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ON THE EXPONENTIAL STABILITY OF SOLUTIONS IN POROUS-ELASTIC WITH DOUBLE POROSITY.

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1- **Abstract:** In the present work we consider a double porous elastic system with viscoelastic and porous dissipations. We use the semigroup approach and the Gearhat-Prüss theorem to prove a well-posedness result and an exponential rate of decay.
2- **Keywords:** semigroup, Exponential decay, double porosity, semigroup, Well posedness
3- **Thesis plan:**

I. Introduction

The double porosity model allows the body to have a double porous structure: a macro porosity connected to pores in the body and a micro porosity connected to fissures in the skeleton. The basic equations for elastic materials with double porosity involve the displacement vector eld, a pressure associated with the pores, and a pressure associated with the ssures. In [?], Cowin and Nunziato developed a linear theory of elastic materials with voids to study mathematically the mechanical behavior of porous solids. In recent paper [?] Iesan and Quintanilla employed Nunziato-Cowin theory to derive a theory of thermoelastic solids which have a double porosity structure.

In this work we consider the following problem

$$\begin{cases} \rho u_{tt} = \mu u_{xx} + b\varphi_x + d\psi_x + \lambda u_{txx} \\ \kappa_1\varphi_{tt} = \alpha\varphi_{xx} + b_1\psi_{xx} - bu_x - \alpha_1\varphi - \alpha_3\psi - \tau_1\varphi_t \\ \kappa_2\psi_{tt} = b_1\varphi_{xx} + \gamma\psi_{xx} - du_x - \alpha_3\varphi - \alpha_2\psi - \tau_2\psi_t \end{cases}$$

Where u is the transversal displacement of a beam of length φ,ψ are porosity variables. To be able to descuss the system we consider that the unknown functions satisfy the following boundary conditions and initial

$$u(x,t) = \varphi_x(x,t) = \psi_x(x,t) = 0, \forall t > 0 \; for \; x \in [0,L]$$
$$u(x,0) = u_0(x), \varphi(x,0) = \varphi_0, \psi(x,0) = \psi_0$$
$$u_t(x,0) = u_1(x), \varphi_t(x,0) = \varphi_1, \psi_t(x,0) = \psi_1$$

II. DECAY

The energy of system (1) is defined

$$E(t)=\int_0^L[\rho|u_{tt}|^2+\mu|u_x|^2+\kappa_1|\varphi_t|^2+\kappa_2|\psi_t|^2+\alpha|\varphi_x|^2$$
$$+\gamma|\psi_x|^2+\alpha_1|\varphi|^2+\alpha_2|\psi|^2]+2b\int_0^L\operatorname{Re}(u_x\overline{\varphi})$$
$$+2\alpha_3\int_0^L\operatorname{Re}(\varphi\overline{\psi})+2b_1\int_0^L\operatorname{Re}(\varphi_x\overline{\psi_x})+2d\int_0^L\operatorname{Re}(u_x\overline{\psi}).$$

We assume that the matrix

$$A=\begin{pmatrix}\mu & b & d\\ b & \alpha_1 & \alpha_3\\ d & \alpha_3 & \alpha_2\end{pmatrix}, B=\begin{pmatrix}\alpha & b_1\\ b_1 & \gamma\end{pmatrix}.$$

are positive difinites. Then, the energy $E(t)$ satisfies

$E(t)=\frac{1}{2}\int_0^L(\rho u_t^2+\kappa_1\varphi_t^2+\kappa_2\psi_t^2)+YAY^T+Z_xBZ_x^T\geq 0$ where, $Y=(u_x,\varphi,\psi)^T$ and $Z=(\varphi_x,\psi_x)^T$.

Moreover, direct calculation gives

$$E'(t)=-\tau_2\int_0^L|w|^2\,dx-\tau_1\int_0^L|\varphi|^2\,dx-\lambda\int_0^L|v_x|^2dx.$$

Theorem 2.1(Gearhart).[ ]

Let $T(t)=e^{At}$ be a C_0-semigroup , then, $T(t)$ is exponentially stable if and only if

$$i\mathbb{R}=\{i\lambda;\lambda\in\mathbb{R}\}\subset\rho(A),$$
$$\overline{\lim_{|\lambda|\to\infty}}\left\|(i\lambda I-A)^{-1}\right\|<\infty.$$

The main result of this work is given by the following

Theorem 2.2. For all $(u_0,u_1,\varphi_0,\varphi_1,\psi_0,\psi_1)\in\mathrm{H}$ the solution (u,φ,ψ) of problem decays exponentially.

Proof.The proof of this theorem is based on Gearhard's Theorem 1 and will be established throught the two following lemmas.

Lemma 2.3. LetAbe the operator associated with the system , then

$$\{i\lambda,\lambda\in\mathbb{R}\}\subset\rho(A).$$

Proof. The proof will be given in three steeps.

i) Shows that $0\in\rho(A)$ and prove that $i\lambda\in\rho(A)$ for $\lambda\in\left(-\left\|A^{-1}\right\|^{-1},\left\|A^{-1}\right\|^{-1}\right)$.

ii) Prove that if $\sup\left\{\left\|(i\lambda I-A)^{-1}\right\|,|\lambda|<\left\|A^{-1}\right\|^{-1}\right\}=M<\infty$, then,

$\left\{i\lambda,|\lambda|<\left\|A^{-1}\right\|^{-1}+M^{-1}\right\}\subset\rho(A)$.

iii) Suppose that $\{i\lambda,\lambda\in\mathbb{R}\}$ is not in $\rho(A)$ then, $\exists\sigma\in\mathbb{R};\left\|A^{-1}\right\|^{-1}\leq|\sigma|<\infty$ such that there exist two sequences $(U_n)\subset D(A),\|U_n\|=1,\ (\lambda_n)\subset\mathbb{R},|\lambda_n|<|\sigma|,\ \lambda_n\to\sigma$ such that

$$\lim_{n\to+\infty}\left\|(i\lambda_nI-A)U_n\right\|=0$$

and obtained a contradiction. □

Lemma 2.4. Let A be the operator defined above, then

$$\overline{\lim_{|\lambda|\to\infty}}\left\|(i\lambda I - A)^{-1}\right\| < \infty.$$

Proof. The proof will be given by contradiction argument. Suppose that Lemma 2.4. is not satisfied, then ther exists a sequence $(i\lambda_n) \subset \rho(A), |\lambda_n| \to 0$ and a sequence of unit vectors $(U_n) \subset D(A)$ such that

$$\left\|(i\lambda_n I - A)U_n\right\| \to 0$$

We prove that $\|U_n\|_H^2 \to 0$

which contradict the fact that $\|U_n\|_H = 1$and the proof of lemma is complete. □

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