Wave propagation behaviour of functionally graded material plates based on neutral surface position

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Abstract— **In this paper, an efficient higher-order shear deformation is developed for dynamic analysis of plate in the presence of thermal environments. By dividing the transverse displacement into bending and shear parts, the number of unknowns and governing equations of the present theory is reduced, The thermal effects and temperature-dependent material properties are both taken into account. The temperature field is assumed to be a uniform distribution over the plate surface and varied in the thickness direction only. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituent's isotropic plates. It can be concluded that the present theory is not only accurate but also simple in predicting the wave propagation characteristics in the functionally graded plate.**

Keywords— **Wave propagation; P-FGM plate; Thermal effects; Higher order theory; Neutral surface position**

I. INTRODUCTION

Functionally graded materials (FGMs) are new materials which are designed to achieve a functional performance with gradually variable properties in one or more directions (Koizumi, 1992) [1]. This continuity prevents the material from having disadvantages of composites such as delamination due to large interlaminar stresses, initiation and propagation of cracks because of large plastic deformation at the interfaces and so on. Typically, FGMs are made of a mixture of ceramics and a combination of different metals (Bennoun et al ., 2016 [2]; Ebrahimi and Dashti, 2015 [3]; Sallai et al., 2015 [4]; Meradjah et al., 2015 [5]; Kar and Panda, 2015 [6]; Pradhan and Chakraverty, 2015 [7]; Bakora and Tounsi, 2015 [8]; Bouchafa et al., 2015 [9]; Arefi, 2015 [10]; Akbaş, 2015 [11]; Mansouri and Shariyat, 2015 [12]; Belabed et al., 2014 [13]; Khalfi et al., 2014 [14]; Mansouri and Shariyat, 2014 [15]; Hadji et al., 2014 [16]; Fekrar et al., 2014 [17]; Tounsi et al., 2013a [18]. So the key point is an accurate description of the variables and the material properties in the thickness direction, to perform a satisfactory analysis of the mechanical behavior of FGM plates. Many works on FGM structures have been studied in literature.

II. FUNDAMENTAL FORMULATIONS

Consider a rectangular plate made of FGMs of thickness h. the analysis of the FGM plates can easily be treated with the homogenous isotropic plate theories, because the stretching and bending equations of the plate are not coupled. In order to determine the position of neutral surface of FGM plates, two different datum planes are considered for the measurement of , namely, and measured from the middle surface and the neutral surface of the plate, respectively, as shown in Fig. 1. The volume fraction of ceramic can be written in terms of and coordinates as:

$$V_C = \left(\frac{z_{ms}}{h} + \frac{1}{2}\right)^n = \left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^n \qquad (1)$$

The material non-homogeneous properties of FG plate, as a function of thickness coordinate, become:

(2)

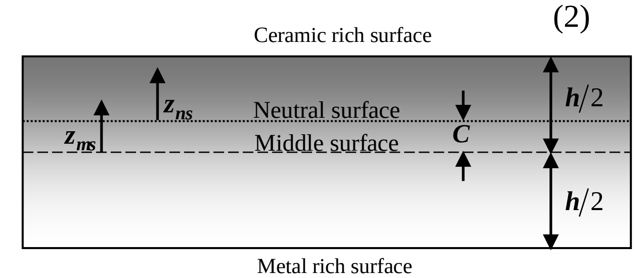


Fig. 1: The position of middle surface and neutral surface for a functionally graded plate.

Consequently, the position of neutral surface can be obtained as:

$$C = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms} dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms}) dz_{ms}} \tag{3}$$

The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components.

III. KINEMATICS AND CONSTITUTIVE EQUATIONS

Based on the assumptions made in the preceding section, the displacement field can be obtained:

$$\begin{aligned} u(x,y,z_{ns},t) &= u_0(x,y,t) - z_{ns}\frac{\partial w_b}{\partial x} + f\left(z_{ns}\right)\frac{\partial w_s}{\partial x} \\ v(x,y,z_{ns},t) &= v_0(x,y,t) - z_{ns}\frac{\partial w_b}{\partial y} + f\left(z_{ns}\right)\frac{\partial w_s}{\partial y} \\ w(x,y,z_{ns},t) &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \tag{4}$$

Where $f(z_{ns}) = \left(z_{ns} + C\right)\left[-\frac{1}{4} + \frac{5}{3}\left(\frac{z_{ns}+C}{h}\right)^2\right]$ (5)

For elastic and isotropic FGMs, the constitutive relations can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha\,\Delta T \\ \varepsilon_y - \alpha\,\Delta T \\ \gamma_{xy} \end{Bmatrix} \tag{6}$$

And

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{7}$$

Using the material properties defined in Eq. (2), stiffness coefficients, Q_{ij}, can be expressed as:

$$Q_{11} = Q_{22} = \frac{E(z_{ns},T)}{1-\nu(z_{ns},T)^2}, \quad Q_{12} = \frac{\nu\,E(z_{ns},T)}{1-\nu(z_{ns},T)^2},$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z_{ns},T)}{2\left[1+\nu(z_{ns},T)\right]}, \tag{8}$$

IV. GOVERNING EQUATIONS

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Reddy, 2002):

$$0 = \int_0^t (\delta U - \delta K)\,dt \tag{9}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate stated as:

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x\,\delta\varepsilon_x + \sigma_y\,\delta\varepsilon_y + \tau_{xy}\,\delta\gamma_{xy} + \tau_{yz}\,\delta\gamma_{yz} + \tau_{zx}\,\delta\gamma_{zx}\right] dA dz_{ns} \\ &= \int_A \left[N_x\,\delta\varepsilon_x^0 + N_y\,\delta\varepsilon_y^0 + N_{xy}\,\delta\varepsilon_{xy}^0 + M_x^b\,\delta k_x^b + M_y^b\,\delta k_y^b + M_{xy}^b\,\delta k_{xy}^b + M_x^s\,\delta k_x^s \right. \\ &\quad \left. + M_y^s\,\delta k_y^s + M_{xy}^s\,\delta k_{xy}^s + S_{yz}^s\,\delta\gamma_{yz}^s + S_{xz}^s\,\delta\gamma_{xz}^s\right] dA \end{aligned} \tag{10}$$

The following equations of motion of the plate are obtained:

$$\begin{aligned} \delta u &: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u} \\ \delta v &: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v} \\ \delta w_b &: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0(\ddot{w}_b + \ddot{w}_s) - I_2\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right) \\ \delta w_s &: \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = I_0(\ddot{w}_b + \ddot{w}_s) - \frac{I_2}{84}\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) \end{aligned} \tag{11}$$

V. DISPERSION RELATIONS

We assume solutions for u_0, v_0, w_b and w_s representing propagating waves in the x-y plane with the form

VI. $$\begin{Bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_b(x,y,t) \\ w_s(x,y,t) \end{Bmatrix} = \begin{Bmatrix} U\exp\left[i(k_1x + k_2y - \omega t\right] \\ V\exp\left[i(k_1x + k_2y - \omega t\right] \\ W_b\exp\left[i(k_1x + k_2y - \omega t\right] \\ W_s\exp\left[i(k_1x + k_2y - \omega t\right] \end{Bmatrix} \tag{12}$$

Where U; V; W_b and W_s are the coefficients of the wave amplitude.

VII. WE OBTAIN: $\left(\left[K\right] - \omega^2\left[M\right]\right)\left\{\Delta\right\} = \left\{0\right\}$ (13)

VIII. WHERE $\left\{\Delta\right\} = \left\{U, V, W_b, W_s\right\}^T$,

IX. $\omega_1 = W_1(k), \omega_2 = W_2(k), \omega_3 = W_3(k)$

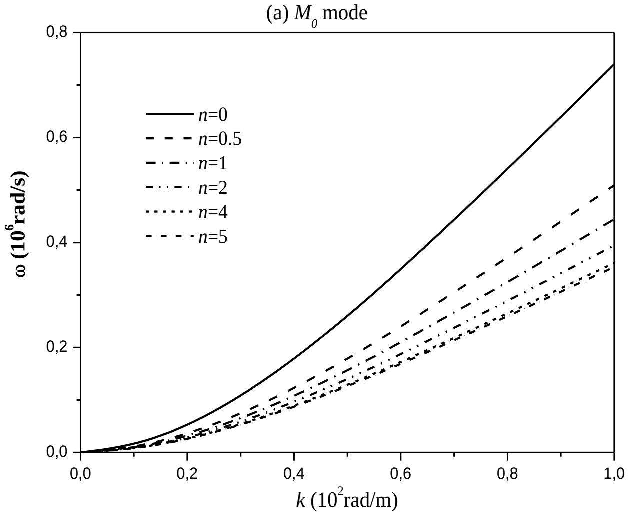
X. AND $\omega_4 = W_4(k)$

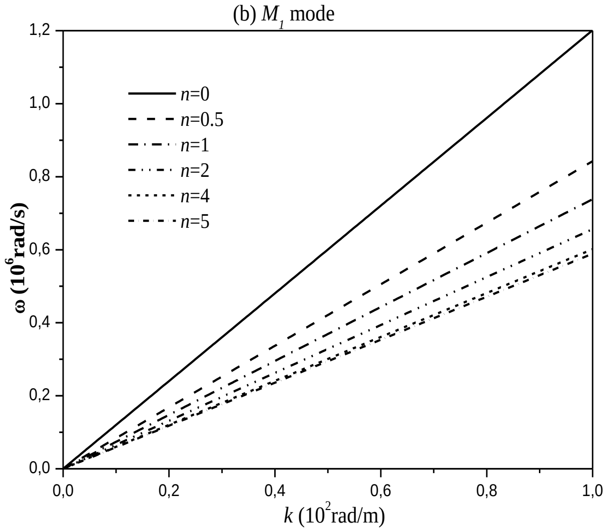
They correspond with the wave M_0, M_1 M_2 and M_3 respectively. The wave modes and correspond to the flexural wave, the wave modes and correspond to the extensional wave.

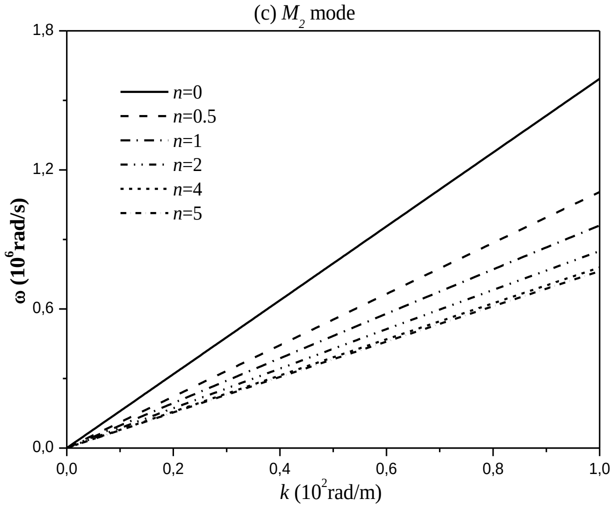
$$\left[K\right] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}, \quad \left[M\right] = \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \tag{14}$$

XI. NUMERICAL RESULTS AND DISCUSSION

In this section, the eigenvalues problem for a Si3N4/SUS304 functionally graded material plate is considered. The thickness of the functionally graded plate is 0.02 m. The Young's modulus E , density ρ , Poisson's ratio ν and thermal expansion coefficient α of these materials $T_M = T_C = 300\,K$, which are taken from reference (Yang and Shen, 2002; Reddy and Chin, 1998).







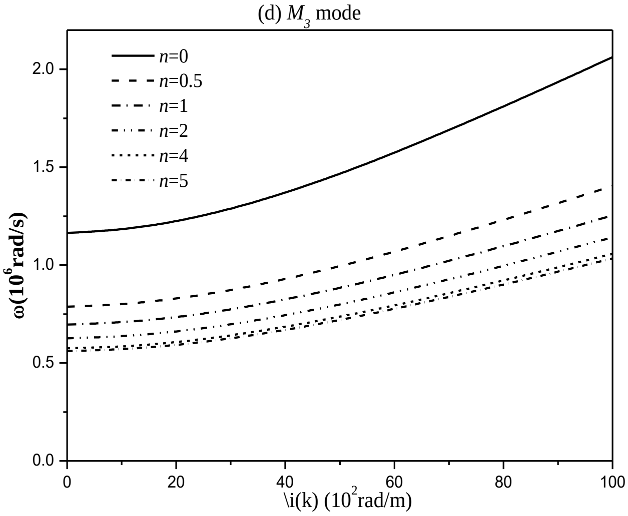


Fig 2: The dispersion curves of the different functionally graded plates ($T_M = T_C = 300\,K$).

The accuracy of the present neutral surface-based model involving only four unknown displacement functions is verified by comparing the obtained results with those computed using Reddy's theory (Sun and Luo, 2011b [19]). Figs. 2 show, respectively, the dispersion curves and the phase velocity for flexural wave mode M_0 of the different functionally graded plates under thermal environmental condition $T_M = T_C = 300\,K$. It can be seen that the results of the present neutral surface-based model (with only four unknown displacement functions) are in excellent agreement with those of Reddy's theory (with five unknown displacement functions) for all values of power law index n. This indicates that the partition of the transverse displacement into the bending and shear parts lead not only to accurate results, but it can improve the computational cost due to reducing the number of unknowns as well as governing equations of the wave propagation in the functionally graded plate.

XII. CONCLUSIONS

The proposed theory has an advantage over the existing higher-order shear deformation theories since they involve less unknowns as well as equations of motion. The computational cost can therefore be reduced. In addition, the partition of the transverse displacement of the proposed theory into the bending and shear parts helps one to see the contributions due to shear and bending to the total one. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. Finally, it can be said that the proposed higher order shear and normal deformation theory is not only accurate but also provides an elegant and easily implementable approach for simulating the characteristics of wave propagation of the functionally graded plate.

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