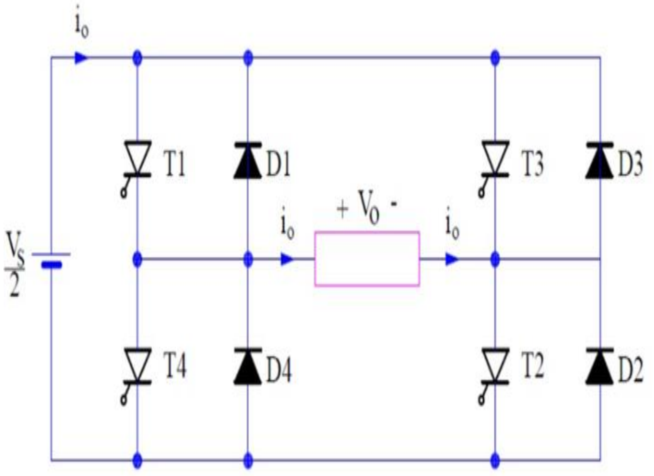

Figure I.6 : **Full-bridge inverter**

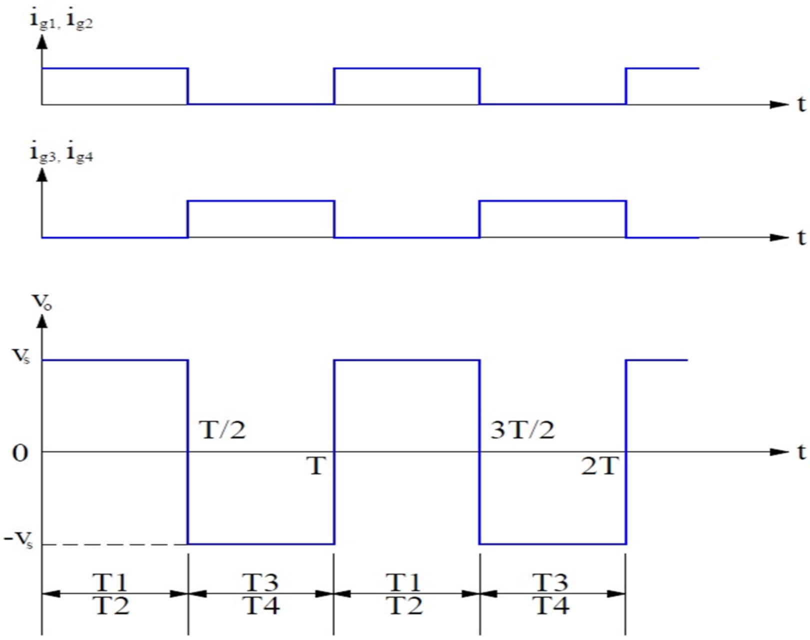

Figure I.7 : Full-brigdge inverter diagram

I.3.3.Three-Phase Inverter:

I.3.3.1. Three-Phase Full-Bridge Inverter:

The three-phase inverter consists of three switching cells, each containing two switches that can be controlled for opening and closing, along with two diodes connected in anti-parallel to these switches to allow bidirectional current flow.

The circuit includes a reversible DC voltage source EEE. The load is typically a rotating field machine and therefore behaves as a three-phase source of alternating currents (which are often assumed to be sinusoidal). For demonstration purposes of the operating principle, a balanced three-phase resistive load is used. [8]

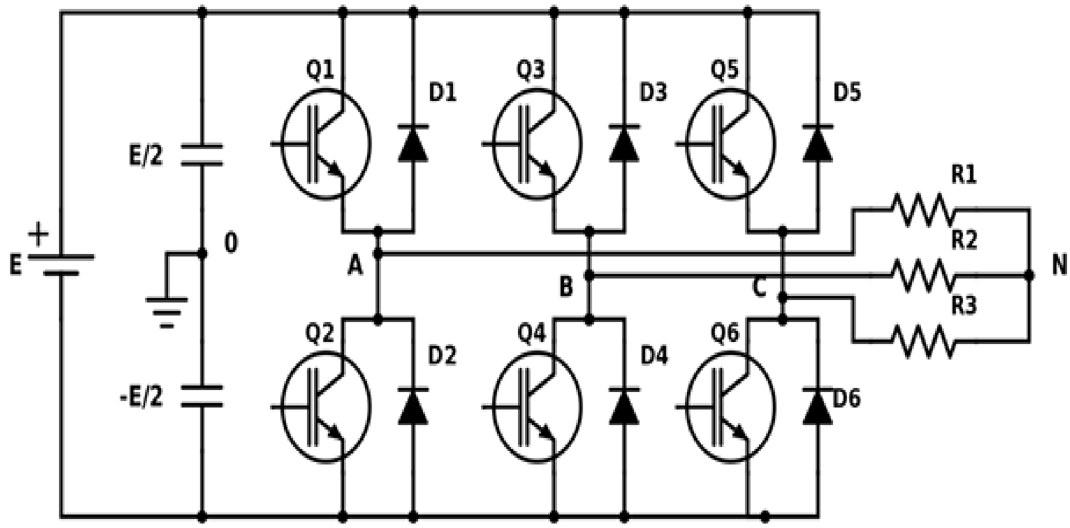

Figure I.8: Three-phase full-bridge inverter

$$\text{Vao=Va-Vo} \begin{cases} +\frac{E}{2}\ when\ closed\ Q1 \\ -\frac{E}{2}\ when\ open\ Q1 \end{cases} \quad \textbf{(I.1)}$$

The same applies to voltage Vbo and Vco.

So we conclude the complex voltage.

$$\begin{cases} Vab = Va - Vb = (Va - Vo) - (Vb - Vo) \\ Vbc = Vb - Vc = (Vb - VO) - (Vc - Vo) \\ Vca = Vc - Va = (Vc - Vo) - (Va - Vo) \end{cases} \quad \textbf{(I.2)}$$

Since we have a balanced system, the sum of the efforts equals zero. We write:

$$\{ia + ib + ic = 0\ so\ Van + Vbn + Vcn = 0 \quad \textbf{(I.3)}$$

We can also write it:

$$\{(Vao + Von) + (Vbo + Von) + (Vco + Von) = 0 \quad \textbf{(I.4)}$$

So,

$$\{Von = \frac{1}{3}(Vao + Vbo + Vco) \quad \textbf{(I.5)}$$

We can also write it:

$$\begin{cases} Vao = Van + Vno \\ Vbo = Vbn + Vno \\ Vco = Vcn + Vno \end{cases} \quad \textbf{(I.6)}$$

We substitute the last two equations and simplify, and we find:

$$\begin{cases} Van = \frac{2}{3}Vao - \frac{1}{3}Vbo - \frac{1}{3}Vco \\ Vbn = -\frac{1}{3}Vao + \frac{2}{3}Vbo - \frac{1}{3}Vco \\ Vcn = -\frac{1}{3}Vao - \frac{1}{3}Vbo + \frac{2}{3}Vco \end{cases} \quad \textbf{(I.7)}$$

We compensate:

$$\begin{cases} Vao = Q1.E \\ Vbo = Q2.E \\ Vco = Q3.E \end{cases} \quad \textbf{(I.8)}$$

We find:

$$\begin{cases} Van = \frac{E}{3}(2Q1 - Q2 - Q3) \\ Vbn = \frac{E}{3}(-Q1 + 2Q2 - Q3) \\ Vcn = \frac{E}{3}(-Q1 - Q2 + 2Q3) \end{cases} \quad \textbf{(I.9)}$$

We write the previous equations in matrix form, so they become as follows:

$$\begin{bmatrix} Van \\ Vbn \\ Vcn \end{bmatrix} = \frac{E}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix} \quad \textbf{(I.10)}$$

Suppose there are three bridges Q1, Q2, Q3. Each bridge has two switches Q1 and Q'1... If Q1 is closed, then Q'1 is open. Two switches cannot be open in one bridge.

The first bridge Q1 $\begin{cases} Q1 = F \\ Q'1 = 0 \end{cases}$

The same applies to the other two bridges Q2 and Q3 .

Table I.1 : Operation of three-phase voltage inverter

N	Q_1	Q_2	Q_3	V_{AB}	V_{BC}	V_{CA}	V_{AN}	V_{BN}	V_{CN}	i_{Q1}	i_{Q2}	i_{Q3}	i
1	F	F	F	0	0	0	0	0	0	i_A	i_B	i_C	0
2	F	0	F	E	-E	0	E/3	-2E/3	E/3	i_A	0	i_C	$-i_B$
3	F	F	0	0	E	-E	E/3	E/3	-2E/3	i_A	i_B	0	$-i_C$
4	F	0	0	E	0	-E	2E/3	-E/3	-E/3	i_A	0	0	i_A
5	0	F	F	-E	0	U	-2E/3	E/3	E/3	0	i_B	i_C	$-i_A$
6	0	0	F	0	-E	E	-E/3	-E/3	2E/3	0	0	i_C	i_C
7	0	F	0	-E	E	0	-E/3	2E/3	-E/3	0	i_B	0	i_B
8	0	0	0	0	0	0	0	0	0	0	0	0	0

I.3.4.Current Source Inverters – CSI

Current Source Inverters is a type of DC-AC Inverter that converts DC input current into AC current at a given frequency. The frequency of the output AC current depends on the frequency of the switching devices such as thyristors, transistors, etc. It is also known as a current-fed inverter (CFI) and the input current of this inverter remains constant. In an ideal CSI, the output current is independent of the load. However, the output voltage is dependent on the nature of the load. Current source inverters are ideal for various applications such as speed control drives for AC motors, synchronous motor starting, plasma generators, induction heaters, UPS units, IM motors, lagging Var compensators, and power factor correction unit [7]

I.3.4.1. Single Phase Current Source Inverter :

The circuit diagram of a single-phase CSI is shown below :

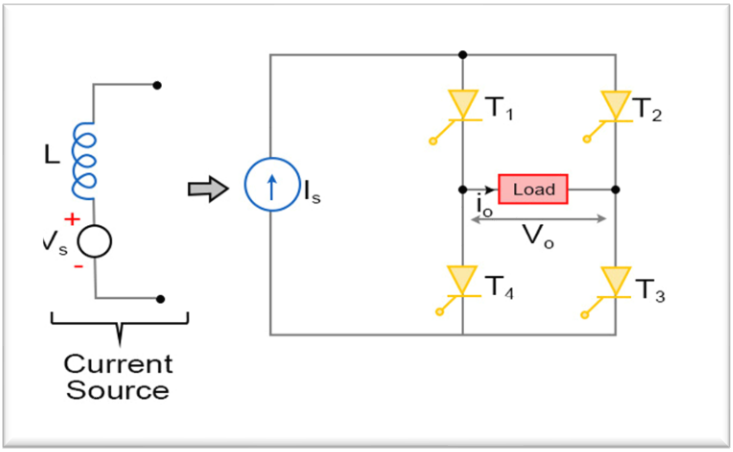

Figure I.9: Single phase iverter circuit

There are two waves when the current is positive(+i0) and when it is negative(-i0).

First wave:

In the first wave, thyristors T1 and T3 are closed, and T2 and T4 are open. This causes a current to pass through T1 and T3, and here the current (i0) is positive because it passes in the same direction as the current (is).

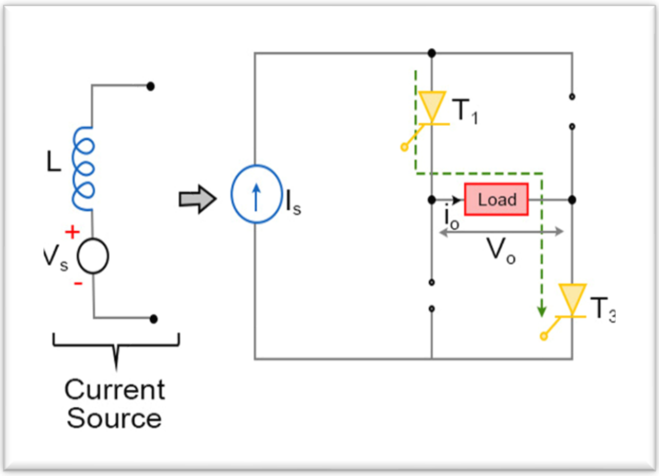

Figure 1.10 First wave of a single phase inverter circuit

the second wave:

In a second wave, which is opposite to the first wave, thyristors T1 and T3 are open, and T2 and T4 are closed. This causes a current to pass through T2 and T4, and here the current (i0) is negative because it passes opposite the direction of the current (is).

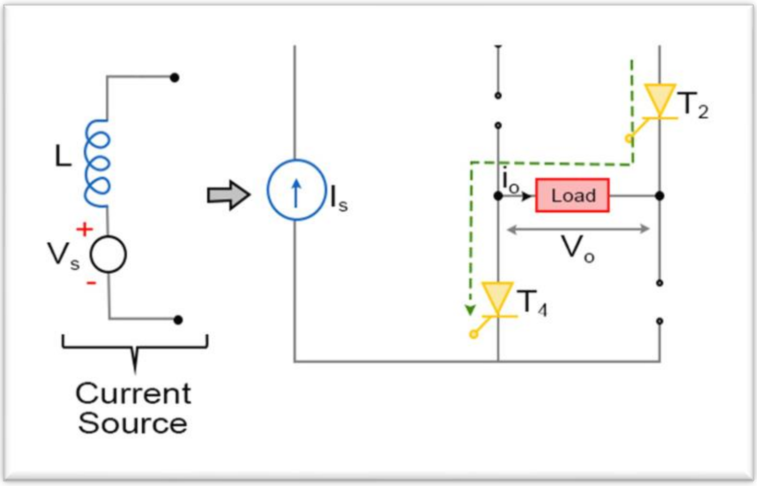

Figure I.11 Second wave single phase inverter

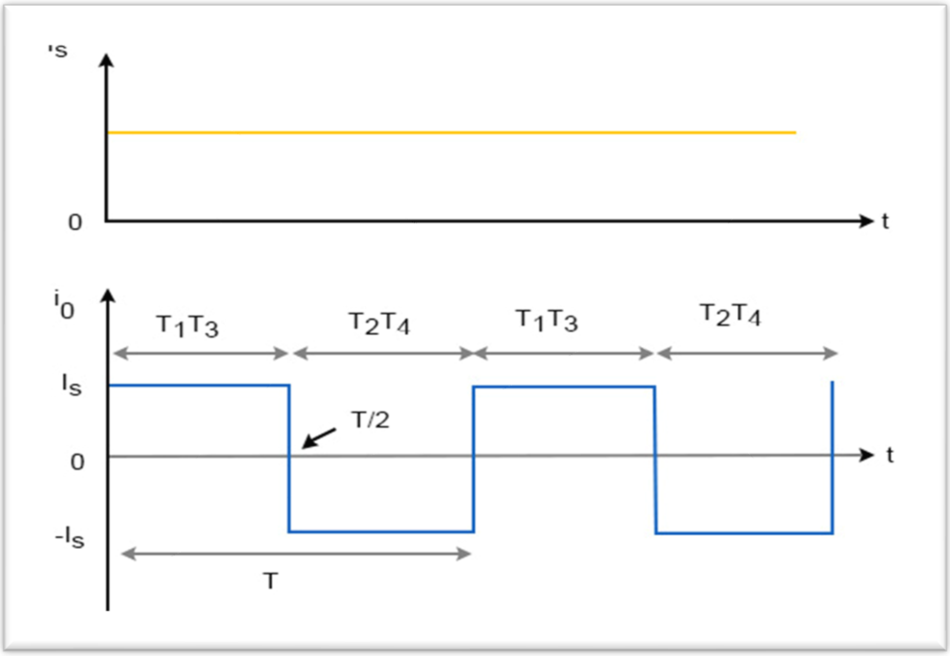

Figure I.12 Second wave single phase inverter circuit working diagram

I.3.4.2. Three-Phase Current Source Inverters :

A three-phase power source inverter is a device that converts direct current into alternating current using suitable modulation techniques so that the input current remains constant while the voltage varies depending on the nature of the load.

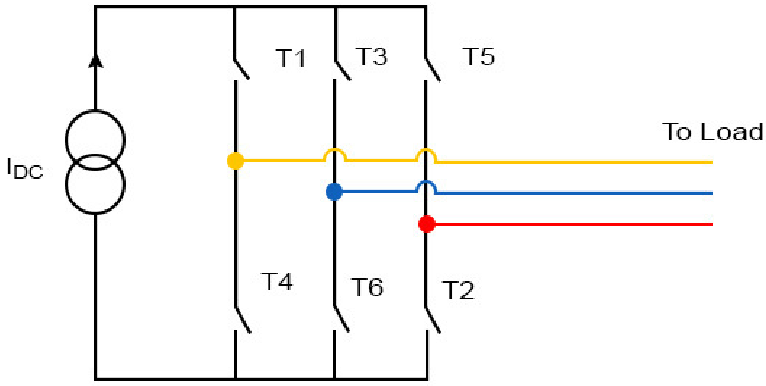

Figure I.12 Three phase inverter circuit

I.3.5.Multilevel Inverters:

The modular multilevel converter (m-MC) is also known as scalable technology where the voltage level verifies the number of sub-modules (SM). This technology is applicable up to the highest transmission voltages. The basic concept of M-MC is the cascaded connection of several cells or SMs with individual control systems. Therefore, the M-MC can act as a controllable voltage source converter (VSC) with large numbers of discrete possible voltage steps . Thus, for multilevel topology, it can prevent generation of any harmonic content.[9]

I.4.Conclusion :

In this chapter, we have provided a comprehensive definition of the inverter, the principle of converting direct current into alternating current, and its types, such as half-bridge, full-bridge, single-phase inverter, and three-phase inverter.

Chapter II:

Inverter Control Techniques

II.1. Introduction

Voltage source inverters (VSIs) play a central role in modern power conversion systems, particularly in applications where reliable and high-quality AC power is required. The performance of these inverters is highly dependent on the control strategy employed to generate the desired output voltage. This chapter presents a comparative study of three widely used inverter control techniques, focusing on their ability to regulate output voltage, minimize total harmonic distortion (THD), and ensure stable operation under dynamic loading conditions [10].

The chapter begins with an analysis of the 180° conduction technique, a classical method based on six-step switching sequences. This technique is well known for its structural simplicity and ease of implementation, making it a common choice in early inverter designs. However, its primary limitation lies in the significant harmonic content present in the output waveform, which can degrade power quality and increase filtering requirements [11].

Following that, the chapter explores Sinusoidal Pulse Width Modulation (SPWM), a method that modulates the pulse width of switching signals in accordance with a sinusoidal reference waveform. SPWM offers considerable improvements over six-step control in terms of waveform smoothness and THD reduction, and it remains a standard in many industrial applications due to its balance of performance and simplicity [12].

The third technique addressed is Space Vector Modulation (SVM), a more mathematically oriented control method that utilizes a vector representation of switching states to synthesize the desired output waveform. SVM provides superior voltage utilization and lower harmonic distortion compared to SPWM, especially at higher modulation indices. However, it comes with increased computational complexity [13].

For each method, the chapter includes:

- A schematic diagram of the inverter circuit.
- Mathematical derivations of phase and line voltages, current waveforms, and switching patterns.
- Analytical calculations of average and RMS values.
- Harmonic spectrum and THD analysis.
- Simulation results that validate theoretical models.

The chapter concludes with a comparative summary that highlights the relative advantages and drawbacks of each method in terms of efficiency, implementation complexity, and power quality. This analysis provides a solid foundation for selecting an appropriate control strategy based on application-specific requirements.[14]

II.2. Pulse Width Modulation control techniques (PWM):

It is a common method for controlling the output voltage and frequency of three-phase inverters in power applications. This method modifies the switching signals from the inverter switches to generate the desired output waveform. [14]

This technique causes distortions in the output waveform, known as THD distortions.

There are four types of signal control:

• full-wave PWM

• Sine-Triangle PWM

• Vector PWM

II.3. full-wave PWM Control Method at 180°

II.3.1. Circuit Diagram and Principle of Operation

Full-wave PWM control method at 180° is one of the fundamental techniques used in the control of voltage-source inverters (VSIs), particularly in three-phase inverters. In this technique, each switch in a leg of the inverter is turned ON for 180 electrical degrees and then turned OFF for the next 180 degrees. This means that at any given moment, three switches are conducting: one from each leg, and their operation overlaps for half a cycle. This creates a line voltage waveform with six steps per cycle, producing a quasi-square waveform.

Below is the basic schematic diagram of a three-phase inverter using 180° conduction mode:

Each phase is connected to a pair of complementary switches (e.g., T1-T4, T3-T6, T5-T2). The switching sequence is designed so that for each 60° interval, a specific combination of three switches is ON, while the others are OFF.

This method does not require a reference sinusoidal waveform or a carrier signal, which simplifies the control logic. However, it results in a high total harmonic distortion (THD) in the output voltage, especially at low modulation indices. [2]

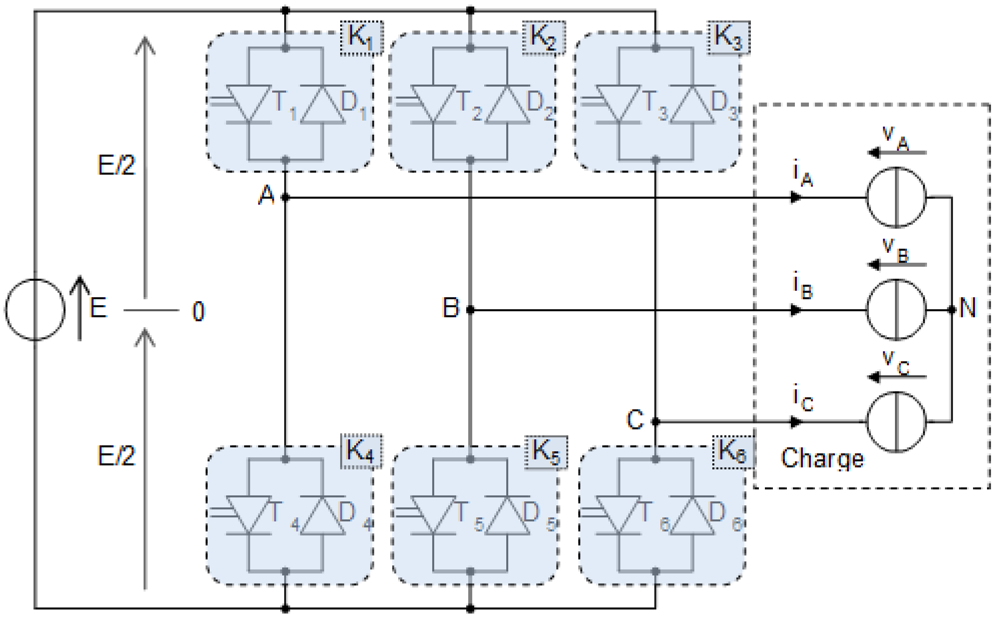

Figure II.1 Balanced three-phase inverter circuit

II.3.2. Representing and calculating simple and complex tensions:

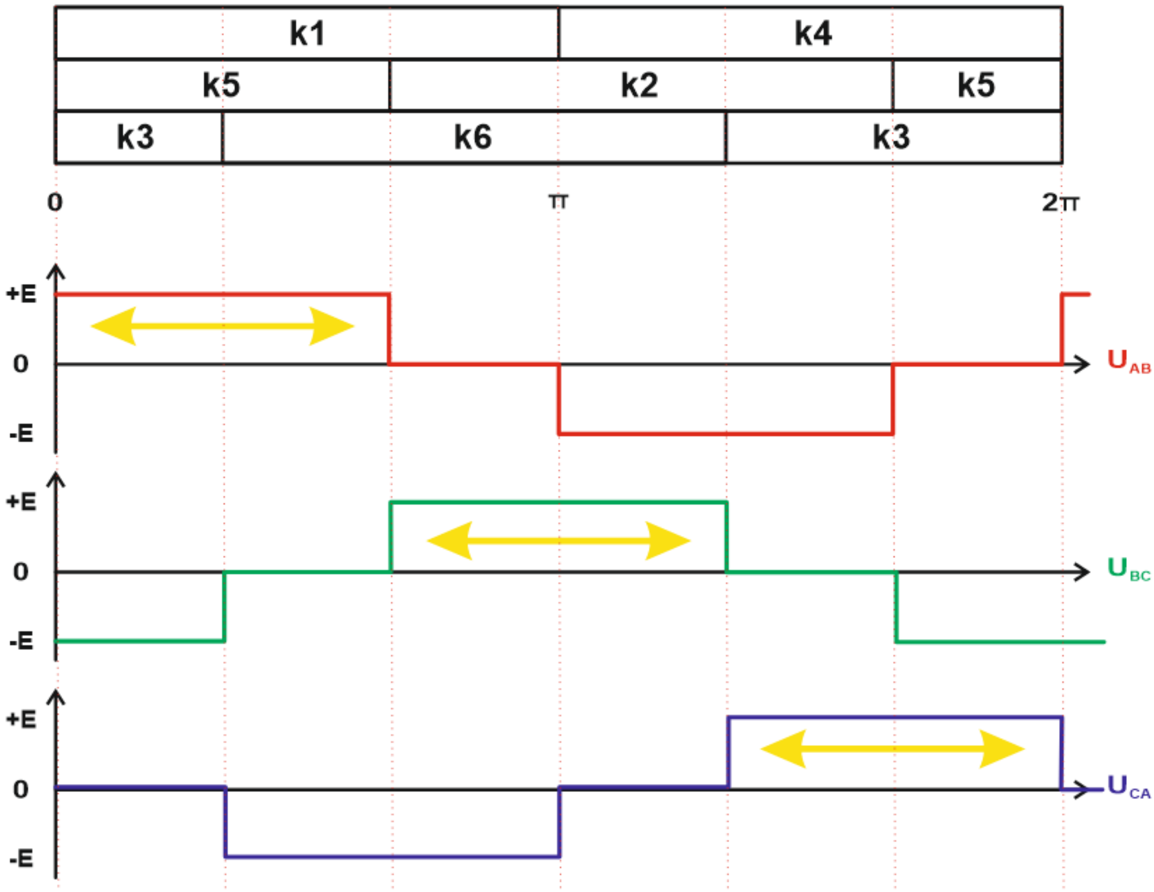

Figure II.2 Complex tension diagram

$$\begin{cases} U_{AB} = V_{AN} - V_{BN} \\ U_{BC} = V_{BN} - V_{CN} \\ U_{CA} = V_{CN} - V_{AN} \end{cases} \qquad \textbf{(II.1)}$$

and $..V_{AN} - V_{BN} - V_{CN} = 0$

We will try to find V_{AN} formula,

$$\begin{cases} U_{AB} - U_{CA} = V_{AN} - V_{BN} - (V_{CN} - V_{AN}) \\ U_{AB} - U_{CA} = 2V_{AN} - V_{BN} - V_{CN} \end{cases} \qquad \textbf{(II.2)}$$

$$(-V_{BN} - V_{CN} = V_{AN})$$

$$U_{AB} - U_{CA} = 3V_{AN}$$

$$\boxed{V_{AN} = \frac{U_{AB} - U_{CA}}{3} \qquad V_{BN} = \frac{U_{BC} - U_{AB}}{3} \qquad V_{CN} = \frac{U_{CA} - U_{BC}}{3}}$$

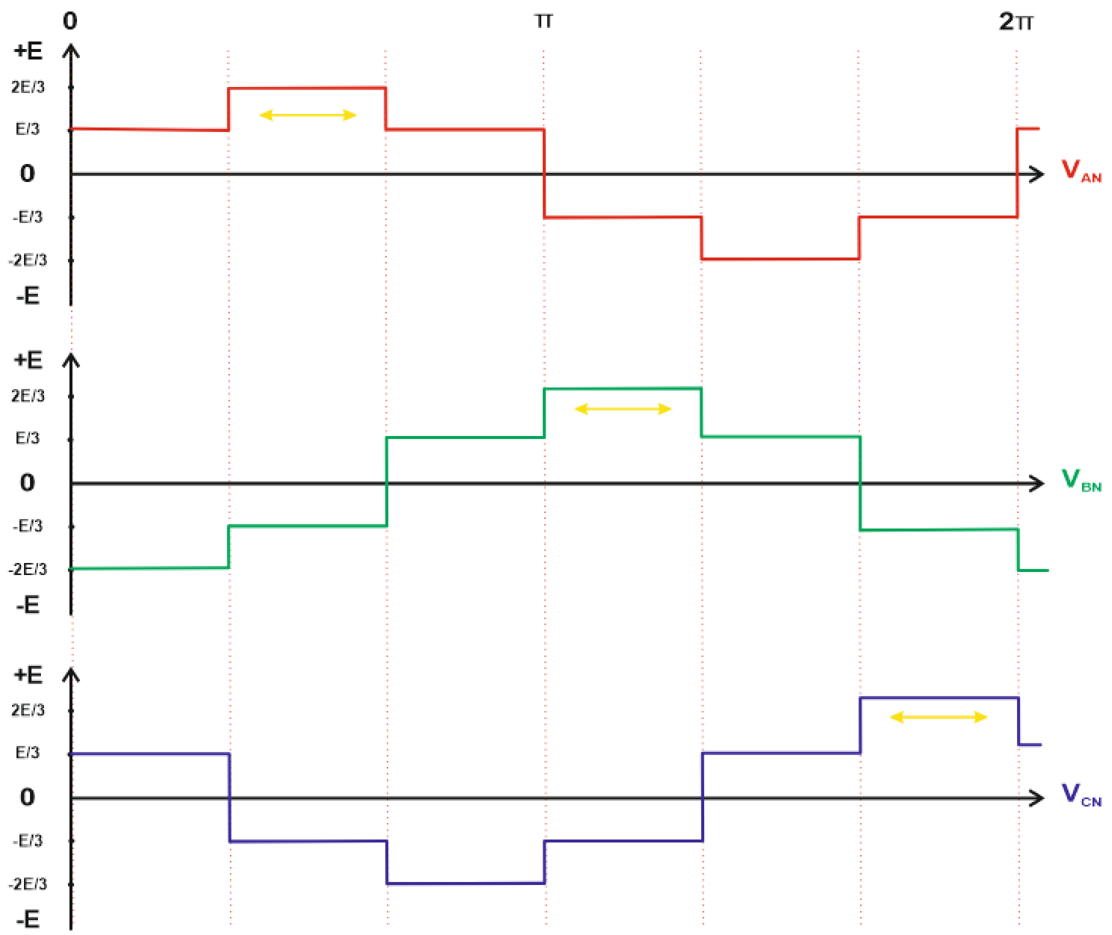

Figure II.3 Simple tension diagram

II.3.3. THD Calculation

$$U_{AB}(t) = \begin{cases} +V_{DC} & 0° < \theta > 180° \\ -V_{DC} & 180° < \theta > 360° \end{cases}$$

Fourier Series of the Output

$$U_{AB}(t) = \frac{4V_{DC}}{\pi}\left[\sin(wt) + \frac{1}{3}\sin(3wt) + \frac{1}{5}\sin(5wt) + \frac{1}{7}\sin(7wt) + \ldots\right] \quad \textbf{(II.3)}$$

$$\mathrm{THD} = \frac{\sqrt{V_3^2 + V_5^2 + V_7^2 + \ldots}}{V_1} \quad \textbf{(II.4)}$$

$$\mathrm{THD} = \frac{\sqrt{\left(\frac{4V_{DC}}{3\pi}\right)^2 + \left(\frac{4V_{DC}}{5\pi}\right)^2 + \left(\frac{4V_{DC}}{7\pi}\right)^2}}{\frac{4V_{DC}}{\pi}}$$

$$\mathrm{THD} = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2} = 0{,}414$$

$$\boxed{\mathrm{THD} \approx 41{,}4\ \%}$$

This value can vary depending on the load type. For example, inductive loads may reduce high-frequency harmonics through filtering action [15].

II.3.4. Simulation in MATLAB (full-wave PWM Control Method at 180°) :

Electrical circuit diagram in the Matlab program :

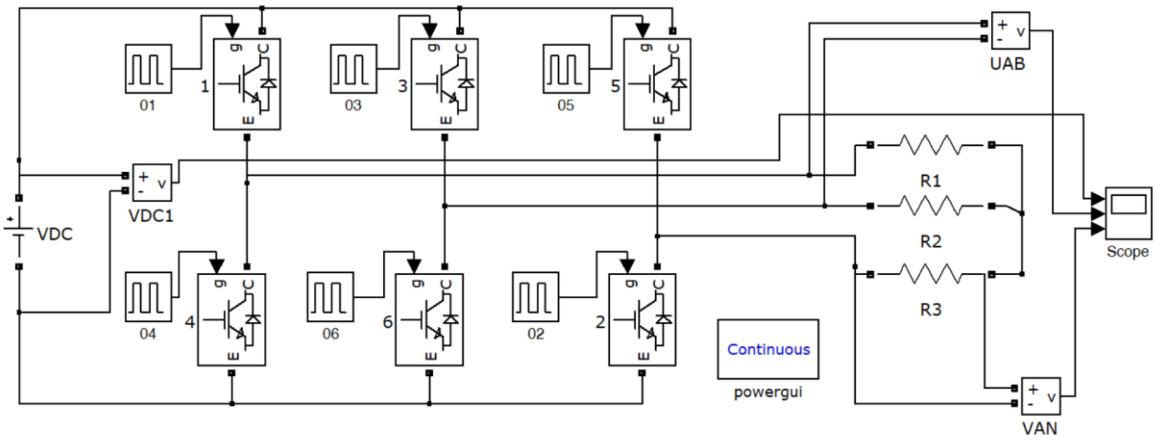

Figure II.4 Simulation in MATLAB full-wave PWM Control Method at 180°

Simulation result: complex and simple stress and source stress :

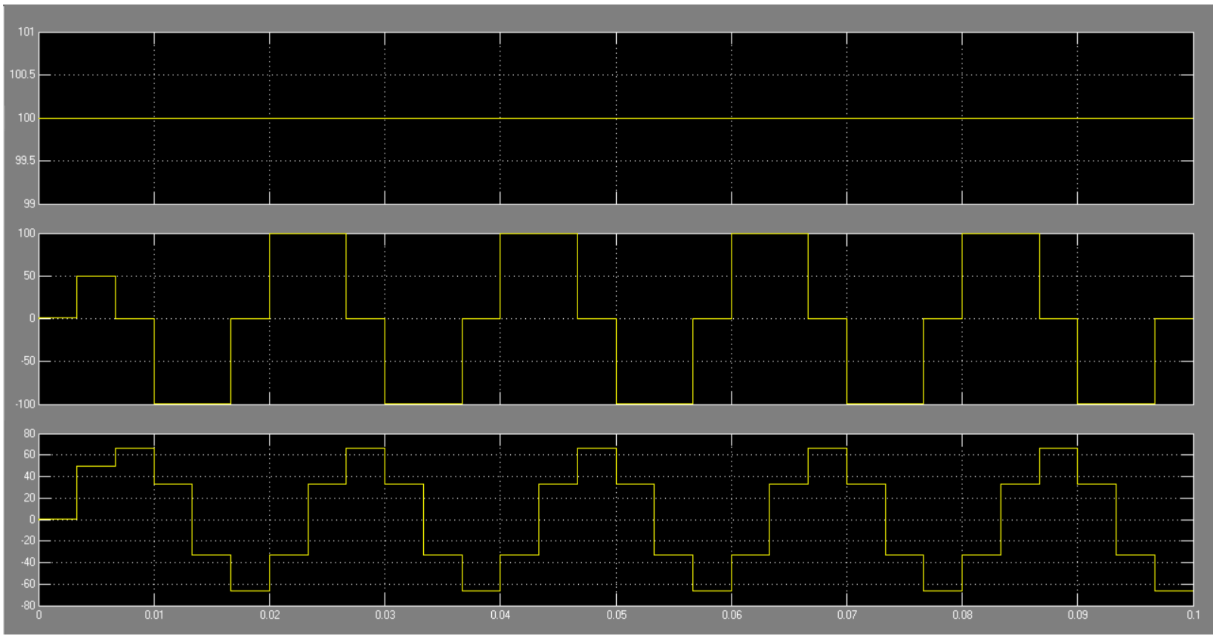

Figure II.5 Simulation result of complex, simple and source stress

II.4. Sine-Triangle PWM(SPWM) :

The basic principle of pulse width modulation is the slicing of a rectangular full wave. Thus, the inverter's output voltage is formed by a succession of pulses of varying width and amplitude equal to the supply voltage (DC). The most common technique for reproducing a PWM signal is to compare a triangular signal called a high-frequency carrier to a reference signal called a modulator, which constitutes the energy of the signal collected at the inverter output [4].

The modulation index

$$ma = \frac{Vcontrol}{Vftri} \tag{II.5}$$

The frequency modulation (mf)

$$mf = \frac{ftri}{fcontrol} \tag{II.6}$$

Sinusoidal pulse width modulation (SPWM) is a technique used to reduce harmonics and improve total harmonic distortion (THD) in the inverter's output voltage. By comparing a sine wave reference wave to a triangular carrier wave, multiple narrow pulses are generated, reducing the harmonic content of the output signal.

High-frequency switching in inverters generates high-frequency harmonics. These harmonics can be easily filtered out using low-pass filters, improving the quality of the resulting AC signa [6].

II.4.1.Types Sin-Triangle PWM:

There are three types used in this type of signal control:

1/*Triangular Signal Figure 2.6(a)

2/*Sawtooth Signal Figure 2.6(b)

3/*Inverted Sawtooth Signal Figure 2.6(c)

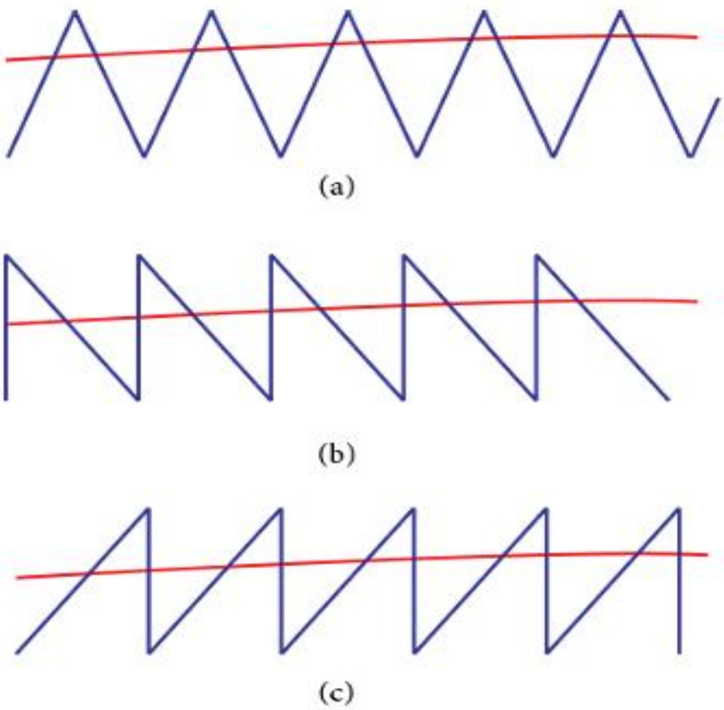

Image II.6 Types Sin--Triangle PWM

But in our study will be on Triangular Sign

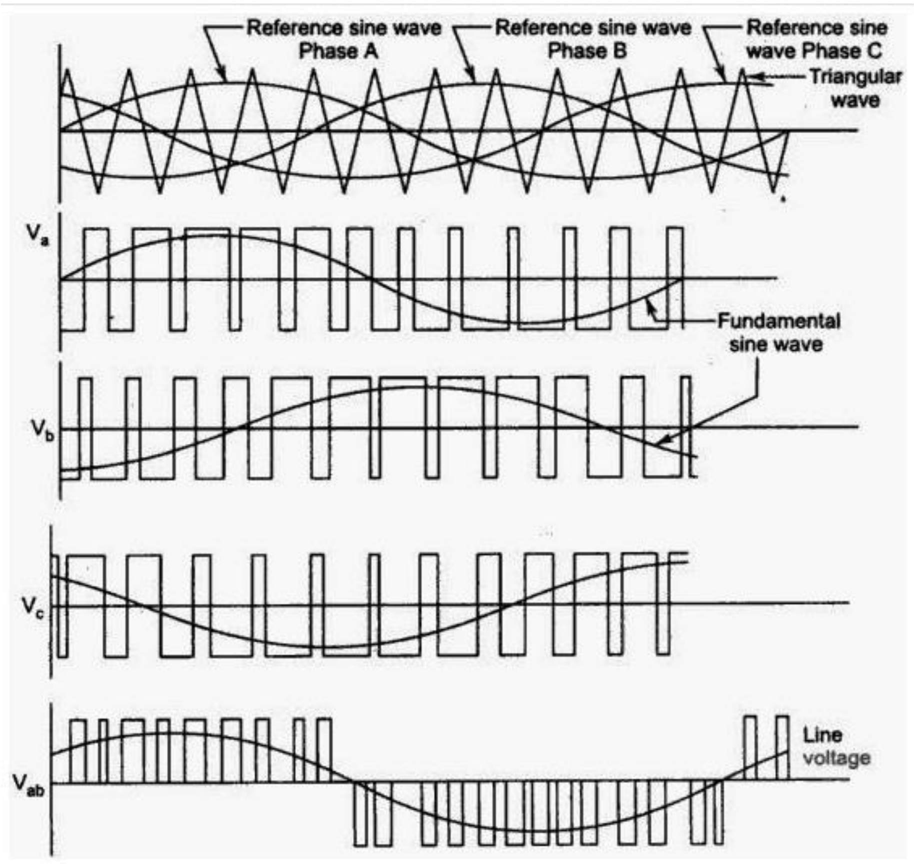

Figure II.7 Vector control technology tensions

We have the reference sine wave equation for phase a, b and :

$$\begin{cases} Va = Vm\sin(wt) \\ Vb = Vm\sin(wt - 120) \\ Vc = Vm\sin(wt - 240) \end{cases} \quad \text{(II.7)}$$

II.4.2. Harmonics in output voltage :

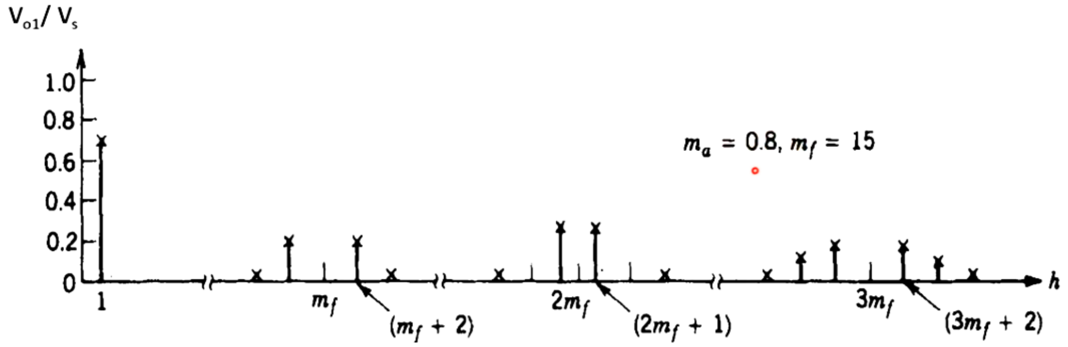

Figure II.8 Harmonics in output voltage **(II.7)**

The harmonics in vo appear as siddebands , cetred around the switching frequency and its multiples,ie,mf,2mf,3mf..

II.4.3. Calculate Total Harmonic Distortion (THDspwm):

It expresses the degree of purity of the electrical wave from harmonics that are multiples of the fundamental frequency of the wave. The distortion factor must be (THD=0) and the relationship of this factor is given as follows:

$$THD\% = \frac{\sqrt{V_{2rms}^2 + V_{3rms}^2 + \cdots + V_{n\,rms}^2}}{V_{1\,rms}} \times 100 \quad \text{(II.8)}$$

The following table shows the permissible ratios for the voltage:

Table II.1 : Permissible voltage ratios

Voltage class	Didicated system THD	General system THD
2.4 – 29 KV 112 KV and above	8 % 1.5 %	5 % 1.5 %

Here is an example of calculating THD for SPWM.

We have :

V1 rms=220v / V3 rms=15v /V5 rms=10v /V7 rms=5v

$$THD = \frac{\sqrt{V_{3rms}^2+V_{5rms}^2+V_{7rms}^2}}{V_{1\,rms}} \tag{II.9}$$

$$THD = \frac{\sqrt{15^2+10^2+5^2}}{220}$$

$$THD = \frac{\sqrt{225+100+25}}{202} = \frac{18.71}{220} = 0.085 = 8.5\%$$

II.4.4. MATLAB simulation :

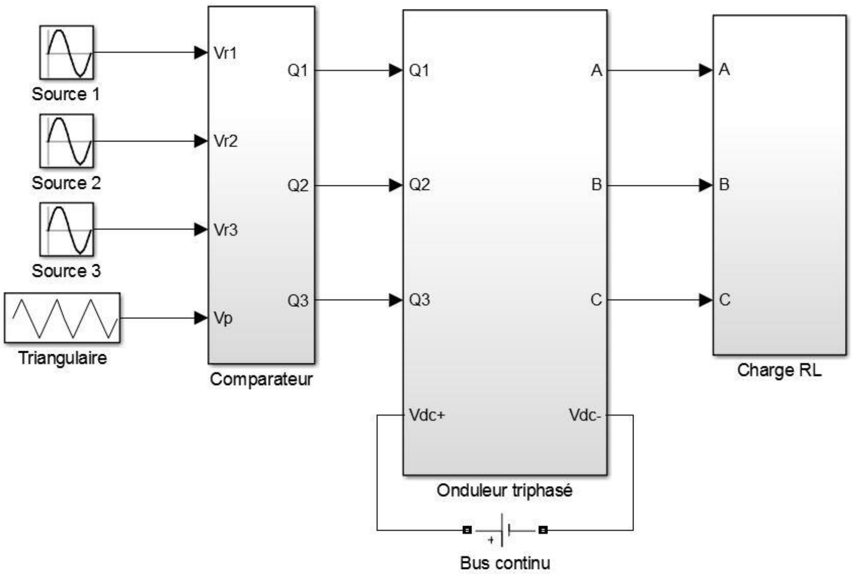

Figure II.9 Simulink block diagram of vector PWM technique

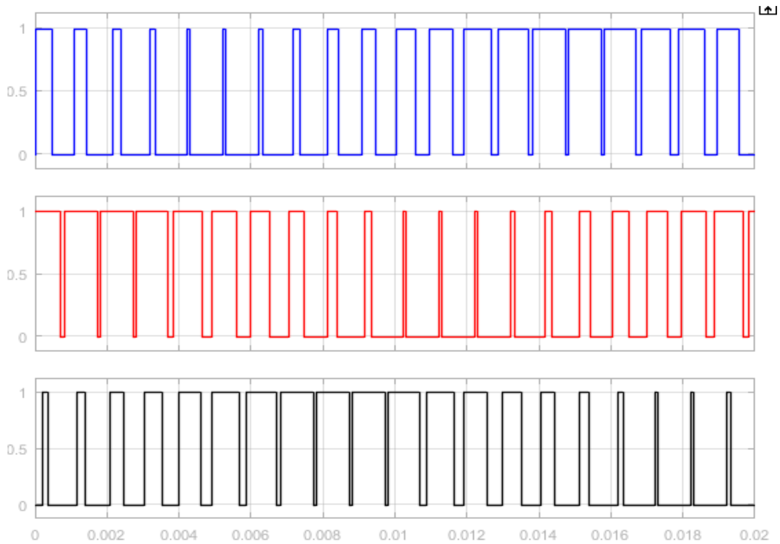

Figure II.10 Control signals Q1, Q2, and Q3

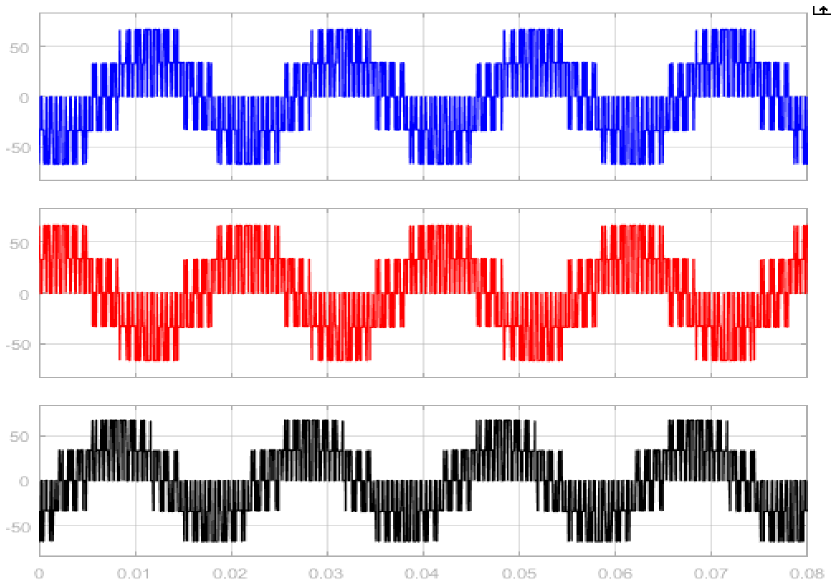

Figure II.11 (d) Simple output voltage VAN,VBN,VCN

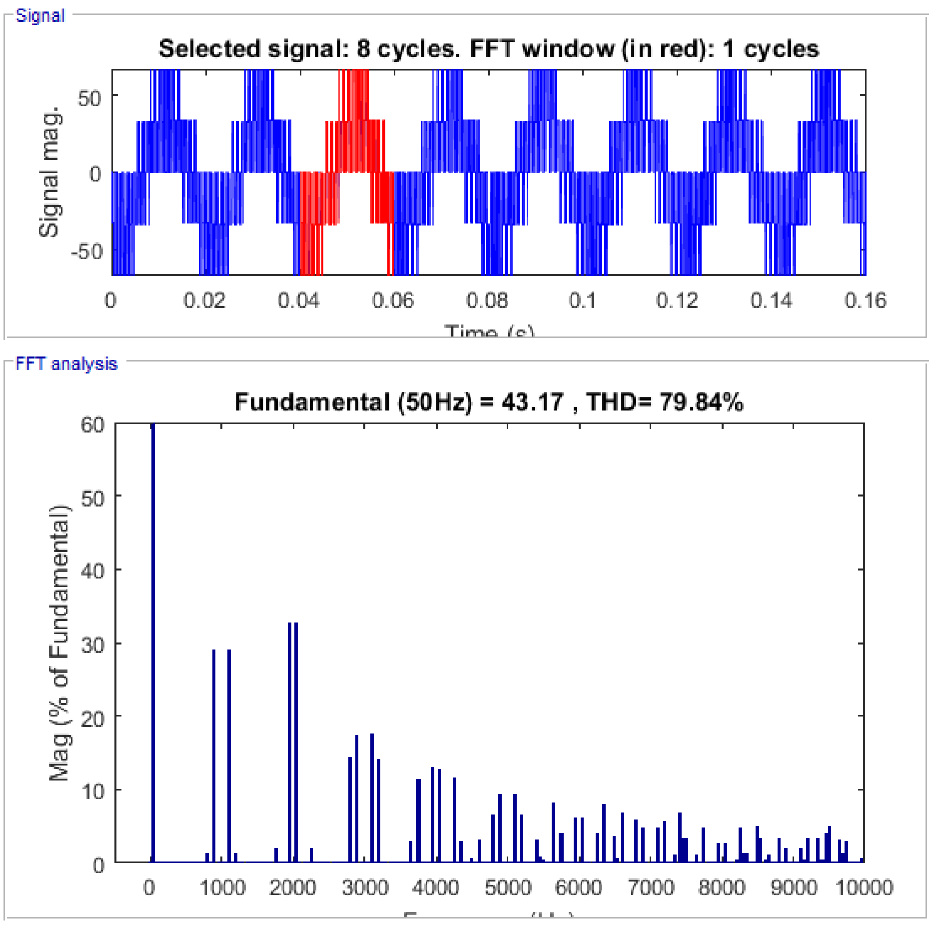

Figure II.12: Spectre Harmonic Tension FROM fMAX=10,000

II.5. Space Vector Pulse Width Modulation(svpwm) :

Space vector modulation (SVM) is a common technique in field-oriented control for induction motors and permanent magnet synchronous motors (PMSM). Space vector modulation is responsible for generating pulse width modulated signals to control the switches of an inverter, which then produces the required modulated voltage to drive the motor at the desired speed or torque. Space vector modulation is also known as Space vector pulse width modulation (SVPWM). [16]

Balanced three-phase sinusoidal voltages are Spaced at an angle of 120° and their equation is written as follows:

$$\begin{cases} Va = Vm\sin(wt) \\ Vb = Vm\,sin(wt-\frac{2\pi}{3}) \\ Vc = Vm\ \sin(wt+\frac{2\pi}{3}) \end{cases} \qquad \textbf{(II.10)}$$

II.5.1 Eight possibilities for modifying a Space vector:

We have three bridges. In each bridge there are two states: 1 or 0. If it is 1, the upper switches are on, and closed if it is 0. The upper switches and the lower switches cannot be on at the same time.

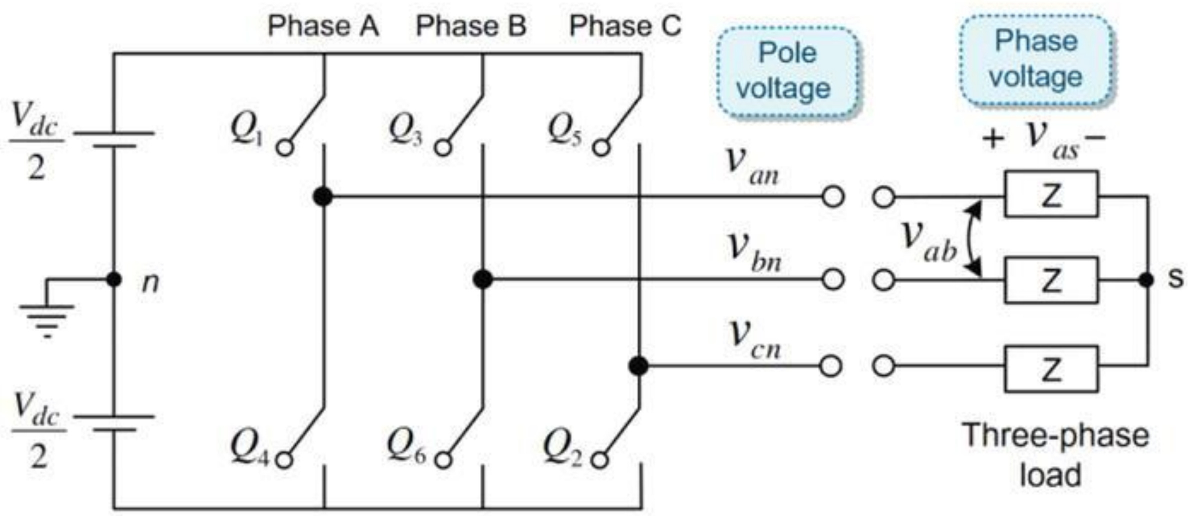

Figure II.13. Three-phase inverter cricuit

II.5.2 Voltage calculation:

State 1 (100) :

Table II.2. Eight possibilities cases in modifying sv

Space Vector	S1	S2	S3
V0	**0**	**0**	**0**
V1	**1**	**0**	**0**
V2	**1**	**1**	**0**
V3	**0**	**1**	**0**
V4	**0**	**1**	**1**
V5	**0**	**0**	**1**
V6	**1**	**0**	**1**
V7	**1**	**1**	**1**

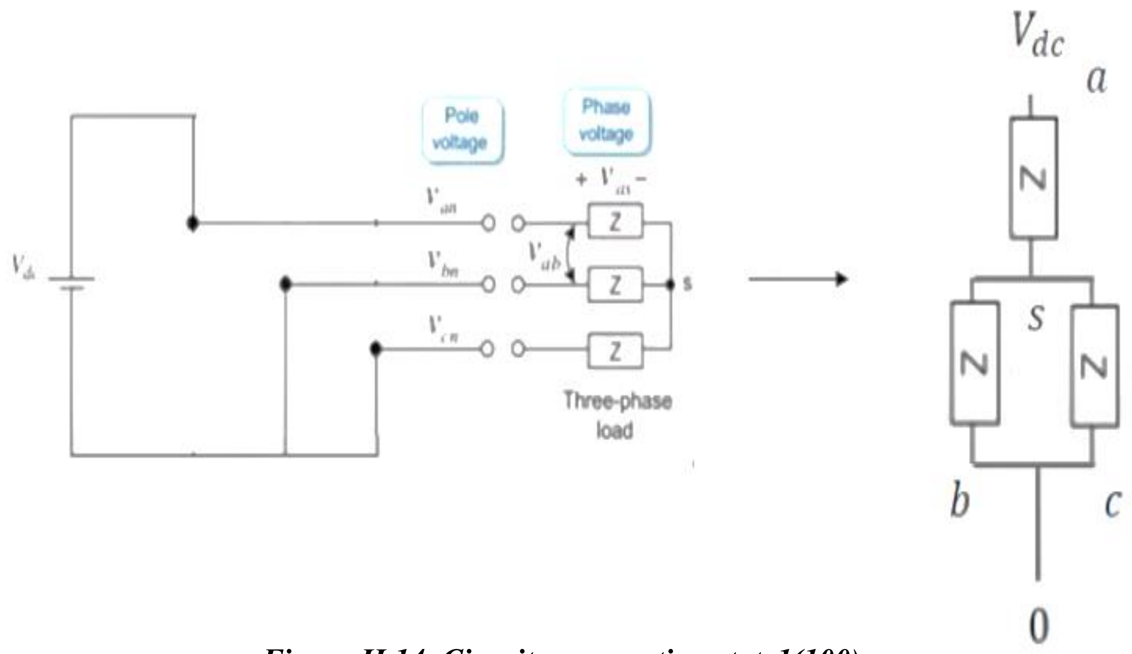

Figure II.14. Circuit representing state1(100)

$$Vas = \frac{z}{z+\frac{z*z}{z+z}}Vdc = \frac{z}{z+\frac{1}{2}z}Vdc \tag{II.11}$$

$$Vas = \frac{2}{3}Vdc$$

$$Vbs = \frac{-\frac{1}{2}z}{z+\frac{z*z}{z+z}}Vds \tag{II.12}$$

$$Vbs = \frac{-1}{3}Vds$$

$$\begin{cases} Vcs = \frac{-\frac{1}{2}z}{z+\frac{z*z}{z+z}}Vds \\ Vcs = -\frac{1}{3}Vds \end{cases} \tag{II.13}$$

In this way we complete the remaining cases of the tables. This summarizes this calculation.

Table II.3 : Space vector and phase voltges

Space Vector	V_{as}	V_{bs}	V_{cs}
V0 (000)	0	0	0
V1(100)	$\frac{2}{3}V_{dc}$	$-\frac{1}{3}V_{dc}$	$-\frac{1}{3}V_{dc}$
V2(110)	$\frac{1}{3}V_{dc}$	$\frac{1}{3}V_{dc}$	$-\frac{2}{3}V_{dc}$
V3(010)	$-\frac{1}{3}V_{dc}$	$\frac{2}{3}V_{dc}$	$-\frac{1}{3}V_{dc}$
V4(011)	$-\frac{2}{3}V_{dc}$	$\frac{1}{3}V_{dc}$	$\frac{1}{3}V_{dc}$
V5(001)	$-\frac{1}{3}V_{dc}$	$-\frac{1}{3}V_{dc}$	$\frac{2}{3}V_{dc}$
V6(101)	$\frac{1}{3}V_{dc}$	$-\frac{2}{3}V_{dc}$	$\frac{1}{3}V_{dc}$
V7(111)	0	0	0

II.5.3 Calculate angle (α) and length (Vr) each state:

We convert Va, Vb, and Vc to V and V by applying Clark transform.

$$\begin{bmatrix} V\alpha \\ V\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} \text{ eq.1} \tag{II.15}$$

We have the following rules to calculate the angle and the length of the state

$$\begin{cases} Vr = \sqrt{(V\alpha)^2 + (V\beta)^2} \\ \alpha = tan^{-1}(\frac{V\beta}{V\alpha}) \end{cases} \text{eq.2} \qquad \textbf{(II.16)}$$

We extract the following from eq1:

$$\begin{cases} V\alpha = \frac{2}{3}Va - \frac{1}{3}Vb - \frac{1}{3}Vc \\ V\beta = \frac{\sqrt{3}}{3}Vb - \frac{\sqrt{3}}{3}VC \end{cases} \text{eq.3} \qquad \textbf{(II.17)}$$

We will calculate the angle and length of state 1(100),From Table 1 we have the values of Va, Vb and Vc as follows:

$$\begin{cases} Va = \frac{2}{3}Vds \\ Vb = -\frac{1}{3}Vds \\ Vc = -\frac{1}{3}Vds \end{cases} \text{eq.4} \qquad \textbf{(II.18)}$$

We substitute eq.4 in eq.3 and find the following:

$$\begin{cases} V\alpha = \left[\frac{2}{3} - \left(\frac{1}{2} * \left(-\frac{1}{3}\right)\right) - \left(\frac{1}{2} * \left(-\frac{1}{3}\right)\right)\right] Vds \\ V\beta = \left[\left(\frac{\sqrt{3}}{3} * \left(-\frac{1}{3}\right)\right) - (\frac{\sqrt{3}}{3} * \left(-\frac{1}{3}\right))\right] Vdc \end{cases} \text{eq.5} \qquad \textbf{(II.19)}$$

$$\begin{cases} V\alpha = 1\ Vdc \\ V\beta = 0 \end{cases} \text{eq.6} \qquad \textbf{(II.20)}$$

We substitute eq.6 into eq.2 :

$$\begin{cases} Vr = \sqrt{(Vds)^2 + (0)^2} \\ \alpha = \tan^{-1}(\frac{0}{Vds}) \end{cases} \text{eq.7}$$

$$\begin{cases} Vr = Vds \\ \alpha = 0 \end{cases} \text{eq.8} \qquad \textbf{(II.21)}$$

In this way we calculate all the remaining states we have. Table 3 below summarizes these calculations.

Table II.4: Angles and lengths for each state

Space Vector	V_r	α
V0 (000)	**0**	**0**
V1(100)	V_{dc}	**0**
V2(110)	V_{dc}	**60**
V3(010)	V_{dc}	**120**
V4(011)	V_{dc}	**180**
V5(001)	V_{dc}	**240**
V6(101)	V_{dc}	**300**
V7(111)	**0**	**0**

II.5.4 Visulizaation of Space vectors :

Figure II.11 shows provides eight possible switching states, made up of six active and two zero switching states. Active vectors divide plane for six sectors, where a reference vector Vref Figure is obtained by switching on (for proper time) two adjacent vectors. It can be seen that vector Vref is possible to implement by the different switching on/off sequence of V1 and V2….

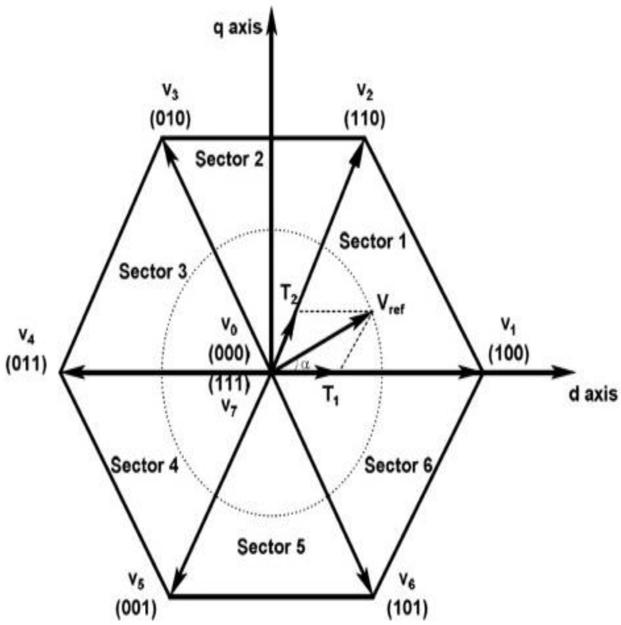

Figure II.15 space vectors whith states

We assume that the reference vector is in sector 1. How can we achieve this through the switching process using Space vector?

We have a vector Va and Vb as shown in Figure 2.12 where Va is in vector V1, Vb is in vector

V2, and Vr is the resultant vector

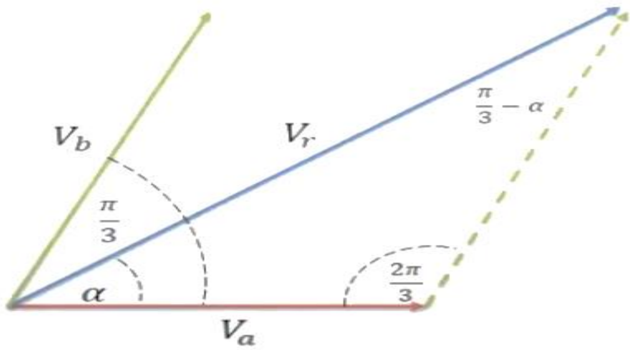

Figure II. 16 Vector Vr : resultant of the vector Va and Vb

Define linear relation between vector time of activation and the voltage

$$\begin{cases} \boldsymbol{Va} = \frac{\boldsymbol{Ta}}{\boldsymbol{Tt}}\boldsymbol{Vdc} \\ \boldsymbol{Vb} = \frac{\boldsymbol{Tb}}{\boldsymbol{Tt}}\boldsymbol{Vdc} \end{cases} \text{eq.9} \qquad \textbf{(II.22)}$$

$$\begin{cases} \boldsymbol{Ta} = \frac{\boldsymbol{Tt}}{\boldsymbol{Vdc}}\boldsymbol{Va} \\ \boldsymbol{Tb} = \frac{\boldsymbol{Tt}}{\boldsymbol{Vdc}}\boldsymbol{Vb} \end{cases} \text{eq.10} \qquad \textbf{(II.23)}$$

Apply sine rule to tringle

$$\begin{cases} \frac{\boldsymbol{Vr}}{\boldsymbol{sin}(\frac{2\pi}{3})} = \frac{\boldsymbol{Va}}{\boldsymbol{sin}(\frac{\pi}{3}-\boldsymbol{\alpha})} = \frac{\boldsymbol{Vb}}{\boldsymbol{sin}(\boldsymbol{\alpha})} \end{cases} \text{eq.11} \qquad \textbf{(II.24)}$$

From eq.10 we extract Va, Vb:

$$\begin{cases} \boldsymbol{Va} = \boldsymbol{Vr}\frac{\boldsymbol{sin}(\frac{\pi}{3}-\boldsymbol{\alpha})}{\boldsymbol{sin}\ (\frac{2\pi}{3})} \\ \boldsymbol{Vb} = \boldsymbol{Vr}\frac{\boldsymbol{sin}(\boldsymbol{\alpha})}{\boldsymbol{sin}(\frac{2\pi}{3})} \end{cases} \text{eq.12} \qquad \textbf{(II.25)}$$

$$\begin{cases} \boldsymbol{Va} = \frac{\mathbf{2}}{\sqrt{\mathbf{3}}}\ \boldsymbol{Vr}\ \boldsymbol{sin}\ (\frac{\boldsymbol{\pi}}{\mathbf{3}} - \boldsymbol{\alpha}) \\ \boldsymbol{Vb} = \frac{\mathbf{2}}{\sqrt{\mathbf{3}}}\ \boldsymbol{Vr}\ \boldsymbol{sin}(\boldsymbol{\alpha}) \end{cases} \text{eq.13} \qquad \textbf{(II.26)}$$

We substitute eq.12 into eq.9.

$$\begin{cases} \boldsymbol{Ta} = \frac{\mathbf{2}*\boldsymbol{Tt}}{\sqrt{\mathbf{3}}*\boldsymbol{Vdc}} * \boldsymbol{Vr}\ \boldsymbol{sin}\ (\frac{\boldsymbol{\pi}}{\mathbf{3}} - \boldsymbol{\alpha}) \\ \boldsymbol{Tb} = \frac{\mathbf{2}\boldsymbol{Tt}}{\sqrt{\mathbf{3}}*\boldsymbol{Vdc}} * \boldsymbol{Vr}\ \boldsymbol{sin}\ (\boldsymbol{\alpha}) \\ \boldsymbol{T0} = \boldsymbol{Tt} - \boldsymbol{Ta} - \boldsymbol{Tb} \end{cases} \text{eq.14} \qquad \textbf{(II.27)}$$

$$\begin{cases} \boldsymbol{Ta} = \frac{\mathbf{2}*\boldsymbol{Tt}}{\sqrt{\mathbf{3}}*\boldsymbol{Vdc}} * \boldsymbol{Vr}\ \boldsymbol{sin}\ (\frac{\boldsymbol{K}*\boldsymbol{\pi}}{\mathbf{3}} - \boldsymbol{\alpha}) \\ \boldsymbol{Tb} = \frac{\mathbf{2}*\boldsymbol{Tt}}{\sqrt{\mathbf{3}}*\boldsymbol{Vdc}} * \boldsymbol{Vr}\ \boldsymbol{sin}(\boldsymbol{\alpha} - \frac{(\boldsymbol{K}-\mathbf{1})*\boldsymbol{\pi}}{\mathbf{3}}) \\ \boldsymbol{T0} = \boldsymbol{Tt} - \boldsymbol{Ta} - \boldsymbol{Tb} \end{cases} \text{eq.15} \qquad \textbf{(II.28)}$$

Eq.13: for angle $\frac{\boldsymbol{\pi}}{\mathbf{3}}$ $(\boldsymbol{\alpha} < \frac{\boldsymbol{\pi}}{\mathbf{3}})$ but: eq.15 for angle any angle and K is sector number .

II.5.5 Calculate Total Harmonic Distortion(THDsvpwm):

Harmonic is non-sinusoïdal currents or voltages that are persent in the normally sinusoidal network

Here is an example of calculating THD for svpwm:

We have:

V1 rms=380v / V9 rms= 10v / V11 rms= 8v

$$\begin{cases} THD = \dfrac{\sqrt{V_{9\,rms}^2 + V_{11\,rms}^2}}{V_{1\,rms}} \\ THD = \dfrac{\sqrt{10^2+8^2}}{380} \\ THD = 0.0336 = 3.36\% \end{cases} \quad \textbf{(II.28)}$$

Note: In SVPWM fundamental harmonics are often multiples of 3, but they self-cancel in balanced loads, so we only count asynchronous harmonics

II.5.6 MATLAB simulation :

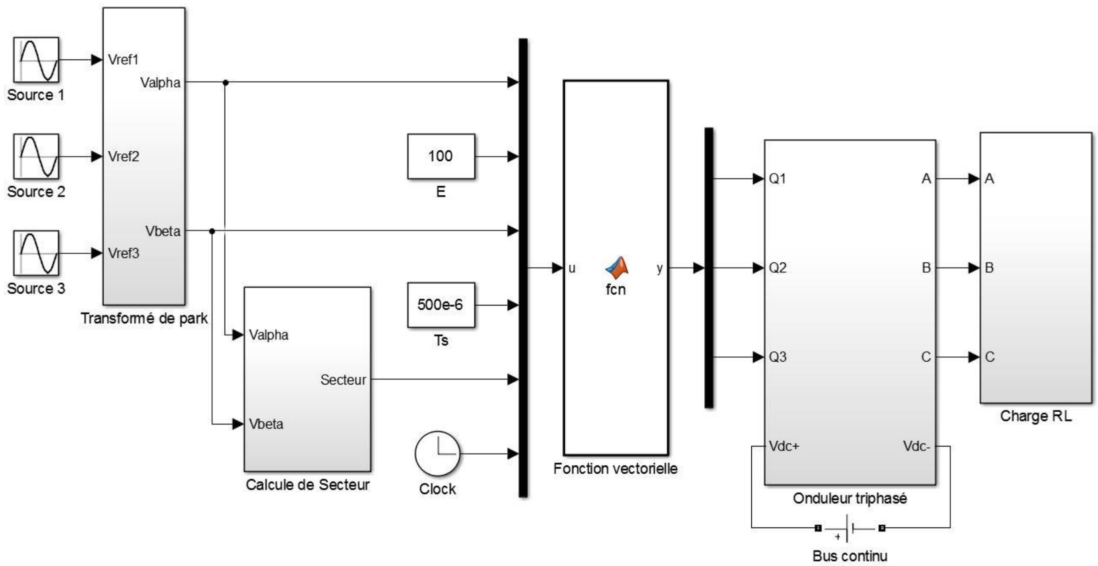

Figure II. 17 Schéma bloc Simulink de technique MLI vectorielle

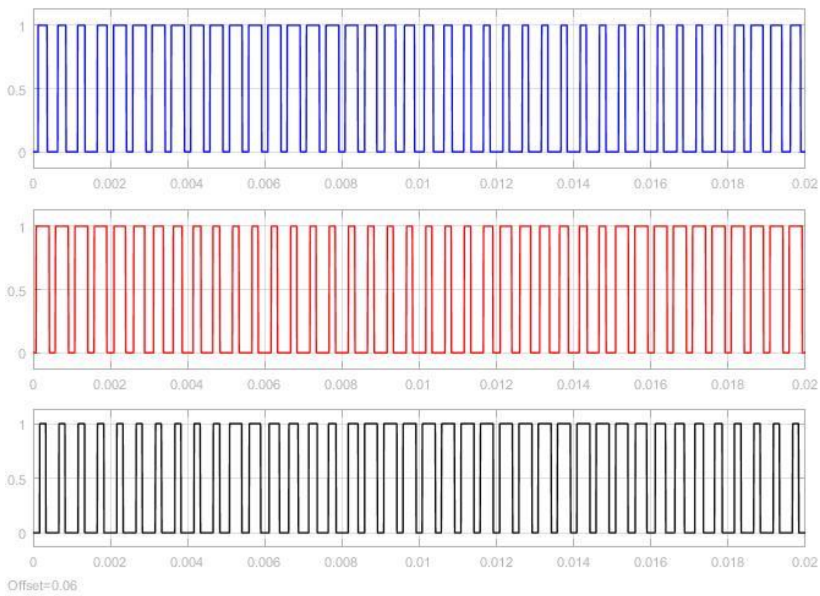

Figure II. 18: Control signals Q1, Q2, and Q3

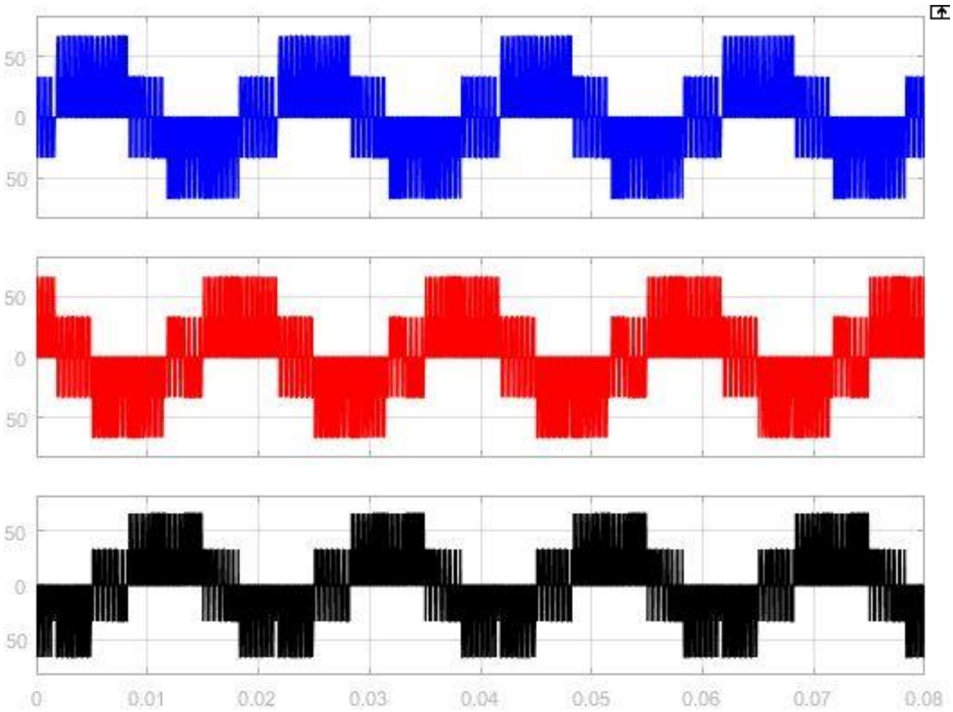

Figure II. 19 Simple output voltage VAN,VBN,VCN

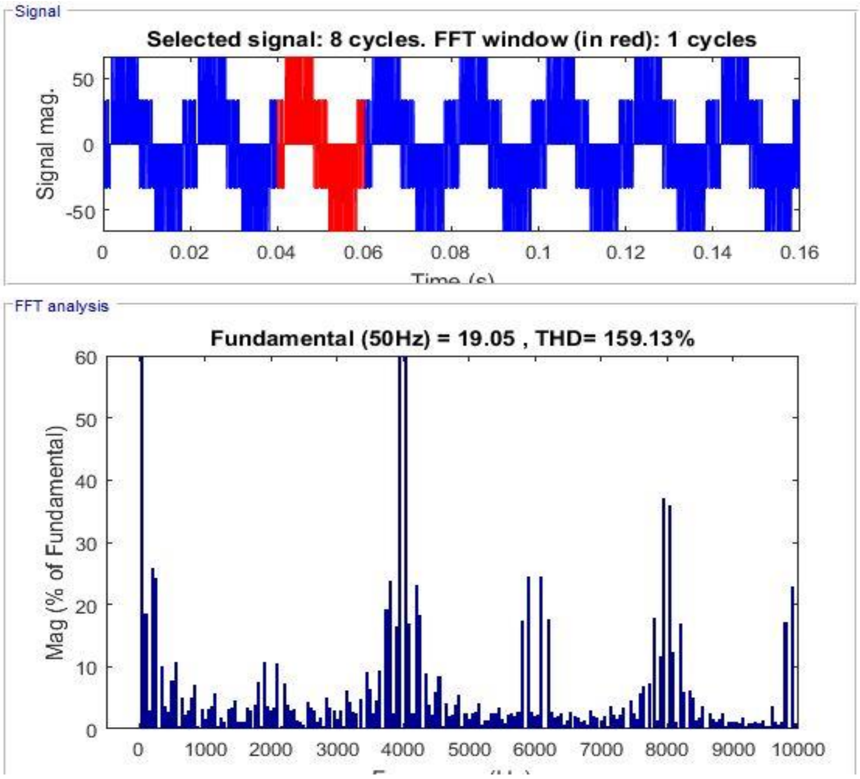

Figure II. 20 : Spectre Harmonic Tension FROM fMAX=10,000

II.6. Conclusion

This chapter provided a comprehensive overview of various Pulse Width Modulation (PWM) techniques used in power electronic systems, particularly voltage source inverters. Emphasis was placed on their role in reducing harmonic distortion, minimizing switching losses, and improving the overall performance of electrical drives. Traditional methods such as sinusoidal PWM and Space vector PWM were discussed in terms of their operational principles and limitations.

Special attention was given to the pre-calculated PWM technique, which stands out for its ability to significantly lower switching frequency, operate efficiently in the overmodulation region, and offer better harmonic elimination through the optimization of switching angles. Despite its higher computational requirements, this technique presents a promising solution for high-performance industrial applications, especially where harmonic control and system efficiency are critical.

The insights gained from this chapter lay the groundwork for the next stage of this study. In Chapter Three, a detailed modeling and simulation of the pre-calculated PWM technique will be conducted using MATLAB/Simulink, with a focus on evaluating Total Harmonic Distortion (THD) and comparing the outcomes with conventional PWM methods.

Chapter III:

Selective Harmonic Elimination

III.1. Introduction

In recent years, the need for high-quality voltage waveforms with minimal harmonic content has driven the development of advanced inverter control techniques. Among them, the pre-calculation control method—also known as Selective Harmonic Elimination (SHE)—has proven to be one of the most effective strategies for reducing total harmonic distortion (THD), particularly in medium- and high-power applications .

Unlike real-time modulation techniques such as SPWM and SVM, the Selective Harmonic Elimination computes switching angles offline in advance of inverter operation. These angles are determined mathematically to eliminate selected low-order harmonics (e.g., 5th, 7th, 11th), while maintaining a specified amplitude for the fundamental component . By eliminating the most significant harmonics at the source, this method minimizes the need for bulky filters and improves overall power quality.

This chapter provides a detailed study of the Selective Harmonic Elimination, beginning with the mathematical formulation of output waveforms using Fourier series. The chapter then presents the nonlinear equation system derived from harmonic elimination criteria, followed by an explanation of numerical solution techniques—particularly the Newton-Raphson method. Analytical results are supported by simulation models that illustrate the effectiveness of this method in suppressing harmonics and maintaining stable inverter output .

Furthermore, the impact of switching angle optimization on THD is explored, and a comparison is made between pre-calculation and traditional modulation strategies to highlight the advantages and limitations of this approach under varying operating conditions .

III.2. Selective Harmonic Elimination (Pre-calculated PWM)

The application of Pulse Width Modulation (PWM) techniques in various industrial systems aims to improve harmonic distortion rates, reduce losses in the inverter or the load, and minimize torque ripple and vibrations in electric machine applications. The pre-calculated PWM technique is based on the offline optimal determination of switching instants in order to eliminate lower-order unwanted harmonics and precisely control the fundamental voltage. Although this approach requires extensive computational effort, it offers several advantages over sinusoidal PWM and Space vector PWM techniques. It can significantly reduce the inverter switching frequency by approximately half, allows direct operation in the overmodulation region to provide higher output voltage, and helps reduce DC current ripple while eliminating oscillations in the output filter. Additionally, it offers a simpler implementation through the use of a predefined lookup table. In this method, notches are introduced into the square wave at predetermined angles. The resulting waveform exhibits double symmetry with respect to both the quarter and half period. The switching angles can be controlled to eliminate specific harmonic components and regulate the fundamental voltage. [17]

III.3. Fourier series decomposition of output signal :

The Fourier transform of a periodic alternating signal is given by:

$$\begin{cases} V(t) = a_0 + \sum_{n=1}^{\infty} a_n.cos(n\omega t) + a_n.sin(n\omega t) \\ a_n = \frac{1}{\pi}\int_0^{2\pi} v(t)cos(n\omega t)d\omega t \\ b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)sin(n\omega t)d\omega t \end{cases} \quad \textbf{(III.1)}$$

The control patterns of the three-phase inverter are assumed to have quarter-period symmetry and half-period antisymmetry as shown in Figure III.1

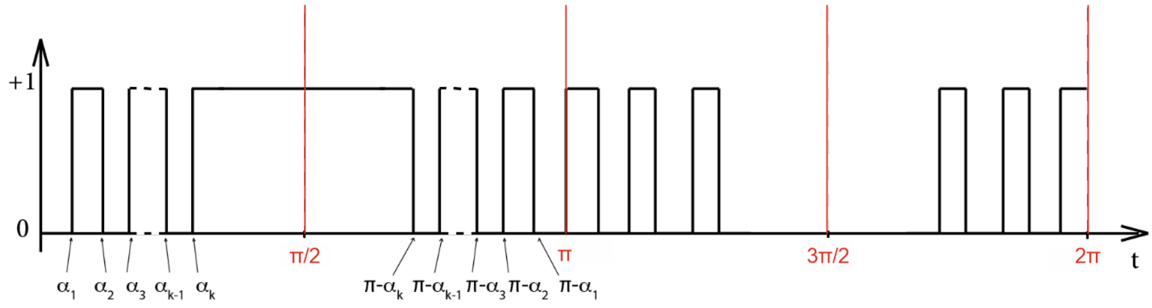

Figure III.1 Quarter period symmetry and anti-half period symmetry

For a wave with quarter-period symmetry, only odd harmonics with sinus component will be present a_n=0 *et* a_0=0, in addition for balanced three-phase systems harmonics that are multiples of three are automatically eliminated, so

$$\begin{cases} V(t) = \sum_{n=1}^{\infty} a_n . sin(n\omega t) \\ b_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v(t) sin(n\omega t) d\omega t \end{cases} \quad \textbf{(III.2)}$$

Assuming the wave has unit amplitude to simplify calculations, bn will be developed as follows:

$$b_n = \frac{4}{\pi}\Big[\int_0^{\alpha_1}(0) sin(n\omega t)d\omega t + \int_0^{\alpha_1}(0) sin(n\omega t)d\omega t + \int_0^{\alpha_1}(0) sin(n\omega t)d\omega t \ldots + \int_{\alpha_{k-1}}^{\alpha_k}(0)^{k-1} sin(n\omega t)d\omega t + \int_{\alpha_k}^{\frac{\pi}{2}}(+1)\, sin(n\omega t)d\omega t\Big] \quad \textbf{(III.3)}$$

Using the general relationship:

$$\int_{\theta_2}^{\theta_1} sin(n\omega t)d\omega t = \frac{1}{n}(cosn\theta_2 - cosn\theta_1) \quad \textbf{(III.4)}$$

The first and last non-zero terms are written:

$$\begin{cases} \int_{\alpha_1}^{\alpha_2} sin(n\omega t)d\omega t = \frac{1}{n}(cos(n\alpha_2) - cos(n\alpha_1)) \\ \int_{\alpha_k}^{\frac{\pi}{2}}(+1) sin(n\omega t)d\omega t = -\frac{1}{n}\, cos(n\alpha_2) - cos(n\alpha_1)) \\ b_n = \frac{4}{n\pi}[-cosn\alpha_1 + cosn\alpha_2 + \ldots. - cosn\alpha_k] \end{cases} \quad \textbf{(III.5)}$$

By integrating the other components of equation Eq2. 29 and substituting equations Eq2.31 and Eq2.32:

$$b_n = \frac{4}{n\pi}\left[\sum_{k=1}^{k}(-1)^k cos(n\alpha_k)\right] \qquad \textbf{(III.5)}$$

Eq. 2.34 contains k variables (angles) and k equations to be solved to eliminate k-1 harmonics and control the fundamental component.

For the three-phase voltage inverter, eliminating low harmonics in the phase-to-neutral voltage implies eliminating low harmonics in the phase-to-phase voltage.

III.4. Pre-calculated five-angle PWM :

The first optimization constraint is to define the desired level of the fundamental component. Cancellation of the 5th, 7th, 11th, and 13th harmonics requires that the appropriate Fourier coefficients be zero. The number of degrees of freedom is given by the number of angles αk. Consequently, the output voltage has five level changes within a 90° interval.

For k=5, we obtain the following nonlinear system of equations:

$$\begin{cases} b_1 = \frac{4}{\pi}[- cos\alpha_1 + cos\alpha_2 - cos\alpha_3 + cos\alpha_4 - cos\alpha_5 + cos\alpha_6 - cos\alpha_7] = M \\ b_5 = \frac{4}{5\pi}[- cos5\alpha_1 + cos5\alpha_2 - cos5\alpha_3 + cos5\alpha_4 - cos5\alpha_5 + cos5\alpha_6 - cos5\alpha_7] = 0 \\ b_7 = \frac{4}{7\pi}[- cos7\alpha_1 + cos7\alpha_2 - cos7\alpha_3 + cos7\alpha_4 - cos7\alpha_5 + cos7\alpha_6 - cos7\alpha_7] = 0 \\ b_{11} = \frac{4}{11\pi}[- cos11\alpha_1 + cos11\alpha_2 - cos11\alpha_3 + cos11\alpha_4 - cos11\alpha_5 + cos11\alpha_6 - cos11\alpha_7] = 0 \\ b_{13} = \frac{4}{13\pi}[- cos13\alpha_1 + cos13\alpha_2 - cos13\alpha_3 + cos13\alpha_4 - cos13\alpha_5 + cos13\alpha_6 - cos13\alpha_7] = 0 \\ \vdots \end{cases} \qquad \textbf{(III.6)}$$

Where M is the modulation rate and VAN the effective value of output voltage for one phase, and VAN1 the effective value of fundamental component.

$$\boxed{M = \frac{V_{AN1}}{V_{AN}}} \qquad \textbf{(III.7)}$$

The desired solution must satisfy the following condition: $\alpha_1<\alpha_2<\alpha_3<\alpha_4<\alpha_5<\frac{\pi}{2}$

III.5. Pre-calculated seven-angle PWM :

For seven angles, we can eliminate 6 harmonics (5, 7, 11, 13, 17 and 19) and control the fundamental component of the output voltage. For k=7, we obtain the following non-linear system of equations:

$$\begin{cases} b_1 = \frac{4}{\pi}[-cos\alpha_1 + cos\alpha_2 - cos\alpha_3 + cos\alpha_4 - cos\alpha_5 + cos\alpha_6 - cos\alpha_7] = M \\ b_5 = \frac{4}{5\pi}[-cos5\alpha_1 + cos5\alpha_2 - cos5\alpha_3 + cos5\alpha_4 - cos5\alpha_5 + cos5\alpha_6 - cos5\alpha_7] = 0 \\ b_7 = \frac{4}{7\pi}[-cos7\alpha_1 + cos7\alpha_2 - cos7\alpha_3 + cos7\alpha_4 - cos7\alpha_5 + cos7\alpha_6 - cos7\alpha_7] = 0 \\ b_{11} = \frac{4}{11\pi}[-cos11\alpha_1 + cos11\alpha_2 - cos11\alpha_3 + cos11\alpha_4 - cos11\alpha_5 + cos11\alpha_6 - cos11\alpha_7] = 0 \\ b_{13} = \frac{4}{13\pi}[-cos13\alpha_1 + cos13\alpha_2 - cos13\alpha_3 + cos13\alpha_4 - cos13\alpha_5 + cos13\alpha_6 - cos13\alpha_7] = 0 \\ b_{17} = \frac{4}{17\pi}[-cos17\alpha_1 + cos17\alpha_2 - cos17\alpha_3 + cos17\alpha_4 - cos17\alpha_5 + cos17\alpha_6 - cos17\alpha_7] = 0 \\ b_{19} = \frac{4}{19\pi}[-cos19\alpha_1 + cos19\alpha_2 - cos19\alpha_3 + cos19\alpha_4 - cos19\alpha_5 + cos19\alpha_6 - cos19\alpha_7] = 0 \end{cases} \quad \textbf{(III.8)}$$

To solve this system of non-linear equations, we use numerical methods such as: Newton-Raphson or linearization by least square method, etc.

III.6. The Newton-Raphson method :

This method is well known for its fast convergence, provided the method is supported by good initial conditions. Newton's method defines the solution procedure for the set of equations given by:

$$\alpha_{k+1} = \alpha_k - \frac{F(\alpha_k)}{F'(\alpha_k)} \quad \textbf{(III.9)}$$

In matrix form, equation Eq2.37 is written

$$\Delta(\alpha) = J(\alpha)^{-1}.f(\alpha) \quad \textbf{(III.10)}$$

Where J(α) is the Jacobian matrix that groups the first-order partial derivatives of the function f(α).

The steps of the Newton-Raphson sum method are as follows:

1. The initial values of α are substituted into the matrices J(α) and f(α).

2. The vector Δ(α) is calculated using the Gaussian elimination method.

3. The resulting vector Δ(α) is compared to the tolerance vector.

4. If the value of Δ(α) is less than the tolerance vector, the loop is exited.

5. Otherwise, the new values of α are found by $\alpha_{k+1} = \alpha_k - \Delta(\alpha)$

6. The new values of α are substituted into the matrices J and f. Then, the loop is repeated from

step (2). The vector f(α) and matrix J(α) for five angles:

$$f(\alpha) = \begin{bmatrix} -\frac{\pi}{4}M - cos\alpha_1 + cos\alpha_2 - cos\alpha_3 + cos\alpha_4 - cos\alpha_5 \\ -cos5\alpha_1 + cos5\alpha_2 - cos5\alpha_3 + cos5\alpha_4 - cos5\alpha_5 \\ -cos7\alpha_1 + cos7\alpha_2 - cos7\alpha_3 + cos7\alpha_4 - cos7\alpha_5 \\ -cos11\alpha_1 + cos11\alpha_2 - cos11\alpha_3 + cos11\alpha_4 - cos11\alpha_5 \\ -cos13\alpha_1 + cos13\alpha_2 - cos13\alpha_3 + cos13\alpha_4 - cos13\alpha_5 \end{bmatrix} \quad \textbf{(III.11)}$$

$$J(\alpha) = \begin{bmatrix} -sin\alpha_1 & sin\alpha_2 & -sin\alpha_3 & sin\alpha_4 & -sin\alpha_5 \\ -5sin5\alpha_1 & 5sin5\alpha_2 & -5sin5\alpha_3 & 5sin5\alpha_4 & -5sin5\alpha_5 \\ -7sin7\alpha_1 & 7sin7\alpha_2 & -7sin7\alpha_3 & 7sin7\alpha_4 & -7sin7\alpha_5 \\ -11sin11\alpha_1 & 11sin11\alpha_2 & -11sin11\alpha_3 & 11sin11\alpha_4 & -11sin11\alpha_5 \\ -13sin13\alpha_1 & 13sin13\alpha_2 & -13sin13\alpha_3 & 13sin13\alpha_4 & -13sin13\alpha_5 \end{bmatrix} \quad \textbf{(III.12)}$$

The vector f(α) and matrix J(α) for seven angles:

$$f(\alpha) = \begin{bmatrix} -\frac{\pi}{4}M - cos\alpha_1 + cos\alpha_2 - cos\alpha_3 + cos\alpha_4 - cos\alpha_5 + cos\alpha_6 - cos\alpha_7 \\ -cos5\alpha_1 + cos5\alpha_2 - cos5\alpha_3 + cos5\alpha_4 - cos5\alpha_5 + cos5\alpha_6 - cos5\alpha_7 \\ -cos7\alpha_1 + cos7\alpha_2 - cos7\alpha_3 + cos7\alpha_4 - cos7\alpha_5 + cos7\alpha_6 - cos7\alpha_7 \\ -cos11\alpha_1 + cos11\alpha_2 - cos11\alpha_3 + cos11\alpha_4 - cos11\alpha_5 + cos11\alpha_6 - cos11\alpha_7 \\ -cos13\alpha_1 + cos13\alpha_2 - cos13\alpha_3 + cos13\alpha_4 - cos13\alpha_5 + cos13\alpha_6 - cos13\alpha_7 \\ -cos17\alpha_1 + cos17\alpha_2 - cos17\alpha_3 + cos17\alpha_4 - cos17\alpha_5 + cos17\alpha_6 - cos17\alpha_7 \\ -cos19\alpha_1 + cos19\alpha_2 - cos19\alpha_3 + cos19\alpha_4 - cos19\alpha_5 + cos19\alpha_6 - cos19\alpha_7 \end{bmatrix} \quad \textbf{(III.13)}$$

$$J(\alpha) = \begin{bmatrix} -sin\alpha_1 & sin\alpha_2 & -sin\alpha_3 & sin\alpha_4 & -sin\alpha_5 & sin\alpha_6 & -sin\alpha_7 \\ -5sin5\alpha_1 & 5sin5\alpha_2 & -5sin5\alpha_3 & 5sin5\alpha_4 & -5sin5\alpha_5 & 5sin5\alpha_6 & -5sin5\alpha_7 \\ -7sin7\alpha_1 & 7sin7\alpha_2 & -7sin7\alpha_3 & 7sin7\alpha_4 & -7sin7\alpha_5 & 7sin7\alpha_6 & -7sin7\alpha_7 \\ -11sin11\alpha_1 & 11sin11\alpha_2 & -11sin11\alpha_3 & 11sin11\alpha_4 & -11sin11\alpha_5 & 11sin11\alpha_6 & -11sin11\alpha_7 \\ -13sin13\alpha_1 & 13sin13\alpha_2 & -13sin13\alpha_3 & 13sin13\alpha_4 & -13sin13\alpha_5 & 13sin13\alpha_6 & -13sin13\alpha_7 \\ -17sin17\alpha_1 & 17sin17\alpha_2 & -17sin17\alpha_3 & 17sin17\alpha_4 & -17sin17\alpha_5 & 17sin17\alpha_6 & -17sin17\alpha_7 \\ -19sin19\alpha_1 & 19sin19\alpha_2 & -19sin19\alpha_3 & 19sin19\alpha_4 & -19sin19\alpha_5 & 19sin19\alpha_6 & -19sin19\alpha_7 \end{bmatrix} \quad \textbf{(III.14)}$$

The tolerance vectors for five and seven angles are:

$$T_5 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} . 10^{-5} \quad et \quad T_7 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} . 10^{-5} \quad \textbf{(III.15)}$$

III.7. MATLAB simulation :

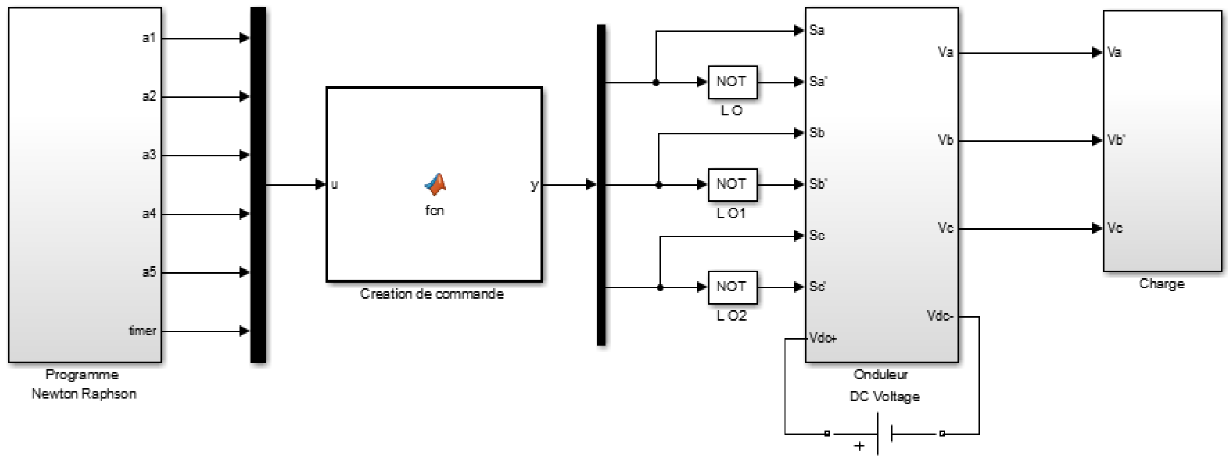

Figure III.2 Simulink block diagram of five-angle pre-calculated PWM technique

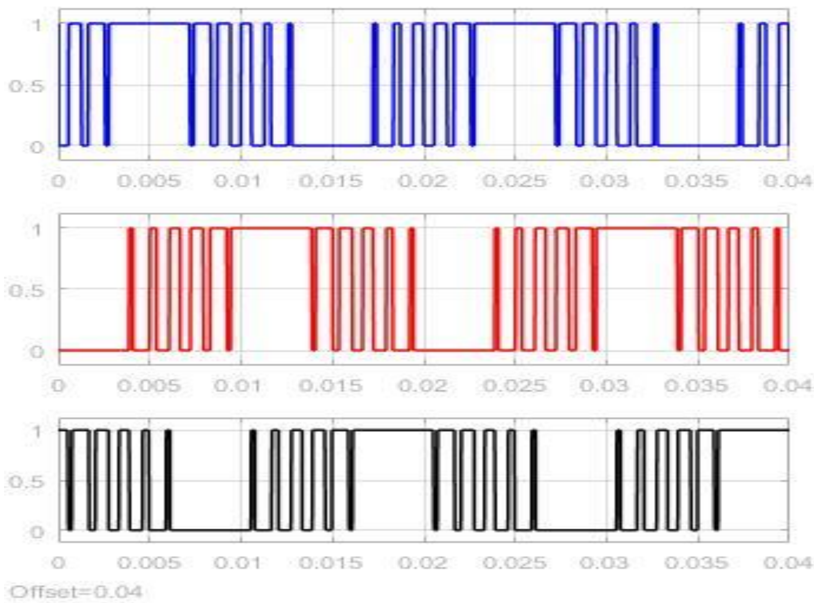

Figure III.3 Control signals Q1, Q2, and Q3

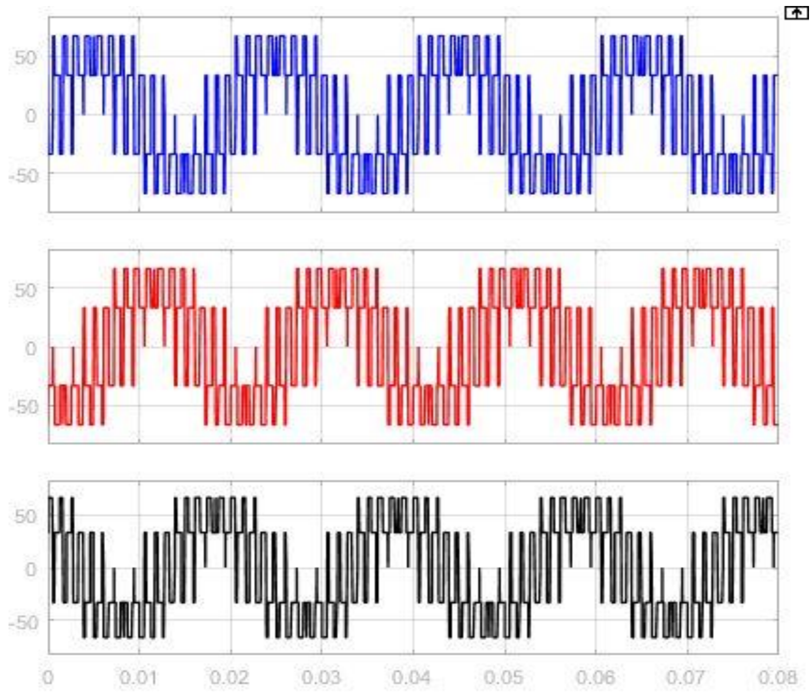

Figure III.4 Simple exit voltage VAN,VBN,VCN

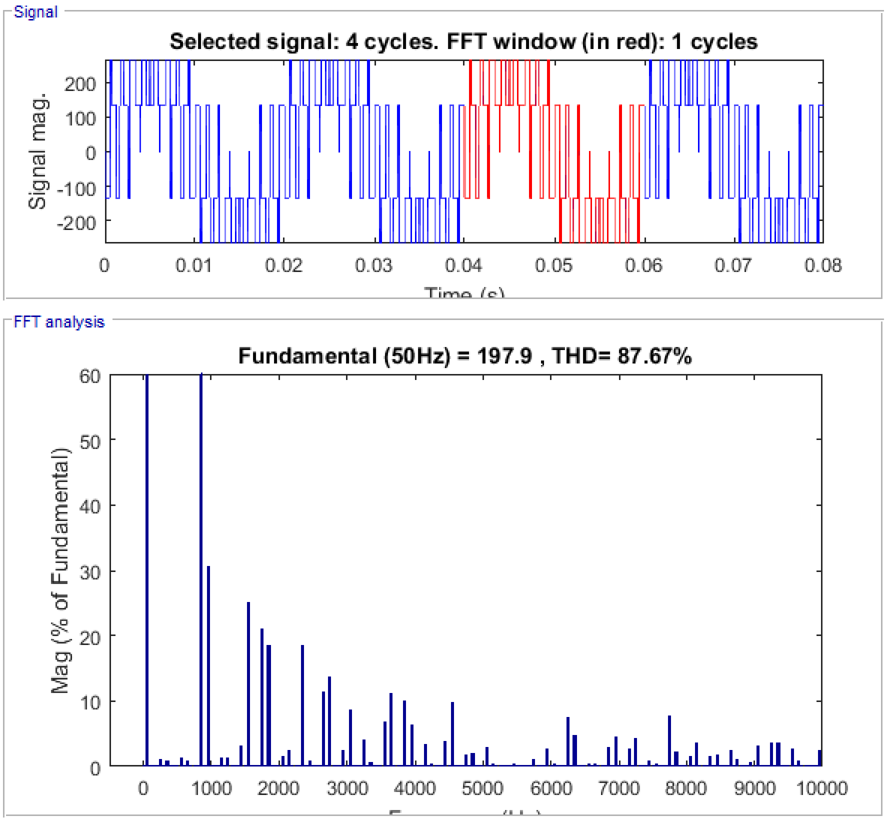

Figure III.5 Spectre Harmonic of valtage VAN with fMAX=10,000

III.7. Comparison of THD results for different control techniques

Table above shows the Comparison of THD results for different control techniques.

Table III.1. Comparison of THD results for different control techniques

Method	**Value**
Selective Harmonic Elimination	**87.67%**
Space Vector Pulse Width Modulation	**159.13 %**
Sine-Triangle PWM	**79.84 %**

III.8. Conclusion

The pre-calculation control method offers a powerful and precise approach to inverter waveform optimization. By selectively eliminating low-order harmonics at the switching level, it achieves remarkably low THD values, making it suitable for high-power applications with stringent power quality requirements. Despite its offline nature and computational demands, this method significantly reduces the need for output filtering and enhances inverter efficiency.

Through mathematical modeling, numerical analysis, and simulation, this chapter has demonstrated the potential of pre-calculated switching strategies to outperform traditional modulation methods. When properly implemented, this technique ensures high-quality AC output, greater waveform purity, and compliance with harmonic standards in modern power systems

General Conclusion

General Conclusion

This dissertation has presented a comparative analysis of various control strategies for voltage source inverters, with a particular emphasis on enhancing output waveform quality and minimizing total harmonic distortion. Traditional methods, including the 180-degree conduction technique, sinusoidal pulse width modulation, and space vector modulation, were first examined to establish a performance baseline.

Through extensive MATLAB/Simulink simulations and mathematical modeling, the results revealed that the Selective Harmonic Elimination technique consistently achieved the lowest levels of total harmonic distortion, often below 10%, compared to 20–30% for the other methods. The waveform generated using this technique closely approximated a sinusoidal shape, even with fewer switching transitions. Additionally, the method demonstrated improved voltage utilization and waveform symmetry. However, the trade-off involved increased computational complexity, as solving nonlinear equations for switching angles required numerical methods and careful initial conditions to ensure convergence.

The study then concentrated on the pre-calculation control technique, also referred to as Selective Harmonic Elimination, which allows for the precise cancellation of specific low-order harmonics by calculating switching angles in advance. Through detailed mathematical modeling and simulation using MATLAB and Simulink, it was demonstrated that this method achieves significantly lower harmonic distortion and offers improved control over the waveform compared to conventional approaches.

We notice that the THD of the precalculated method is a little high compared to the sine triangle method. This is due to the harmonics of the low angles, but if we use a low-pass filter, the THD of the precalculated method will be improved.

Despite requiring offline computation and being sensitive to variations in the load, the pre-calculation technique remains a promising candidate for high-power and quality-critical industrial applications. Future research directions may include the development of real-time adaptive versions of this method, the integration of artificial intelligence to dynamically optimize switching angles, and hardware-level implementation using digital signal processors or field-programmable gate arrays.

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