Simulations of Two-Phase Flows Based on the Riemann Problem.

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Abstract

In these papers we present mathematical and analytical simulation techniques to solve the two-phase drift flow model. The purpose is to provide a detailed presentation of the complete and exact solution to the Riemann problem and using the Fortron program to compute the exact solution .

Keyword: Drift-flux model, riemann problem, finite volume, godunov centred methods, compressibility factor.

1 Introduction

The drift-flux model is one of the most basic models of two-phase fluid flow problems. Such a model is commonly used in wellbore, chemical and process industry and in the safety analysis of nuclear plants. The model is a two-phase flow model arising from averaging the single-phase flow equations [Ishii (1975); Drew and Passman (1999)]. It results in a continuity equation for each phase and a mixture momentum equation along with supplementary closure laws [Ishii (1975); Zuber and Findlay(1965)]. The mathematical study of flow of fluid in networks of pipes is a very active field of research. In this paper, we suggest an algorithm to obtain an accurate analytical solution to the floating flow model. In particular, the exact solution suggested here is based on the Riemann problem in the unstable form of this model.

2 Main results

The model of drift-flux is expressed by a set of partial differential equations based on the physical laws of mass and momentum conservations, can be written in differential form.
Let the Riemann problem of the current drift-flux model

$$\frac{\partial \mathbb{U}}{\partial t} + \frac{\partial \mathbb{F}(\mathbb{U})}{\partial x} = 0, \qquad t \in \mathbb{R}^+, \quad x \in \mathbb{R}, \qquad (1)$$

with the following initial conditions:

$$\mathbb{U}(x,0) = \begin{cases} \mathbb{U}^l & if \quad x < x_0 \\ \mathbb{U}^r & if \quad x > x_0 \end{cases} \qquad (2)$$

The solution of the Riemann problem (1) and (2) has three waves, which are associated with the eigenvalues λ_1, λ_2 *and* λ_3. The three waves separate four constant states. So the complete solution define in the left and right side (x/t) for each of the three waves.

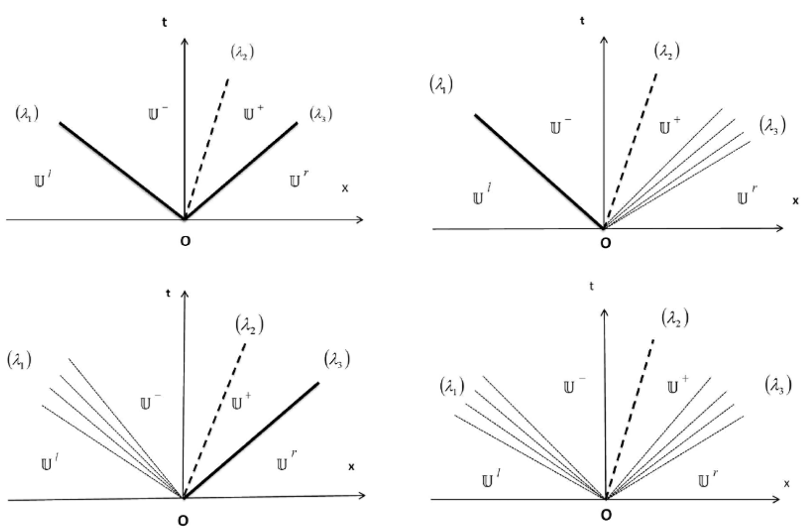

Fig. 1. Structure of the solution of the Riemann problem on the $x - t$ plane. There are three wave families associated with the eigenvalues λ_1, λ_2 and λ_3.

A listing of a FORTRAN program to compute the exact solution to the Riemann problem (1),(2) is now given. The main program calls two routines . The first routine solves iteratively for the density $\mathbb{U}_1 = \rho 1$, $\mathbb{U}_2 = \rho 2$ and then computes the particle speed u the guessed value for the iteration is provided by the subroutine GUESSP. The second routine finds, for given $\mathbb{U}_1 = \rho 1$, $\mathbb{U}_2 = \rho 2$, u, p and S = x/t, the solution of the Riemann problem at the point (x,t). This routine can then be utilised in numerical methods to solve the general initial boundary value problem for the Euler equations.

2.1 High-Resolution Numerical Implementation

This problem are solved numerically using finite volume methods. Two different numerical methods, namely, the FORCE and the Lax-Friedrichs methods are employed to validate the current solutions.

table 1:

test	ρ_1^l	ρ_2^l	u^l	ρ_1^r	ρ_2^r	u^r	a_g	a_l
1	50	1000	-100	50	1000	100	500	100
2	6	1	0	1	1	0	10	7

table 2:

test	ρ_1^-	ρ_2^-	u^l	ρ_1^+	ρ_2^+
1	25.25169	505.0340	0.0	25.25169	505.0337
2	2.987665	0.4979441	6.782284	2.168898	2.168898

Table 2. Solution of the Riemann problem with the initial data from Table 1.

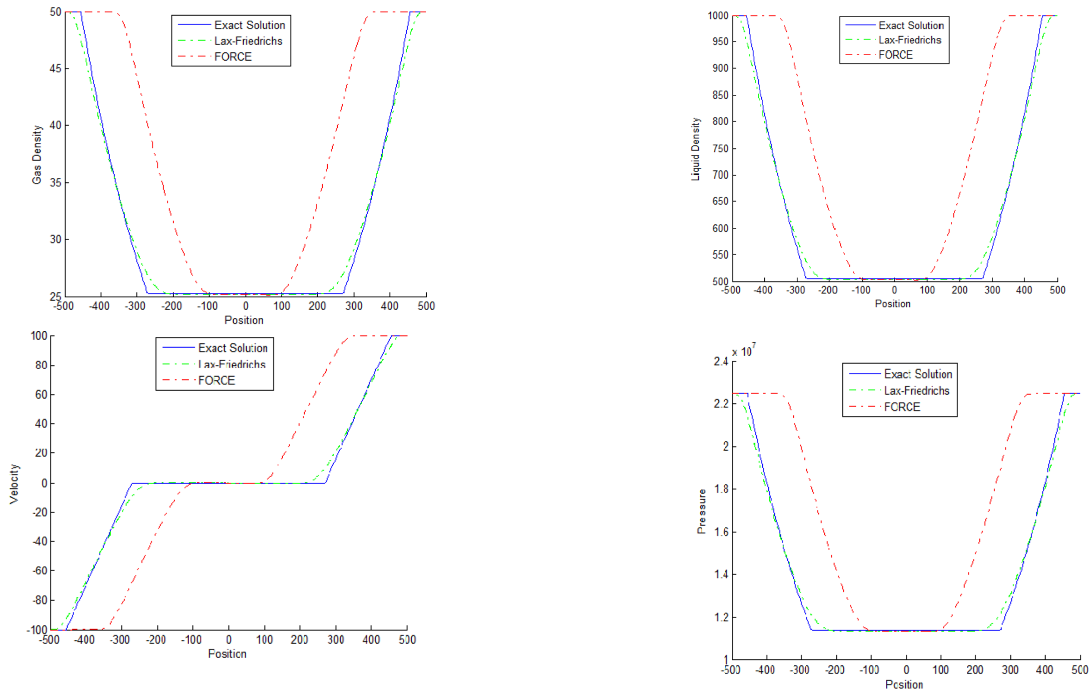

Figure 1: Both rarefaction solutions of the Riemann problem. Solid line is the exact solution, whereas stars, dotted and squares are numerical results FORCE and Lax-Friedrichs respectively, at time t = 1.85.

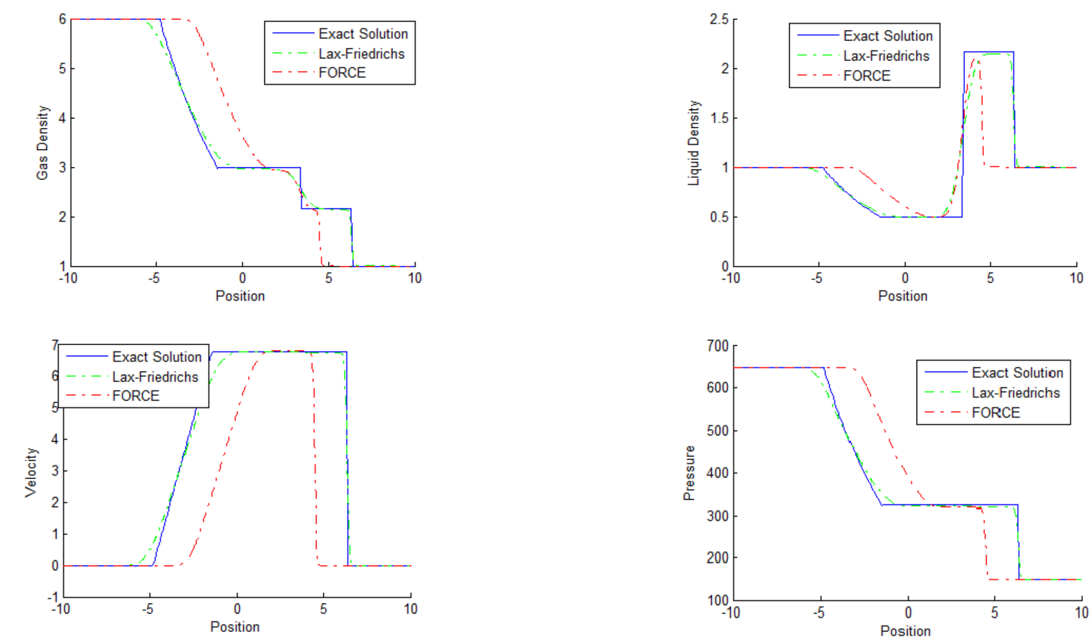

Figure 2: Solution of the Riemann problem for the Left rarefaction/contact discontinuity/right shock solutions at time t = 0.5

3 Conclusion

We obtain accurate analytical solutions based on the Riemann problem from the model equations program by main program. These are also evaluated numerically against the methods in which the Riemann solution is completely digital. An excellent agreement was indicated between analytical results and numerical forecasts.

References

[1] , S. and Flatten, T. [2007]. "On the wave structure of two-phase flow models," *SIAM J. Appl. Math.* **67**, 487-511.

[2] , M. [1975]. Thermo-Fluid Dynamics Theory of Two-Phase Flow (Eyrolles, Paris).

[3] Pan, L., Webbb, S. W. and Oldenburg, C. M. [2011b]. "Analytical solution for two-phase flow in a wellbore using the drift-flux model," *Adv. Water Resour.* **34**, 1656-1665.

[4] Raja Sekhar, T. and Sharma, V. D. [2012]. "Solution to the Riemann problem in a onedimensional magnetogasdynamic flow," *Int. J. Comput.* Math. **89**, 200-216.

[5] , D. [2011]. "Numerical resolution for a compressible two-phase flow model based on the theory of thermodynamically compatible systems," *Appl. Math. Comput.* **217**, 5023-5040.