الجمهورية الجزائرية الديمقراطية الشعبية

People's Democratic Republic of Algeria

وزارة التعليم العالي والبحث العلمي

Ministry of Higher Education and Scientific Research

جامعة الشهيد حمه لخضر الوادي

Echahid Hamma Lakhdar University of El Oued

Faculty of Technology

Hydraulic & Civil Engineering Department

كليـــة التكنولوجيا

قســـم الري والهندسة المدنية

Course of:

Topography

Dr. Mega Nabil

Level 2nd Year Sciences & Technology

Specialty: Public Works, Hydraulic & Civil Engineering

Semester 4

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INTRODUCTION

This course of Topography is intended for the students of 2nd year of the degree in the fourth semester (S4) specialty "**Civil Engineering, Public Works, and Hydraulics**".

Required skills:

The learning of the course "Topography" requires some previous knowledge in Mathematics. In Fact, it is indispensable for students of this subject to have the pre-requisites for the following concepts :

- Geometry
- Trigonometry
- Statistics
- Physical
- Technical drawing

Course objectives :

The student will be able to know the basics of the topography and allowing him to carry out and monitor the subsequent implantation of a building, leveling, measurement of angles and distance, determination of the coordinates of the topographic points, and plot plans, topographic.

Plan of the course

Chapter 1. General, Definitions, and concepts
Chapter 2. Measuring distances
Chapter 3. Leveling direct and Indirect
Chapter 4. Measurement of Angles
Chapter 5. Determination of surfaces

Each chapter provides basic definitions depend on the contained, the formulas necessary for the calculation and control according to the tolerances required, and finalize by digital applications in the form of exercises and real-world case dealt with in the practical work.

Table of contents

Chapter IV Measurement of angles

Chapter IV Area calculation

<u>Chapter I</u>

General Definitions and Concepts

I. 1. Definitions

- **Topography**

Topography (from Greek graphien = draw) is the art of representing graphically a place in the form of plans or maps, The preparation itself of these maps or plans is /mapping. A map or a plan is a graphical representation, to a certain scale, the orthogonal projection of details of the surface of the earth, they are natural (rivers, mountains, forests, etc), artificial (building, roads etc) or conventional (administrative boundaries).

Another definition of the topography: It is a relatively faithful representation of a land surface, through direct and indirect measures (angles, distances, altitude, GPS, etc.) on a piece of flat to a given scale.

Scale: This is the ratio between a distance on the map/map and its counterpart on the ground.

- **Surveying**

Surveying (from the Greek topos = place, and metron = measure) is a set of techniques for measuring geometrical used to determine the shape and dimensions of objects and places, without taking account of the curvature of the earth.

- **Geodesy**

Geodesy takes its name from the Greek words Geo (Earth) and Desy (form and dimensions).

Geodesy is the science of the qualitative and quantitative study of the shape of the earth and its physical properties (gravity, magnetic field, etc.).

Geodesy allows you to locate, with high-precision geodetic points serve as framing for topographic surveys.

- **The topographic survey**

The topographic survey is the set of operations designed to gather on the ground of the necessary elements for the establishment of a plan or a map.

The rise of the details is the set of operations involved in a topographic survey and determining from the points the canvas of the whole polygon or details, the position of the various objects of natural or artificial origin is found in the field. The survey, the name given to the resulting document of a lift, is intended, possibly after digital processing, to the establishment of plans, charts, or digital: it is the phase delay.

A survey is conducted from comments: actions to observe the way an instrument for action; by extension, "the observations" refers to the results of these measures.

The phase of a topographic survey, or an establishment, which provides and / or uses the numerical values of all the elements planimetric and altimetric is called surveying; generally, the surveying is the technique of survey and / or implementation implementation to large and very large scales graphics or digital: it is the phase delay.

- **The topometric calculations**

They treat numerically the observations of angles, distances and height differences to provide the rectangular coordinate plane : x-coordinate ***(E***, ordered ***N*** and the altitude ***H*** of the points of the terrain, as well as the areas ; in return, the topometric calculations exploit these values to determine the angles, distances, height differences not measured, to allow the settlements.

- **The graphical maps**

A map is a conventional representation, generally flat relative positions, of phenomena, real or abstract, localizable in space. The map allows showing the changes and the developments of the phenomena in the time as well as their factor of movement and traveling in space.

A plan or drawing of topography is a conventional representation of the large-scale field.

There are three types of scales:

- Small scale : $100\ 000 \leq E$
- Medium scale : $10\ 000 \leq E \leq 100\ 000$
- Major scale: $E < 10\ 000$, in general, 1/5000, 1/2000, 1/1000, the designation of “large scale” applies rather, 1/500, 1/200, 1/100, 1/50.

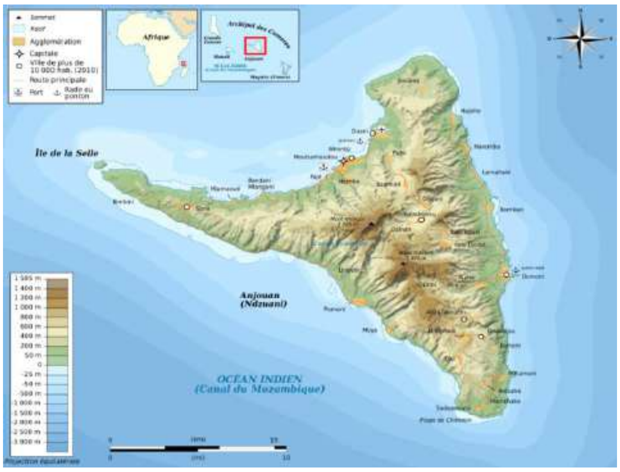


Figure I. 1. Example of the topographic map

Figure I. 2. Example of urban plan

A scale can be represented in two (02) forms:

- ***Digital form*:** it is generally expressed in the form of a fraction whose numerator is 1, the denominator giving the field measurement of the length is taken as the unit on the map. This digital scale is of the form $\frac{1}{E}$.

Since the adoption of the metric system, it was recognized that the denominator should be of the form " n *103 ", this makes the common use, as in this case, "1 mm" represents the "no feet" on the ground ; it is therefore notes : $\frac{1}{\mathrm{n x}\, 10^3}$

To achieve the scale explicitly and to facilitate the measurements, it is customary to include on the map in a graphical form.

- ***Graphical form:*** it usually has a single line divided into equal parts, representing on the map, the field unit is chosen, for example the km. The scale goes from left to the right and to the left of the zero of a party called a "heel" is subdivided into sub-multiples of the unit, for example, in hundred meters (hm).

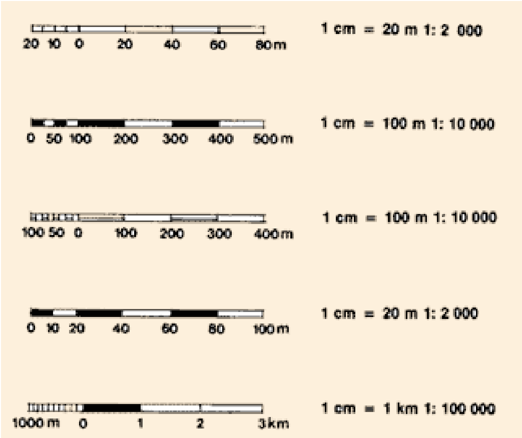


Figure I. 3. Examples of charts scale

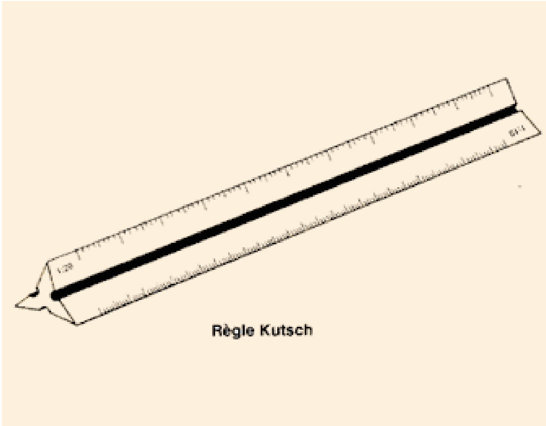


Figure I. 4. Rule Kutch

The appellation *topographic plan* generally applies to the plan that represents the elements planimetric apparent, natural or artificial, the terrain, and carries the conventional representation of the altimetry ; it has a *geometric quality*, that is to say, a degree of adequacy to the needs expressed or implied by the geometry of an image relative to the reference system used. Topographical plans have different objectives; it is often their destination, which will impose the accuracy of the sunrise and the choice of details.

There are different types of maps and plans as summarized in the table below according to the scale of the drawing:

Scale	**Purpose**
1/1 000 000 to 1/500 000	Maps
of 1/250, 000 to 1/100 000	topographic Maps on a small scale
1/50000,1/25, 000 to 1/20 000	topographic Maps at medium scales
1/10 000	topographic Maps of large-scale
1/5000	Plans, topographical studies, urban plans,
1/2 000	Plans occupancy of land, descriptive parcel
1/1000, 1/500	plots of land, land registries urban
1/200	Plans, road, settlement, subdivision
1/100	ownership Plans, plan mass

1/50	Plans architecture, formwork...

On the plans or maps, you must always search for or to put the following information:

- The ***name*** of the area or the field represented and/or the designation of the type of project in which it is to be used;

- ***The exact location*** of the field;

- ***The projection and ellipsoid used***

- The ***name of the person or persons*** who made the sunrises, topographical, on which are based the plan or the card;

- The (the) ***date of issue or*** (s) ***of establishment*** surveys, an indication of the type of survey, and the source of the data;

- ***The grid*** (geographic: giving the geographic coordinate of a point ; longitude, latitude. Mileage: giving the rectangular coordinate plane-X or E, and Y, or N of a point in a system of UTM projection and ellipsoid WGS84, for example).

- **The direction of the magnetic north**; the magnetic declination (in function of time and place).

- ***The scale*** of the plan or of the card (it can be digital or graphic)

- ***Relief*** shown by contour lines, the point's sides and special lines (slope and escarpment...), the contour interval level, if the card indicates the vertical relief.

- ***The legend*** (giving the meanings of the symbols used, in short, is a description of the symbols of graphical representation.

Table 01. Comparison between a map and a plan map.

Criterion	Plans, Topographic	Maps, Topographic
Scale,	Usually on a large scale (e.g. 1:1 000 1:25 000)	Often on a small scale (e.g. 1:50 000 1:500 000)
Details	Representation very detailed items, including buildings, roads and terrain	Representation less detailed, focusing on areas that are more wide
Use	that are Used for specific projects, such	Used for navigation, education and

Criterion	Plans, Topographic	Maps, Topographic
	as engineering or urban planning	studies, general geographical
Elements Represented	Include details such as curves, levels, infrastructure, and property limits	Shows the general geographical features such as rivers, mountains and cities
Symbology	to Use symbols standardized, often with a caption, detailed	Use of symbols simplified, usually accompanied by a legend
Orientation	Can be oriented according to the specific need of the project	Oriented according to the north, with indications of the direction
the Target Audience	Primarily of professionals (engineers, planners)	General public, hikers, students

Orientation of a map

- **The orientation of The plan/map**

The orientation of a topographic map refers to the way in which the card is aligned with respect to the cardinal points (North, South, East, West). This orientation is essential for understanding the relationship between the elements represented on the map, and their actual position on the field. Found:

- Standard Convention : North up

Most topographic maps are oriented with true North (or "true North", Fig. 04) at the top of the card. This convention makes it easier to:

  - The navigation with a compass.
  - The comparison between different cards.
  - The consistency with the systems of coordinates (e.g. latitude/longitude).

- Key elements to determine the orientation

Several graphic elements help to shape a topographic map:

- The graticule (Grid): lines of latitude (parallels) and longitude (meridians) indicate the geographic North.
- The North of the Grid (Grid North): On the maps using a projection system (e.g. UTM), a grid, rectangular, is bunk. The North of the grid may be slightly different from the geographic North because of the projection.
- The Rose of the Winds or Diagram of Declension: A symbol indicates the three types of North:

- Geographic north (true North): the direction of the North Pole.
- Magnetic north: the direction indicated by a compass (influenced by the earth's magnetic field).
- North of the Grid: orientation of the grid of the map.

- **Magnetic declination**

Magnetic declination is the angle between the geographic North and magnetic North. It varies according to location and can change over time. On a topographic map:

- The declination is indicated in the legend (e.g. "Declination: 5° West in 2023").
- It must be taken into account for navigating with a compass (Fig. 04).

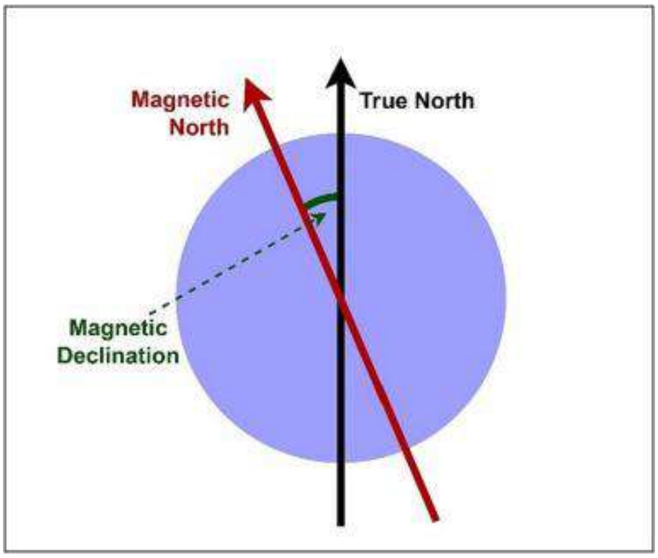


Figure I. 5. Orientation of a topographic map

Application 1:

- If we measure a distance of 2.5 cm on a map and the distance on the ground is 25 m, the scale will be: $\frac{2.5}{2500} = \frac{1}{1000}$

- If we measure a length of 7, 4 cm on a scale of 1/500, the actual length will be : **7.4 x 500 3 700 cm 37 mr.**
- Conversely, if the length measured on the ground is : 85 m, it will be represented on a plan to 1/200 by :

$\frac{85}{200}$ **= 0, 425 m = 42, 5 cm**

Application 2:

A map has a scale of 1:100 000. If the measured distance on the map between two cities is 5 cm, what is the actual distance between these two cities on the ground ?

Scale 1:100 000 means that 1 cm on the map represents 100 000 cm (1 km) in reality. To find the actual distance, we use the following formula:

Actual Distance = Distance on the map × scaling Factor.

In this case:

Actual Distance = 5 cm × 100 000 cm.

Actual Distance = 5 00 000 cm.

Convert kilometers: The actual distance between the two cities is **5 km away**.

In summary, the two types of scales (numerical and graphical) are complementary and facilitate the understanding of the distances on a map.

I. 2.The applications of the topography

The topography is involved in a number of areas and activities. The most answered are given by the following points:

I. 2.1.Development projects

These are the projects that alter the planimetry and altimetry of a field: improvements to land such as land consolidation with the related work, housing estates with the study of roads and utility networks (VRD), paths, road and rail, water management : drainage, irrigation, canals, ditches, etc

I. 2.2.Locations

The development projects are “intellectual products”, drawn usually from topographic data, which must be achieved on the ground. To do this, the topographer implants, in other words, puts on the field, the planimetric and altimetric elements necessary for this achievement.

I. 2.3.The monitoring and control of the works

The works of art once constructed often ask for a follow-up, that is to say, a physical examination, at intervals more or less regular according to destination ; dams, bridges, subsidence, etc., The topographic works correspondents, generally resulted in the measurement of the variation of the Cartesian coordinates (x, y, z)- or the plane coordinates (E, N, H)- points rigorously defined, followed by digital processing various declaring a state and providing for a change.

The engineer of civil Engineering, Hydraulics, and Public Works must be able to understand any document prepared by a surveyor, to be able to communicate with a topographer, know-how of the operations of the topography, manipulate devices topographic, recognize, eventually, the work carried out.

I. 3. The tools of the topography

The topography requires the knowledge of many of the tools in the technical fields (measuring instruments), mathematics (geometry, calculations), regulations, statistics (calculation error)

3.3.1. Devices

- Theodolites, total stations, total stations ;
- Levels ;
- IMEL : Measuring Instrument Electronics Lengths ;
- GNSS receivers ;
- Chains, brackets, optical, sight, landmark, etc.

3.3.2. Balance sheet errors

Errors encountered in measurement can generally be classified into two main categories:

- **Systematic errors**: these are reproducible errors that affect the measurements in a consistent way. They are generally caused by defects in the instrument, poor calibration, unsuitable observation methods, or environmental influences. Since their origin can often be identified, systematic errors are, in principle, measurable and can be reduced or eliminated through proper calibration and correction procedures.
- **Accidental (random) errors**: these are unpredictable variations that occur during measurement, even when the same procedure is repeated under similar conditions. They are related to the inherent precision of the instrument, the observer, and the measurement conditions. Although they cannot be completely eliminated, they can be estimated and quantified using statistical methods. The use of more precise instruments helps reduce their magnitude.
- **Instrument-related errors**: every measuring device is characterized by a certain level of precision, which is mainly associated with accidental errors, but it may also

introduce systematic errors if it is poorly adjusted or incorrectly calibrated.

- For this reason, it is essential not only to understand the correct use of each instrument, but also to master the methods of **control, verification, and calibration** in order to ensure the reliability and accuracy of measurements.

3.3.3. The methods of measurement

In planimetry: the radiation (angles, distances), the x and y coordinate GNSS; and in altimetry: leveling direct, leveling indirect, GNSS measurements.

3.3.4. The regulatory aspect

Regulatory texts: laws, codes, orders, decrees, recommendations, professional quality Control: quality standards.

I. 4. Concepts of geodetic base

The implementation of geodesy and techniques that are derived require the existence of a set of basic parameters:

- A geodetic reference system
- A geodetic network of monumented points

I. 4.1. The geodetic system

A geodetic system (or datum) is an affine coordinate system with the following characteristics:

- The center O is close to the center of mass of the Earth
- The axis OZ is closer to the axis of rotation of the earth
- The plane OXZ is close to the meridian plane origin

The geodetic coordinates of the point M are not objective values but dependent of a theoretical model. A point on the earth's crust is considered fixed with respect to the geodetic system, despite the small displacement, it can undergo (tide land, overload oceanic tectonic movements). Thus, it appears the need to have a reference surface: the ellipsoid.

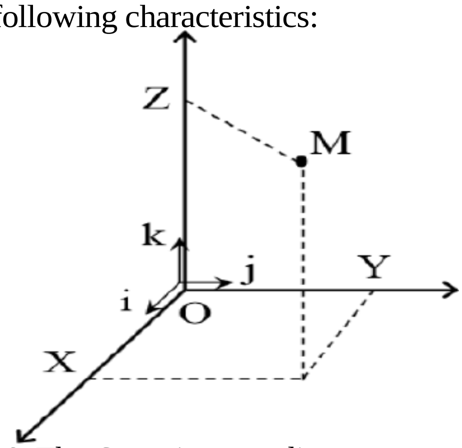


Figure I. 6. The Cartesian coordinate system

I. 4.2. The geodetic network

A geodetic network is a set of points of the costs of the earth (such as piers, terminals,...) whose coordinates are defined, estimated relative to a geodetic datum. Several types of networks are distinguished:

- networks plane
- networks leveling
- the three-dimensional networks geocentric

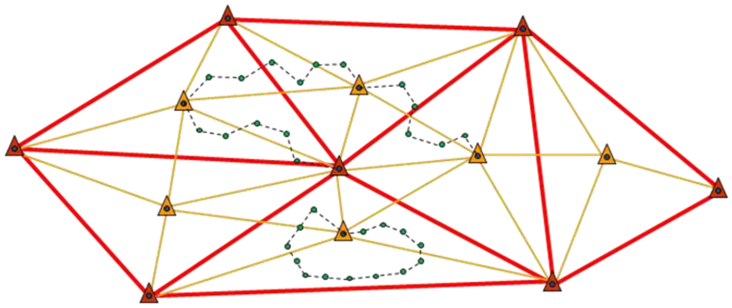


Figure I. 7. Example of a Geodetic network

I. 4.3.The surface

Several surfaces are to be considered when one is interested in positioning geodesic.

The first is, of course, the **surface topography**. It is she who plays the role of interface between part solid and part liquid, or gas from the Earth. It is what we know best, a point of view sensory and physical, it is the subject of numerous science and technology.

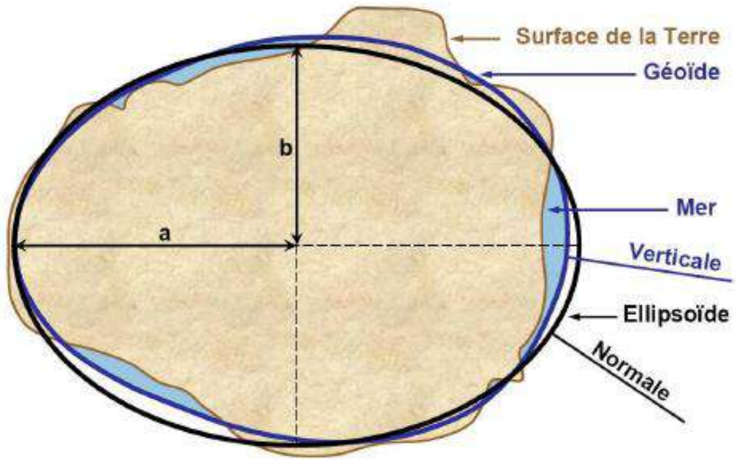


Figure I. 8. The representative surfaces of the earth

The **geoid** is the second area to consider. It is defined as the equipotential surface of the gravity field. The acceleration of gravity (g) he is so normal in every way. An excellent physical realization of this net is the average area of the seas and oceans. But under the continents, access to this area can only be indirect. Thus the physical reality undeniable that surface while keeping in mind the difficulties that requires determination.

Finally, the **ellipsoid** of revolution is the latest surface. Mathematical model defined in order to facilitate the calculations and for it to be as close as possible to the geoid, it can be local or global, depending on the application desired datum to which it is associated (global coverage or territory only).

I. 4.4. Different types of coordinates

The coordinates of a point can be expressed in different ways, depending on the reference system used and the purpose of the representation:

• **Geographic coordinates**: a point is located by its latitude and longitude, which are angular values measured on the Earth's surface. Latitude indicates the position north or south of the Equator, while longitude indicates the position east or west of the Greenwich meridian. This system is well suited for global positioning and navigation.

• **Cartesian coordinates**: a point is expressed in a geocentric reference system using metric values, generally along three axes (X, Y, Z). In this case, the origin is usually taken at the center of the Earth. This representation is particularly useful for mathematical calculations, satellite positioning, and geodetic analysis.

• **Projected coordinates**: a point is represented on a plane map by metric values, usually as Easting and Northing coordinates. These coordinates are obtained by projecting the curved surface of the Earth onto a flat surface using a map projection. This system is widely used in cartography, engineering, and geographic information systems because it allows distances, areas, and positions to be handled more easily on maps.

Chapter II

Distance Measurements

II.1.Introduction:

In this chapter, we discuss the main techniques of distance measurement used in the field of topography by comparing the details of each. The goal is not to determine the best, but to choose one depending on the hardware available and the required accuracy.

II.2.Measures of the distances using a chain

The measurement chain is the most classic and used to determine the distances. Its main disadvantage is to be dependent on the terrain (hilly or not, with a steep slope or not, etc) and be limited in scope (the ribbons used are commonly limited to 100 m). The accuracy of the measurement is also limited and strongly depends on the operators.

*Today, one uses the **gauge**, single, double, triple, or quintuple, and much easier to manipulate. We kept the name **string** , which becomes the general term encompassing the gauge, dual gauge, etc, We can also use the term ribbon.*

The tapes are divided into three classes of accuracy : the following table gives the **tolerances of accuracy** set by a european standard European Economic Community (EEC).

	10 m	20 m	30 m	50 m	100 m
CLASSE I	± 1,1 mm	± 2,1 mm	± 3,1 mm	± 5,1 mm	
CLASSE II	± 2,3 mm	± 4,3 mm	± 6,3 mm	± 10,3 mm	± 20,3 mm
CLASSE III	± 4,6 mm	± 8,6 mm	± 12,6 mm	± 20,6 mm	

The values of the table being tolerances, if one wants to obtain the standard deviation simply divide by 2.7. For example, a ribbon, 50 m class II, the standard deviation for a measurement is ± 10,3 / 2,7 = ± 3,8 mm.

The length of a ribbon is given to an ambient temperature data (at 20 °C in general) and for a voltage given. The tension force to be respected is usually indicated on the ribbon. The ribbons in soft materials are very sensitive to this tension.

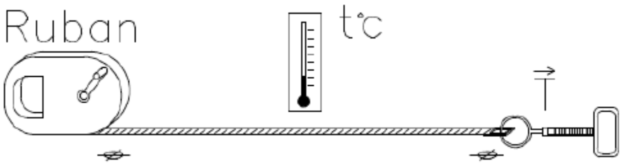


Figure II.1. Measuring tape

When **fine measurements**, the accuracy of which must be in the neighbourhood of the tolerance of the tape, and if the sequence requires many litters of string, we need to align the different litters of either view, or with sheets of survey, or milestones. An alignment error of 30 cm on a ribbon, 50 m gives an error on the measured distance of 1 mm. In this case, the measurement read is greater than the actual value.

II.2.1. Measures in regular field

In topography, the essential information is the horizontal distance between two points. Depending on the configuration of the terrain, it is more or less difficult to get precisely to the chain.

II.2.1. 1. Regular and horizontal ground

If the field is steady and steep the low (less than 2 %), it is possible to **reassure them** of tape on the floor and consider that the horizontal distance is read directly (fig II.2). The precision that it is possible to get a measurement is, at best, of the order of ± 5 mm to 50 m for a ribbon, class I.

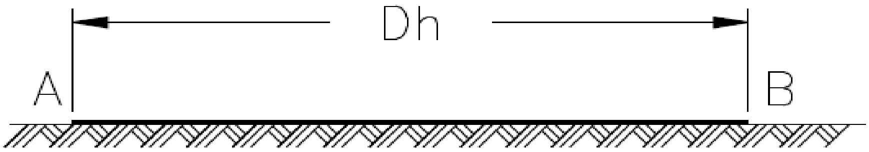


Figure II.2. Measuring tape in a horizontal site

Application: Show that from 2 % slope, an error of 1 cm appears on a measure of 50 m.

Answer: Dp = 50 m, ΔH = 0,02. 50 = 1 m, so Dh = 49,99 m.

Alignment methods and details

The methods of alignment are crucial to ensure the accuracy of the distance measurements. This involves the use of reference points and instruments referred to ensure that the measurements are made in a straight line. Alignment errors can lead to material misstatement.

Example: During the construction of a road, an engineer uses pegs to mark reference points along the route as planned. Using a theodolite to check the alignment between these points, it ensures that the road will be built according to the specifications, thus avoiding corrections costly later.

Practice measurement

The practice of measurement involves the implementation of suitable techniques for measurement of distance. This includes the choice of the appropriate instrument, the taking into account of the environmental conditions and the application of correction methods to compensate for potential errors.

Example: A surveyor takes measurements on rough terrain. He chooses a laser range finder to get accurate measurements, despite the obstacles. Taking into account of visibility conditions and performing multiple measurements, it is possible to correct the potential errors due to movement or obstructions.

II.2.1. 2. Regular slope

If the ground is not perfectly horizontal, it is necessary to consider that we measure the distance according to the slope. To determine the horizontal distance precisely, it is thus necessary to measure the level ΔH between A and B, or the slope p of AB (fig II.3).

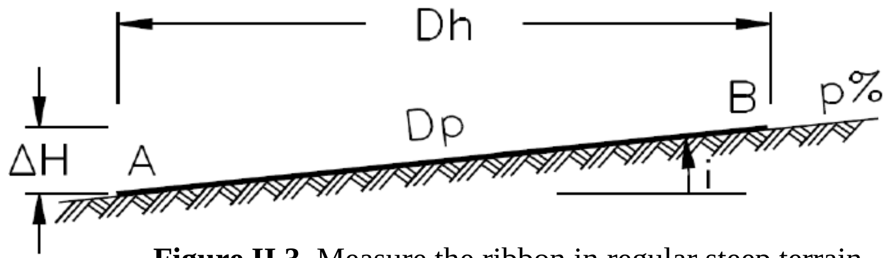


Figure II.3. Measure the ribbon in regular steep terrain

Either: $D_H = \sqrt{D_P^2 - \Delta H^2}$

$$Dh = Dp \cdot \cos i = Dp \cdot \sqrt{\frac{1}{1+\tan^2 i}} = \frac{Dp}{\sqrt{1+p^2}}$$

The accuracy is of the same order as previously, that is to say 10 mm to 50 m.

Application: You measure a distance according to the slope of 37,25 m and you measure a slope of 2.3%. What are the values of Dh and ΔH ?

Answer:

$$Dh = 37{,}25 \,/\, \sqrt{1 + 0{,}023^2} = 37{,}24 \text{ m} \quad \text{et} \quad \Delta H = \sqrt{37{,}25^2 - 37{,}24^2} = 0{,}86 \text{ m.}$$

II.2.2. Measures in rough terrain or steep slope

You can't hold the ribbon on the ground because of its undulations. In addition, the slope (or the distance to daisy-chain) is such that one cannot directly measure the distance *Dh*.

II.2.2. 1. Extent by the difference in height between the horizontal

Let us mention the called method to measure by **drops horizontal** or **cultellation**. Illustrated in figure II.4, it requires the use of a bubble level and two sons to lead in most of the chain and sheets

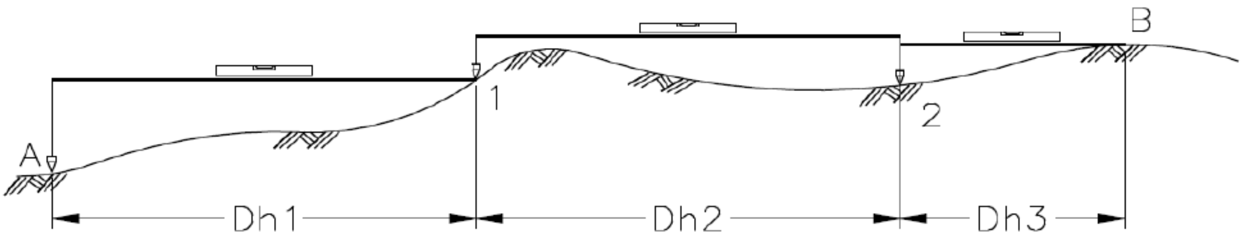
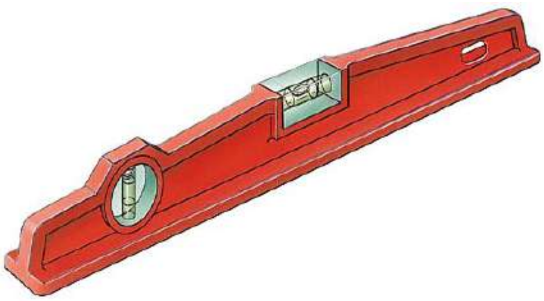


Figure II.4. Measure the ribbon by projections, horizontal

of survey (or milestones). Its implementation is long and the process is not very precise. It may be noted that: Dh **= *Dh*1+*Dh*2+*Dh*3**

bubble Level



The chain

Sheets surveying milestones

Figure II.5. Materials used for direct measurement of a distance

Note

When the operator must refer to several times the ribbon to measure a length, it is necessary to **align the bearings**. This alignment is usually performed to view using sheets of survey, or milestones. The misalignment must be less than 20 cm to 30 m (which is relatively easy to be met) to obtain a precision of one millimeter.

II.2.2.2. Measurement in suspended mode

A wire material that is stable (Invar[1]) is **stretched above the ground**. The voltage is maintained **constant** by weight (fig II.6). The operator must measure the altitude difference ΔH between the peaks Has′ and B′ of the tripod suspension of the wire to be able to calculate the length of *Dh* as a function of the distance angled *Di* measured :$D_H = \sqrt{D_I^2 - \Delta H^2}$

[1] Invar is an alloy of iron (64%) and nickel (36%) whose main property is to have a very low coefficient of expansion.

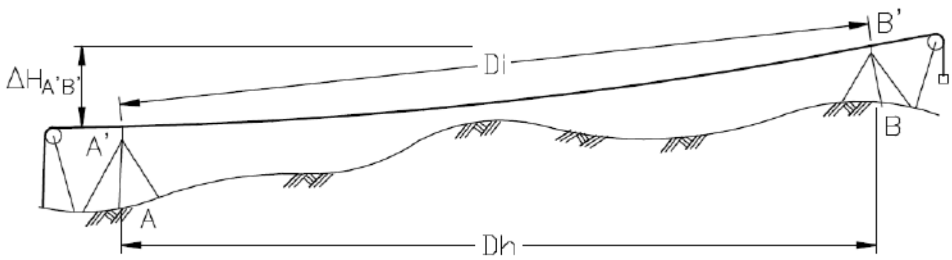


Figure II.6. Measurement in suspended mode

This method gives satisfactory results in the measurement of precision, but it is a long time to implement. We obtain a precision down to the millimeter for ranges of a few hundred meters. It is applicable to a ribbon.

2.3. Indirect measures of distance

The indirect measures of distance are used when direct measurement is impossible or impractical. This may involve methods trigonometric or calculations based on the angles measured. These techniques are used to estimate distances with an acceptable accuracy.

Example: A surveyor wishes to measure the distance between two points, A and B, separated by a ravine. It takes place at a point C and uses a theodolite to measure the angles ∠ACB∠ACB and ∠ABC∠ABC. By applying the law of cosine, it is possible to calculate the distance AB, without having to cross the ravine, ensuring the safety of his team.

Exercise: indirect measurement with trigonometry

Statement

A surveyor must measure the distance between two points, A and B, which are far away so that a direct measurement is impossible due to an obstacle (for example, a building). The topographer is at a point C, where he can see the points A and B. measure the following angles :

- Angle ACB = 60°
- The distance from C to A (CA) : 100 meters

Questions

1. To calculate the distance between points A and B (AB) using the law of cosine.
2. Identify the potential sources of error in this measurement.
3. Propose solutions to minimize these errors in the next steps.

Fix

1. Calculation of the distance between the points A and B

To calculate the distance AB, we can use the law of cosines, which is given by the formula:

$C^2 = a^2 + b^2 - 2ab \cdot \cos(C)$,

In our case:

- $a = CA = 100$ meters
- $b = CB$ (we still don't know the distance)
- $C = ACB = 60°$

To find BC, we must assume that it is also 100 meters (to simplify the example). Thus, we have:

$AB^2 = 100^2 + 100^2 - 2x100x100x\cos(60°)$

Knowing that cos(60°) = 0.5, then we have :

$AB^2 = 10000+10000-10000 = 10000$

Therefore:

$AB = (10000)^{0.5} = 100$ meters

2. Identification of the sources of errors

The sources of potential errors in this measurement may include:

- **Systematic errors:**
  - If the measured angle is not accurate (for example, because of an instrument not calibrated), this can distort the calculation of the distance.
  - The distances CA and CB may be poorly measured.
- **Random errors:**
  - Environmental factors, such as ground motion, or vibration, can affect the accuracy of the measurement.
  - The reading of the angles can be influenced by human error, such as a bad interpretation of the data.
  -

3. Solutions to minimize errors

To reduce the errors during the next steps, it is recommended to:

- Check the calibration of the instruments before starting the measurement.
- Use of the methods of measurement repeated for the angles and distances, and then takes an average of the results.

- Work in a team, where a person measures the angles and other measure distances, to reduce reading errors.
- Measurements in calm conditions to avoid disturbances caused by wind or other environmental factors.

This exercise highlights the importance of the indirect measurement by topography, in particular when it is impossible to directly measure the distances. By applying rigorous methods and taking into account potential errors, surveyors can improve the accuracy of their statements.

II.3. Parallactic Measures (Measurement with stadia)

This type of parallactic measurement requires the use of a Theodolites and stadia. A Stadia is a rule with two leds (triangular or circular), of which the distance is known (usually 2 m). There are Stadias Invar, for high-accuracy measurements. The Stadia is equipped with a circular bubble, and a viewfinder to adjust the perpendicularity relative to the line of sight Has′ B′ (fig II.7).

The operator has Had a Theodolite (or a circle alignment) and B a stadia horizontal perpendicular to the distance AB.

The height adjustment is unnecessary: the measured angle is the angle projected on the horizontal plane.

In projection on the horizontal plane passing through the point A, we get:

$\tan\frac{\alpha}{2} = \frac{L}{2Dh} \Rightarrow Dh = \cot\frac{\alpha}{2}$ with L =2 m

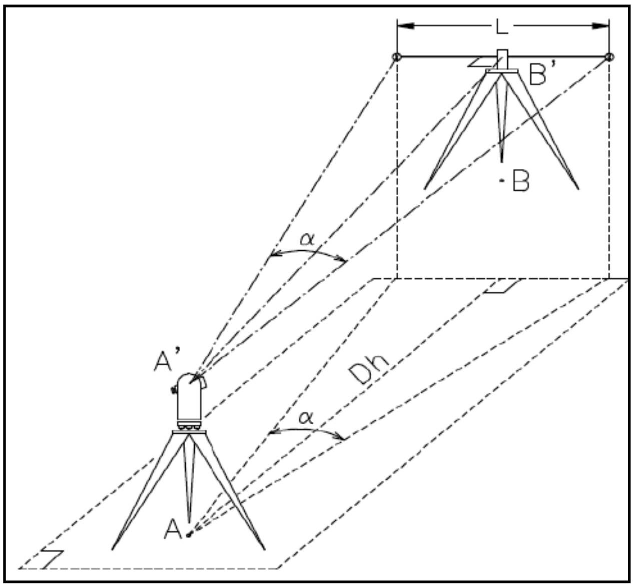


Figure II.7. Measuring using Stadia

II.4. Stadimetric measurements

The stadimetry is a less accurate method than the previous. It allows the indirect measurement of horizontal distance by reading the length intercepted on a pattern by Stadimetric threads of the aiming reticle.

The point (A, optical center of a Theodolite, is located at the vertical of the point stationed in the S ; the operator aims at a target placed in P and performs the reading intercepted by each wire on the target is $m1$ and $m2$.

The horizontal distance can be expressed as: $Dh = \frac{m_2 - m_1}{2\tan\left(\frac{\alpha}{2}\right)}$

If the angle α is a constant in the device used, we have: $Dh = K\,(m_2 - m_1)\,\sin^2 V$.

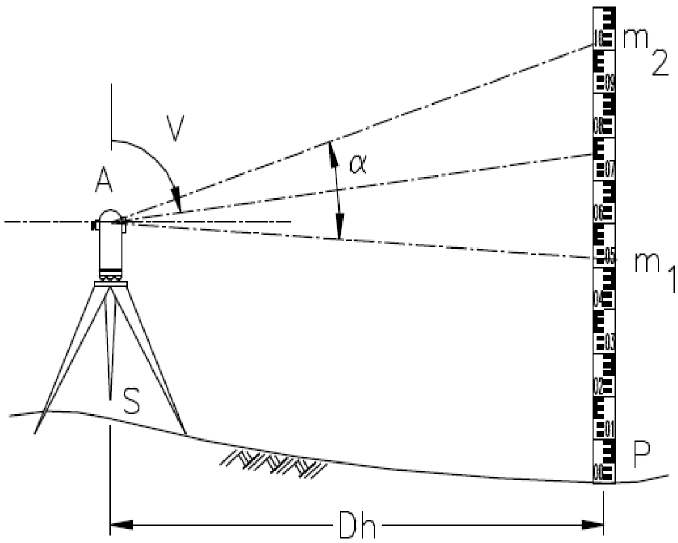


Figure II.8. Stadimetric measurements

The constant $K = \frac{1}{2\tan\left(\frac{\alpha}{2}\right)}$ is called **Stadimetric constant**. It is usually worth 100; this is why the expression of *Dh* becomes: $Dh = 100(m_2 - m_1).\sin^2 V$

Application

A measure of metrology requires knowledge to the millimeter the distance between two Theodolites. A Theodolite is stationed at each of the points A and B and referred to angular on a stadia Invar; the operator seeks to determine the

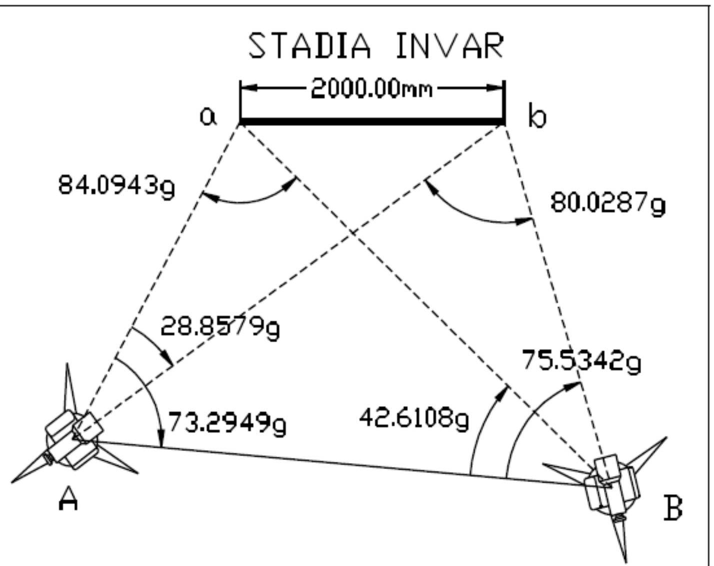


Figure II.9. Application of Parallactic measurements precision of a knowledge base AB. (fig .II.7).

The operator able to tape the base AB and then performs the specified angular detail (figure 4.10 b). It calculates finally the length of stadia inferred from these measurements and from a **factor of scale** that will provide the basis of AB with an accuracy of less than a millimeter.

AB is measured 4, 09 m to the ribbon.

Chapter III

Direct and Indirect Leveling

III.1. Direct Leveling

The **direct leveling**, also called **geometric leveling**, is to determine the **level** ΔH_{AB} between two points A and B with the help of a device: the **level** instrument and a vertical scale called the **focus**.

The focus is placed successively on the two points. The operator reads the value of ***La*** on the pattern laid in A and the value of ***Lb*** on the pattern laid in B. The difference of the readings on sight is equal to the difference in level between A and B. This difference in level is a value that is algebraic in which the sign indicates if B is higher or lower than A (if ΔH_{AB} is negative, then B is lower than A).

Figure III.1. Device level and the pattern

The altitude from A to B is: $\Delta H_{AB} = La - Lb$

The altitude from B to A is: $\Delta H_{BA} = Lb - La$

The altitude H_a of the point A is the distance calculated according to the vertical, which separates it from the geoid (the level surface 0). If the elevation of point A is known, we can deduce that the point B by:

$H_B = H_A + \Delta H_{AB}$

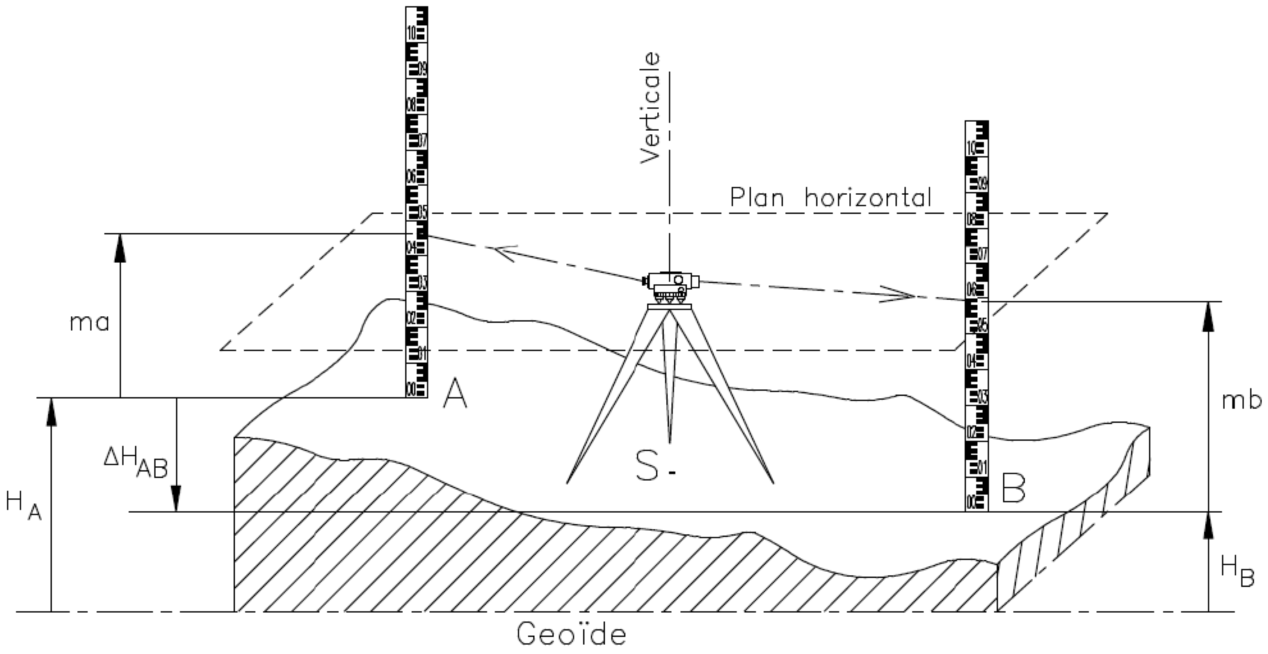


Figure III.2. Basic principle of the leveling direct

Note

- The altitude is often denoted *Z* instead of *H*.
- The **range** is the distance from the level to the target ; it differs according to the material and the accuracy sought, and must be a maximum of 60 m in ordinary leveling and 35 m in precise leveling. To the extent possible, the operator sets the level at roughly the same distance from A and B so as to achieve **equality of the bearings.**

Level: The level that is not (or very rarely) parked on a given point, the tripod is placed on any one point. The operator must go back after you have positioned the tripod to ensure the horizontal position of the top plate. When the tray is approximately horizontal, the operator sets the level.

Readings on target: The target is a linear scale, which must be held vertically (it has a circular bubble) on the point involved in the level to be measured. The pattern classic is usually calibrated in cm.

The cross-hairs of a level typically consists of four wires :
The superior stadimetric wire (s'), which gives a read-m1 on the target;
The lower stadimetric wire (s), which gives the reading m2 on the target;
The leveler wire (n), which gives the reading m on the target;
The vertical wire (v), which allows the pointing of the target, or of an object

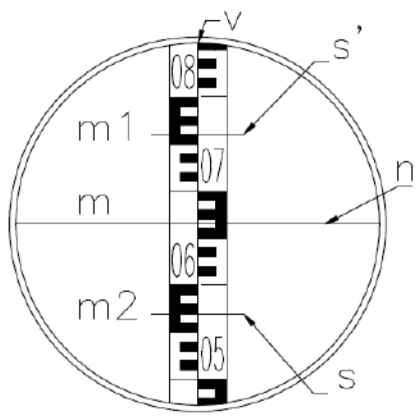


Figure III.3. Crosshairs

The reading on each wire is estimated visually to the nearest millimeter (6,64 dm on figure III.3, wire leveler). The stadimetric wire used to obtain a value closer to the scope. For each reading, it is a good idea to read the three horizontal wires so as to avoid the mistakes of reading: one checks directly on the ground, as: $m = \frac{m_1+m_2}{2}$

For example, figure III.3: 6,64 dm ≈ (5,69 + 7,60)/2.

III.1.1. Mistakes and Errors

Discussed below are the various faults and possible sources of error.

III.1.1.1.Faults: There is a distinction between mistakes of

- **Timing**: forgetting to set the bubble, clearing blocked ;
- **Read**: confusion of the line leveler with a stadimetric dash; confusion graduation or unit ;
- **Transcription book**: a bad rendition of the value read.

III.1.1.2. Systematic errors: systematic errors are:

- The calibration error of the mire ;
- The lack of verticality of the pattern: deregulated bubble;
- The error of inclination of the optical axis: optical axis not perpendicular to the main axis ;
- The default operation of the compensator.

III.1.2. Simple pathways

When the points A and B are situated so that only one station of the level is not sufficient to determine their level (remoteness, mask, level too high, etc), it is necessary to decompose the difference in level total height differences elementary with the help of intermediate points (I1, I2, ..., see fig III.4). All of these decompositions is called the leveling journey.

A routing box , of a "point of origin" - known in altitude, goes through a number of intermediate points and is closing in on a " point end point different from the point of origin and also known altitude. The journey of the figure. 5.16 is framed between A and B.

When one seeks to determine the altitude of a point end B from that of a known marker, it generally makes a **journey round trip** from A to A via B. This allows you to calculate the elevation of B and check the validity of the measures by finding the altitude from A.

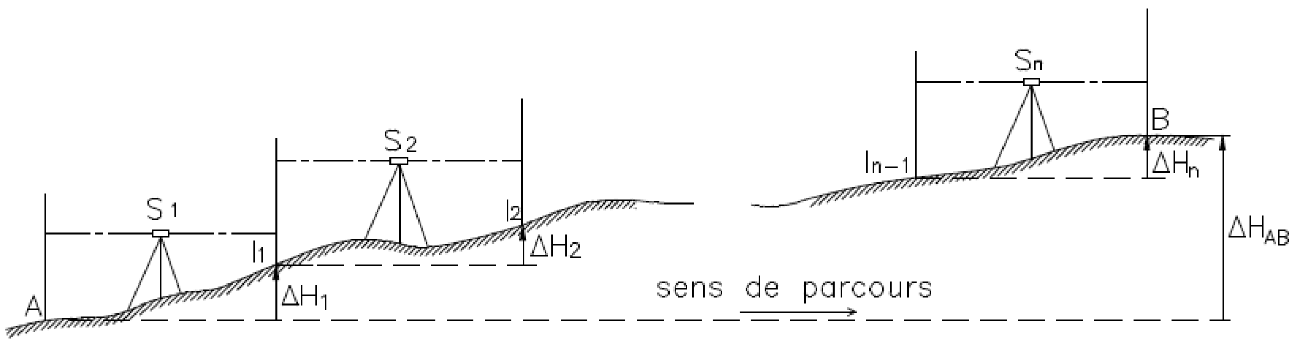


Figure III.4. Leveling journey

When a path is a loop, returning to its starting point A, it is called a **path closed**. It is widely used for the following reasons :

- It allows for the determination of altitudes even when you do not know a single marker ;
- It is possible to compute such a path by assigning an altitude arbitrary to a fixed starting point and sustainable ; a simple translation will spend altitudes from this local system to the altitudes true ;
- It allows a close control that is independent of the accuracy of knowledge of the elevation of the point of reference.

III.1.2.1. Practice of the leveling journey

A leveling path is performed by the following operations :

1. the focus is on the origin point A, the operator is parked on level S $-_1$, of which it determines the distance by counting the number of steps separating A of S_1, so that it does not exceed the maximum range of 60 m. The operator uses a **read-back**, that is to say, in the direction of the route you have chosen, on the point A, denoted $\mathbf{L_{ar\ A}}$;
2. the rodman moves to come on the first intermediate point **(I_1** the most stable possible (stone, metal base, called the " toad ", piquet, etc), and to determine the distance by counting itself the number of steps separating A of S_1 in order to reproduce the number of no's$_1$ to I $-_1$;
3. always parked in S_1, the operator reads the aim of the **reading before** He noted **The$_{av\ I1}$** ;
it is then possible to calculate the difference in level A-to-I_1 in the following way :
$\Delta H_1 = L_{ar\ A} - L_{av\ I1}$ = read back on – reading before the I_1.
4. The operator must read the stadimetric wire and verify that $m \approx (m_1 + m_2)/2$;

The operator moves to choose a station S2 and so on ;

5. the gradients of partial are the following :

$L_{ar\ \mathbf{A}} - L_{av\ \mathbf{I1}} = \Delta H_1$ level of **A** to $\mathbf{I_1}$
The$_{ar\ \mathbf{I1}}$ $- L_{av\ \mathbf{I2}} = \Delta H_2$ level from $\mathbf{I_1}$ to $\mathbf{I_2}$

....
The$_{ar\ \mathbf{(n-1)}}$ $- L_{av\ \mathbf{B}} = \Delta H_n$ level of **I (n-1)** to **B**

$$\sum_{i=1}^{i=n} \text{Lar} - \sum_{i=1}^{i=n} \text{Lav} = \sum_{i=1}^{i=n} \Delta \boldsymbol{H_i} = \sum_{i=1}^{i=n} \Delta H_{\boldsymbol{AB}}$$

The altitude difference total ΔH_{AB} from A to B is equal to the sum of the readings back less the sum of the readings before.

Note

If the path is closed, the altitude difference total must be equal to zero.

III.1.2.2. Closure of the journey

Knowing the altitude of A, we can calculate (again) from measurements of terrain, the altitude of B : we call this value of $H_{B\ is}$ **the observed value**, denoted $H_{(B\ obs}$.

If the measures were to be error-free, we would find exactly the known altitude H_B. In reality, there is a gap called the **error of closure of the path** (or simply the **closure**) that is subject to tolerance. This closure is denoted $\boldsymbol{f}_H$ is equal to : $\boldsymbol{f_H = H_{obs} - H_B}$

If we call $\boldsymbol{T_{\Delta H}}$ **tolerance** regulatory closure of the journey, so we must verify:

$|\boldsymbol{f}_{-H}| < \boldsymbol{T_{\Delta H}}$. If this is not the case, the measurement must be redone.

III.1.2.3. Clearing the path

The compensation is the operation that is to spread the closure on all measures.

The compensation, denoted C_H , is, therefore, opposite the closure, that is to say : $C_H = -f_H$

This adjustment is to modify the gradients of partial apportioning the compensation total *HP* on each of them. This distribution can be done in several ways :

1- In proportion to the number N of gradients: we will choose this type of compensation in the case where the closure is very low (less than the standard deviation), the compensation on every level is: $CHi = \frac{CH}{N}$.

2- In proportion to the scope: (the closure is between standard deviation and tolerance), it is considered that the greater the scope is important, and the difference in level can be subject to error. This requires about an order of magnitude of the range, which is obtained by stadimetry.

The compensation on each level is then: $\boldsymbol{C_{Hi} = C_H \frac{L_i}{\sum L_i}}$.

III.2. Indirect trigonometric leveling

Leveling indirect trigonometric allows you to determine the height difference ΔH from the station T a Theodolite and a point P referred. This is done by measuring the distance from the inclined along the line-of-sight Di and the zenith angle (denoted V in figure III.5).

We can write, from the schema: ΔHTP =

ht+Di.cosV-hv

With:

ΔH_{TP} is the level of T to P.

ht is the height of the station.

hv is the height of light or, more generally, the desired height above the point sought (it can also be a focus in P).

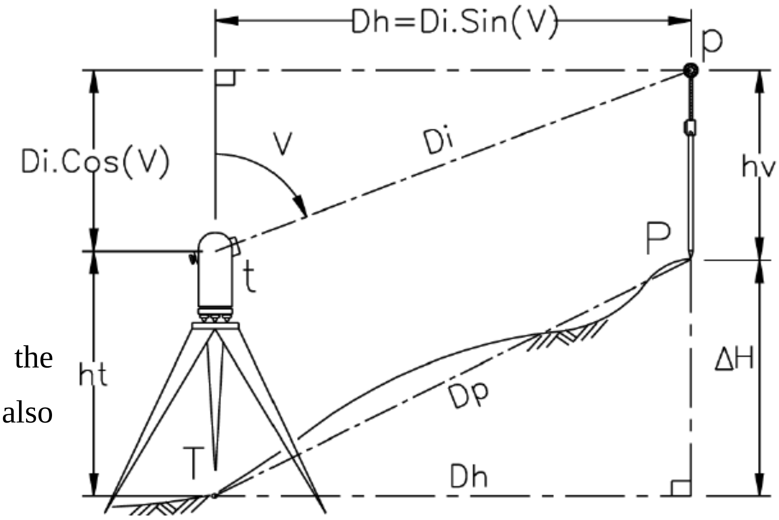


Figure III.5. Indirect trigonometric leveling

We calculate the horizontal distance Dh: **Dh_{TP} = Di.sinV**

We calculate the distance according to the slope Dp: $\boldsymbol{D_p} = \sqrt{\boldsymbol{D_H^2} + \Delta H^2}$

Note

The term Di.cosV is called “level *instrumental”*. It is noted **Δh_i.** This is the difference in level between the axis of the trunnions (t) of the Theodolite and the point p referred to (see chapter measurement of angles).

- Dh does not depend on hv and ht: it is a function of Di and V.
- The appellation "t*rigonometric*" comes from the calculations of trigonometry simple giving ΔH and Dh.

Application : In order to determine the altitude of the intermediate points (I_1, I_2, I_3, I_4), a leveling path was made between the starting point **A (**89,113 m**)** and the point **B(**90,714 m) ; where : T=18 mm. In these conditions you are asked to complete the table.

station	Point referred to	Readings rear (mm)			Readings before (mm)			Range (m)		Gradients ΔH (mm)		Compen (mm)	Altitude (m)
		S' "m1"	n "m"	S "m2"	S' "m1"	n "m"	S "m2"	Rear -	front	+	-		
St1	**Has**	2055	1967	1880									**89.113**
St2	I1	2008	1906	1804	1004	915	827						
St3	I2	1908	1755	1600	1821	1715	1608						
St4	I3	1794	1669	1544	2070	1919	1769						
St5	I4	1617	1562	1506	2003	1879	1755						
St6	**B**				901	841	780						**90.714**
						sum							Error =

Chapter IV

Measurement of Angles

IV.1. Introduction: The measurement of angles is always essential in topography. Compared to the measurements of distance, by means of modern technologies, the angular measurements to keep the advantage of being much more accurate than the litters of measures are long.

Figure IV.1. Angular measuring Instruments

A Theodolite is an instrument for measuring angles in the horizontal (angles projected on a horizontal plane) and vertical angles (the angles projected in a vertical plane). The term Theodolite includes all devices to read "mechanical" by vernier graduated in comparison to the devices, "electronic", which is read on a digital display screen. The basic mechanical Theodolites electronic is often the same as the classic Theodolites.

IV.2. Principle of operation

Figure IV.2 shows the schematic diagram of the operation of a Theodolite.

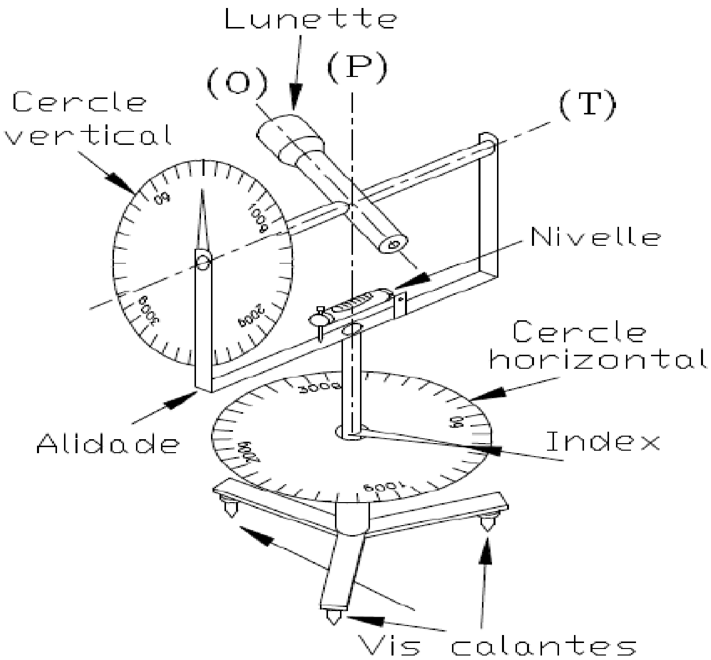


Figure IV.2. Principle of operation of a Theodolite

(P): main axis, it should be vertical after the station Theodolite and must pass through the centre of the scale is horizontal (and the point parked).

(T): secondary axis (or axis of the trunnions), it is perpendicular to (P) and must move to the center of the scale vertically.

(O): optical axis (or line of sight), it should always be perpendicular to (T), the three axes (P), (T) and (O) to be convergent.

Alidade: this is a set of mobile around the main axis (P) consisting of the vertical circle, the bezel, the level ring alidade and playback devices (symbolized here by the index).

The vertical circle (scaling vertical). It is attached to the bezel and rotates around the axis of the trunnions (T).

The horizontal circle or blade (scaling horizontally). It is most often fixed with respect to the base but it can be simply attached to the alidade by a clutch system (T16): we speak of a general movement of the alidade and the circle around (P);

IV.3.To set up a Theodolite

The setting-up of a theodolite is to set the main axis vertical to a point of station given. therefore, it is to place the camera to the vertical of the station and of the other hand in a horizontal plane.

IV.3.1.Update height of the tripod

The upgrade to height of the tripod is as follows:

- After you open and unfolded the tripod, install it on top of the station. A simple principle is intersected projections on the ground of the 3 branches of the tripod on the point station.
- Attach the device on the tripod, taking care to check that the three setting screws are almost at the halfway point.

(For a station more easily, the plateau of the tripod (metal portion on which is placed the tachymeter scale) should be as horizontal as possible).

- Adjust the eyepiece to the eye level of the operator (or better, slightly below the height: it is easier to reduce than increase). Make sure to adjust the sharpness of the aiming reticle. To do this, use the tick marks in diopters of the eye.

IV.3.2. The timing coarse with the circular bubble

once the tripod in place, come in and fix the camera on the set of the tripod with the aid of the clamping screw tripod/tachymeter (for safety, tighten the screw with one hand and with the other firmly hold the device).

The device is fixed on the plate of the tripod. We now need to center it vertically from the station, in a horizontal plane (in a first time, the circular bubble can fit roughly).

While looking through the lead optical, come place the tachymeter scale (Theodolite) on the point of station by moving first, the branches of the tripod (2-by-2).

Stabilize the tripod by pushing each branch in the ground (press on each foot by putting all the weight of his body). Recheck the lead perspective if one is always on point. If not, replace the screws calantes.

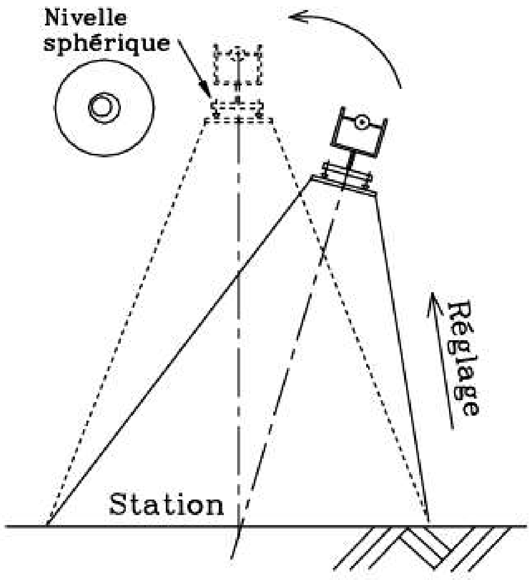


Figure IV.3. Timing of the circular bubble

The circular bubble located on the device is probably out of balance. The wedge by sliding the branches of the tripod. For this operation, it is easier to maintain the branch that is behind the scenes with the help of the foot.

When the level is adjusted, it is important to verify that each branch of the tripod is tight.

If one control the lead in mind, it is very likely that the device is not perfectly centered on the point station. To correct, loosen the clamping screw, tripod Theodolite drag the tachymeter scale on the plate of the tripod up to the right position (*centering forced).*

IV.3.3. The setting to an end with the ring level

The tachymeter and the tripod are completely interdependent, the branches of the tripod are stable and tight, the camera is perfectly vertical to the station and the circular bubble is set. It's now time to finish this resort through the levels-ring.

Following the horizontal rotation axis of the total station, place the ring level parallel to the two setting screws. Adjust the level by turning the two setting screws at the same time, the same amount and in opposite direction (1).

When, in this first position, the level is correctly set, to give the camera (and thus to the level as it is in solidarity with the tachymeter) a rotation of 100 gr. (right-angle, 90°) and adjust to the new level by using the third setting screw (2).

When in this second position, the level is well-adjusted, return to first position and adjust the level.

Repeat this operation in two positions until the perfect result of the nivelle ring.

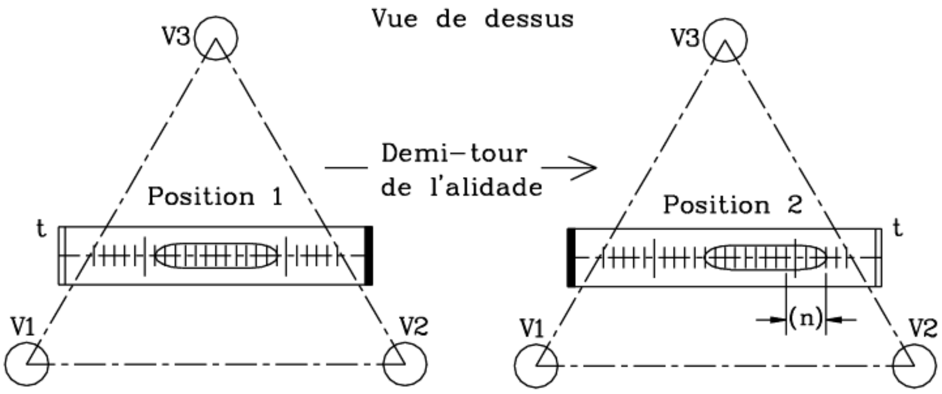


Figure IV.4. Timing of the level ring

Important :

- ✓ When the device is perfectly set in the station, it is strictly forbidden to touch the tripod (even with the hands) and setting screws**.**
- ✓ At the end of each operation of the set-up, control with the help of the lead optical position of the tachymeter scale and the need to repeat the tuning to a perfect result.

Currently, the lead laser replaces the lead optical. If its use is more comfortable than that of lead optics, the principle set-up remains the same.

also, on the newer devices, the nivelle electronic replaces the nivelle ring. In this case, the clamping end to 100 gr. near explained previously, is no longer necessary. The setting of the nivelle electronic is done in a single position using three screws calantes.

IV.4. Horizontal Angles

The horizontal angle observed with the aid of a Theodolite is a plane angle, counted positive in the clockwise direction. The spotting scope rotates in a vertical plane, regardless of the position in height of A and B, the angle observed is identical to «Hz ".

In practice, this angle is calculated by the difference of readings on a horizontal circle graduated from 0 to 400 degrees in the direction of the needles of a watch called the "blade".

$Hz\ (BA) = L_{(B} - L_{HAS}$

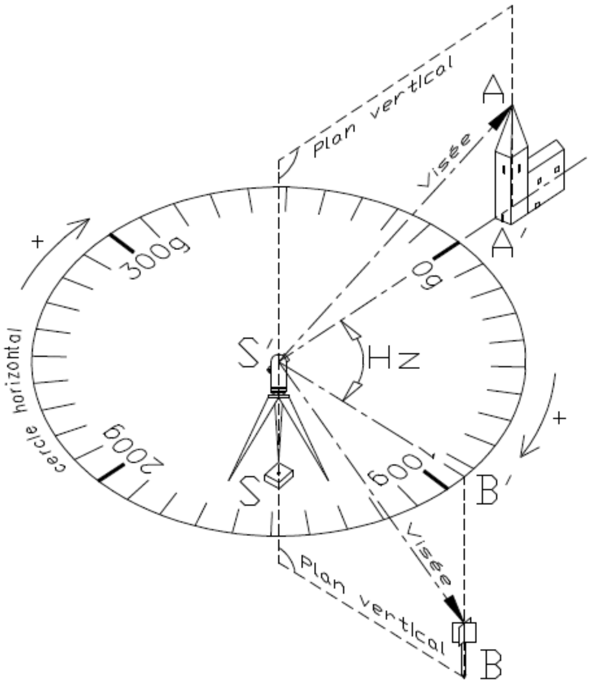


Figure IV.5. Angle measurement

IV.4.1. The double flip

It is a manipulation consisting of a half-simultaneous-turn of the bezel and the alidade. This measurement technique allows you to eliminate some of the systematic errors and to limit the errors of reading. During measurement of horizontal angle, this allows :

- double the readings and therefore reduce the risk of foul play ;
- not always read on the same area of the blade, so as to limit the error due to the defects of graduation of the blade ;
- to eliminate the defects of collimation horizontal (default of perpendicularity of the optical axis) and tourillonnement (lack of perpendicularity of the axis trunnions).

The error of centering on the point of the station, and the error of calibration of the vertical axis are not eliminated by this operation.

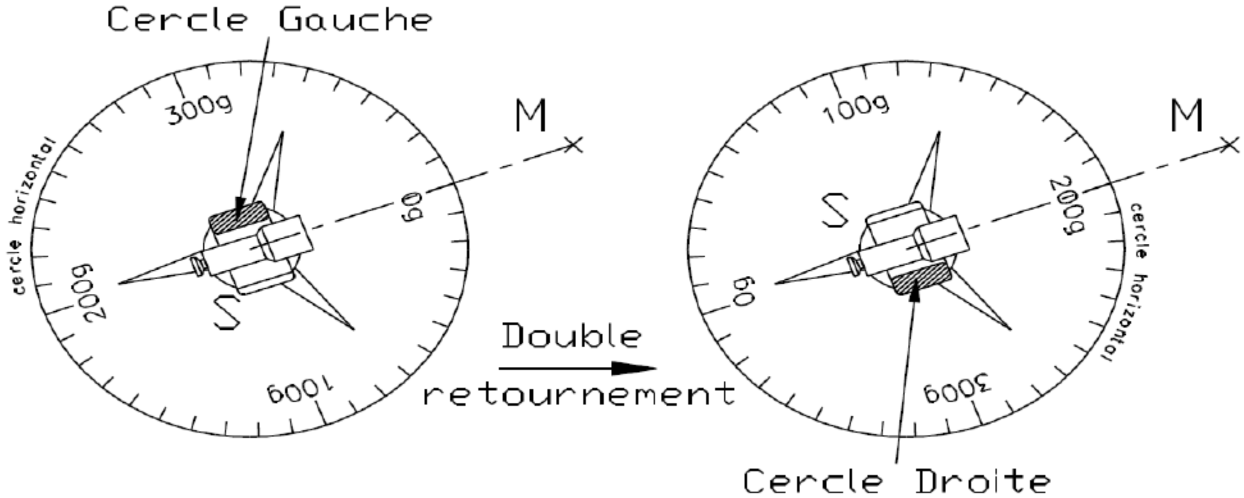


Figure IV.6. Double flip

Practically, it is done:

1. a reading circle to the left (the vertical circle of the instrument to the left of the operator, more generally, in the reference position) ;

2. a double reversal ;

3. a new reading of the same angle in a circle right (vertical circle to the right).

If you are calling Hz_{CG} the read value in the left circle, and Hz_{CD} that played back in right circle, it must be noted : $Hz_{CD} = Hz_{CG} + 200$

In fact, the double-reversal shifts the zero of the graduation of 200 Gra this allows a simple and immediate control of the readings on the field.

The difference between the values Hz_{CG} and (Hz_{CD} – 200) represents the combination of the errors of collimation, set-up, playback, etc

The horizontal angle Hz measured is then:

$$H_Z = \frac{HZ_{CG} + (HZ_{CD} - 200)}{2} \text{ if } HZ_{CD} > 200 \; Respectively$$

$$H_Z = \frac{HZ_{CG} + (HZ_{CD} - 200 + 400)}{2} = \frac{HZ_{CG} + (HZ_{CD} + 200)}{2} \text{ if } HZ_{CD} < 200 \; Respectively$$

IV.4.2. Terminology of the horizontal angles measurement

- **Simple playback** $Hz(BA)= L_{(B}-L_{HAS}$.

- **Sequence** called sequence a set of (n + 1) readings taken from a station on n different directions with the same position of the circles, vertical and horizontal (it will reduce the angles to zero).

For example, in the figure, the reference point is the point **R** on which the operator performs the first read **- L_{R1}**, there is a reading on each point by turning in the clockwise direction and a final reading of closing in on the point **R L_{R2}**

By calculation, the readings are then reduced [1] to the reference R by subtracting the other readings, the average of the two readings on the reference.

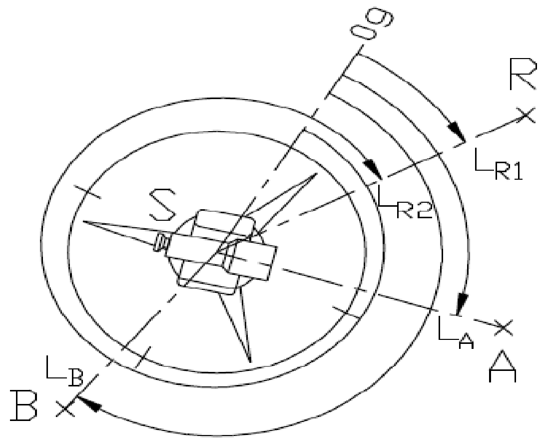


Figure IV.7. Reading horizontal angle

- **Overview**

The round up is the final result of the combination of observations angular (sequences) in the same station and **reported to the same reference** (example the point R in figure IV.7).

In the calculation determines the average value of **the gap on the reference**: it is the algebraic sum of all the variances to read a same pair divided by (n + 1), n being the number of branches covered including the reference.

This deviation is subject to tolerances regulatory:

- **0,7 mgrd canvas accuracy** for four pairs (0,8 mgrd to eight pairs);
- **0,8 mgrd canvas ordinary** for two pairs (0,9 mgrd four pairs).

[1] Reducing readings to the reference is equivalent to assigning the reference reading zero.

IV.4.3. Calculation of Deposit

IV.4.3.1. Definition of the deposit (steering Angle)

The deposit of one direction AB is the horizontal angle measured positive in the clockwise direction between the axis of the projection system used, and the direction AB (figure IV.8).

We note ***ATM*** (or ***VAB).***

Mathematically, it is the positive angle in the clockwise direction between the y-axis of the coordinate system and the vector $\overrightarrow{AB}$. G is between 0 and 400 Gr.

For example (figure 3.26) : G_{AB} is the angle between the North (the-axis) and the direction AB.

G -$_{BA}$ is the angle between North and the direction BA.

The relationship between G_{AB} and G -$_{BA}$ is: $\mathbf{G_{BA} = G_{AB} + 200}$

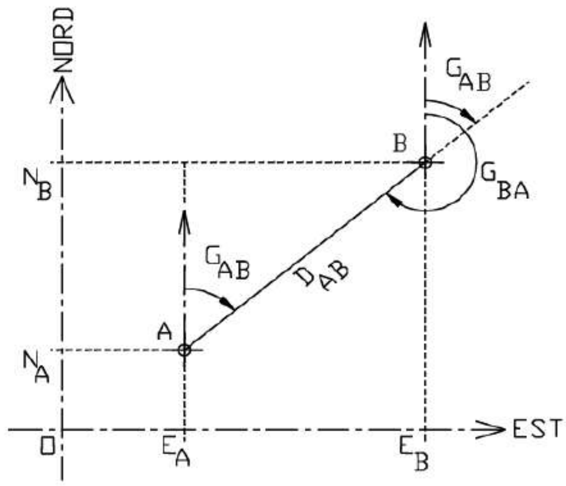


Figure IV.8. Deposit management AB

IV.4.3.2. Calculating deposit, and distance between 2 points

Consider the coordinates of two points A (X_A, Y_A) and B (X_B, Y_B). The following relationship allows to compute G_{AB}: $\mathbf{\tan G_{AB} = \frac{X_B - X_A}{Y_B - Y_A}}$.

In fact, the calculator gives the value of the angle of the auxiliary g (fig IV.9). To obtain G_{AB}, it is thus necessary to take into account the position of point B relative to point A ; it speaks of quadrants :

<u>*Quadrant 1*</u> : B is to the east and north of A ($\Delta X > 0$ and $\Delta Y > 0$). $\Rightarrow$ $\boldsymbol{G_{AB} = g}$

<u>*Quadrant 2*</u> : B is to the east and south of A ($\Delta X > 0$ and $\Delta Y < 0$). $\Rightarrow$ $\boldsymbol{G_{AB} = g+200}$ (with $g < 0$)

<u>*Quadrant 3*</u> : B is to the west and south of A ($\Delta X < 0$ and $\Delta Y < 0$). $\Rightarrow$ $\boldsymbol{G_{AB} = g+200}$ (with $g > 0$)

<u>*Quadrant 4*</u> : B is to the west and north of A ($\Delta X > 0$ and $\Delta Y > 0$). $\Rightarrow$ $\boldsymbol{G_{AB} = g+400}$ (with $g < 0$)

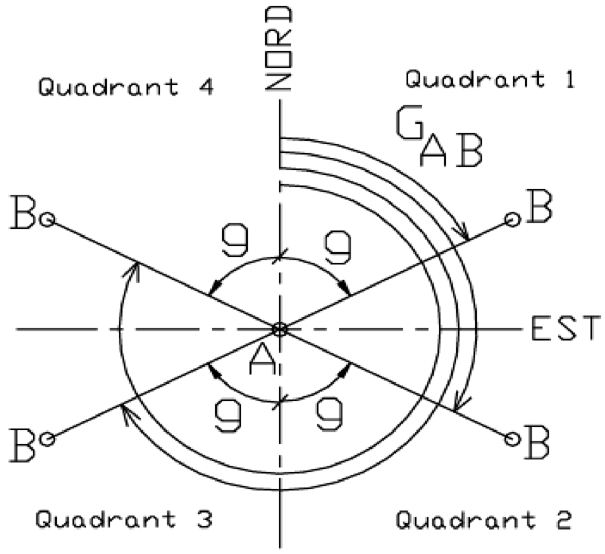


Figure IV.9. Differents quadrants

IV.4.3.3.Calculation of G0 Station

The G0 station (also noted, V0) is a constant orientation of the station S which, when added to a reading of the horizontal angle to a point P referred to, gives the bearing of the direction SP. This is also the reservoir of the zero of the limb, is the angle between the direction of the y-axis of the reference Lambert, and the zero of the limb of the apparatus is parked. The bearing of the direction MS is defined by :

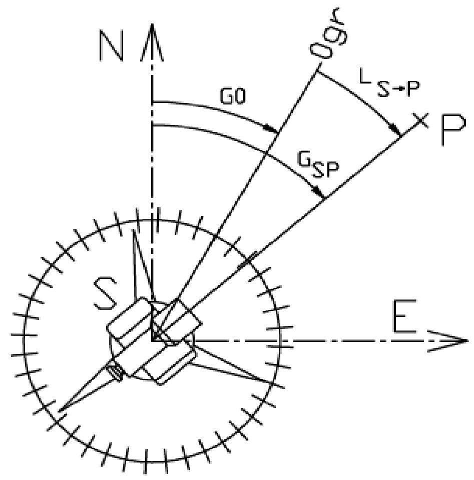


Figure IV.10. G0 Station

$$\mathbf{G_{SP} = G_0 + L_{sp}}$$

$$\mathbf{G_0 = G_{SP} - L_{sp}}$$

- **Definition of G0 through Station**

To improve the accuracy of the orientation of the station, several readings on known points in the coordinates are determined: these points are called points "old". To obtain a correct direction, it takes at least two described (three or four are preferable) distributed in the four quadrants around the point of station S.

Four points old M, N, O, and P referred from station S, we obtain four values of G0:

Referred to M: $G01 = G_{SM} – L_{S \rightarrow M}$ Referred to on O : $G03 = G_{NA}\ L_{S \rightarrow O}$

Referred to on N: $G02 = G_{SN} – L_{S \rightarrow N}$ Referred to on P : $G04 = G_{SP} – L_{S \rightarrow P}$

The average **G0** is the weighted average of the G_{0i}. It is not weighted as if the designs are of unequal length (the weighting is proportional to the length of each referred to as more of a target is long over his guidance theoretical angular is accurate).

$$\mathbf{G0_{way}} = \frac{\sum_{i=1}^{i=n} \mathbf{p_i} * \mathbf{G0_i}}{\sum_{i=1}^{i=n} \mathbf{p_i}}$$

With: n is the number of specified orientation (number of G0i calculated).

Pi represents the weight of each target, that is to say, its length in kilometers:

Pi = Di (km). The weights pi are rounded to the nearest meter in manual calculations.

The analysis of the differences between the G0 individual and the deposit means of guidance to help identify any errors in calculations and observations, but also to show a possible displacement of the known points in the coordinates (terminal moved, poor identification of points referred to...).

The bearing of a direction to be determined is calculated simply then:

$$\mathbf{G_{SP} = G0_{medium} + L_P}$$

IV.4.4.Conversion of coordinates

IV.4.4. 1. Conversion from polar to Rectangular

Direct problem $(X_{\mathrm{Has}},\ Y_A,\ G_{A\mathrm{B}},\ D_{A\mathrm{B}}) \Longrightarrow (X_{\mathrm{B}},\ Y_{\mathrm{B}})$

Calculation of the coordinates of a point B unknown by the given coordinates of a point Is known, and the extent of the deposit and the distance AB.

$$\mathbf{X_B = X_A + D_{AB} * SING_{AB}}$$

$$\mathbf{Y_B = Y_A + D_{AB} * COSG_{AB}}$$

IV.4.4. 2. Converting rectangular to polar

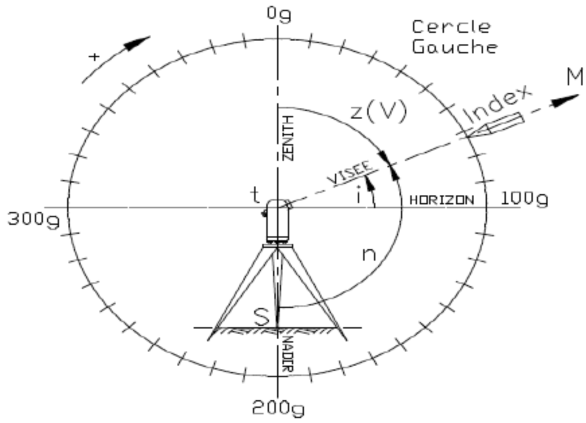
Problem indirect or reverse $(X_A,\ Y_A,\ X_B,\ Y_B) \Longrightarrow (G_{AB},\ D_{AB})$

Calculation of the deposit and the distance AB from the coordinates of the points A and B known.

$$D_{AB} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$$

$$G_{AB} = \tan^{-1}\frac{(X_B - X_A)}{(Y_B - Y_A)}$$

IV.5. Vertical Angles



n = 200 – V

i = 100 – V

100 = n – i

- The angle **i** is counted positive in the counterclockwise so as to obtain an angle of site positive for a target above the horizon and the angle of site to be negative for a target below the horizon.

- The angle **n** is counted positive in the counterclockwise; it is 0 Gra nadir, and 200 Gra at the zenith,

- The angle z is equal to 100 Gra on the horizontal.

Application

1. It is proposed to determine the rectangular coordinates of the points (1, 2, 3, 4) from these Cartesian coordinates present in the array :

	Rectangular coordinates		to polar coordinates	
POINTS	**X(m)**	**Y(m)**	**Dist I(...)**	**Gis I (...)**
St	**0.00**	**0.00**	/	/
1	0.00	12.72		
2	2.36	12.72		
3	8.63	13.17		
4	2.07	3.36		

Complete the table

Indication:

- **St** : station
- Gis : Deposit
- Dist : Distance
- i = 1, 2, 3, 4

2. In order to determine the coordinates of the new points 80 and 81 from geodetic points 50, 51, 52, 53 and 54, a technician surveyor stationed on the point 50, and performs the following observations:

Point Referred to	readings reduced (degrees)	distances horizontal (m)
80	0.0000	300.460
52	52.7859	
81	156.62	216.612
53	232.59	
51	350.38	
54	125.5665	

The coordinates of the geodetic points mentioned in the following table:

Points	X(m)	Y(m)
50	982 591.010	3 155 242.710
51	983 111.450	3157891.810
52	986 130.980	3 154 407.730
53	979 758.400	3 154 999.820
54	982 679.857	3 154 794.980

- Calculate, for each point known as the G0 individual
- Calculate the G0 way to the station 50
- Calculate the plane coordinates of the points 80 and 81

Chapter V

Determination of Surfaces

V. 1. Introduction

The regular polygon consists of a surface bounded by several sides, all of the same length. The angles between the sides are all the same. This polygon is contained in a circle says 'limited' and contains a circle that says 'registered'. The regular polygon is formed by the same number of identical triangles as there are sides. Each triangle sharing a vertex opposite to the side with the other triangles in the center of the regular polygon.

V. 2. Area of any form of polygon

To calculate the area of a polygon, we divide it into squares, rectangles, trapezoids, triangles rectangles; then you add up the partial areas obtained and it was the total surface area.
We can also surround the irregular polygon by drawing a rectangle circumscribed, through all its vertices ; it calculates the area of various figures, triangles, rectangles, trapezoids that are outside of the polygon, and subtracts the sum of the area of the rectangle circumscribed.
The area of a polygon can be obtained either by decomposition into elementary areas (sum = sum of the areas) or by difference (area of a rectangle minus the sum of the areas excluded)

V. 2.1. Principle used for the decomposition of an irregular insurveying polygon

When the ground that we must measure an irregular shape it needs to be split into rectangles, trapezoids, triangles, rectangles, of which it is easy to find the surface area. The total of these partial areas is the total surface area.
uses *the square of the surveyor* or any other instrument that can determine on the ground in a straight line and taken to the base and perpendicular, which lead to the various angles of the field (*right angle optic for example*), the dividing and polygons which it is easy to calculate the area. We still use the milestones variously coloured, planted on the ground form the outline of the polygons.

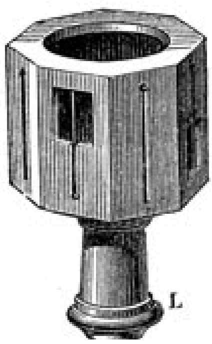


Figure V. 1. Square of Surveyor

Finally, to measure the distances are used, the land surveyor chain with a length of 10 meters and worksheets that help them bite into the earth as the starting points of each measure 10 meters.

V. 2.2. The vertices of the polygon are known to Cartesian coordinates

Is a polygon of n vertices each of which is known by its Cartesian coordinates (Xi ; Yi). Figure V. 1 shows an example with n = 4. The surface area of this polygon is expressed in two equivalent ways:

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \sum_{\mathbf{i=1}}^{\mathbf{i=n}} \mathbf{X_i} . (\mathbf{Y_{i-1}} - \mathbf{Y_{i+1}})$$

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \sum_{\mathbf{i=1}}^{\mathbf{i=n}} \mathbf{Y_i} . (\mathbf{X_{i-1}} - \mathbf{X_{i+1}})$$

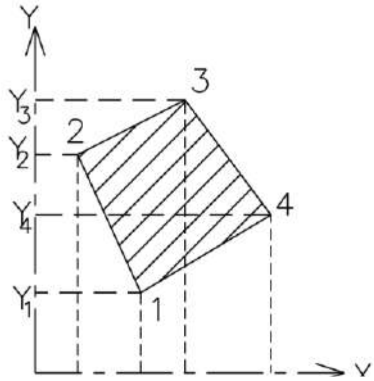


Figure V. 2. Peaks in Cartesian coordinates

If the surface S is positive, then the surface S' is negative, and vice versa. We must, therefore, always verify that S' + S = 0.

During rotation of the indices i, we apply the following convention : $X_0 = X_n$; $Y_0 = Y_n$; $X_{n+1} = X_{-1}$; $Y_{n+1} = Y_{-1}$.

This is equivalent to consider the vertices as being on a loop described by rotating around the surface; the top 1 is the following vertex n and, therefore, the vertex n is the last of the top 1.

V. 2.3. The vertices of the polygon are known in Polar coordinates

A device of the type theodolite stationed at point S, you can perform the readings of the angles $\boldsymbol{\alpha_i}$, on the vertices of the polygon. If we measured the horizontal distance from the point S to each of the vertices, we know these vertices in polar coordinates, topographic (**Dh** , $\boldsymbol{\alpha}$) in the coordinate system (S, X, Y), the y-axis Is the position of the zero of the horizontal circle of the theodolite .

these polar coordinates are a specialpipeline to the topography since the zero of the angles is placed on the y-axis, Y-and their sense of rotation is clockwise. The polar coordinates math place the zero of the angles on the X-axis X with angles turning positively in the trigonometric direction.

We divide the total surface area of the polygon of n sides triangles from all of the top S. One can infer the surface projection horizontale of a polygon of n sides by the following formula :

$$S = \frac{1}{2} . \sum_{i=1}^{i=n} Dh_i . Dh_{i+1} . sin(\alpha_{i+1} - \alpha_i)$$

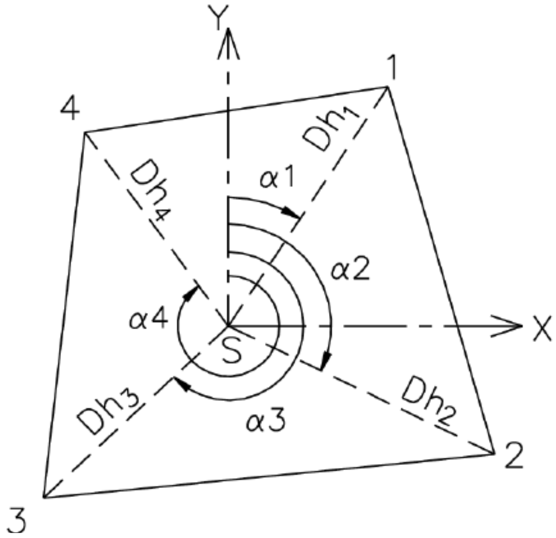


Figure V. 3. Vertices in Polar coordinates

- By convention, $\boldsymbol{\alpha}_{n+1} = \alpha_{-1}$ and $Dh_{n+1} = Dh_1$; the top 1 is the next summit the summit n.
- The area of the triangle (1S4) of the example in figure V. 3 involves the angle $(\alpha_1 - \alpha_4)$ that is negative. To get its value in the table, simply add 400 grd. This is not necessary in the calculation since the sine does not change, nor value, nor sign (sin α = sin(400 + α)).
- If the station S is located at the outside of the polygon, the formula is also applicable. It will then appear in the calculation of the surface negative-it is necessary to keep the sign in the sum of the general formula.

V. 2.4. Sarron Formula

Is a polygon of n sides. If one knows the length of n–1 sides and the measurement of n–2 angles between these sides, we can calculate the area of the polygon by the formula :

$$S = \frac{1}{2}\begin{bmatrix} ab \cdot \sin\widehat{ab} + ac \cdot \sin\widehat{ac} + ad \cdot \sin\widehat{ad} + ... \\ + bc \cdot \sin bc + \widehat{bd} \cdot \sin\widehat{bd} + ... \\ + cd \cdot \sin\widehat{cd} + .. \end{bmatrix}$$

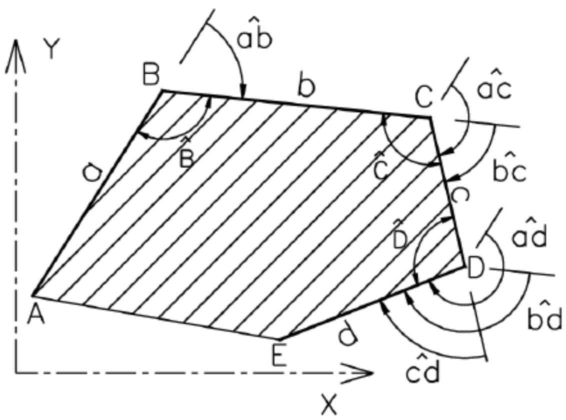


Figure V. 4. Surface lifted by polygon

- a, b, c, d, ... represent the n – 1 sides are known.
- $\widehat{ab}$ is the angle between the vectors $\overrightarrow{AB}$and $\overrightarrow{BC}$.$\widehat{ab}$ is also the complement of the internal angle at the vertex B (noted $\widehat{B}$) : $\widehat{ab}$= 200 – $\widehat{B}$

- $\widehat{ab}$, $\widehat{ac}$, $\widehat{ad}$, $\widehat{bc}$, $\widehat{bd}$, etc, represent the n – 2 angles directed between the n-1 sides of known length.
- The formula applies to the triangle : 2.S = a. b. sin $\widehat{ab}$ = a. b. sin(200 – $\widehat{ab}$).

Application

1. The polygon following is defined by the local coordinates of its vertices expressed in meters in the following table. Calculate its area in square centimeters near.

Point	*A*	*B*	*C*	*D*	*E*
Xi(m)	120,41	341,16	718,59	821,74	297,61
Yi(m)	667,46	819,74	665,49	401,60	384,13

2. Calculate the area of the polygon (A-B-C-Of) raised in polar coordinates, topographic from the station S (figureV.5). These coordinates are gift- born in the following table:

Points	*Dh (m)*	*Angles (grd)*
Has	48,12	53,12
B	51,33	100,03
C	48,71	147,41
D	57,48	261,53
E	47,93	380,37

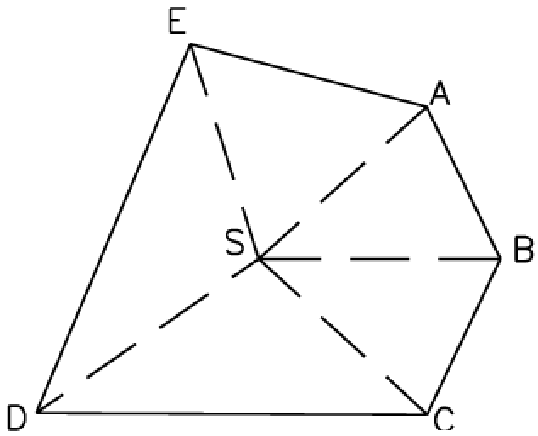


Figure V. 5. Application 1

3. Calculate the area of the polygon ABCDE (figure, V. 6) on which the following measurements were made :

Side AB = 12,32 m Angle ABC = 113,656 grd

Side BC = 28,46 m Angle BCD = 97, 127 grd

Side CD = 25,52 m Angle CDE = 116,632 grd

Side = 31,59 m

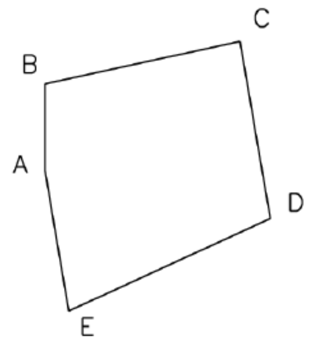


Figure V. 6. Application 2

4. Calculate the area of a polygon

The calculation of the area of a polygon is an essential step in surveying and engineering. The most common method to determine the area of a polygon is to use the formula for the area based on the coordinates of the vertices. For a polygon whose vertices are known, the formula for the area is given by :

Area = $\frac{1}{2}\left|\sum_{i=1}^{n}(x_i y_{i+1} - y_i x_{i+1})\right|$

Where $(x_{n+1}, y_{n+1}) = (x_1, y_1)$

Example : let us Consider a polygon with the vertices A(1, 1), B(4, 1), C(4, 3), and D(1, 3). Applying the formula, we calculate :

Area=1/2∣ (1⋅ 1+4⋅ 3+4⋅ 3+1⋅ 1)−(1⋅ 4+1⋅ 4+3⋅ 1+3⋅ 1)∣ =1/2∣ 1+12+12+1−(4+4+3+3)∣ =1/2∣ 26-14=1/2x12= 6

Thus, the area of the polygon is 6 square units.

5. Determination of the surfaces of the contours represented on the map

The determination of the surfaces of the contours on a plane is a method used to calculate areas of land using contour lines. The surfaces between the contours can be estimated using graphical methods or calculations based on the coordinates of the points of the contour.

Example: On a topographic plan, there are contours at intervals of 2 meters. To estimate the surface area between the two curves of level, say at 0 m and 2 m, we can use the method of trapezoids. If the lengths of the segments of the contour lines are, respectively, 100 m and 80 m, the surface area between these two curves can be approximated by :

Area≈(100+80)2×2=180 m2Aire≈2(100+80)×2=180 m2

This method allows obtaining a quick estimate of the land surface in function of the level curves.

6. Planimeter and measurement of surfaces

The Planimeter is an instrument used for measuring surfaces on a plan or map. It works by tracing the outline of the surface to be measured, and the instrument automatically calculates the area as a function of the distance travelled. The Planimeter may be mechanical or digital, offering a high accuracy in the measurement of surfaces.

Example: A surveyor uses a Planimeter to measure the surface of a lake represented on a map. By placing the tip of the instrument on a starting point and following the contour of the lake, the Planimeter indicates a measured surface area of 250 hectares. This allows the surveyor to provide accurate data for environmental studies, or management of water resources.

This chapter discusses the different methods of determination of the surfaces, either by direct calculation of the polygons, the estimation of surfaces between contours on a plane, or the use of a Planimeter. Each of these methods is essential for various applications in surveying, engineering and management of natural resources.

Background: A land surveyor wishes to measure the surface area of a field is represented on a map using a Planimeter. The outline of the shape field has an irregular shape. After having traced the outline with the Planimeter, it obtains a measure of distance of 150 meters.

Steps to follow:

1. **Preparation :**
    - Make sure that the Planimeter is correctly calibrated before starting the measurement.
    - Place the card on a flat and stable surface.
2. **Measurement of the contour:**
    - Follow the contour of the field with the tip of the Planimeter.
    - Note the total distance traveled by the cursor (150 m).
3. **Calculation of the area:**
    - Use the formula of the Planimeter to convert the distance traveled on the surface. Suppose that the map scale is 1:5000, which means that 1 cm on the map corresponds to 5000 cm (50 meters) in reality.
    - The area measured by the Planimeter is given by the formula:

Area = Distance × scale Factor^2

- In this case, the scale factor is 5000.

4. **Final calculation :**
    - Use the formula :

Range = 150 m × $(5000)^2$ = 150 × 25 000 000 = 3 750 000 m^2

Correction

1. **Preparation:**
    - The calibration of the Planimeter is crucial to ensure accurate measurements. Make sure that the instrument is in good condition before you start.
2. **Measurement of the contour:**
    - Carefully following the outline, you must ensure that the tip remains in contact with the contour throughout the measurement to avoid errors.
3. **Calculation of the area:**
    - The formula used is correct. Taking into account the scale, you have applied the scale factor of the square to convert the measured distance in the real surface.
4. **Final calculation:**
    - The final calculation gives a surface area of 3 750 000 m^2 or 375 hectares. This represents an accurate measurement of the surface of the field on the ground.

Conclusion

These exercises demonstrate how to use a Planimeter for surfaces to be measured on a map. By following the stages of preparation, measurement and calculation, the surveyor can obtain an accurate estimate of the area of a field.

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