Mechanical response of functionally graded beams with porosities

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***Abstract*— This work presents a free vibration analysis of functionally graded metal–ceramic (FG) beams with considering porosities that may possibly occur inside the functionally graded materials (FGMs) during their fabrication. A new displacement field containing integrals is proposed which involves only three variables. Based on the suggested theory, the equations of motion are derived from Hamilton's principle. This theory involves only three unknown functions and accounts for parabolic distribution of transverse shear stress. In addition, the transverse shear stresses are vanished at the top and bottom surfaces of the beam. The Navier solution technique is adopted to derive analytical solutions for simply supported beams. The accuracy and effectiveness of proposed model are verified by comparison with previous research. A detailed numerical study is carried out to examine the influence of the porosity on the free vibration responses of functionally graded beams.**

***Keywords*— Free vibration; Functionally graded materials; Integral; Hamilton's principle.**

I. INTRODUCTION

Functionally graded materials (FGMs) have many advantages for use in engineering structural components. The concept of FGMs was initially introduced in the mid-1980s by Japanese scientists. The FGMs are widely used in mechanical, aerospace, nuclear, and civil engineering. Consequently, studies devoted to understand the static and dynamic behaviors of FGM beams and plates have being paid more and more attentions in recent years.

This work aims to develop a new simple higher order shear deformation theory for the free vibration analyses of FG beams with considering porosities that may possibly occur inside the functionally graded materials (FGMs) during their fabrication. Analytical solutions are obtained for FG beam and its accuracy is verified by comparing the obtained results with those reported in the literature. The effects of various variables, such as span-to-depth ratio, gradient index and the volume fraction of porosity on the free vibration of FG beam are all discussed.

II. PROBLEM FORMULATION

Consider a functionally graded beam with length L and rectangular cross section bxh , with b being the width and h being the height as shown in Fig. 1. The beam is made of isotropic material with material properties varying smoothly in the thickness direction

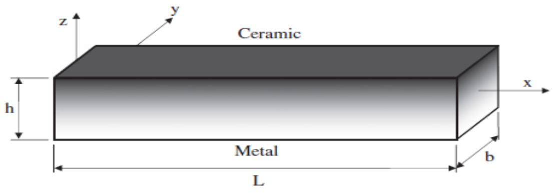

Fig.1. Geometry and coordinate of a FG beam

A. *Kinematics and constitutive equations*

The displacement field can be obtained

$$u(x,z,t)=u_0(x,t)-z\frac{\partial w_0}{\partial x}+k_1\, f(z)\int\theta(x,t)\,dx \tag{1a}$$

$$w(x,z,t)=w_0(x,t) \tag{1b}$$

In this work, the present higher-order shear deformation beam theory is obtained by setting :

$$f(z) = \frac{1}{2} z \left(\frac{1}{4} h^2 - \frac{1}{3} z^2 \right) \quad (2)$$

B. *Equations of motion*

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Thai and Vo 2012)

$$\delta \int_{t_1}^{t_2} (U - T) dt = 0 \quad (3)$$

The equations of motion associated with the present shear deformation theory.

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_0}{\partial x^3} + B_{11}^s k_1 \frac{\partial \theta}{\partial x} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + J_1 A' k_1 \frac{\partial \ddot{\theta}}{\partial x} \quad (4a)$$

$$B_{11} \frac{\partial^3 u_0}{\partial x^3} - D_{11} \frac{\partial^4 w_0}{\partial x^4} + D_{11}^s k_1 \frac{\partial^2 \theta}{\partial x^2} = I_0 \ddot{w}_0 + I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_2 \frac{\partial^2 \ddot{w}_0}{\partial x^2} \quad (4b)$$
$$+ J_2 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2}$$

$$- B_{11}^s k_1 \frac{\partial u_0}{\partial x} + D_{11}^s k_1 \frac{\partial^2 w_0}{\partial x^2} - H_{11}^s k_1^2 \theta + A_{55}^s (k_1 A')^2 \frac{\partial^2 \theta}{\partial x^2}$$
$$= - J_1 k_1 A' \frac{\partial \ddot{u}}{\partial x} + J_2 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} - K_2 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \quad (4c)$$

C. *Analytical solution*

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_0, θ can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\alpha x) e^{i \omega t} \\ W_m \sin(\alpha x) e^{i \omega t} \\ X_m \sin(\alpha x) e^{i \omega t} \end{Bmatrix} \quad (5)$$

where ω is the frequency of free vibration of the beam, $\sqrt{i} = -1$ the imaginary unit.

Substituting Eq. (3) into Eqs. (4), the following problem is obtained:

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \right) \begin{Bmatrix} U_m \\ W_m \\ X_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ 0 \end{Bmatrix} \quad (6)$$

III. RESULTS AND DISCUSSION

In this section, various numerical examples are presented and discussed to verify the accuracy of the present theory in predicting the bending and free vibration of simply supported FG beams. The FG beam is taken to be made of aluminum and alumina with the following material properties:

Ceramic (Alumina, Al2O3): $E_c = 380$ GPa; $\nu = 0.3$; $\rho_c = 3960$ kg/m^3.

Metal (Aluminium, Al): $E_m = 70$ GPa; $\nu = 0.3$; $\rho_m = 2702$ kg/m^3.

For convenience, the following dimensionless form is used:

$$\bar{\omega} = \frac{\omega L^2}{h} \sqrt{\frac{\rho_m}{E_m}}$$

Table 1 show the non dimensional fundamental frequencies $\bar{\omega}$ of perfect and imperfect FG beams of FG beams for different values of power law index p and span-to-depth ratio L/h. The calculated frequencies are compared with those given by Simsek. (2010) using various beam theories. An excellent agreement between the present theory and results of Simsek. (2010) is found. The results reveal that the frequency results decrease as the volume fraction of porosity (α) increases.

Table 1. Variation of fundamental frequency $\bar{\omega}$ with the power-law index for FG beam for $L/h = 5$.

Theory	α	p=0	0.2	0.5	1	5	10	Metal
CBT [1]	$\alpha = 0$	5.3953	5.0206	4.5931	4.1484	3.5949	3.4921	2.8034
FSDBT [1]	$\alpha = 0$	5.1525	4.8066	4.4083	3.9902	3.4312	3.3134	2.6772
ESDBT [1]	$\alpha = 0$	5.1542	4.8105	4.4122	3.9914	3.4014	3.2813	2.6781
PSDBT [1]	$\alpha = 0$	5.1527	4.8092	4.4111	3.9904	3.4012	3.2816	2.6773
Present	$\alpha = 0$	5.1527	4.8081	4.4107	3.9904	3.4012	3.2816	2.6773
	$\alpha = 0.1$	5.2223	4.8498	4.4042	3.9070	3.1478	3.0292	2.3554
	$\alpha = 0.2$	5.3048	4.8995	4.3928	3.7865	2.6961	2.5718	1.8433

Figure 2 shows the non-dimensional fundamental natural frequency $\bar{\omega}$ of perfect and imperfect FG beams versus the power law index for different values of span-to-depth ratio using the present theory. It is observed that an increase in the value of the power law index p leads to a reduction of frequency. In addition, the porosity leads to a decrease of the frequency of the beam.

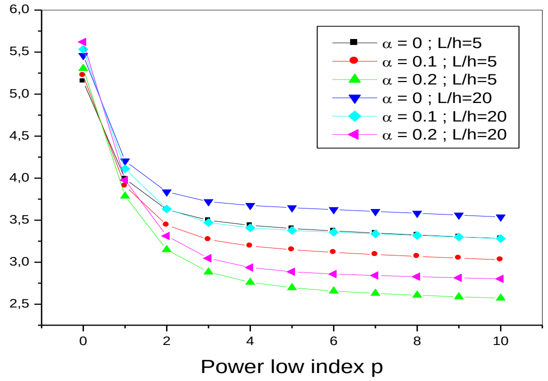

Fig. 2 Variation of the fundamental frequency $\overline{\omega}$ of FG beam with power-law index p .

IV. CONCLUSIONS

Free vibration analysis of perfect and imperfect FG beams under uniform load is carried out in the present study by a new shear deformation beam theory. The theory inherently satisfies the condition of zero transverse shear stresses on the top and bottom surfaces of the beam. The results generated in the present work for various analyses are compared with the existing published results. The comparison proves the accuracy of the presently considered shear deformation theory, and hence it can successfully be employed for the structural analyses of FG beam.

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