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Ministry of Higher Education and Scientific Research
University of El Oued
Faculty of Technology

Course Guide for the Module:
Modeling and Simulation of Electrical Machines
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Chapter I:
General Modeling of Electrical Machines

I.1- Introduction

In the field of electrical engineering and machine design, understanding the structures of machines and the representation of magnetic phenomena is fundamental. This knowledge forms the basis for efficient and effective machine design and operation. This introduction will delve into key concepts such as machine structures, magnetic phenomenon representation, equivalent circuits, magnetomotive force, permeance, induction distribution, winding flux, coupling effects, dispersion flux, and the specific case of sinusoidal distributions. Additionally, we will explore the calculation of torque using the virtual work method. These topics are crucial for comprehending the behavior and performance of electrical machines in various applications.

I.2- Types of Electrical Machines

Electric machines come in three primary categories: transformers, generators, and motors.

Transformer: A transformer is a type of electric machine where both the input and output involve electrical power.

Generator: In the case of a generator, it operates by converting mechanical power into electrical power.

Motor: On the other hand, a motor functions by transforming electrical power into mechanical power.

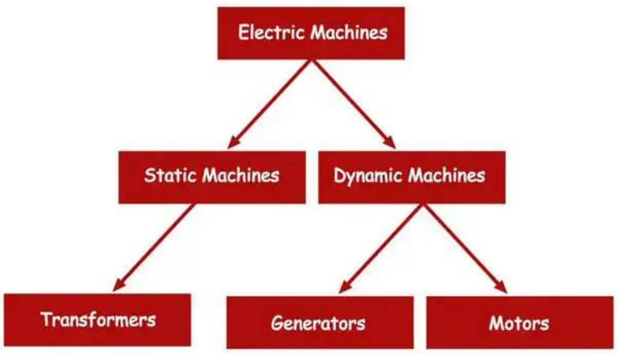


Figure I.1: Types of Electrical Machines

Electrical machines are typically categorized into two broad groups: static machines and dynamic machines. Static machines, exemplified by transformers, primarily deal with electrical power in both input and output. In contrast, dynamic machines, such as motors and generators, are characterized by the conversion of energy between mechanical and electrical forms. Motors transform electrical power into mechanical power, while generators do the opposite by converting mechanical power into electrical power. This classification helps differentiate machines based on their fundamental operational principles and energy conversion processes.

In the realm of electric motors, two primary types stand out: AC motors and DC motors. AC motors offer flexibility in speed control and exhibit a low power demand during startup. Conversely, DC motors are favored for their cost-effectiveness, particularly in lower power applications, as they often involve lower initial costs compared to AC motors and boast ease of installation.

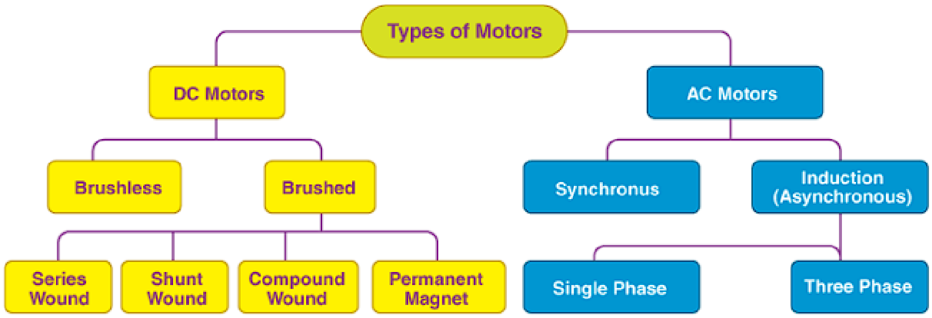


Figure I.2: Types of AC and DC motors

Types of AC Motors:

- Synchronous AC Motor
- Asynchronous AC Motor (Induction AC Motor)

Types of DC Motors:

- Brushed DC Motor
- Brushless DC Motor

Synchronous Motors: In synchronous motors, the speed remains constant even with varying loads, as the supply current frequency synchronizes with rotor rotation. They are commonly used in applications that require constant speed, particularly in machinery.

Asynchronous (Induction) Motors: These motors operate based on the principle of electromagnetic induction generated by the rotor's magnetic field. They come in two main types: single-phase and three-phase. Single-phase induction motors are suitable for smaller loads like household appliances, while three-phase induction motors find use in industrial applications, such as conveyor belts and lifting gears.

Brushed Motors: Traditional DC motors, they are employed in basic applications with simple control systems. Brushed motors come in four varieties: Series Wound, Shunt Wound, Compound Wound, and Permanent Magnet. Series wound DC motors have rotor winding connected in series with the field winding and are used in applications like lifts, cranes, and hoists. Shunt wound DC motors connect the rotor winding in parallel with the field winding, offering higher torque without sacrificing speed and are employed in applications like conveyors, grinders, and vacuum cleaners. Compound wound DC motors combine the polarity of the shunt winding with that of the series fields, providing high starting torque and smooth operation even with varying loads, making them suitable for elevators, circular saws, and centrifugal pumps. Permanent magnet DC motors are used in applications that require precise control and lower torque, such as robotics.

Brushless Motors: Featuring a simpler design, brushless motors have a longer lifespan when used in high-demand applications. They require little maintenance and are highly efficient. These motors are commonly found in appliances that necessitate speed and position control, such as fans, compressors, and pumps.

I.3- Machine Structures

➢ Synchronous Motors Structure:

1. **Stator:** The stator of a synchronous motor consists of a stationary set of coil windings. When an alternating current (AC) is applied to these windings, they generate a rotating magnetic field.

2. **Rotor:** The rotor of a synchronous motor contains electromagnets or permanent magnets. These magnets are designed to align with the magnetic field created by the stator. The synchronization between the stator's rotating magnetic field and the rotor's magnets ensures a constant speed of rotation.

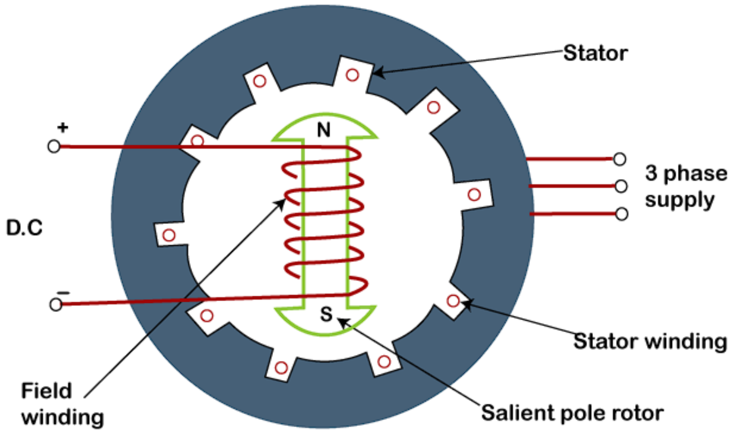


Figure I.3: Synchronous Motors Structure

➢ Asynchronous (Induction) Motors Structure:

1. **Stator:** Similar to synchronous motors, the stator in an induction motor comprises a stationary set of coil windings. When AC power is supplied, these windings generate a rotating magnetic field.

2. **Rotor:** The rotor of an induction motor is typically constructed from laminated iron cores. It induces currents by electromagnetic induction as it moves within the rotating magnetic field generated by the stator. The type of rotor may vary between single-phase and three-phase motors. Single-phase motors may include additional components like starting windings or capacitors for initiating rotation.

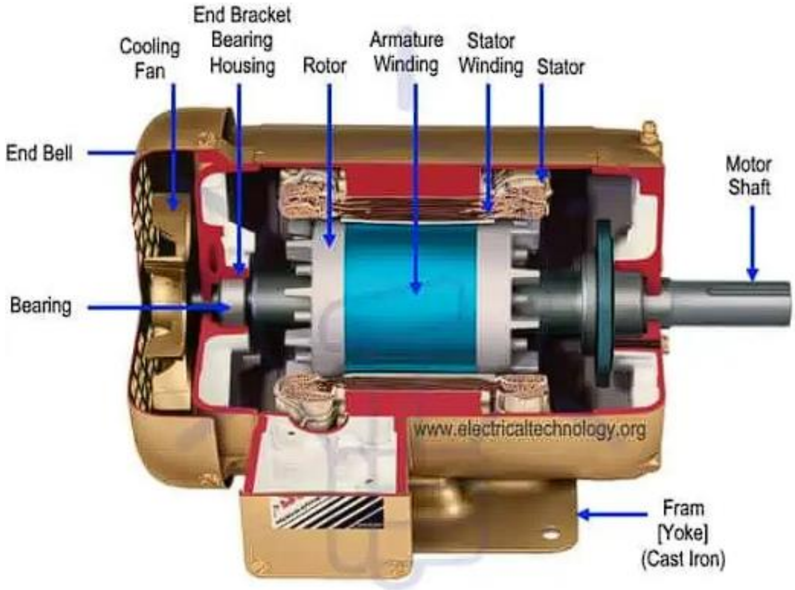


Figure I.4: Induction machine Structure

- **Brushed DC Motors Structure:**

1. **Stator:** The stator in brushed DC motors contains the stationary field windings. These windings produce a magnetic field when energized with DC current.
2. **Rotor:** The rotor of brushed DC motors consists of an armature with coils. Attached to the rotor is a commutator, which is a split ring, and brushes that make physical contact with the commutator. The commutator serves to reverse the direction of current in the rotor coils as the rotor turns. This reversal generates mechanical motion.
3. **Commutator:** The commutator is a critical component in brushed motors. It effectively switches the polarity of the current in the rotor coils as the rotor rotates, ensuring continuous motion.

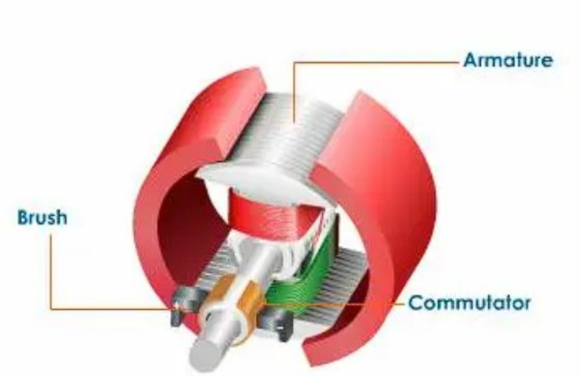


Figure I.5: Brushed DC Motors Structure

Brushless Motors Structure:

1. **Stator:** The stator of a brushless motor houses stationary coil windings. These windings produce a rotating magnetic field when energized with a sequence of electrical signals.

2. **Rotor:** The rotor in a brushless motor contains permanent magnets. These magnets are arranged to interact with the rotating magnetic field produced by the stator. The design eliminates the need for brushes and a commutator.
3. **Control Electronics:** Brushless motors include control electronics that are often an integral part of the motor structure. These electronics manage the timing and sequence of energizing the stator windings. By controlling the stator windings' energization, they determine the direction and speed of rotation.

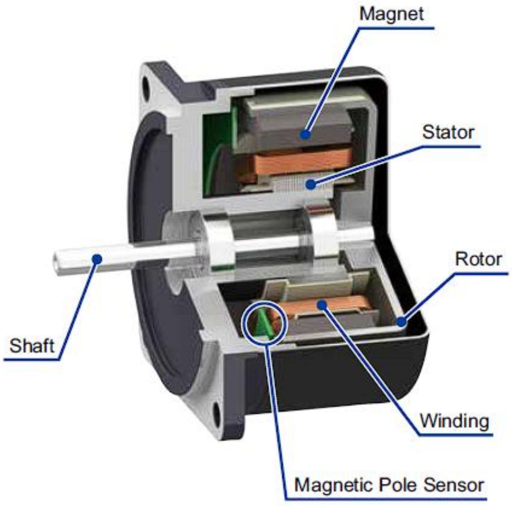


Figure I.6: Brushless Motors Structure

I.4- Magnetic circuits

Electromagnetic principles play a pivotal role in the design and operation of electric machines and power electronics systems. Maxwell's equations, a set of fundamental equations that describe the behavior of electric and magnetic fields, have been instrumental in understanding the interplay between electric and magnetic phenomena. These equations provide valuable insights into the behavior of transformers, motors, and generators, allowing engineers to optimize their performance and efficiency. Furthermore, the concept of magnetic saturation, which occurs when the magnetic material of a core is fully magnetized, is a critical consideration in the design of transformers and inductors, as it can affect their performance under different operating conditions.

- ✓ **Magnetic Material Properties**

Magnetic materials exhibit complex behaviors, characterized by nonlinearity and a range of properties that change with factors such as magnetic field strength, operating frequency, and temperature. This complexity adds challenges to the design of magnetic components. One fundamental property of magnetic materials is their relative permeability (μr), which establishes the relationship between magnetic flux density (B) and magnetic field intensity (H), as expressed by the equation:

$$\boldsymbol{B} = \mu_r \mu_0 \boldsymbol{H} \tag{I.1}$$

Here, μ_0 represents the permeability of free space, equal to $4\pi \times 10^{-7}$ H/m. However, it's crucial to note that the value of μ_r is not constant but varies with the magnetic flux density B, as illustrated in the B-H curve of a magnetic material in Figure 7. This variation in μr indicates that magnetic materials display hysteresis, wherein the interplay between B and H relies not just on their relative magnitudes but also on the rate of change of B concerning H.

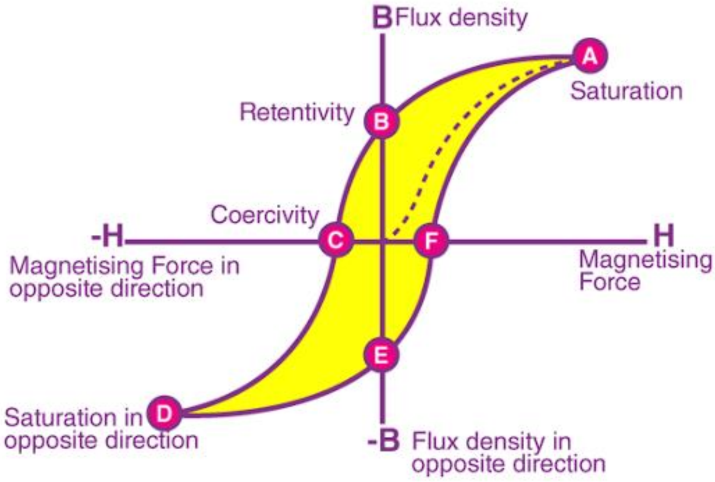


Figure I.7: B-H curve of a magnetic material

- **Faraday's Magnetic Circuit Law:**

Faraday's Magnetic Circuit Law is a fundamental principle in electromagnetism that underpins the operation of electrical devices and power systems. This law establishes a direct connection between a time-varying magnetic flux, denoted as ø(t), and the generation of an induced voltage, v(t), in a conducting loop. As illustrated in Figure 8, when a time-varying magnetic flux passes through the interior of a wire loop, a voltage is induced in the closed loop that encloses the changing flux. The cross-sectional area of the loop, denoted as Ac, plays a crucial role in determining the magnitude of the induced voltage.

Faraday's law can be expressed mathematically as

$$v(t) = \frac{d\phi(t)}{dt} \tag{I.2}$$

where v(t) represents the induced voltage, and dϕ(t)/dt is the rate of change of magnetic flux with respect to time. This equation demonstrates that the induced voltage is directly proportional to the rate of change of magnetic flux. For a winding with N turns, the total induced voltage can be expressed as the product of N and the rate of change of flux, making it a pivotal concept in transformer design and inductor applications. Faraday's Magnetic Circuit Law not only provides the foundation for understanding electromagnetic induction but is also central to the design and analysis of electrical systems, ensuring their efficient and reliable operation.

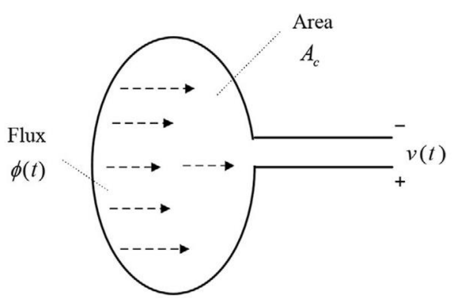


Figure I.8: Faraday's Induction Principle

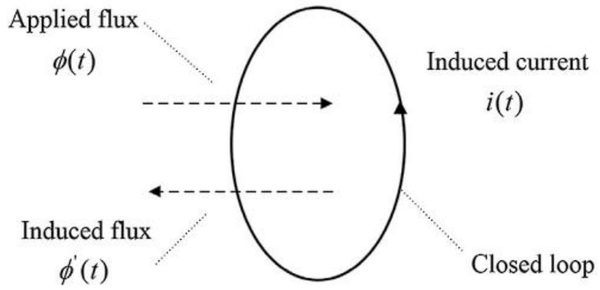


Figure I.9: Lenz's Law Demonstrated in a Closed Loop

Lenz's Law is a fundamental concept in electromagnetism that provides a precise understanding of the behavior of induced currents in response to changing magnetic fields. It can be expressed mathematically as:

$$v(t) = -N \cdot \frac{d\phi(t)}{dt} \tag{I.3}$$

where v(t) is the induced voltage, N represents the number of turns in the winding, and dϕ(t)/dt signifies the rate of change of magnetic flux with respect to time. The negative sign in the equation signifies that the induced voltage opposes the change in magnetic flux, as stipulated by Lenz's Law. This opposition serves to maintain the principle of conservation of energy.

The concept of magnetic flux linkage, λ, is a crucial element in understanding Lenz's Law. It is expressed as:

$$\lambda = N \cdot \phi \tag{I.4}$$

where λ represents the magnetic flux linkage, N is the number of turns in the winding, and ϕ denotes the magnetic flux.

For cases where the magnetic flux distribution is uniform, Equation I.3 can be rewritten as:

$$v(t) = -N .Ac.\frac{dB(t)}{dt} \tag{I.5}$$

where Ac is the cross-sectional area of the winding and dB(t)/dt is the rate of change of magnetic flux density passing through the winding. This equation highlights how the induced voltage is directly proportional to the rate of change of magnetic flux density, and it underlines the significance of Lenz's Law in various electromagnetic applications.

Lenz's Law is further illustrated in Figure 9, where a time-varying magnetic flux ø(t) induces a current i(t) in a closed loop. This induced current subsequently generates its own magnetic flux, ø'(t), with the purpose of opposing changes in the original magnetic flux, thus preserving the law of energy conservation.

Furthermore, Lenz's Law is a crucial tool for understanding and managing eddy current effects in magnetic cores and winding conductors. Eddy currents, which are secondary currents induced in conducting materials due to changes in the magnetic field, can result in energy losses and undesired heating in electrical devices. By adhering to Lenz's Law, engineers can implement strategies to mitigate these eddy current effects, optimizing the efficiency and performance of electrical systems.

- ✓ **Ampere's Magnetic Circuit Law**

Ampere's Magnetic Circuit Law is a foundational principle in electromagnetism, elucidating the relationship between time-varying currents and the magnetic field they induce. As illustrated in Figure 10, when a current i(t) varies over time, it results in a time-varying magnetic field H(t). This law is particularly significant in the design and analysis of magnetic circuits, such as those found in transformers and inductors.

The magnetomotive force (MMF) in the core of a magnetic circuit is determined by the total current passing through the interior of the core loop. Ampere's law is mathematically expressed as a line integral of H around any closed contour, which is equal to the total current enclosed by that contour. This integral form of Ampere's law is crucial for understanding the distribution of magnetic fields in various magnetic components and is expressed as:

$$\oint \mathbf{H} \cdot dl = \sum I \qquad \text{I.6}$$

where $\oint H \cdot dl$ represents the line integral of H around a closed contour, and $\sum I$ is the total current enclosed by that contour. This equation illustrates the fundamental connection between currents and the magnetic fields they generate, serving as a cornerstone for the analysis and design of magnetic circuits in electrical and electronic systems.

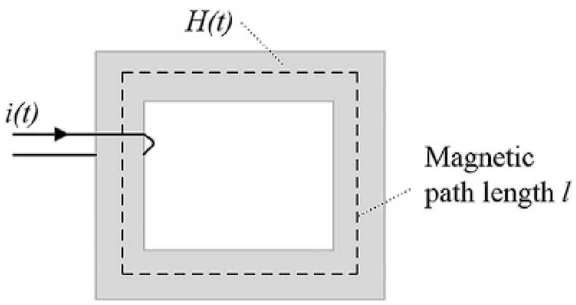


Figure 10: Visual Representation of Ampere's Magnetic Circuit Law

For a winding with N turns, Ampere's law can be expressed as:

$$\oint H \cdot dl = N \cdot i \qquad \text{I.7}$$

In this equation, N represents the number of turns in the winding, H is the magnetic field intensity, and i is the current passing through the winding. Ampere's law is fundamental in understanding the distribution of magnetic fields in magnetic circuits, and it quantifies the relationship between the magnetic field intensity and the total current.

The magnetic field intensity, H, in turn, leads to the development of a magnetic flux density, B, based on the B-H characteristics of the core materials. In this section, we simplify the analysis by neglecting the nonlinearity in the B-H characteristics. Instead, we use the relation provided by Equation (1) to describe the relationship between magnetic flux density and magnetic field intensity, allowing for a more straightforward analysis and understanding of the magnetic properties of the materials involved.

I.5- Reluctance and Inductance

When drawing an analogy between magnetic circuits and electric circuits, it is natural to draw parallels between certain key properties. **The magnetomotive force** (MMF), denoted as F and expressed as:

$$F = N \cdot i \qquad \text{I.8}$$

can be likened to voltage in an electric circuit. Magnetic flux, ø, is analogous to current. As depicted in Figure 11, the MMF behaves like a force that propels magnetic flux through the magnetic circuit. Just as electrical resistance (R) exists in an electric circuit, magnetic core reluctance (R) serves as its magnetic counterpart. The relationship ø = F/R directly mirrors Ohm's law in electricity. Reluctance, similar to resistance, opposes the flow of magnetic flux in much the same way that resistance opposes the flow of electric current.

Magnetic circuits, described by Equation (I.9), can be approached and solved in a manner akin to how electric circuits are addressed using Ohm's law:

$$F = H \,.\, l = N \,.\, i = \phi \,.\Re \qquad \text{I.9}$$

In Figure 11, a coil with N turns is wound around a closed magnetic core with a mean length, lc, and a cross-sectional area, Ac. The reluctance of a basic magnetic circuit element is determined by:

$$\Re = \frac{l_c}{\mu_0 \cdot \mu_r \cdot Ac} \qquad \text{I.10}$$

Here, μ_0 represents the permeability of free space, μ_r is the relative permeability of the core material, and At is the total cross-sectional area. This equation characterizes how reluctance resists the flow of magnetic flux. Furthermore, self-inductance (L) connects the flux linkage generated by a coil to the current in that coil:

$$L = \frac{\lambda}{i} = \frac{N \cdot ø}{i} = \frac{N^2}{\Re} = \frac{\mu_0 \cdot \mu_r \cdot Ac \cdot N^2}{l_c} \qquad \text{I.11}$$

This relationship between self-inductance and the components of the magnetic circuit helps in understanding and modeling the behavior of magnetic circuits, much like in the realm of electric circuits.

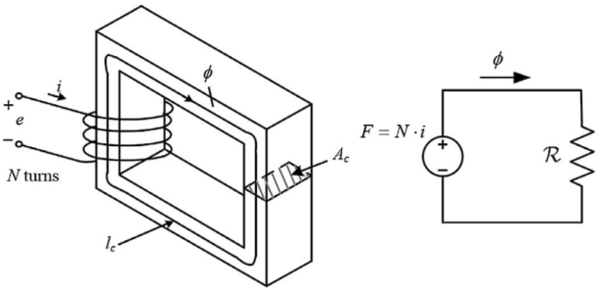


Figure I.11: Representation of Magnetic Field Patterns in a Closed Magnetic Core

In ferromagnetic materials, the B-H characteristic is inherently nonlinear, which means that the relationship between magnetic flux density (B) and magnetic field intensity (H) is nonlinear. In the magnetic core illustrated in Figure 11, the magnetic flux density (B) is limited by the properties of the core material. A small current with a sufficient number of turns can easily drive the core material into saturation, where the core is no longer responsive to increases in magnetic field intensity. To prevent core saturation, engineers often introduce an air gap into the core, as shown in Figure 12. The presence of the air gap allows a larger current to pass through the winding without reaching saturation because it increases the reluctance of the magnetic circuit. Additionally, it's important to note that the introduction of an air gap results in greater energy storage within the system.

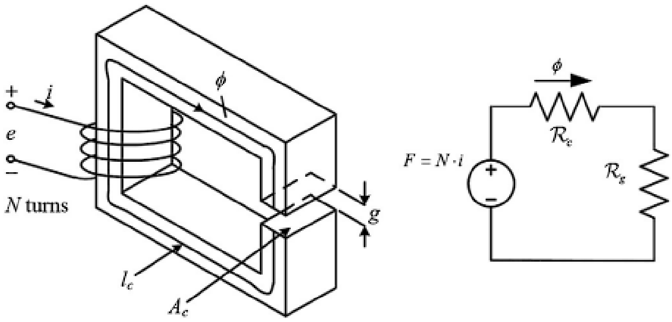


Figure I.12: Representation of Magnetic Field Patterns with air gap

In the structural configuration depicted in Figure 12, two reluctances are connected in series. Therefore, the total MMF, F, is the sum of the MMFs required to establish magnetic flux through the core (Rc) and air gap (Rg), expressed as:

$$F = N \cdot i = \phi \cdot R_c + \phi \cdot R_g \qquad \text{I.12}$$

The individual reluctances are determined by the physical properties of length and cross-section area. Subscript 'c' refers to the core, and subscript 'g' refers to the air gap. For the core reluctance, Rc is defined as:

$$R_c = \frac{l_c}{\mu_0 . \mu_r \cdot Ac} \tag{I.13}$$

And for the air gap reluctance, assuming μ_r=1 for air, R_g is given by:

$$R_g = \frac{g}{\mu_0 . Ag} \tag{I.15}$$

Here, 'g' represents the length of the air gap, and since the relative permeability (μr) for air is much less than that of the core material ($\mu_r \gg \mu_0$), the reluctance of the core, Rc, is typically significantly smaller than that of the air gap, Rg. Therefore, in practical applications, Rc can often be ignored compared to Rg, leading to the simplified equation:

$$F = N \cdot i = \phi \cdot R_g \tag{I.16}$$

This simplification is especially useful in the analysis of magnetic circuits involving air gaps and is instrumental in designing systems that effectively utilize the benefits of air gaps to avoid core saturation.

Hence, the gapped inductance L is:

$$L = \frac{N^2}{\Re_g} = \frac{\mu_0 \cdot Ag \cdot N^2}{g} \tag{I.17}$$

Mutual Inductance (M): Mutual inductance between two coils, Coil 1 and Coil 2, can be expressed as:

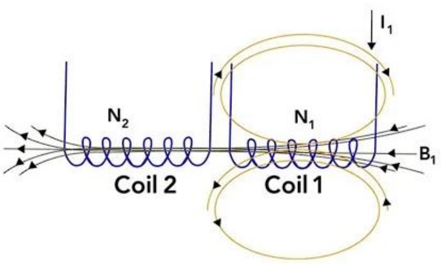


Figure I.13: Mutual Inductance of two coils

$$M = \frac{N_1 N_2}{\Re_{12}} \tag{I.18}$$

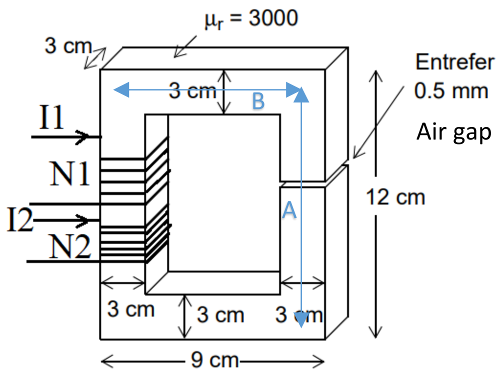
Where:

- M is the mutual inductance.
- N_1 is the number of turns in Coil 1.

- N_2 is the number of turns in Coil 2.
- R_{12} is the mutual reluctance between the two coils.

I.6- Application Example: Self-Inductance and Mutual Inductance

Consider the following magnetic circuit: Current I1 = 1.2A, I2 = 0 A, relative permeability of the magnetic material μr = 10000, the number of turns N1 = 300, N2 = 150. Determine:



1- The average length of the magnetic circuit.

2- The equivalent reluctance of the circuit.

3- The magnetic flux φ and the total flux ϕ1 for circuit N1.

4- Deduce the inductance of circuit N1.

5- Calculate the inductance of circuit N2.

When the current I2 = 0.5A. Determine:

6- The magnetic flux φ and the total flux ϕ1 for circuit N1 and compare it with the total flux ϕ1 from question 3.

7- Deduce the mutual inductance M_{12} in this case.

8- Calculate the mutual inductance M_{21}.

Solution:

1- The average length of the magnetic circuit

This would typically be provided as part of the magnetic circuit geometry (denoted as lc).

lc= (2.A+2.B-E $_{\text{Air gap}}$)

(A.n) A=12-3=9 cm

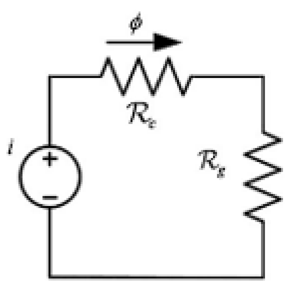
B=9-3=6 cm

E $_{\text{Air gap}}$=0.05 cm

lc= (2.9+2.6-0.05)=(18+12-0.05)=29.95 cm=0.3 m

2- The equivalent reluctance of the circuit

The reluctance Rc of a magnetic circuit is given by:



$$R_c = \frac{lc}{\mu_0.\mu_r.Ac} \text{ Also } R_c = \frac{E_{AirGap}}{\mu_0.\mu_r.Ac}$$

We have equivalent circuit as :

Re=Rc+Rg

(A.n) $R_c = \frac{0.3}{4.\pi.10^{-7}.10000.(0.03*0.03)} = 26.52\ .\ 10^3 H^{-1}$

$$R_g = \frac{0.5.10^{-3}}{4.\pi.10^{-7}.1.(0.03*0.03)} = 44.21\quad 10^4 H^{-1}$$

Re=(26.52 + 442.09). $10^3 = 468.61\ 10^3 H^{-1}$

3- The magnetic flux ϕ and the total flux Φ1 for circuit N1

$\varphi = \frac{MMF}{R_e} = \frac{N1.I1+N2.I2}{R_e}$ (N.A) $\varphi = \frac{300*1.2+150*0}{468.61\ 10^3} = 0.77 * 10^{-3}\ Wbr$

Φ1= φ. N1 (N.A) Φ1=0.77. 10^{-3} 300=99. 10^{-3}=0.23 Wbr

4- Deduce the inductance of circuit N1.

We have Φ1=L_1.I_1 so $L_1 = \frac{\emptyset_1}{I_1}$ (N.A) $L_1 = \frac{0.23}{1.2} = 0.19\ H$

5- Calculate the inductance of circuit N2.

$L_2 = \frac{{N_2}^2}{R_e}$ (N.A) $L_2 = \frac{150^2}{468.61\ 10^3} = 0.048\ H$

6- The magnetic flux φ and the total flux ϕ1 for circuit N1 and compare it with the total flux ϕ1 from question 3.

When the current I2 = 0.5A. Determine:

$\varphi = \frac{MMF}{R_e} = \frac{N1.I1+N2.I2}{R_e}$ (N.A) $\varphi = \frac{300*1.2+150*0.5}{468.61\ 10^3} = 0.93 * 10^{-3}\ Wbr$

Φ1= φ. N1 (N.A) Φ1=0.93. 10^{-3} 300=0.28 Wbr

When the second coil is activated, the total flux in the first coil increases due to the effect of mutual inductance.

7- Deduce the mutual inductance M_{12} in this case.

We have Φ1=L_1.I_1 + M_{12}.I_2 So $M_{12} = \frac{\emptyset_1 - L_1 I_1}{I_2}$ (N.A) $M_{12} = \frac{0.28-0.23}{0.5} = 0.1H$

8- Calculate the mutual inductance M_{21}.

$M_{12} = \frac{N_2 N_1}{R_e}$ (N.A) $M_{12} = \frac{300*150}{468.61\ 10^3} = 0.1\ H$

I.7- Conclusion

In this course, we've embarked on a journey through the fundamental concepts and principles that underlie electrical machines. We started by exploring the diverse world of electrical machines, categorizing them into various types, each with its unique applications and characteristics. Understanding these distinctions is crucial for anyone aspiring to work with electrical machines in any capacity.

With a solid foundation in the types of electrical machines, we delved into their structures. This knowledge helps us comprehend how these machines are built, which, in turn, aids us in their operation and maintenance.

Magnetic circuits were another crucial topic that we covered. We discussed the properties of magnetic materials, which are essential to understanding how materials interact with magnetic fields. We also explored Faraday's Magnetic Circuit Law and Ampere's Magnetic Circuit Law, which provide the fundamental rules governing magnetic circuits. This knowledge is pivotal in the design and analysis of electrical machines.

Chapter II:
Modeling Machines for Dynamic States

II.1- Introduction

The modeling of electrical machines plays a critical role in their development and control, particularly in modern applications that require high precision and efficiency. Accurate models allow for better understanding and optimization of machine performance under various operating conditions. In this chapter, we will explore the fundamental concepts behind machine modeling, focusing on dynamic regimes that are essential for advanced control strategies.

A key aspect of this process is the transformation of the machine's mathematical models, which simplifies the complexity of analysis. Two major transformations are widely used in this context: the Clarke-Concordia transformation and the Park transformation. These transformations facilitate the analysis of both stationary and dynamic behaviors of electrical machines, such as induction machines and synchronous machines.

Clarke-Concordia Transformation: Primarily applied to stationary reference frames, this transformation reduces the complexity of three-phase systems, making it easier to analyze and model machines during steady-state operations.

Park Transformation: This dynamic transformation is crucial for simplifying the mathematical model of machines during transient operations. By transforming the model into a rotating reference frame, it allows for more efficient analysis and control of machines, especially under varying load conditions.

Through these transformations, the mathematical models of machines like induction and synchronous machines can be significantly simplified, enhancing our ability to simulate, control, and optimize their dynamic performance.

II.2- The Role of the Rotating Magnetic Field in Electrical Machines

The concept of the rotating magnetic field is fundamental to the operation of rotating machines, such as induction machines and synchronous machines. In both types of machines, the rotating field is generated by the stator's three-phase winding, which plays a crucial role in producing torque and enabling machine rotation. The stator design in both induction machines and synchronous machines follows the same principles, consisting of three identical coils placed 120 degrees apart. This symmetrical arrangement of coils, connected to a three-phase power supply, creates the rotating magnetic field. Regardless of whether it's an induction machine or a synchronous machine, the stator's construction is essentially identical in conception, allowing for the creation of this rotating magnetic field, which is the core mechanism driving the rotor. The rotor in the case of an induction machine rotates at a speed slightly less than the synchronous speed, while in a synchronous machine, the rotor locks in and rotates at the same speed as the magnetic field. However, despite these operational differences, the role of the stator and its ability to generate a consistent rotating magnetic field remains the same in both machines, highlighting its fundamental importance in electrical machine design.

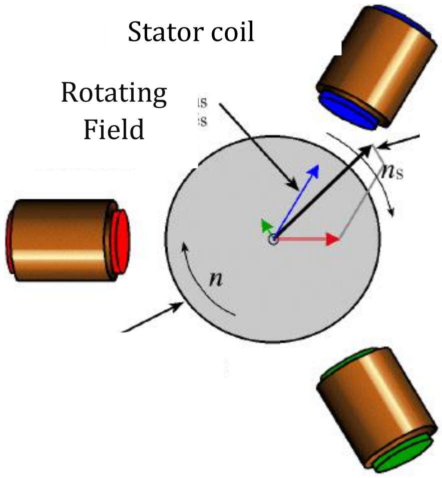


Figure II.1: Rotating Field in Induction and Synchronous Machines

- **Importance of the Rotating Magnetic Field**

1. **Torque Generation**: The interaction between the rotating magnetic field generated in the stator and the rotor currents (in induction machines) or the rotor's magnetic field (in synchronous machines) produces the necessary torque to drive mechanical loads. Without this rotating field, the machine would not be able to function as a motor or generator.
2. **Synchronization**: In synchronous machines, the rotor rotates at the same speed as the magnetic field generated by the stator. This synchronization ensures stable operation, making these machines suitable for applications requiring constant speed under varying loads.
3. **Slip in Induction Machines**: In induction machines, the rotor lags behind the rotating field by a factor known as slip. This difference in speed induces current in the rotor, which interacts with the rotating field to produce torque. The rotating field is, therefore, critical for the induction process and torque production in these machines.

- **Connection to Clarke and Park Transformations**

The mathematical modeling of rotating fields can be complex, especially in dynamic regimes where the machine's operating conditions are changing. This is where the Clarke and Park transformations come into play.

- **Clarke Transformation**: This transformation simplifies the analysis of three-phase systems by converting the three-phase stator currents into a two-axis system (α-β frame), representing the stationary components of the rotating magnetic field. This transformation is particularly useful for steady-state analysis, where the field rotates at a constant speed.
- **Park Transformation**: The Park transformation further simplifies the model by transforming the rotating components into a rotating reference frame (d-q frame) that moves with the rotor. By doing so, the rotating magnetic field becomes constant in this frame, making it easier to analyze dynamic behavior and control the machine, especially during transients or changes in load.

II.3- Clark Concordia Transformation

The purpose of using this transformation is to convert a three-phase ***abc*** system into a two-phase ***α β*** system, simplifying the analysis and control of electrical machines and power systems. Two primary transformations are commonly used for this purpose: the Clarke transformation and the Concordia transformation. The Clarke transformation focuses on preserving the amplitude of the variables during the conversion, making it useful for certain types of analysis. However, it does not preserve the power or torque directly, requiring a multiplication by a factor of 3/2 to account for the correct power representation. On the other hand, the Concordia transformation, which is standardized and widely used in control applications, ensures that power is preserved during the transformation process. While it provides an accurate representation of power, the amplitudes of the transformed variables are not maintained in this case. Both transformations serve distinct purposes depending on whether the preservation of amplitude or power is more critical to the specific application at hand. By reducing the complexity of the three-phase system into two components, these transformations make it easier to analyze dynamic behaviors, especially in electrical machine control and power system simulations

Edith Clarke proposed the transformation in 1951. Let a, b, and c be the initial coordinates of a three-phase system. α, β, and o are the destination coordinates:

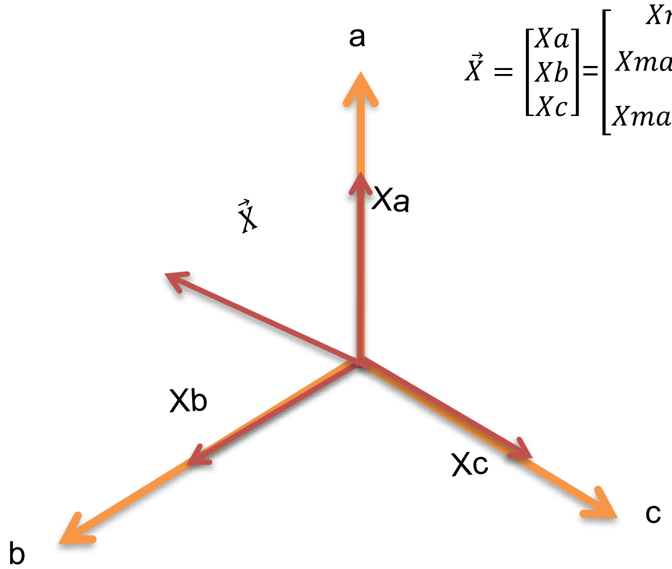
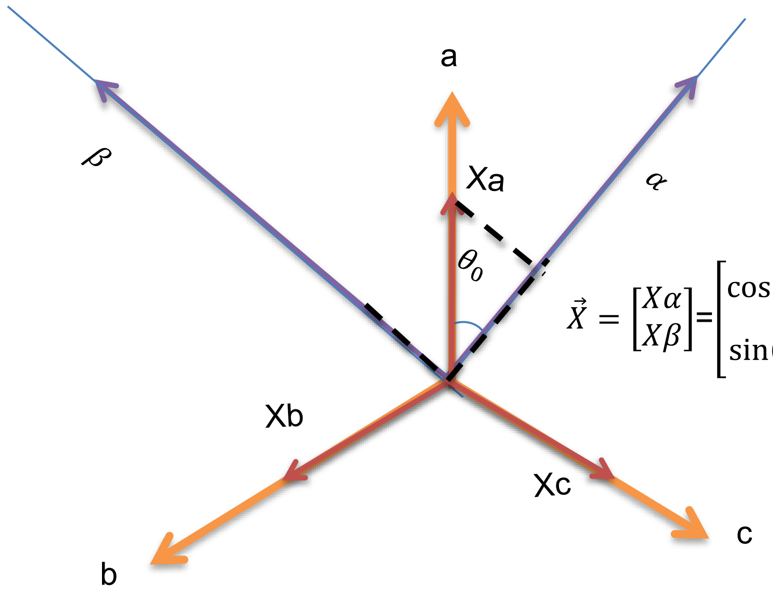


Figure II.2: The image of vector x on a b c

$$\vec{X} = \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} = \begin{bmatrix} Xmax * \sin(\omega t) \\ Xmax * \sin(\omega t - \frac{2\pi}{3}) \\ Xmax * \sin(\omega t + \frac{2\pi}{3}) \end{bmatrix} \quad \text{....II.1}$$

To understand this transformation for a three-phase system, let's assume that we have a vector $\vec{X}$, this vector rotated at a speed ωt. We can represent it in the fixed coordinates a, b, c as Xa, Xb, and Xc, as in equation 1.

The Clark transformation is the image of the vector $\vec{X}$ = $[Xa\ Xb\ Xc]^T$ onto the fixed reference axes α, β. So we have:



$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \end{bmatrix} = \begin{bmatrix} \cos(\theta_0) & \cos\left(\theta_0 - \frac{2\pi}{3}\right) & \cos\left(\theta_0 + \frac{2\pi}{3}\right) \\ \sin(\theta_0) & \sin\left(\theta_0 - \frac{2\pi}{3}\right) & \sin\left(\theta_0 + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \text{...II.2}$$

Figure II.3: The image of vector x on α, β

We assume that θ_0=0, and the matrix will be simpler as follows

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \end{bmatrix} = \begin{bmatrix} \cos(0) & \cos\left(-\frac{2\pi}{3}\right) & \cos\left(\frac{2\pi}{3}\right) \\ \sin(\theta_0) & \sin\left(-\frac{2\pi}{3}\right) & \sin\left(\frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad ...II.3$$

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \text{...II.4}$$

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \\ X0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots.\text{II.5}$$

Thanks to Concordia, the Clark Matrix will be a square matrix with det[X]=1 to have the inverse of the matrix equal to the transmission matrix:

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \\ X0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots.\text{II.6}$$

II.4- Park Transformation

The difference between the Park transformation and Clark Concordia lies in the Park coordinates having rotational axes (see Figure 4), unlike the Clark Concordia transformation. The Park transformation, often confused with the dqo transformation, is a mathematical tool used in electrical engineering, particularly for vector control, to model a three-phase system with a two-phase model. It involves a change of coordinates. The first two axes in the new base are traditionally named d, q. The transformed variables are typically currents, voltages, or fluxes.

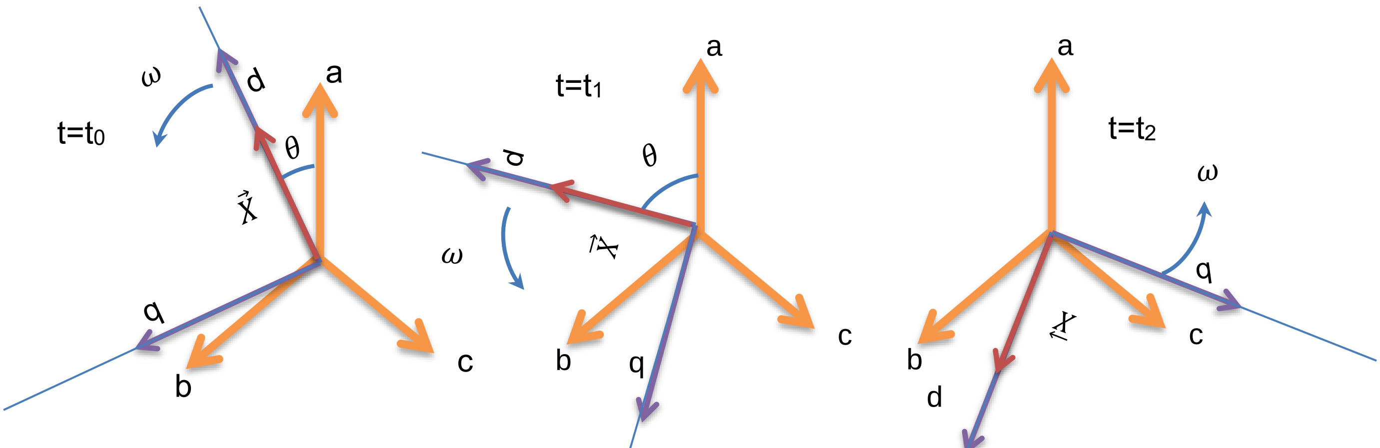


Figure II.4: The image of vector x on the d q axes at different moments.

According to the Park transformation, we can write:

$$\begin{bmatrix} Xd \\ Xq \\ X0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots\text{II.7}$$

So, the Park matrix is:

$$[P(\theta)] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \ldots \text{II}.8$$

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = P(\theta) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \ldots.\text{II}.9$$

II.5- Application of the Park Transformation to the Three-Phase Stator

In this application, we employ the Park transformation on the three-phase stator because it is a widely shared approach between both induction machines (IM) and synchronous machines (SM). This transformation is especially valuable for modeling electrical machines as it converts the three-phase abc system into the dq rotating reference frame, aligning with the rotor flux. This simplifies the control and analysis of these machines by decoupling the torque and flux components, which is crucial for advanced control techniques like vector control. Since both IM and SM rely on similar principles of the rotating magnetic field in their stators, the Park transformation proves to be an indispensable tool for representing machine dynamics in a more manageable form. Its versatility makes it applicable across different types of machines, enhancing its relevance in various modeling and control scenarios

For all the stator phases:

$$\begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{sa} \\ \varphi_{sb} \\ \varphi_{sc} \end{bmatrix} \ldots \text{II}.10$$

The stator resistance being the same for the three phases, there is no need to write a resistance matrix.

Where :

$$[V] = [R][I] + \frac{d}{dt}[\varphi] \ldots \text{II}.11$$

To eliminate θ from the matrix [Lss] and to enable control algorithms to handle continuous electrical quantities, the stator windings (a, b, c) are replaced by two windings (d, q) in quadrature. The conversion of electrical quantities from the stator (a, b, c) to the electrical quantities (d, q), ensuring the conversion of the mmf (magnetomotive force) and instantaneous power, is achieved through the Park transformation.

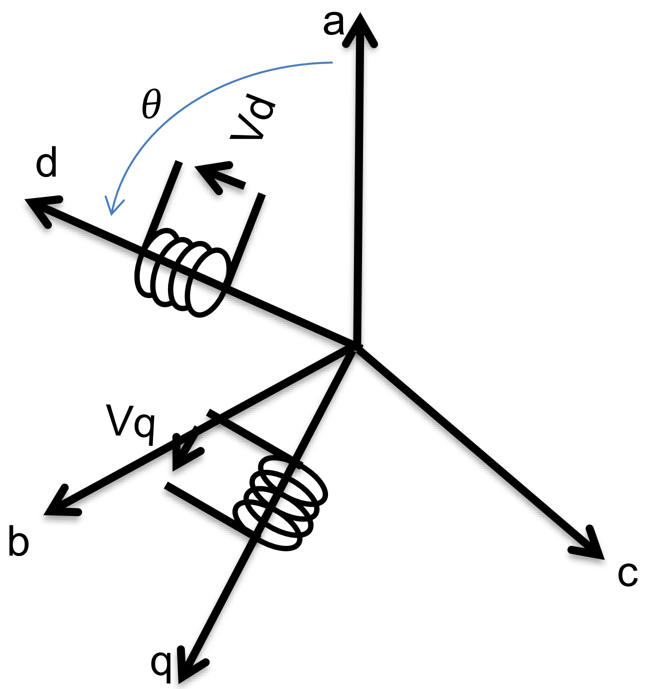


Figure II.5: Equivalent Diagram of the Synchronous Machine in the Park reference frame.

The passage matrix, denoted as equation 16,

So, the matrix $[P(\theta)]^{-1}$ is given by:

$$[P(\theta)]^{-1} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta & -\sin\theta & \frac{1}{\sqrt{2}} \\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\theta-\frac{4\pi}{3}) & -\sin(\theta-\frac{4\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix} \quad \ldots\text{II.12}$$

The transition from the three-phase system to the (d, q) system linked to the rotor is done using the following relationship:

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = [P(\theta)]\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \ldots \text{II.13}$$

Applying the transformation to the system:

$$[V] = [R][I] + \frac{d}{dt}[\varphi] \ldots \text{II. 14}$$

$$[P]^{-1}[V_{dq0}] = [R][P]^{-1}[I_{dq0}] + \frac{d}{dt}([P]^{-1}[\varphi_{dqo}])$$

Then, by multiplying by $[P(\theta)]^{-1}$:

$$[V_{dq0}] = [R][I_{dqo}] + \frac{d}{dt}[\varphi_{dqo}] + [P](\frac{d}{dt}[P]^{-1})[\varphi_{dqo}] \quad \text{....II.21}$$

For the demonstration of the model, we can use :

$$[P](\frac{d}{dt}[P]^{-1}) = \frac{d\theta}{dt}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{....II.22}$$

II.6- Dynamic Park Transformation and Flux Orientation in Field-Oriented Control(FOC):

The dynamic reference of Park transformation can be considered based on the speed of the d-q axes. When the reference frame is synchronized with the rotating magnetic field, we can observe the behavior of the flux components ϕdr and ϕqr . In this case, the orientation of the flux towards the d-axis is crucial for Field-Oriented Control (FOC), as it allows the decoupling of torque and flux control, and eliminates the ϕqr or ϕdr flux component.

- **Figure 6** shows the flux vector $\overrightarrow{\varphi_s}$ on the d-q axes at different time instances, demonstrating the natural variations in flux over time.
- **Figure 7** presents the flux vector $\overrightarrow{\varphi_s}$ after being oriented towards the d-axis, illustrating how FOC achieves this alignment.
- **Figure 8** depicts the rotor flux vector $\overrightarrow{\varphi_r}$ after orientation towards the d-axis, ensuring optimal torque control by aligning the rotor flux with the stator field.

This method simplifies the control of AC motors by focusing on flux alignment, a key principle in FOC.

The choice of Park model reference frame allows us to determine the speed of the d q axes and for the frame fixed relative to the rotating field, the magnitude and argument of φ_s will be constant, consequently φ_{ds} and φ_{qs} the image of the rotating field on the d q axes will also be constant.

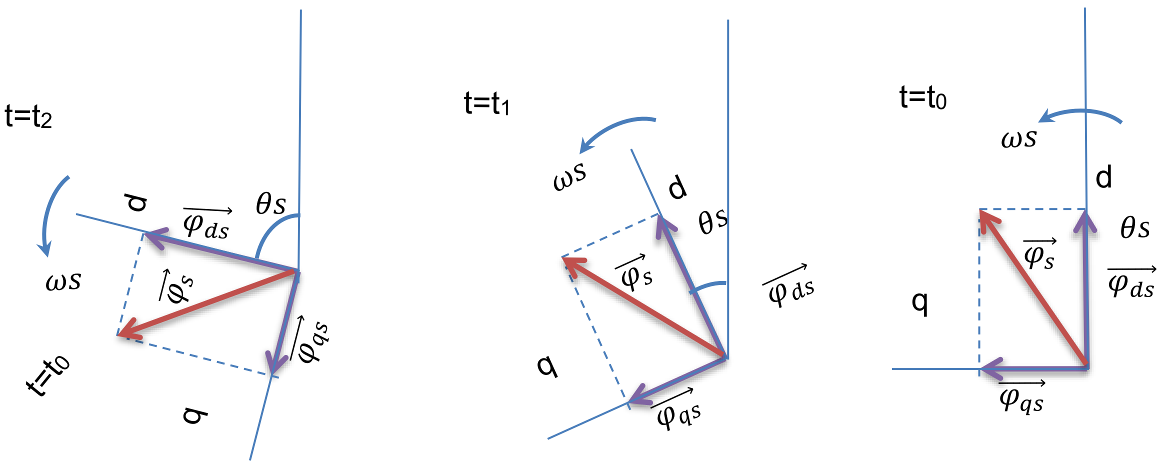


Fig.6- The flux image $\overrightarrow{\varphi_s}$ vector on the d q axes at different moments

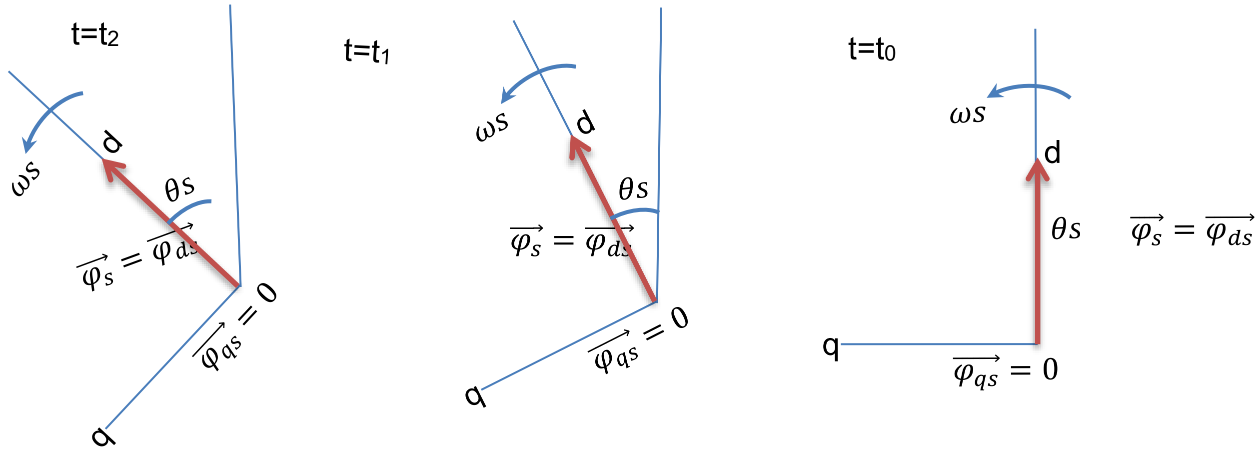


Fig.7- The flux image $\overrightarrow{\varphi_s}$ vector on the d q axes after flux orientation towards the d axis

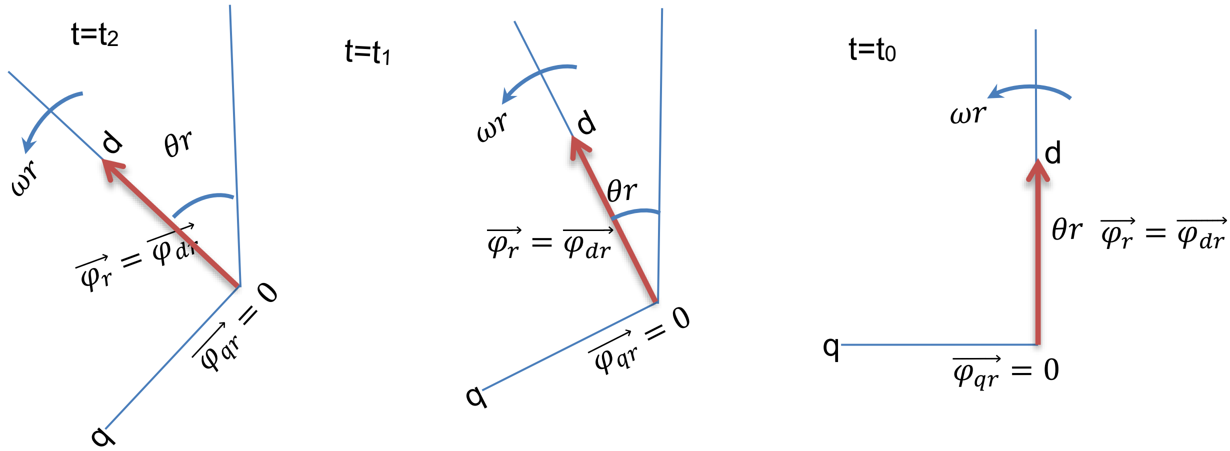


Fig.8- The flux image $\overrightarrow{\varphi_r}$ vector on the d q axes after flux orientation towards the d axis

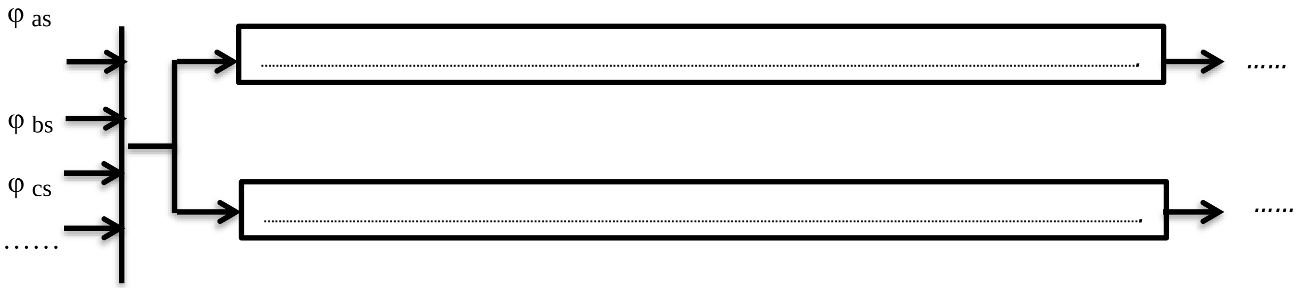
II.7- Application Example

The stator fluxes in steady state of an IM (asynchronous machine) presented by the equations:

$$\begin{cases} \varphi_{as}(t) = 1.5 * \sin(2\pi 50t) \\ \varphi_{bs}(t) = 1.5 * \sin(2\pi 50t - 2\pi/3) \\ \varphi_{cs}(t) = 1.5 * \sin(2\pi 50t + 2\pi/3) \end{cases}$$

1- Find the formulas of the equations $\varphi_d(t)$ et $\varphi_q(t)$ by the Park transformation for φ_{as}, φ_{bs} and φ_{cs} where the speed of the d and q axes equal to the rotating field speed $\overrightarrow{\varphi_s}$ which represents $[\varphi_{as}\ \varphi_{bs}\ \varphi_{cs}]^T$

Complete the simulation diagram of the Park transformation under Simulink according to the following diagram



2- Find the formulas of the equations $\varphi\alpha(t)$ et $\varphi\beta(t)$ by the **Clark Concordia** transformation for φ_{as}, φ_{bs} and φ_{cs}

3- Determine the module of the rotating field vector then its positions for the times t= 0.001 s, 0.005s and 0.01s.

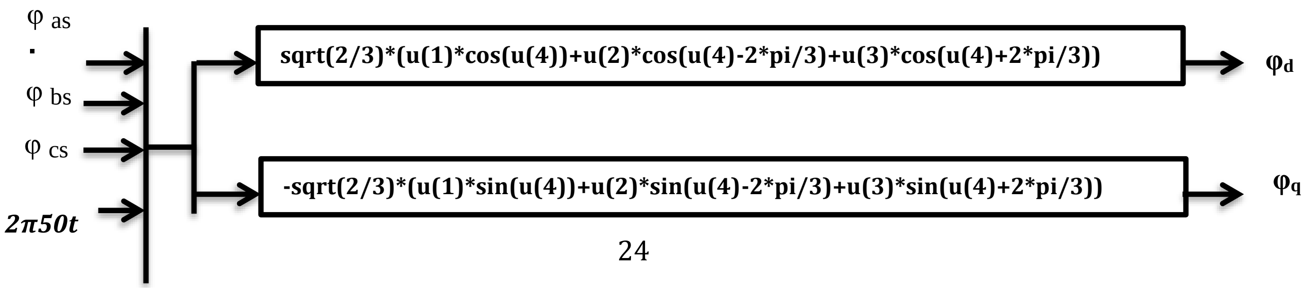
Solution

1- Find the formulas of the equations $\varphi_d(t)$ et $\varphi_q(t)$ by the Park transformation for φ_{as}, φ_{bs} and φ_{cs} where the speed of the d and q axes equal to the rotating field speed $\overrightarrow{\varphi_s}$ which represents $[\varphi_{as}\ \varphi_{bs}\ \varphi_{cs}]^T$

$\varphi_d(t)=\sqrt{\frac{2}{3}}\left(1.5 * \sin(2\pi 50t) * \cos(2\pi 50t) + 1.5 * \sin\left(2\pi 50t - \frac{2\pi}{3}\right) * \cos\left(2\pi 50t - \frac{2\pi}{3}\right) + 1.5 * \sin(2\pi 50t + 2\pi/3) * \cos(2\pi 50t + 2\pi/3)\right)$

$\varphi_q(t)=-\sqrt{\frac{2}{3}}\left(1.5 * \sin(2\pi 50t) * \sin(2\pi 50t) + 1.5 * \sin\left(2\pi 50t - \frac{2\pi}{3}\right) * \sin\left(2\pi 50t - \frac{2\pi}{3}\right) + 1.5 * \sin(2\pi 50t + 2\pi/3) * \sin(2\pi 50t + 2\pi/3)\right)$

Complete the simulation diagram of the Park transformation under Simulink according to the following diagram:



2- Find the formulas of the equations φα(t) et φβ(t) by the **Clark Concordia** transformation for φ_{as}, φ_{bs} and φ_{cs}

φα (t)= $\sqrt{\frac{2}{3}}(1.5 * \sin(2\pi 50t) - \frac{1}{2} * 1.5 * \sin\left(2\pi 50t - \frac{2\pi}{3}\right) - \frac{1}{2} * 1.5 * \sin(2\pi 50t + 2\pi/3))$

φβ (t)= $\sqrt{\frac{2}{3}}(-\frac{\sqrt{3}}{2} * 1.5 * \sin\left(2\pi 50t - \frac{2\pi}{3}\right) * \sin\left(2\pi 50t - \frac{2\pi}{3}\right) + \frac{\sqrt{3}}{2} * 1.5 * \sin(2\pi 50t + 2\pi/3) * \sin(2\pi 50t + 2\pi/3))$

3- Determine the module of the rotating field vector then its positions for the times t= 0.001 s, 0.005s and 0.01s..

Time	Arg($\varphi_{\alpha s}, \varphi_{\beta s}$)	Module($\varphi_{\alpha s}, \varphi_{\beta s}$)
t=0.001	-72°	1.84
t=0.005	0°	1.84
t=0.01	90°	1.84

II.8- Conclusion

In conclusion, the course on Modeling of Machines for Dynamic Regimes has provided a comprehensive understanding of the fundamental transformations, particularly Clarke and Park transformations, which are essential tools for simplifying the analysis and control of electrical machines. By applying these transformations to the stator of Induction Machines (IM) and Synchronous Machines (SM), we can convert three-phase systems into two-phase equivalents, facilitating the analysis in a stationary or rotating reference frame.

The Park transformation, in particular, plays a critical role in achieving a decoupled model of the machine, which is central to Field-Oriented Control (FOC). This decoupling allows for independent control of torque and flux, enabling precise and efficient control of machine dynamics. Ultimately, the application of these transformations not only simplifies the mathematical representation of machine behavior but also enhances the performance and control of machines in dynamic regimes, making them indispensable in modern control strategies.

Chapter III: Modeling and Simulation of Direct Current Machines (DCM)

III.1- Introduction

A direct current machine is an electrical machine: an electromechanical converter that allows bidirectional energy conversion between an electrical installation supplied by direct current and a mechanical device. In motor operation, electrical energy is transformed into mechanical energy. In generator operation, mechanical energy is transformed into electrical energy. In this course, we will cover the construction, operating principle, modeling, and simulation. Then, the DC machine model in the dq reference frame.

III.2- Construction of DC Machine

The direct current machine consists of the following main parts:

- A stationary part called STATOR, which functions as the inductor.
- A moving part called ROTOR, which functions as the armature.
- A connection between the rotor and external components of the machine called the COLLECTOR."

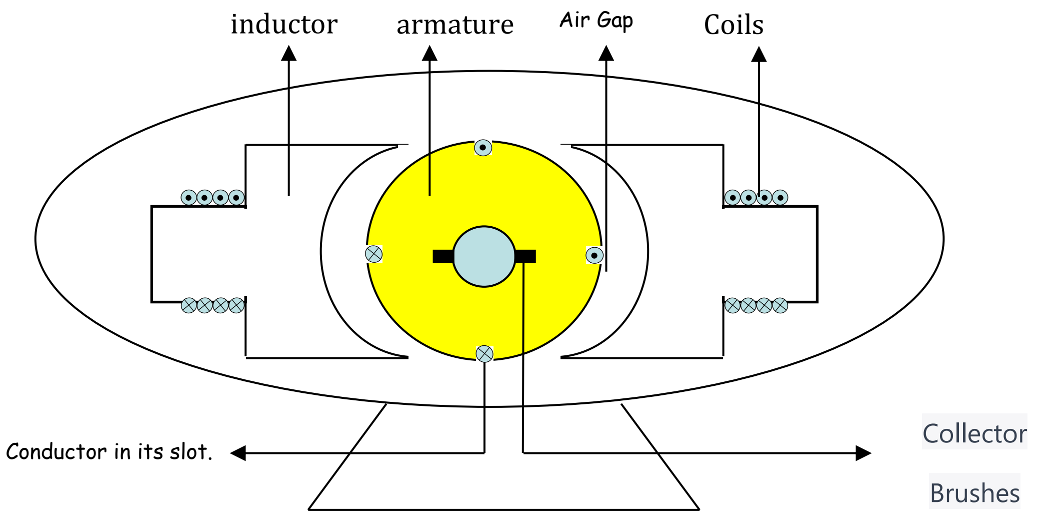


Figure III.1: Construction of the Direct Current Motor

- **The inductor :**

It is formed either by permanent ferrite magnets or coils placed around the polar cores. When the coils carry a direct current, they create a magnetic field in the magnetic circuit of the machine, especially in the air gap, the space separating the stationary and moving parts, where the conductors are located.

- The armature: The armature core is made of iron to channel the field lines, and the conductors are housed in slots on the rotor, with two conductors forming a coil.

- Collector and brushes: The collector is a set of insulated copper segments arranged at the end of the rotor, and the brushes, carried by the stator, make contact with the collector.

III.3- Operating Principle

A direct current machine has N active conductors, the useful flux under a pole created by the inductor is Φ, expressed in webers, and n represents the rotational speed of the rotor shaft, in revolutions per second.

Two cases may occur:

- If a conductor is both carrying an electric current and immersed in a magnetic field, it is then subjected to an electromagnetic force.
- If a conductor is both in rotational motion and immersed in a magnetic field, it generates an electromotive force.

These two cases can be described by the following diagram:

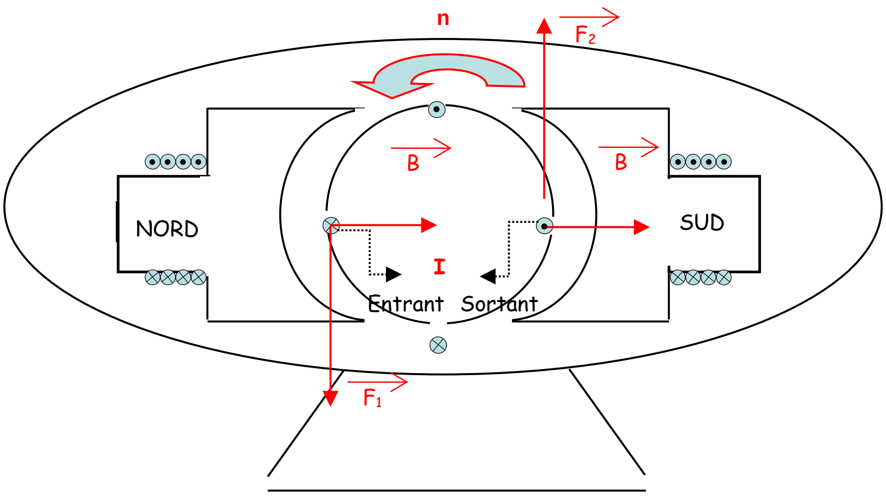


Figure III.2: Operating Principle of the DC Motor

Current + Magnetic Field → Electromagnetic Force

Force + Magnetic Field → Electromotive Force

The active conductors, numbering N, cut through the lines of the magnetic field; therefore, they are subject to induced electromotive forces. The resultant electromotive force (EMF) of all these N coils:

$$E = N.n.\Phi$$

E	(Electromotive Force) EMF in volts [V]
n	(Rotational frequency in revolutions per second) [rev/s]
Φ	(Flux in webers) [Wb]
N	N (Number of active conductors)

This relationship is crucial for the machine because it connects the magnetic flux Φ, an electromagnetic quantity, the voltage E, an electrical quantity, and the rotational frequency n, a mechanical quantity.

Given that $\Omega = 2\pi * n$, another relationship involving these three types of quantities is frequently used, taking into account the angular velocity Ω expressed in radians per second:

$$E = K.\Phi.\Omega$$

E	(Electromotive Force) EMF in volts [V]
Ω	(Angular velocity in radians per second) [rad/s]
Φ	(Flux in webers) [Wb]
K	(Constant)

III.4- Dynamic Model of the Direct Current Motor on the d-q Axes

To study and express the dynamic equation, we make the following simplifying assumptions: The following simplifying assumptions must be adopted:

- ✓ Constant air gap.
- ✓ Neglect of slot effect.
- ✓ Unsaturated magnetic circuit with constant permeability.
- ✓ Negligible iron losses.
- ✓ The influence of skin effect and heating on characteristics is not considered.
- ✓ Negligible armature reaction.

4.1- Modeling and Simulation of Separately Excited DCM

n a separately excited direct current motor, the armature winding and the field winding each have separate sources of direct current.

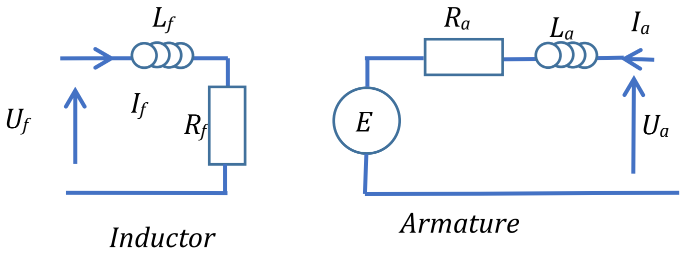


Figure III.3: Electrical Diagram of Separately Excited DCM

The dynamic model represents both transient and steady-state operation. During the transient regime, the speed, torque, and current are variable and closely related to the motor's parameters (resistance, inductance, inertia, friction, flux, and supply voltage). Figure 4 shows the armature windings and separate excitation.

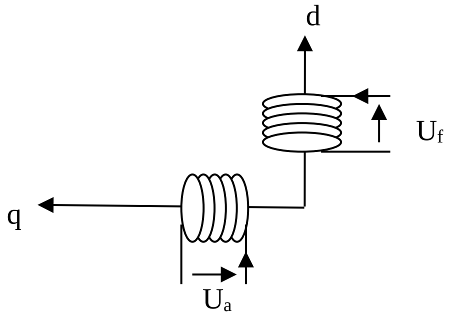


Figure III.4: Simplified Model of the DCM on the d and q Axes

Ua: The voltage of the armature winding;

Uf: The voltage of the separate excitation winding;

In the context of these assumptions and for a balanced machine, the machine's equations are written in the following form:

Electrical Model

$$\begin{cases} U_f = R_f . I_f + \frac{d\phi_f}{dt} \\ U_a = R_a . I_a + \frac{d\phi_a}{dt} + e_a \end{cases} \quad \ldots\text{III.1} \qquad e_a = \omega . K . \phi_f = \omega . Mfd . I_f \quad \ldots\text{III.2}$$

Flux Equations

$$\begin{cases} \phi_f = L_f . I_f \\ \phi_a = L_a . I_a \end{cases} \quad \ldots\text{III.3}$$

The mechanical equation is written as:

$$C_e = \frac{dW_{méc}}{d\theta} = K . \phi_f . I_a \qquad \ldots\text{III.4}$$

et $J \frac{d}{dt}(\Omega) = C_e - C_r - F\omega_m$ ...III.5

The equations in matrix form:

$$\begin{bmatrix} U_f \\ U_a \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_a \end{bmatrix} \begin{bmatrix} I_f \\ I_a \end{bmatrix} + \begin{bmatrix} L_f & 0 \\ 0 & L_a \end{bmatrix} \begin{bmatrix} \dot{I_f} \\ \dot{I_a} \end{bmatrix} + \omega \begin{bmatrix} 0 & 0 \\ K . L_f & 0 \end{bmatrix} \begin{bmatrix} I_f \\ I_a \end{bmatrix} \qquad \ldots\text{III.6}$$

$$U = \begin{bmatrix} U_f \\ U_a \end{bmatrix}, R = \begin{bmatrix} R_f & 0 \\ 0 & R_a \end{bmatrix}, I = \begin{bmatrix} I_f \\ I_a \end{bmatrix}, L = \begin{bmatrix} L_f & 0 \\ 0 & L_a \end{bmatrix}, M = \begin{bmatrix} 0 & 0 \\ Mfd & 0 \end{bmatrix} \qquad \ldots\text{III.7}$$

- Uf: Field voltage applied to the field winding.
- Ua: Armature voltage applied to the armature winding.
- Ra: Armature resistance.
- Ia: Armature current.
- ω: Rotational speed (rad/s).
- Φ: Useful flux per pole (webers).
- Ce: Electromagnetic torque.
- k: Construction characteristic constant.
- ea: Back electromotive force of the motor.
- Cr: Resisting torque.
- J: Moment of inertia.
- f: Friction coefficient.

And it seems like you're referring to a block diagram in SIMULINK for this model.

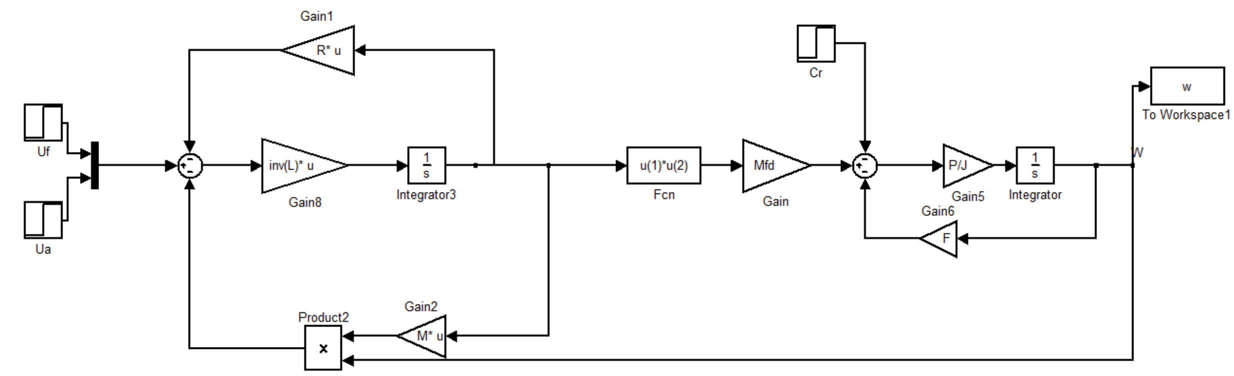


Figure III.5: Functional Diagram in Simulink of the DCM Motor

4.2- The Effect of Armature Reaction in DCM

The current flowing in the armature creates an undesirable flux, affecting the total load flux. The flux lines without the effect of armature reaction, as shown in Figure 6, are always perpendicular to the armature, maximizing the torque generated by the Lorentz force. On the other hand, when the motor is under load, the armature current becomes significant, and as a result, the armature magnetic flux affects the field flux lines and they are no longer perpendicular, as shown in Figure 7.

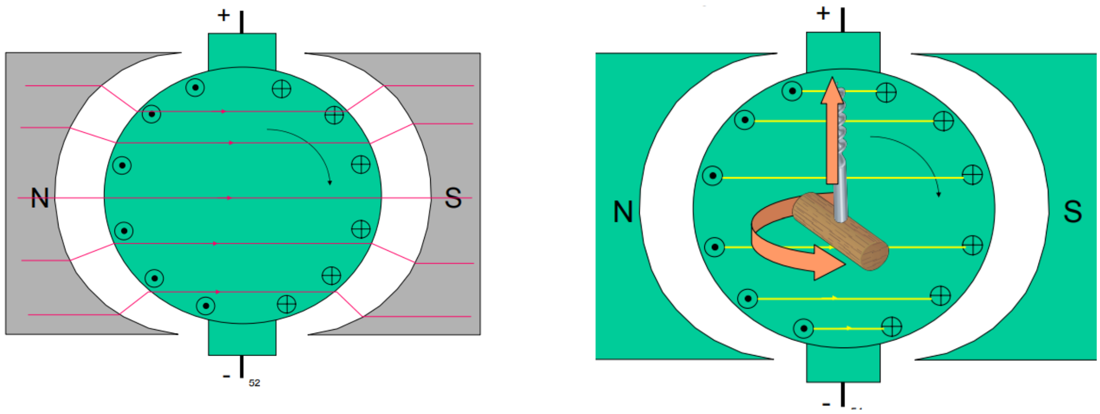


Figure III.6: Field Flux Lines without Armature Reaction Effect

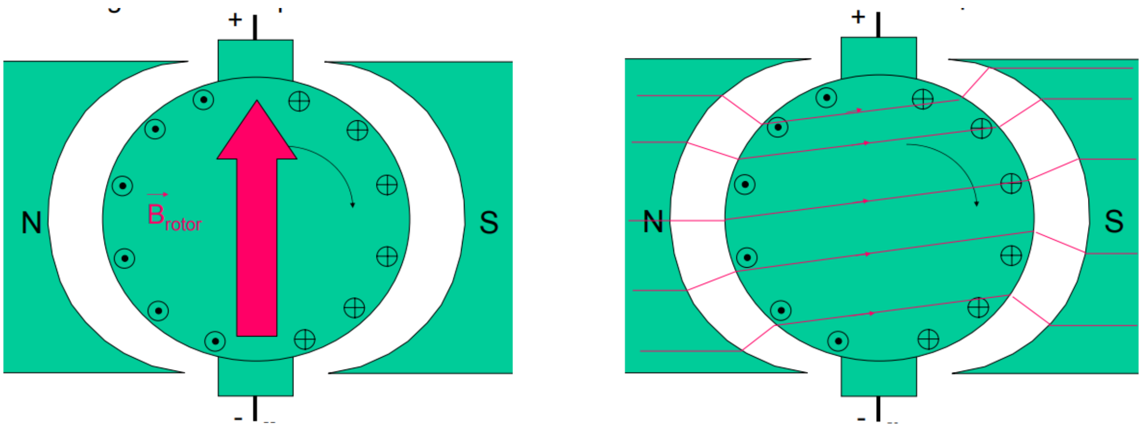


Figure III.7: Armature Magnetic Reaction and Its Effect on Field Flux Lines

To eliminate armature magnetic reaction, there are three methods in the literature as follows:

- ✓ **Dynamically adjusting the position of the brushes**

This compensation is even more necessary as it leads to a displacement of the neutral line, requiring the brushes to be adjusted to capture the maximum voltage (which also helps prevent excessive voltage between adjacent collector segments).

- ✓ **Auxiliary Commutation Poles**

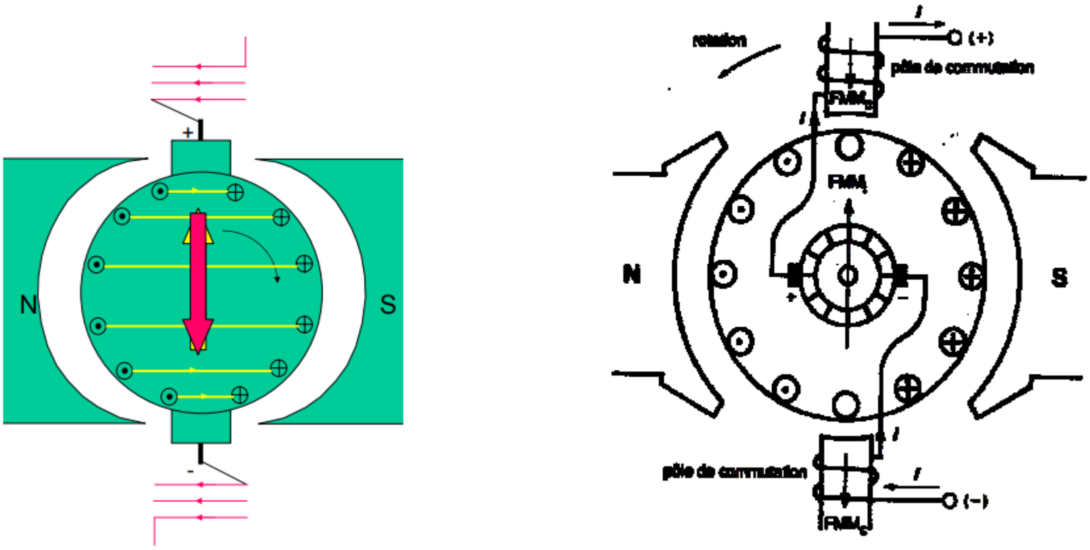


Figure III.8: Auxiliary Poles to Reduce the Armature Magnetic Reaction Effect

The auxiliary or commutation poles are placed in the interpole axis. Their windings carry the current I supplied or drawn by the armature. The induction in the air gap under the auxiliary poles must have a value such that it generates the commutation voltage in the commutation section.

- ✓ **DCM Compensation Windings**

When it is not possible to dynamically adjust the position of the brushes, the brushes are set in a static operation (without a load), and compensation windings are used to vary the field position based on the load. These compensation windings are located precisely opposite to the armature windings and carry exactly the same current. These compensation windings are connected in series and counteract the magnetic field's shifting.

Compensation windings are integrated into the main poles in very large generators. They are not used in medium-power generators.

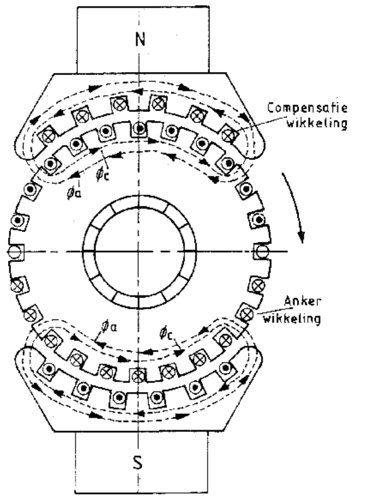


Figure III.9: Compensation Windings to Reduce the Armature Magnetic Reaction Effect

4.3- Separately Excited DCM Model with Auxiliary Poles and Compensation Windings

Figure III.10 shows the armature windings, auxiliary poles, compensation windings, and separate excitation on the d-q axes.

Ua: Voltage of the armature winding;

Uf: Voltage of the separate excitation winding;

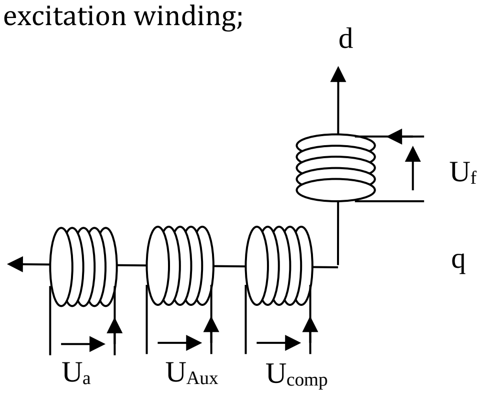


Figure III.10: Model of the DCM with Auxiliary Poles and Compensation Windings on the d and q Axes

In the context of these assumptions and for a balanced machine, the machine's equations are written in the following form:

Electrical Model

$$\begin{cases} U_d = U_f \\ U_{qa} = U_a + U_{Aux} + U_{comp} \\ I_{qa} = I_a = -I_{Aux} = -I_{comp} \end{cases} \quad \text{…III.8}$$

$$\begin{cases} U_f = R_f . I_f + \dfrac{d\phi_f}{dt} \\ U_{qa} = R_{qa} . I_a + \dfrac{d\phi_{qa}}{dt} + \omega . M_{fd} . I_f \end{cases} \quad \text{…III.9}$$

Flux Equations

$$\begin{cases} \phi_d = \phi_f = L_f.I_f \\ \phi_a = L_a.I_a + M_{a\,Aux} I_{Aux} + M_{a\,Comp} I_{Comp} = L_a.I_{qa} - M_{a\,Aux} I_{qa} - M_{a\,comp} I_{qa} \\ \phi_{Aux} = L_{Aux}.I_{Aux} + M_{a\,Aux} I_a + M_{Aux\,comp} I_{comp} = -L_{Aux}.I_{qa} + M_{a\,Aux} I_{qa} - M_{Aux\,comp} I_{qa} \\ \phi_{comp} = L_{comp}.I_{comp} + M_{a\,comp} I_a + M_{Aux\,comp} I_{Aux} = -L_{comp}.I_{qa} + M_{a\,comp} I_{qa} - M_{Aux\,comp} I_{qa} \\ \phi_{qa} = \phi_a + \phi_{Aux} + \phi_{comp} \end{cases} \quad \ldots\text{III.10}$$

So the fluxes on the d-q axes are:

$$\begin{cases} \phi_d = \phi_f = L_f.I_f \\ \phi_{qa} = I_{qa}(L_a - L_{Aux} - L_{comp} - 2.M_{Aux\,comp}) \end{cases} \quad \ldots\text{III.11}$$

So

$$L_{qa} = L_a - L_{Aux} - L_{comp} - 2.M_{Aux\,comp} \quad \ldots\text{III.12}$$

And the resistance on the q-axis.

$$R_{qa} = R_{qa} + R_{Aux} + R_{comp} \quad \ldots\text{III.13}$$

The mechanical equation is written as:

$$C_e = M_{fd}.I_a \quad \ldots\text{III.14}$$

And $J\frac{d}{dt}(\Omega) = C_e - C_r - F\omega_m$ ...III.15

The equations in matrix form;

$$\begin{bmatrix} U_f \\ U_a \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_{qa} \end{bmatrix} \begin{bmatrix} I_f \\ I_{qa} \end{bmatrix} + \begin{bmatrix} L_f & 0 \\ 0 & L_{qa} \end{bmatrix} \begin{bmatrix} \dot{I_f} \\ \dot{I_a} \end{bmatrix} + \omega \begin{bmatrix} 0 & 0 \\ K.L_f & 0 \end{bmatrix} \begin{bmatrix} I_f \\ I_a \end{bmatrix} \quad \ldots\text{III.16}$$

$$U = \begin{bmatrix} U_f \\ U_a \end{bmatrix}, R = \begin{bmatrix} R_f & 0 \\ 0 & R_{qa} \end{bmatrix}, I = \begin{bmatrix} I_f \\ I_a \end{bmatrix}, L = \begin{bmatrix} L_f & 0 \\ 0 & L_{qa} \end{bmatrix}, M = \begin{bmatrix} 0 & 0 \\ M_{fd} & 0 \end{bmatrix} \quad \ldots\text{III.17}$$

4.4- Series-Wound DCM

The field winding of this motor is in series with the armature, so the armature current is equal to the field current.

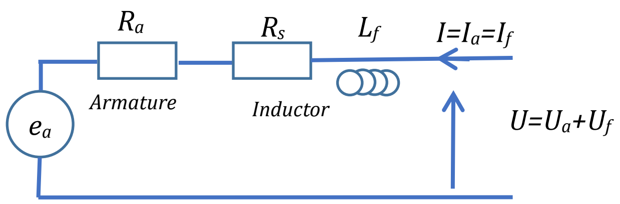


Figure III.11: Equivalent Electrical Diagram of Separately Excited DCM

So in transient regime, we have::

$$U = (R_s + R_a).I + e_a + \frac{d(L_s + L_a)}{dt} \qquad \text{III.18}$$

The equation for back electromotive force is written as:

$$e_a = K.\phi_f.\omega \ = K.L_s.I.\omega = M_{sd}.I.\omega \qquad \text{III.19}$$

When considering that $K.L_s$=M_{sd}, equation (19) will be:

$$e_a = M_{sd}.I.\omega \qquad \text{III.20}$$

By substituting equation (20) into equation (18), we find:

$$U = (R_s + R_a).I + M_{sd}.I.\omega + \frac{d(L_s + L_a)}{dt} \qquad \text{III.21}$$

And the mechanical model is written as:

$$Ce = K_s.I^2$$
$$J\frac{d}{dt}(\omega_m) = C_e - C_r - F\omega \qquad \text{III.22}$$

4.5- DCM with Constant Excitation

Today, direct current motors primarily use permanent magnets to generate the field flux, and the direct current supply is applied only to the armature winding.
In this section, the DCM model is the simplest, and the equivalent electrical diagram of this model can be presented in Figure (11) below:

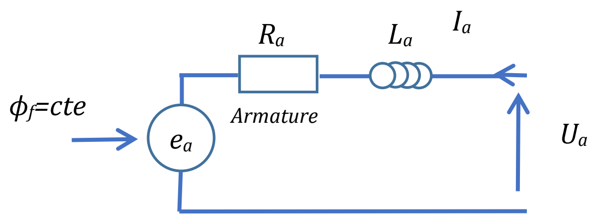


Figure III.12: Equivalent Electrical Diagram of DCM with Constant Excitation

So, the equation for the back electromotive force remains as follows:

$$e_a = \omega.K.\phi_f \ \text{ and } K.\phi_f = cte = \ K' \ \text{ so } \ e_a = K'.\omega \qquad \text{III.23}$$

In the transient regime, the electrical model is as follows:

$$U_a = R_a.I_a + \frac{d\phi_a}{dt} + e_a \qquad \text{III.24}$$

The flux equation:

$$\phi_a = L_a.I_a \qquad \text{III.25}$$

By substituting the flux equation and the back electromotive force equation into the electrical model, we find:

$$U_a = R_a.I_a + L_a\frac{dI_a}{dt} + \omega.K' \qquad \text{III.26}$$

And the mechanical model is written as:

$$C_e = K.\phi_f.I_a \;\; and$$

$$K.\phi_f = cte = \; K' \;\; \Rightarrow C_e = K'.I_a \qquad \text{III.27}$$

$$J\frac{d}{dt}(\omega_m) = T_e - TL - F\omega \qquad \text{III.28}$$

Now we will examine the transient response of reaching the final speed by the applied armature voltage for a direct current motor. A simple way to examine the transient response is to use the system's transfer function.

We can obtain the transfer function for the direct current motor drive system by taking the Laplace transform of the differential equations as follows:
By equation 27

$$U_a = R_a.I_a + L_a\frac{dI_a}{dt} + \omega\,.K' \Rightarrow \; U_a = I_a(R_a + L_a.s) + \omega\,.K' \qquad \text{III.29}$$

And by equation 18

$$J\frac{d}{dt}(\omega_m) = T_e - TL - F\omega_m \Rightarrow T_e = \omega_m(s.J + F) - \; T_L = K'.I_a \qquad \text{III.30}$$

So, with the Laplace operator and equations 29 and 30, they can be represented by the closed-loop functional diagram as illustrated in Figure 13.

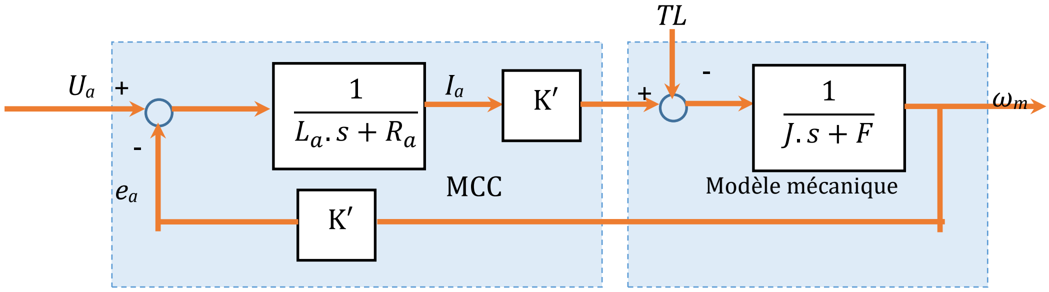


Figure III.13: Functional Block Diagram of the Constant-Flux DCM

According to the functional diagram in Figure 12, the transfer function from the input armature voltage Ua to the output angular velocity ω_m is given by:

$$\frac{\omega_m}{U_a} = \frac{\frac{1}{L_a.s + R_a}.K'.\frac{1}{J.s + B}}{1 + \left(\frac{1}{L_a.s + R_a}.K'.\frac{1}{J.s + B}.K'\right)} \qquad \text{III.31}$$

$$\frac{\omega_m}{U_a} = \frac{\frac{K'}{J.L_a}}{s^2 + \left(\frac{R_a}{L_a} + \frac{B}{J}\right).s + \left(\frac{R_a}{L_a}.\frac{B}{J} + \frac{K'^2}{J.L_a}.\right)} \qquad \text{III.32}$$

This transfer function is a second-order system with two poles, which are important factors in determining the transient response of the system.

III.5- State Model of a Direct Current DCM

In this section, we consider a direct current machine with constant excitation $\phi_f = Cte$, on the one hand, and zero load Cr = 0;

Therefore, the model will be simpler as follows:

$\phi_f = \mathrm{Cte} \Rightarrow \mathrm{K}.\phi_f = \mathrm{Cte} = \mathrm{K_m} \Rightarrow \mathrm{E} = \mathrm{K_m}.\omega$ …III.33

The electrical equations (I.1, I.2) are written as:

$$U_a = R_a.I_a + L_a\frac{dI_a}{dt} + \omega\,.K_m \qquad \text{…III.34}$$

Continuing with the said proposition, and using equations 27 and 28, the mechanical equation becomes:

$$J\frac{d\omega}{dt} = T_e - F\omega \qquad \text{…III.35}$$

$$T_e = K_m.I_a \qquad \text{…III.36}$$

To present the state model of the DCM, we assume that:

Input: Ua, Output: y(t) = ω

State vector: : $x_1(t)= I_a$, $x_2(t)= \omega$, $x(t)=[\, x_1(t)\ x_2(t)]^T$

We use equations I.21, I.22, I.23, and I.24 to determine the state model of the DCM, thus:

So :

III.34 $\rightarrow \frac{dI_a}{dt} = -\frac{R_a}{L_a}.I_a - \frac{K_m}{L_a}.\omega + \frac{1}{L_a}U_a$ …III.37

III.35 $\rightarrow \frac{d\omega}{dt} = \frac{K_m}{J}.I_a - \frac{F}{J}\omega$

The equations in matrix form;

$$\begin{cases} \begin{bmatrix} \dot{I}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_m}{L_a} \\ \frac{K_m}{J} & -\frac{F}{J} \end{bmatrix} \begin{bmatrix} I_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_a \\ 0 \end{bmatrix} \\ y(t) = [0 \quad 1] \begin{bmatrix} I_a \\ \omega \end{bmatrix} \end{cases} \qquad \text{…III.38}$$

$X(t)=\begin{bmatrix} I_a \\ \omega \end{bmatrix}$, $A=\begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_m}{L_a} \\ \frac{K_m}{J} & -\frac{F}{J} \end{bmatrix}$, $B=\begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & 0 \end{bmatrix}$, $C=[0 \quad 1]$

The block diagram for this model in SIMULINK

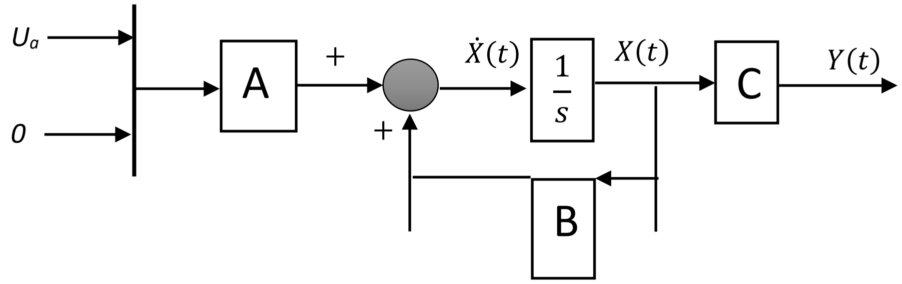


Figure III.14: Simulation of the DC Motor

III.6- Conclusion

The modeling and simulation of the direct current machine have been presented in this course. Initially, the construction and operating principles of the DCM were introduced. Subsequently, we demonstrated the dynamic model of the direct current motor in the d-q axes with a block diagram in SIMULINK, and then the separately excited DCM model was considered, taking into account auxiliary poles and compensation windings.

Finally, the DCM model with series excitation and constant flux was presented, along with an introduction to the state variables of a dynamic system and their adaptation to the DCM model.

Chapter IV:

Modeling and Simulation of Synchronous Machines

IV.1- Introduction

The synchronous machine is an AC machine whose rotor's rotational speed is equal to the speed of the rotating field created by the stator. This machine is primarily used in power plants as a generator. Unlike the asynchronous machine, the synchronous machine cannot start in an open-loop configuration. For this reason, it has been traditionally used in high-power, fixed-speed applications. Nevertheless, with advancements in power electronics and control, it has become possible to use it in variable-speed applications. Furthermore, with the discovery of highly efficient permanent magnets, synchronous machines have the potential to be used as motors in high-power and high-precision applications.

IV.2- Synchronous Machine Structures

Synchronous machines consist of two main parts:

The exciter, made up of either electromagnets carrying direct current or permanent magnets, located on the rotor.

The armature, carried by the stator, carrying alternating currents (single or three-phase.(

a) Stator (Armature)

For a three-phase synchronous machine, it consists of three sets of conductors housed in the stator slots, forming three circuits offset at a suitable angle to each other. They are traversed by three currents that create a three-phase system to produce a rotating field.

b) Rotor (Exciter)

The rotor carries magnets or excitation coils with direct current flowing through them, creating North and South poles. It generates a rotating field in the air gap of the machine. There are two types:

Smooth-pole rotor, which has high mechanical robustness, and is adopted for high-power alternators with high rotational frequencies (nuclear power plant alternators.(

Salient-pole rotor (used for machines rotating at low speeds). It is simpler to construct and is used for generator sets

Figure IV.1: (a) Smooth-Pole Rotor (b) Salient-Pole Rotor

IV.3- Operating Principle

In alternating current machines, the magnetic field rotates at the same angular velocity as the rotor in synchronous machines, and at a slightly higher speed in asynchronous machines. This field is created both by the rotating rotor and by the stator windings carrying three-phase alternating currents. We have a U-shaped magnet that is rotated at speed ns and a small magnet or a magnetized needle capable of rotating around its axis.

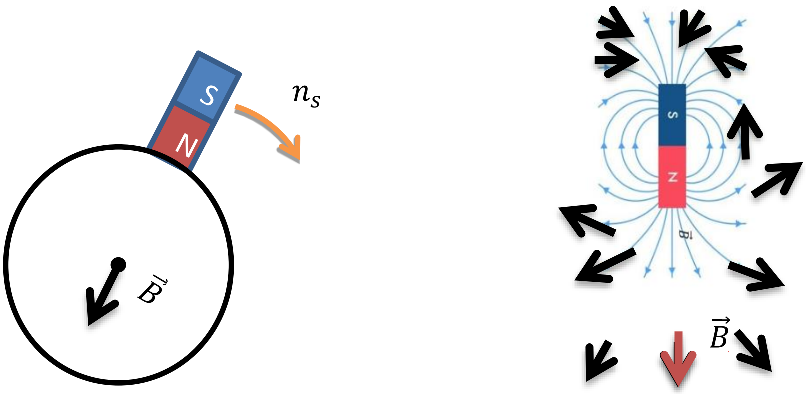


Figure IV.2: Generation Principle of Rotating Field by Permanent Magnet

The three-phase winding, supplied by a balanced three-phase voltage system, creates a rotating magnetic field at a speed of ns, such that:

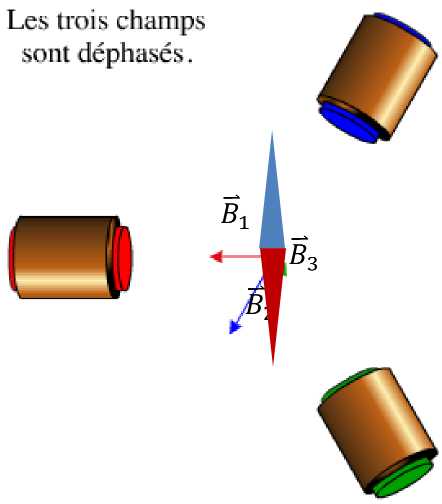
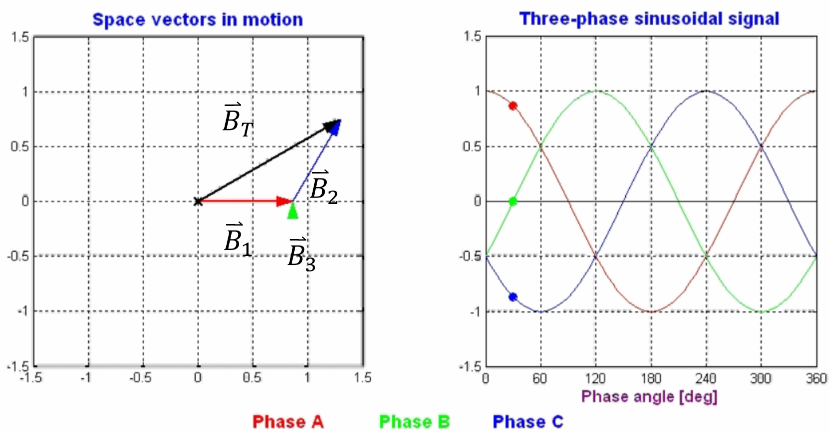


Figure IV.3: Generation of a Rotating Field by a Balanced Three-Phase System



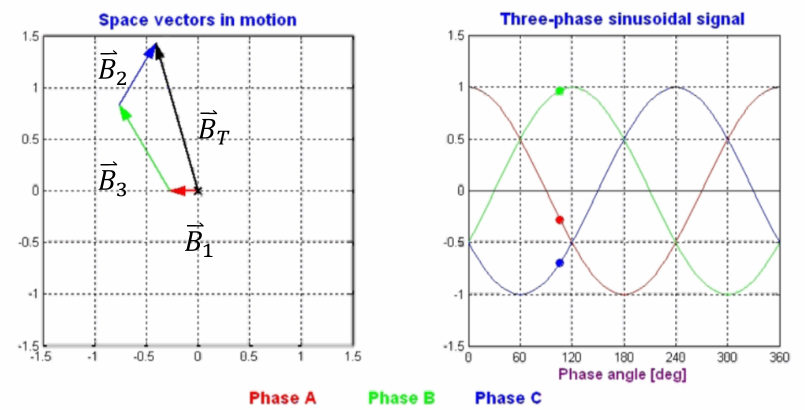


Figure IV.4: Rotating Field Produced by the Sum of Three Sinusoidal Vectors

The rotor, supplied with direct current through a system of sliding contacts (brushes or slip rings), creates a rotor magnetic field that follows the stator's rotating field with an angular delay related to the load (the heavier the load, the greater the delay). Since the rotor rotates at the same speed as the rotating field, this motor cannot be started directly on the grid. A frequency converter can be used, gradually increasing the frequency during the startup phase (ramp). Alternatively, this motor can be started "asynchronously" by using the exciter winding as a secondary winding. This motor can also be used to improve the power factor of an installation. In this case, it must be "overexcited." It then provides reactive power to the grid (capacitive load).

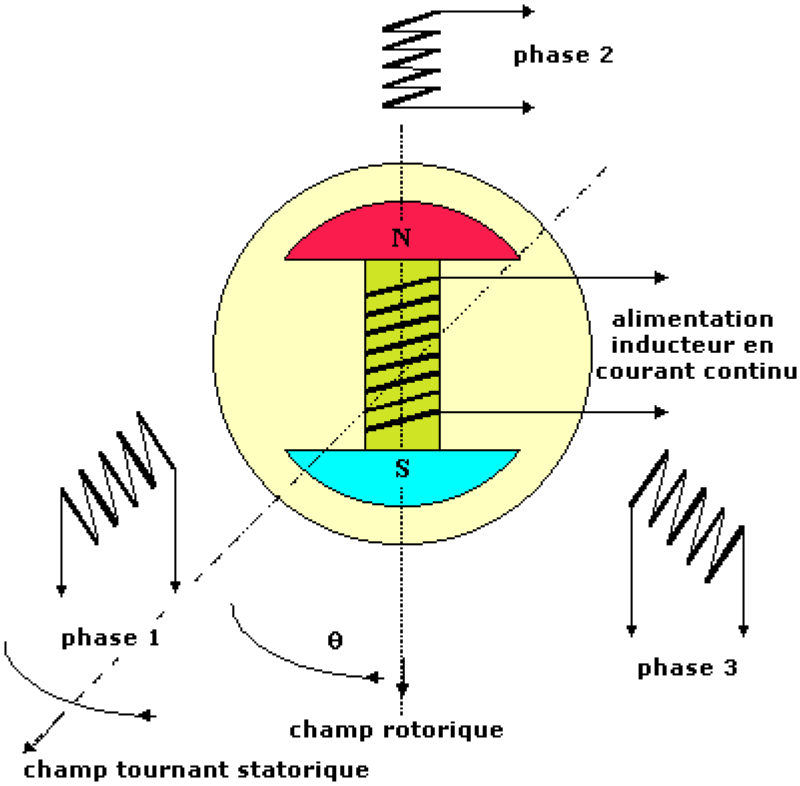


Figure IV.5: Wound-Rotor Synchronous Machines

The synchronous motor consists of a stator carrying a three-phase winding with 2p poles connected to the source supplying voltages and currents at frequency f. The rotor or exciter carries a winding supplied with direct current excitation, which creates 2p poles successively North and South relative to the rotor.

The currents at frequency f provided by the three-phase source generate a magnetomotive force in the air gap, rotating at the synchronous speed Ns

$$N_s = \frac{f}{p}(tr/s) \quad \text{....(IV.1)}$$

The poles of this magnetomotive force (EMF) attract the poles of the rotor and make it rotate at synchronous speed; hence the name 'synchronous motor' given to the motor.

The synchronous motor, like the asynchronous machine, consists of a wound stator whose power supply gives rise to a fundamental component of magnetomotive force Fa rotating at the angular speed ωs. The rotor or the polar wheel, with the same number of poles as the stator, is excited by a winding carrying direct current or by magnets to produce an excitation magnetomotive force F0. In the case of a synchronous motor with permanent magnets, the exciter is replaced by magnets, which has the advantage of eliminating brushes and rotor losses, as well as the need for a source to supply the excitation current. However, rotor flux control is not possible

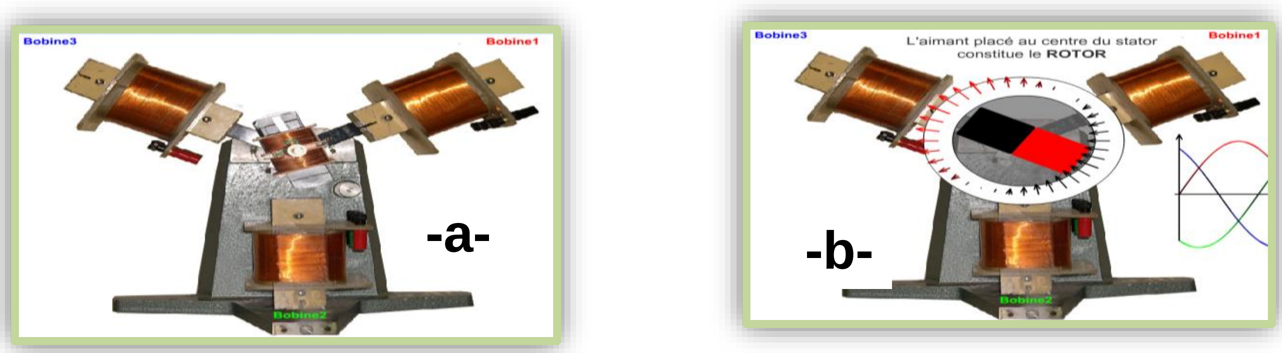


Figure IV.6 : (a) Wound-Rotor / (b) Permanent Magnet Rotor

IV.4- Permanent Model of Synchronous Machine

There are several equivalent models for the synchronous machine, depending on the number of parameters one wishes to consider.

IV.4.1 Behn-Eschenburg Equivalent Model

The **Behn-Eschenburg** model applies only to non-saturated machines with smooth poles. To study the sinusoidal steady-state operation of a stator phase supplied with voltage v', drawing a current i', and in which the exciter generates an electromotive force e', we use the equivalent circuit shown in Figure (7).

V', I', E' represent the sinusoidal values of the pulsation $\omega = 2\pi f$, X is the synchronous reactance. (In a first approximation, resistance can be neglected.)

The vector diagram (Figure (II-3 b)) illustrates the relationship:

In the diagram, three angles can be observed:

- φ, the phase shift of I' with respect to V',
- θ, the phase shift between E' and V', referred to as the internal angle of displacement,
- ψ, the phase shift of I' with respect to E', allowing for the rotor pole position relative to the stator to be determined.

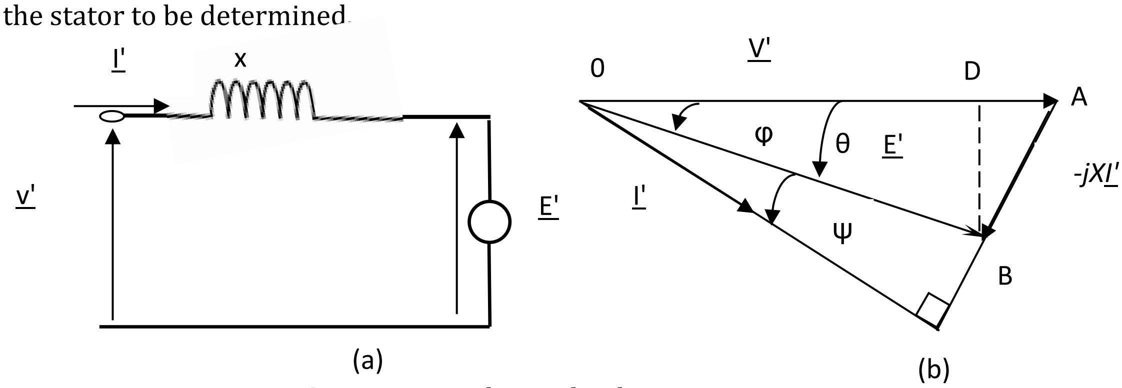


Figure IV.7: Behn-Eschenburg Diagram

The electromotive force (f.e.m) E' is proportional to the speed and the excitation flux, and, as long as saturation can be neglected, it is also proportional to the excitation current.

The power absorbed by the motor is: P = 3V'I'cos(φ)

If losses are neglected, P represents the developed mechanical power.

By denoting Ω as the synchronous angular speed, the torque C is given by: ω/p

$$C = \frac{P}{\Omega} = \frac{3V'I'Cos\varphi}{\Omega} \quad \text{....(IV.2)}$$

- In the OAB triangle of the vector diagram, the angle at A is equal to $\pi/2 - \phi$

So : $\dfrac{XI'}{Sin\theta} = \dfrac{E'}{Cos\varphi}$

This results in: $P = \dfrac{3V'E'}{X} Sin\theta \quad , C = \dfrac{3V'E'}{\Omega X} Sin\theta$

IV.4.2 Potier Equivalent Model

This model is more comprehensive than the Behn-Eschenburg model. It takes into account saturation by varying the excitation current according to the current passing through the stator coils. This modification of the excitation current causes the electromotive force to fluctuate.

In this model, we have:

$$\begin{cases} \underline{V} = \underline{E_r} - R\underline{I} - j\lambda\omega\underline{I} \\ \underline{J_e} = \underline{J_{eo}} - \alpha\underline{I} \end{cases} \quad \text{....(IV.3)}$$

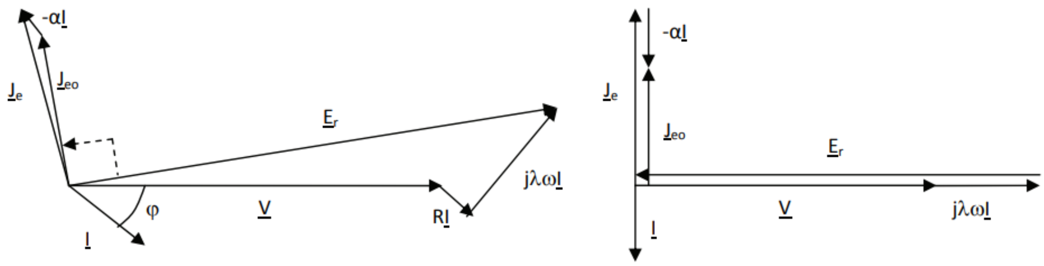


Figure IV.8: Vector Diagrams of a Synchronous Machine Supplying (a) R-L Load, (b) Purely Inductive Load

IV.5- Dynamic Models of Synchronous Machines:

IV.5.1. Simplifying Assumptions:

- Constant air gap.
- Negligible cogging effect.
- Negligible saturation, hysteresis, and eddy currents.
- Resistance of windings does not vary with temperature.
- Negligible skin effect.

IV.5.2 Formulation of the Machine Equations:

- **Electrical Equation:**

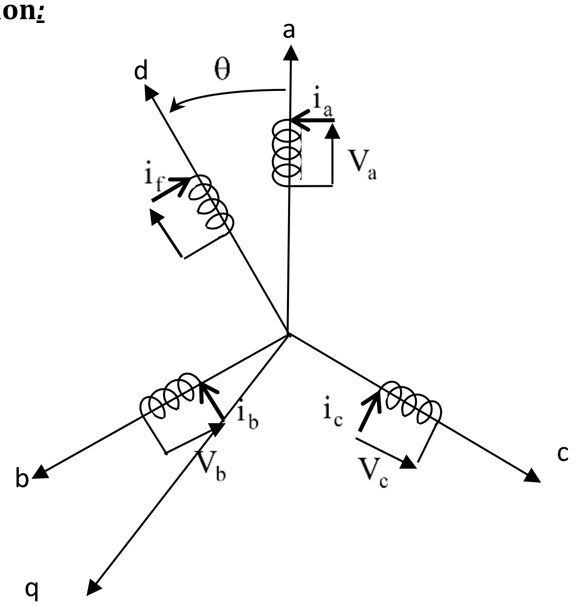


Figure IV.**8:** Representation of the windings of a synchronous machine without dampers

a) At the stator:

$$\begin{cases} V_{sa} = R_s I_{sa} + \dfrac{d\varphi_{as}}{dt} \\ V_{sb} = R_s I_{sb} + \dfrac{d\varphi_{bs}}{dt} \\ V_{sc} = R_s I_{sc} + \dfrac{d\varphi_{cs}}{dt} \end{cases} \quad \text{....(IV.4)}$$

The variables are at the stator, and we define V, I, φ as the voltages, currents, and flux of the stator of the machine.

b) At the rotor :

$V_f = R_f I_f + \frac{d\varphi_f}{dt}$ ....(IV.5)

V_f , I_f , φ_f : Voltages, currents, and flux of the rotor of the machine.

➢ ***Magnetic equations:***

a) Stator flux:

$$\begin{cases} \varphi_{sa} = L_s I_{sa} + M_{ss} I_{sb} + M_{ss} I_{sc} + M_{sf} I_f \cos(\theta) \\ \varphi_{sb} = L_s I_{sb} + M_{ss} I_{sc} + M_{ss} I_{sa} + M_{sf} I_f \cos(\theta - 2*\pi/3) \\ \varphi_{sc} = L_s I_{sc} + M_{ss} I_{sa} + M_{ss} I_{sb} + M_{sf} I_f \cos(\theta + 2*\pi/3) \end{cases} \quad \text{....(IV.6)}$$

b) Rotor flux :

$\varphi_f = L_f I_f + M_{fs} I_{sa} \cos(\theta) + M_{fs} I_{sb} \cos(\theta - 2*\pi/3) + M_{fs} I_{sc} \cos(\theta + 2*\pi/3)$ ....(IV.7)

The model can be grouped in matrix form as follows:

$[V] = [R][I] + [L]\frac{d}{dt}[I] + [M]\frac{d}{dt}[I]$....(IV.8)

After replacing the flux equations 6 and 7 in the stator and rotor model of equations IV.4 and IV.5, the matrix form of the MS model will be :

$$\begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \\ V_f \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_s & 0 \\ 0 & 0 & 0 & R_f \end{bmatrix} \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \\ I_f \end{bmatrix} + \begin{bmatrix} L_s & 0 & 0 & 0 \\ 0 & L_s & 0 & 0 \\ 0 & 0 & L_s & 0 \\ 0 & 0 & 0 & L_f \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \\ I_f \end{bmatrix} + \quad \text{..(IV.9)}$$

$$\begin{bmatrix} 0 & M_{ss} & M_{ss} & M_{sf} cos(\theta) \\ M_{ss} & 0 & M_{ss} & M_{sf} cos(\theta - 2*\pi/3) \\ M_{ss} & M_{ss} & 0 & M_{sf} cos(\theta + 2*\pi/3) \\ M_{fs} cos(\theta) & M_{fs} cos(\theta - 2*\pi/3) & M_{fs} cos(\theta + 2*\pi/3) & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \\ I_f \end{bmatrix}$$

IV.6- Three-phase to two-phase transformation

The purpose of using this transformation is to switch from a three-phase abc system to a two-phase α β system. There are mainly two transformations: Clarke and Concordia. The

Clarke transformation preserves the amplitude of the variables but not the power or torque (a multiplication by a coefficient of 3/2 is necessary). In contrast, the Concordia transformation, which is standardized, preserves power but not amplitudes.

IV.6.1 Clark Concordia Transformation

Edith Clarke proposed the transformation in 1951. Let a, b, and c be the initial coordinates of a three-phase system. α, β, and o are the destination coordinates:

$$\vec{X} = \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} = \begin{bmatrix} Xmax * \sin(wt) \\ Xmax * \sin(wt - \frac{2\pi}{3}) \\ Xmax * \sin(wt + \frac{2\pi}{3}) \end{bmatrix} \quad ....(\text{IV.10})$$

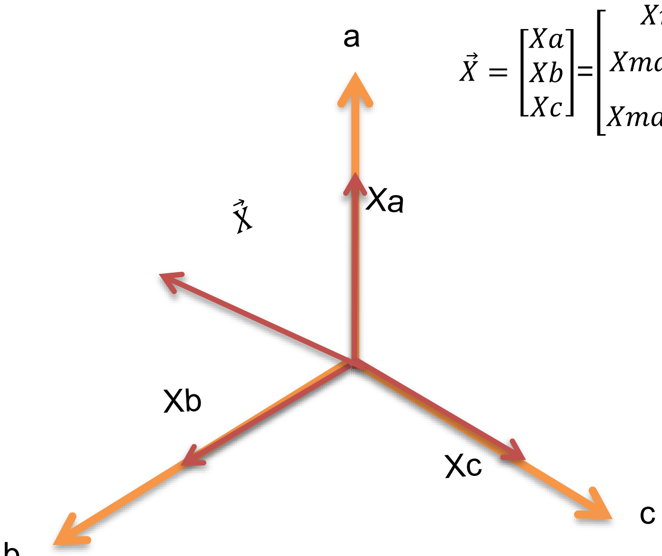
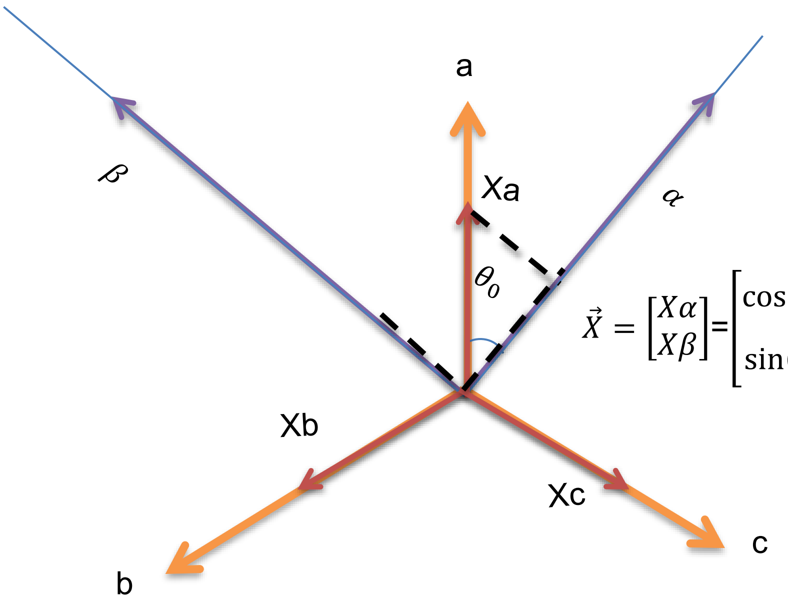


Figure IV.9: The image of vector x on a b c

To understand this transformation for a three-phase system, let's assume that we have a vector X $\vec{}$, this vector rotated at a speed wt. We can represent it in the fixed coordinates a, b, c as Xa, Xb, and Xc, as in equation 10.

The Clark transformation is the image of the vector $\vec{X}$ = [Xa Xb Xc]^T onto the fixed reference axes α, β. So we have:



$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \end{bmatrix} = \begin{bmatrix} \cos(\theta_0) & \cos\left(\theta_0 - \frac{2\pi}{3}\right) & \cos\left(\theta_0 + \frac{2\pi}{3}\right) \\ \sin(\theta_0) & \sin\left(\theta_0 - \frac{2\pi}{3}\right) & \sin\left(\theta_0 + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad ...(\text{IV.11})$$

Figure IV.10: The image of vector x on α, β

We assume that θ_0=0, and the matrix will be simpler as follows

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \end{bmatrix} = \begin{bmatrix} \cos(0) & \cos\left(-\frac{2\pi}{3}\right) & \cos\left(\frac{2\pi}{3}\right) \\ \sin(\theta_0) & \sin\left(-\frac{2\pi}{3}\right) & \sin\left(\frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots (\mathrm{IV.12})$$

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots (\mathrm{IV.13})$$

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \\ X0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots (\mathrm{IV.14})$$

"Thanks to Concordia, the Clark Matrix will be a square matrix with det=1 to have the inverse of the matrix equal to the transmission matrix:

$$\vec{X} = \begin{bmatrix} X\alpha \\ X\beta \\ X0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \quad \ldots (\mathrm{IV.15})$$

IV.6.2. Park Transformation

The difference between the Park transformation and Clark Concordia lies in the Park coordinates having rotational axes (see Figure IV.11), unlike the Clark Concordia transformation. The Park transformation, often confused with the dqo transformation, is a mathematical tool used in electrical engineering, particularly for vector control, to model a three-phase system with a two-phase model. It involves a change of coordinates. The first two axes in the new base are traditionally named d, q. The transformed variables are typically currents, voltages, or fluxes.

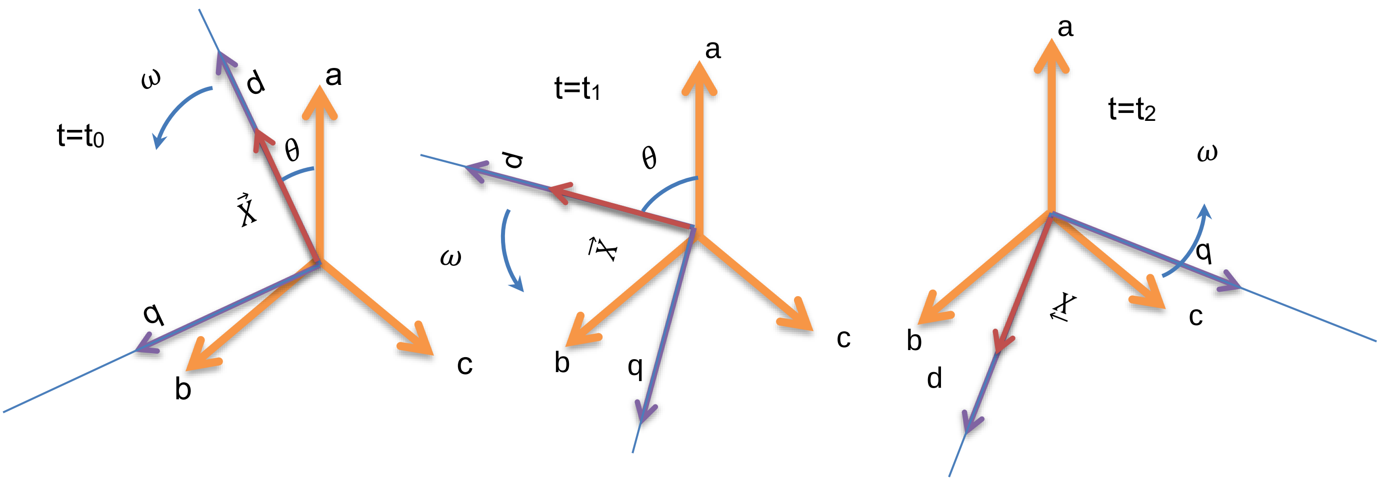


Figure IV.11: The image of vector x on the d q axes at different moments.

According to the Park transformation, we can write:

$$\begin{bmatrix} Xd \\ Xq \\ X0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} Xa \\ Xb \\ Xc \end{bmatrix} \ldots (\text{IV}.16)$$

So, the Park matrix is:

$$[P(\theta)] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = P(\theta) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \ldots . (\text{IV}.17)$$

IV.7- Park Equations for Synchronous Machines

To eliminate θ from the matrix $[Lss]$ and to enable control algorithms to handle continuous electrical quantities, the stator windings (a, b, c) are replaced by two windings (d, q) in quadrature. The conversion of electrical quantities from the stator (a, b, c) to the electrical quantities (d, q), ensuring the conversion of the mmf (magnetomotive force) and instantaneous power, is achieved through the Park transformation.

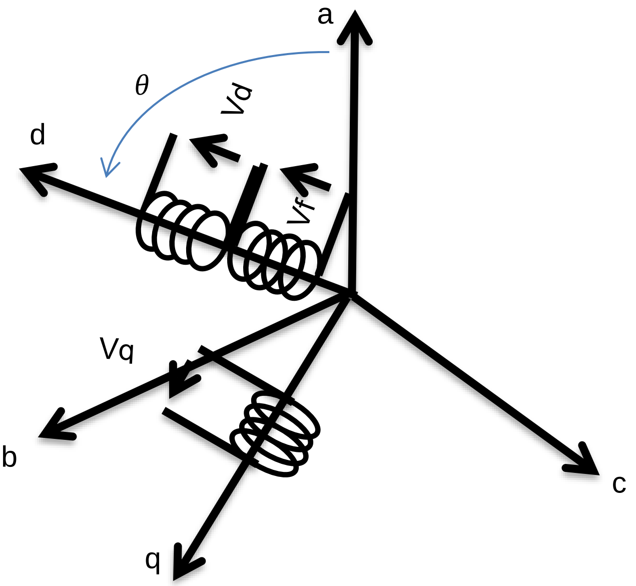


Figure IV.12: Equivalent Diagram of the Synchronous Machine in the Park reference frame.

The equivalent machine in Figure (IV.12) is identical to a DC machine with winding f as the inductor and having two induced windings, winding d aligned with the f inductor axis, and winding q in quadrature with f. The passage matrix, denoted as in equation IV.16. So, the matrix is given by:

The passage matrix, denoted as equation IV.16,

So, the matrix $[\mathrm{P}(\theta)]^{-1}$ is given by:

$$[P(\theta)]^{-1}=\sqrt{\frac{2}{3}}\begin{bmatrix}\cos\theta & -\sin\theta & \frac{1}{\sqrt{2}}\\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & \frac{1}{\sqrt{2}}\\ \cos(\theta-\frac{4\pi}{3}) & -\sin(\theta-\frac{4\pi}{3}) & \frac{1}{\sqrt{2}}\end{bmatrix} \quad \ldots(\text{IV.18})$$

The transition from the three-phase system to the (d, q) system linked to the rotor is done using the following relationship:

$$\begin{bmatrix}x_d\\ x_q\\ x_0\end{bmatrix}=[P(\theta)]\begin{bmatrix}x_a\\ x_b\\ x_c\end{bmatrix} \ldots (\text{IV.19})$$

Applying the transformation to the system:

$$[V]=[R][I]+\frac{d}{dt}[\varphi]$$

$$[P]^{-1}[V_{dq0}]=[R][P]^{-1}[I_{dq0}]+\frac{d}{dt}([P]^{-1}[\varphi_{dqo}]) \quad \textbf{....} \ (\text{IV. 20})$$

Then, by multiplying by $[P(\theta)]^{-1}$:

$$[V_{dq0}]=[R][I_{dqo}]+\frac{d}{dt}[\varphi_{dqo}]+[P](\frac{d}{dt}[P]^{-1})[\varphi_{dqo}] \quad \ldots.(\text{IV.21})$$

For the demonstration of the model, we have::

$$[P](\frac{d}{dt}[P]^{-1})=\frac{d\theta}{dt}\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \quad \ldots.(\text{IV.22})$$

In the synchronous motor, the flux generated by the inductor is constant. The model of this motor is obtained by assuming the constancy of I_f, and we obtain the following equations:

a) Magnetic Equations:

On the d-axis : $\phi_d = L_d I_d + M_f I_f$

$$\phi_f = L_f I_f + M_f I_d \quad \text{....(IV.23)}$$

On the q-axis: $\phi_q = L_q I_q$

b) Electrical Equations:

According to the system:

$$\left.\begin{aligned} V_d &= RI_d + \frac{d\varphi_d}{dt} - \left(\frac{d\theta}{dt}\right)\varphi_q \\ V_q &= RI_q + \frac{d\varphi_q}{dt} + \left(\frac{d\theta}{dt}\right)\varphi_d \\ V_f &= R_f I_f + \frac{d\varphi_f}{dt} \end{aligned}\right\} \quad \text{....(IV.24)}$$

It becomes:

$$\left.\begin{aligned} V_d &= RI_d + L_d\frac{dI_d}{dt} + M_f\frac{dI_f}{dt} - \omega L_q I_q \\ V_q &= RI_q + L_q\frac{dI_q}{dt} + \omega(L_d I_d + M_f I_f) \\ V_f &= R_f I_f + L_f\frac{di_f}{dt} + L_d\frac{dI_d}{dt} \end{aligned}\right\} \quad \text{.... (IV. 25)}$$

So, the model in matrix form will be::

$$\begin{bmatrix}\varphi_d\\ \varphi_q\\ \varphi_f\end{bmatrix} = \begin{bmatrix}L_d & 0 & M_f\\ 0 & L_q & 0\\ M_f & 0 & L_f\end{bmatrix}\begin{bmatrix}I_d\\ I_q\\ I_f\end{bmatrix} \text{...(IV.26)}$$

$$\begin{bmatrix}V_d\\ V_q\\ V_f\end{bmatrix} = \begin{bmatrix}R & 0 & 0\\ 0 & R & 0\\ 0 & 0 & R_f\end{bmatrix}\begin{bmatrix}I_d\\ I_q\\ I_f\end{bmatrix} + \begin{bmatrix}L_d & 0 & M_f\\ 0 & L_q & 0\\ M_f & 0 & L_f\end{bmatrix}\frac{d}{dt}\begin{bmatrix}I_d\\ I_q\\ I_f\end{bmatrix} + \omega\begin{bmatrix}0 & -L_q & 0\\ L_d & 0 & M_f\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}I_d\\ I_q\\ I_f\end{bmatrix} \quad \text{....(IV.26)}$$

IV.8- Expression of Power and Electromagnetic Torque:

The expression for electromagnetic torque is given by

$$T_e = \frac{\partial W_e}{\partial \theta_{geo}} = p\,\frac{\partial W_e}{\partial \theta_{ele}} \quad \text{....(IV.27)}$$

In the theory of the field of electrical machines, the electromagnetic torque involved in the equation is expressed by the partial derivative of electromagnetic energy storage with respect to the geometric angle of rotation of the rotor.

Where:

We: energy stored in the magnetic circuit.

θ_{geo} : Angular deviation of the movable part (rotor relative to the stator).

With:

$$\theta_{geo} = \frac{\theta_{erle}}{p} \quad \text{....(IV.28)}$$

According to Park, the expression of power is written as follows:

$$p(t) = \frac{3}{2}(V_d I_d + V_q I_q) \quad \text{....(IV.29)}$$

By replacing Vd and Vq with their expressions, it comes:

$$p(t) = \frac{3}{2}[(R_s(I_d^2 - I_q^2)] + \frac{3}{2}[I_d \frac{d\phi_d}{dt} + I_q \frac{d\phi_q}{dt}] + \frac{3}{2}[\phi_d I_d - \phi_q I_q]\omega \quad \text{....(IV.30)}$$

Where:

The 1st term: represents the ohmic losses (Joule effect losses)

The 2nd term: represents the variation of stored magnetic energy

The 3rd term: represents the power transferred from the stator to the rotor through the air gap (electromagnetic power).

Given that:

$$P_e = T_e W_r \quad \text{....(IV.31)}$$

One can also write:

$$\begin{aligned} T_e &= \frac{3}{2} p \left[(L_d - L_q) I_d I_q + M_f I_q I_f \right] \\ T_e &= \frac{3}{2} p \left[(\varphi_d I_q - \varphi_q I_d) \right] \end{aligned} \quad \text{... (IV.32)}$$

IV.9- Equation of Motion:

The equation of motion for the machine is:

$$T_e - T_L - f W_r = J \frac{d\omega_r}{dt} \quad \text{.... (IV.33)}$$

With:

J: Moment of inertia of rotating masses;

TL: Load torque imposed on the machine shaft;

Te: Electromagnetic torque;

ωr: Mechanical rotation speed (ω=P.ωr);

f: Viscous friction coefficient

IV.10- Formulation as a State Equation for Control:

The generalized form of the system equations in state-space form is:

$$\begin{cases} \dot{X}(t) = A\,X(t) + B\,U(t) \\ Y(t) = C\,X(t) + D\,U(t) \end{cases} \quad \text{...(IV.34)}$$

– X(t): state vector of the system with dimension n.
– U(t): input vector or control vector of the system with dimension l.
– Y(t): output vector of the system with dimension m.
– A: state matrix of the system with dimension n × n.
– B: input or control matrix of the system with dimension n × l.
– C: output matrix of the system with dimension m × n.
– D: direct transmission matrix of the system with dimension m × l.

The state vector, input vector, or output vector will be chosen based on control requirements or modeling needs. Thus, we have the following model:

IV.10.1 State-space model of the Synchronous Machine:

Considering the voltages Vd, Vq, and the excitation flux φ_f as control variables, the stator currents Id, Iq as state variables, and the torque T_L as disturbance. From equations (IV.35), we can write the following system of equations:

With:

$$[x]=\lfloor I_d \quad I_q \rfloor^T$$

$$[U]=\lfloor V_d \quad V_q \quad \phi_f \rfloor$$

and :

$$\begin{bmatrix} \dot{I}_d \\ \dot{I}_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \omega\frac{L_q}{L_d} \\ -\omega\frac{L_d}{L_q} & -\frac{R_s}{L_q} \end{bmatrix}\begin{bmatrix} I_d \\ I_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix}\begin{bmatrix} V_d \\ V_q \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\phi_f}{L_q} \end{bmatrix}\omega \quad \text{....(IV.35)}$$

And we set:

$$[A]=\begin{bmatrix} -\frac{R_s}{L_d} & \omega\frac{L_q}{L_d} \\ -\omega\frac{L_d}{L_q} & -\frac{R_s}{L_q} \end{bmatrix} \text{ ' } [B]=\begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix} \text{ et } [c]=\begin{bmatrix} 0 \\ -\frac{\phi_f}{L_q} \end{bmatrix}$$

We can write the matrix [A] as follows:

$$[A]=\begin{bmatrix} -\frac{R_s}{L_d} & 0 \\ 0 & -\frac{R_s}{L_q} \end{bmatrix} + \begin{bmatrix} 0 & \frac{L_q}{L_d} \\ -\frac{L_d}{L_q} & 0 \end{bmatrix} \quad \text{....(IV.36)}$$

In the form: $[A]=[A_{01}]+[A_{02}]$

$$[A_{01}]=\begin{bmatrix} -\frac{R_s}{L_d} & 0 \\ 0 & -\frac{R_s}{L_q} \end{bmatrix} \quad [A_{02}]=\begin{bmatrix} 0 & \frac{L_q}{L_d} \\ -\frac{L_d}{L_q} & 0 \end{bmatrix} \quad \text{....(IV.37)}$$

To arrive at the same system, [C]= [1 1] [D]]= 0 must be added:

IV.10.2 State-space model of PMSM

The model of the Permanent Magnet Synchronous Machine (PMSM) is simpler than the one presented above; we can say that this model considers a specific case of wound-rotor synchronous machines. According to equation IV.38:

According to equation IV.38:

$$\left.\begin{aligned} V_d &= RI_d + \frac{d\varphi_d}{dt} - \left(\frac{d\theta}{dt}\right)\varphi_q \\ V_q &= RI_q + \frac{d\varphi_q}{dt} + \left(\frac{d\theta}{dt}\right)\varphi_d \\ &\cancel{V_f = R_f I_f + \frac{d\varphi_f}{dt}} \end{aligned}\right\} \quad \text{....(IV.38)}$$

$$\left.\begin{aligned} \varphi_d &= L_d I_d + \cancel{M_f I_f} + \varphi_f \\ \varphi_q &= L_q I_q \\ &\cancel{\varphi_f = L_f I_f + M_f I_d} \end{aligned}\right\} \quad \text{....(IV.39)}$$

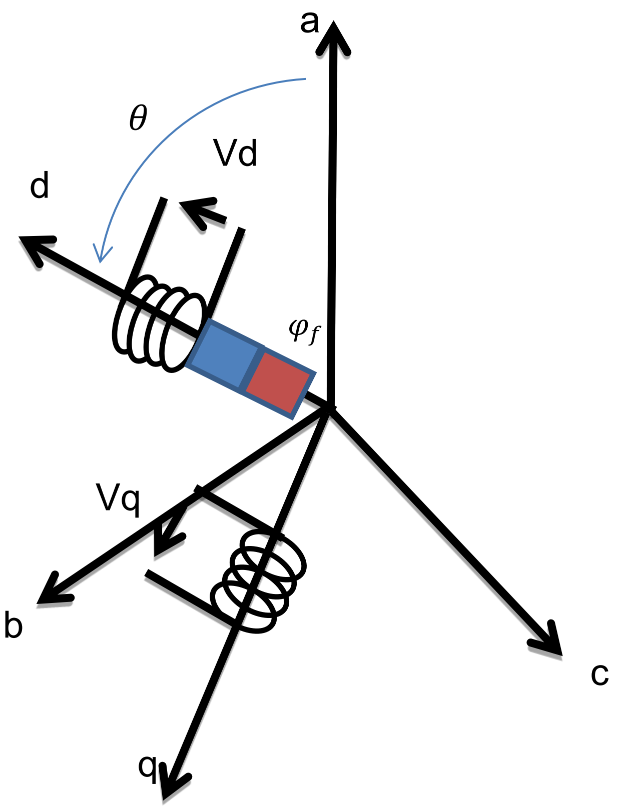


Figure IV.13: Equivalent diagram of the PMSM in the Park reference frame..

So, the electrical model of PMSM will be:

$$\left.\begin{aligned} V_d &= RI_d + \frac{d\varphi_d}{dt} - \left(\frac{d\theta}{dt}\right)\varphi_q \\ V_q &= RI_q + \frac{d\varphi_q}{dt} + \left(\frac{d\theta}{dt}\right)\varphi_d \end{aligned}\right\} \quad \text{....(IV.40)}$$

$$\left.\begin{aligned} \varphi_d &= L_d I_d + \varphi_f \\ \varphi_q &= L_q I_q \end{aligned}\right\} \quad \text{....(IV.41)}$$

So, the model in matrix form:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \end{bmatrix}\begin{bmatrix} I_d \\ I_q \\ \varphi_f \end{bmatrix} + \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \end{bmatrix}\frac{d}{dt}\begin{bmatrix} I_d \\ I_q \\ \varphi_f \end{bmatrix} + \omega\begin{bmatrix} 0 & -L_q & 0 \\ L_d & 0 & 1 \end{bmatrix}\begin{bmatrix} I_d \\ I_q \\ \varphi_f \end{bmatrix}$$

The mechanical model remains the same as the wound-rotor synchronous machine.

$$\left.\begin{aligned} T_e &= \frac{3}{2}P\left(\varphi_d I_q - \varphi_q I_d\right) \\ T_e - T_r - f\omega_m &= J\frac{d\omega_m}{dt} \\ \omega &= \omega_m p \end{aligned}\right\} \quad \text{....(IV.42)}$$

So, where we consider the voltages Vd, Vq, and the excitation flux φ_f as control variables, the stator currents Id, Iq as state variables, we formulate the model as follows :

$$\left.\begin{aligned} \frac{dI_d}{dt} &= \frac{V_d}{L_d} - \frac{R}{L_d}I_d + \omega\frac{L_q}{L_d}I_q \\ \frac{dI_q}{dt} &= \frac{V_q}{L_q} - \frac{R}{L_q}I_q - \omega\frac{L_d}{L_q}I_d - \frac{\omega}{L_q}\varphi_f \end{aligned}\right\} \quad \text{....(IV.43)}$$

In the end, the matrix form is written as:

$$\frac{d}{dt}\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_d} & \omega\frac{L_q}{L_d} \\ -\omega\frac{L_d}{L_q} & -\frac{R}{L_q} \end{bmatrix}\begin{bmatrix} I_d \\ I_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & -\frac{\omega}{L_q} \end{bmatrix}\begin{bmatrix} V_d \\ V_q \\ \varphi_f \end{bmatrix} \quad \text{....(IV.44)}$$

$$[A] = \begin{bmatrix} -\frac{R}{L_d} & \omega\frac{L_q}{L_d} \\ -\omega\frac{L_d}{L_q} & -\frac{R}{L_q} \end{bmatrix}$$

$$[B] = \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & -\frac{\omega}{L_q} \end{bmatrix}$$

$$[A] = [A_1] + \omega[A_2] = \begin{bmatrix} -\frac{R}{L_d} & 0 \\ 0 & -\frac{R}{L_q} \end{bmatrix} + \omega\begin{bmatrix} 0 & \frac{L_q}{L_d} \\ -\frac{L_d}{L_q} & 0 \end{bmatrix}$$

$$[B] = [B_1] + \omega[B_2] = \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & 0 \end{bmatrix} + \omega\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_q} \end{bmatrix}$$

We can summarize the model:

$$[\dot{I}] = ([A_1] + \omega[A_2])[I] + ([B_1] + \omega[B_2])[V] \quad \text{....(IV.45)}$$

On the other hand, we consider the voltages Vd Vq, φ_f, T_L, and Te as control variables, the stator currents Id Iq and ω_m as state variables, we formulate the model as follows::

$$\left.\begin{aligned} \frac{dI_d}{dt} &= \frac{V_d}{L_d} - \frac{R}{L_d}I_d + \omega\frac{L_q}{L_d}I_q \\ \frac{dI_q}{dt} &= \frac{V_q}{L_q} - \frac{R}{L_q}I_q - \omega\frac{L_d}{L_q}I_d - \frac{\omega}{L_q}\varphi_f \\ \frac{d\omega_m}{dt} &= \frac{1}{J}(T_e - T_L - f\omega_m) \end{aligned}\right\} \quad \text{...(IV.46)}$$

$$T_e = \frac{3}{2}P\left(\varphi_f I_q + L_d I_d I_q - L_q I_q I_d\right)$$

The matrix form of this statement is:

$$\frac{d}{dt}\begin{bmatrix} I_d \\ I_q \\ \boldsymbol{\omega}_m \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_d} & \omega\frac{L_q}{L_d} & 0 \\ -\omega\frac{L_d}{L_q} & -\frac{R}{L_q} & 0 \\ 0 & 0 & \frac{f}{J} \end{bmatrix}\begin{bmatrix} I_d \\ I_q \\ \boldsymbol{\omega}_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{L_q} & -\frac{\omega}{L_q} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{J} & \frac{1}{J} \end{bmatrix}\begin{bmatrix} V_d \\ V_q \\ \varphi_f \\ Tr \\ Te \end{bmatrix}$$

IV.11- Simulation of Synchronous Machines:

After applying the Park transformation to the Vabc sources to convert them into two-phase Vdq components, the model described in **Equation 27** is used in conjunction with the mechanical model. The mechanical dynamics are represented by the torque equation (Equation 31) and the mechanical model equation (Equation 32). These equations are implemented in MATLAB Simulink to simulate the behavior and performance of Synchronous Machines under dynamic conditions, allowing us to observe and analyze the system's response.

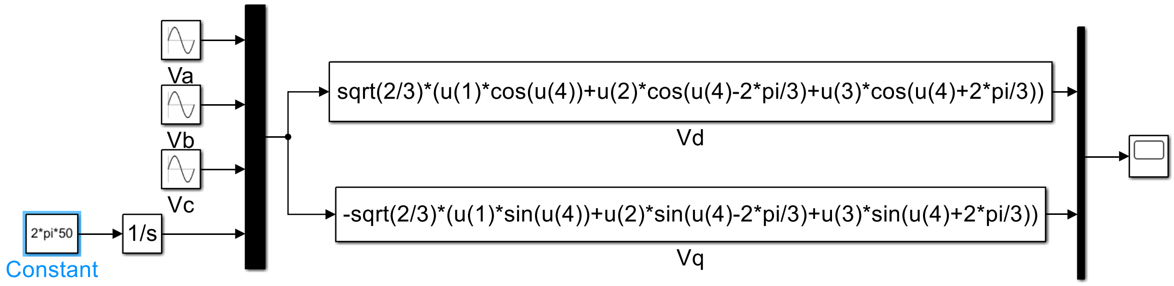


Fig.14 Park Transformation for the three-phase system

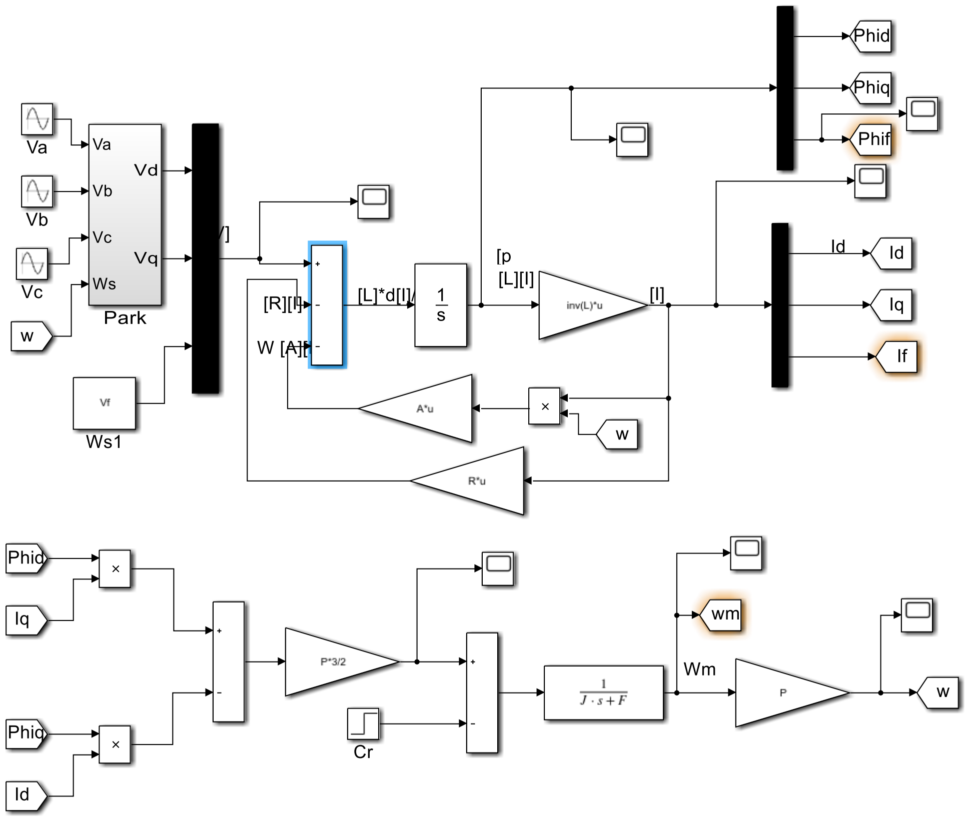


Figure IV.14 Simulation of the synchronous machine with wound rotor

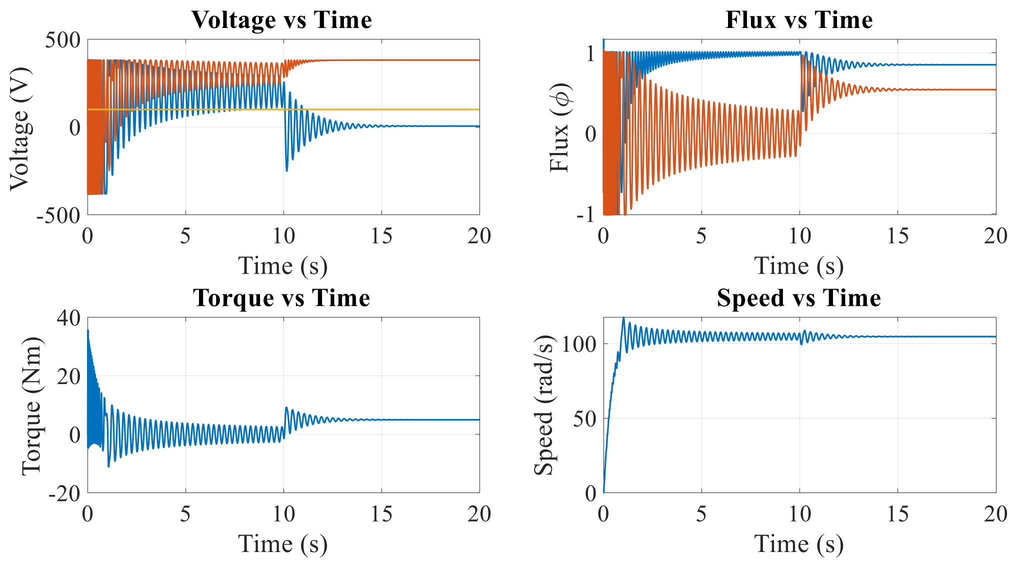


Figure IV.15 Results of the synchronous motor simulation under a 5 Nm load

The results of the synchronous motor simulation under a 5 Nm load at 5 seconds reveal several key dynamics. The applied voltage remains constant throughout the operation, unaffected by the mechanical load, which is expected given that the power supply is stable and regulates the stator voltage independently of load changes. In contrast, the magnetic flux increases when the load is applied, reflecting the motor's response to the additional mechanical demand by enhancing the stator current, thus increasing the flux linkage. This behavior indicates the motor's ability to maintain synchronization between the rotor and the stator's magnetic field. The torque rises appropriately to 5 Nm, corresponding to the load applied at 5 seconds, demonstrating the motor’s capability to generate sufficient torque in response to the load. Furthermore, the motor speed remains constant, which is characteristic of synchronous motors, but a damping of oscillations is observed, indicating that the system stabilizes quickly after the initial load is applied. This damping effect suggests effective control over the oscillatory behavior, ensuring smooth operation under load conditions.

The validity of the presented mathematical model is evident from its ability to accurately predict the motor's behavior under load conditions, as demonstrated in the simulation results. The model effectively captures key dynamics, such as the constant voltage, increasing flux, torque response, and stabilized speed with damped oscillations. This accuracy is crucial for real-time control of the machine, as the model can be used to design controllers that ensure precise torque and speed regulation, enhance stability, and optimize performance under varying loads in practical applications.

IV.12- Conclusion:

Various structures, the rotating field, the operating principle, and the various mathematical models of the synchronous machine have been presented in this course.

The three-phase to two-phase transformation of Clark Concordia or Park has been demonstrated to understand the basic principles of stationary or rotational transformations. Then, the mathematical model of the synchronous machine has been presented following the Park transformation, along with various state-space models for control purposes.

Chapter V:

Modeling and Simulation of Induction Machines (Asynchronous Machines)

V.1- Introduction

The modeling of electrical machines represents a crucial phase in their development, and recent strides in computer science and software engineering have empowered the creation of high-performance models. This chapter focuses on unveiling the mathematical model of the asynchronous machine, utilizing the Park transformation. This transformative approach not only simplifies the model's complexity but also converts the three-phase system into a more manageable two-axis orthogonal system.

This mathematical model serves as an indispensable tool for studying various aspects of machine behavior. Beyond its abstraction, it becomes a powerful instrument for in-depth analyses, particularly in flux control, torque regulation, voltage stability, and speed dynamics. It forms the bedrock for exploring the machine's operational characteristics, providing crucial insights for refining and optimizing performance.

As we delve into the intricacies of this model, its significance becomes apparent. The study of the control regime, involving the interplay of flux, torque, and speed, becomes more accessible through this mathematical framework. Additionally, the comprehension of voltage regulation, vital for stability and reliability, finds its roots in the structured representation provided by the model.

In essence, this chapter serves as a gateway to the world of modeling and simulation, where theoretical abstractions intersect with the tangible realities of electrical machines. The mathematical elegance of the asynchronous machine model, coupled with the transformative capabilities of the Park transformation, lays the groundwork for a deeper understanding of the complex dynamics governing practical applications of electrical machines.

V.2- Operating Principle

It stems from the very principle of the asynchronous motor that the rotor, subject only to its electromagnetic torque, cannot rotate at an angular speed equal to that of the inducing rotating field (known as synchronous speed). If, by any means, it were brought to this speed, it would no longer be swept by the stator field and, consequently, would no longer be the site of induced currents. As a result, it would cease to be subject to the resulting torque and would tend to slow down until the induced currents reach a sufficient amplitude to create a torque equal in magnitude but opposite in direction to the mechanical torque opposing rotation. To characterize the rotor speed, we define the slip (g), which is the relative difference between the synchronous speed (Ns) and its actual speed (N).

Therefore, the operating principle of an asynchronous machine is based on the electromagnetic interaction of the rotating field created by the three-phase current supplied to the stator winding by the grid and the induced currents in the rotor winding when the rotor conductors are intersected by the rotating field. This electromagnetic interaction between the stator and rotor of the machine is only possible when the speed of the rotating field differs from that of the rotor. In this way, we can say that the operation of an asynchronous machine is comparable to that of a transformer whose secondary winding is rotating.

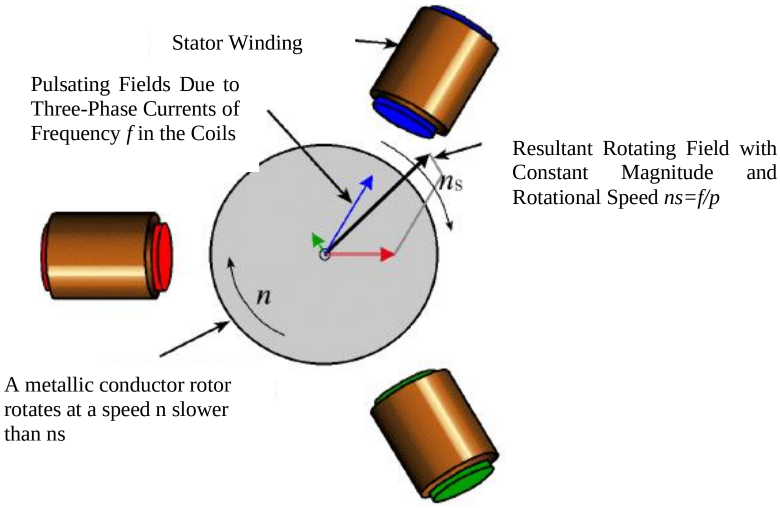


Figure V.1: Operating Principle of IM.

V.3- Model of the three-phase IM

The model of the three-phase induction machine is illustrated by the diagram in Figure 2 with the stator and rotor armatures each provided with a three-phase winding, there are three stator windings: S_A, S_B and S_C, and for the three rotor windings: Ra, Rb and Rc, and θ: Angle between the axis of the stator phase and the rotor phase.

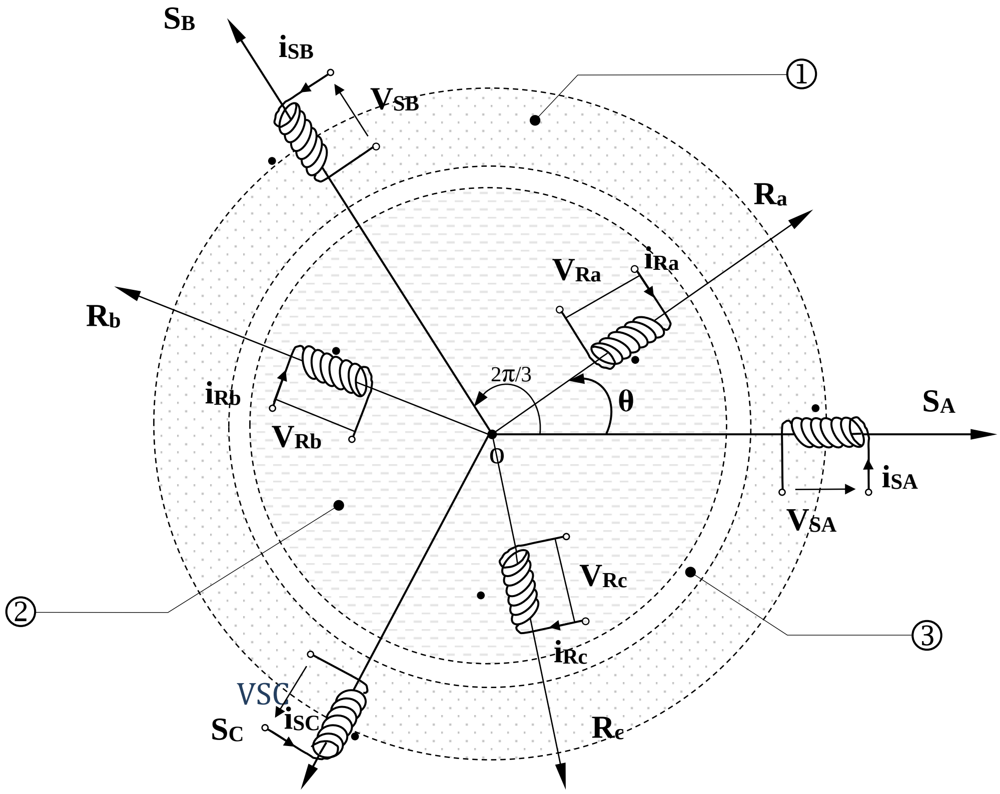


Figure V.2: Representation of the stator and rotor windings of a three-phase IM (asynchronous machine). ①Fixed part: Stator. ②Moving part: Rotor. ③Constant air gap.

V.3.1 Simplifying Assumptions

To streamline the presentation of the fundamental relationships governing the operation and control strategy of the asynchronous motor, the following simplifying assumptions can be made:

1. Consistency of Self-Inductances
   - Non-saturated magnetic circuit with constant permeability.
   - Constant air gap.
2. Reducing the Rotor to the Stator
   - Assuming the rotor is three-phase, similar to the stator.
   - Equal number of turns in their windings.
3. Assuming Bipolar Windings
   - Assuming both stator and rotor windings are bipolar.
   - Presuming that their phases generate sinusoidally distributed flux.
4. Consideration of Fundamental Alternating Quantities Only
   - Taking into account only the fundamental components of alternating quantities.
5. Neglecting Slot Effects
   - Disregarding the influence of slots.
6. Negligible Ferromagnetic Losses
   - Assuming ferromagnetic losses are negligible.
7. Disregarding Skin Effect and Heating Effects
   - Neglecting the influence of skin effect and temperature rise on the characteristics.

V.3.2 Equation of the machine model

The behavior of the machine is entirely defined by three types of equations, namely:

- Electrical equations.
- Magnetic equations.
- Mechanical equations.

V.3.2.1 Electrical equation

For all the stator phases:

$$\begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{sa} \\ \varphi_{sb} \\ \varphi_{sc} \end{bmatrix} \qquad \text{(V.1)}$$

The stator resistance being the same for the three phases, there is no need to write a resistance matrix.

Where :

$[V_s] = [R_s][I_s] + \frac{d}{dt}\ [\varphi_s]$ (V.2)

Likewise, at the rotor:

$$\begin{bmatrix} V_{ra} \\ V_{rb} \\ V_{rc} \end{bmatrix} = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} I_{ra} \\ I_{rb} \\ I_{rc} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{ra} \\ \varphi_{rb} \\ \varphi_{rc} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(V.3)}$$

Where :

$[V_r] = [R_r][I_r] + \frac{d}{dt}\ [\varphi_r] = [0]$ (V.4)

The rotor being short-circuited, its voltages are zero.

V_s : The voltage matrix per phase at the stator.

V_r : The voltage matrix per phase at the rotor.

I_s : The stator current matrix.

I_r : The rotor current matrix.

V.3.2.2 Magnetic equation

Each flow involves an interaction with the currents of all phases and understands its own (notion of flow / own inductance).

$$\begin{bmatrix} [\varphi_s] \\ [\varphi_r] \end{bmatrix} = \begin{bmatrix} [L_s] & [M_{sr}] \\ [M_{rs}] & [L_r] \end{bmatrix} \begin{bmatrix} [I_s] \\ [I_r] \end{bmatrix} \quad \text{(V.5)}$$

With :

$[L_s] = \begin{bmatrix} L_s & M_s & M_s \\ M_s & L_s & M_s \\ M_s & M_s & L_s \end{bmatrix}$ Stator winding matrix

L_s : is the inductance of a single winding

M_s : The mutual coupling inductance between the stator windings.

$[L_r] = \begin{bmatrix} L_r & M_r & M_r \\ M_r & L_r & M_r \\ M_r & M_r & L_r \end{bmatrix}$ Rotor winding matrix

L_r : is the inductance of a single winding

M_r : The mutual coupling inductance between rotor winding

$$\text{(V.6)}[M_{sr}] = [M_{rs}]^t = M_{sr} \begin{bmatrix} \cos\theta & \cos(\theta + \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) \\ \cos(\theta - \frac{2\pi}{3}) & \cos\theta & \cos(\theta + \frac{2\pi}{3}) \\ \cos(\theta + \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & \cos\theta \end{bmatrix}$$

θ: Electrical angle defines the instantaneous relative position between the rotor axes and the stator axes which are chosen as reference axes.

We will finally have:

$[V_{sabc}] = [R_s][I_{sabc}] + \frac{d}{dt}\ ([L_s][I_{sabc}] + [M_{sr}][I_{rabc}]$ (V.7)

$[V_{rabc}] = [R_r][I_{rabc}] + \frac{d}{dt}\ ([L_r][I_{rabc}] + [M_{rs}][I_{sabc}])$ (V.8)

V.3.2.3 Mechanical equation

The study of the dynamic characteristics of asynchronous machines introduces variations not only in electrical parameters (voltage, current flow, EMF) but also in mechanical parameters (torque, speed). The equation of motion of the machine is written:

$$T_e - T_{st} = J\frac{d\Omega}{dt} + f\Omega \qquad \text{(V.9).}$$

T_e :The electromagnetic torque of the machine. [Nm]

T_{st} : The resistive (static) torque at the machine shaft. [Nm].

J **:** The moment of inertia. [Kgm2]

Ω : The angular speed of the rotor, or the mechanical speed of the rotor..

f : Friction coefficient [Nm / rad/s].

The electrical speed of the rotor: $\omega_r = P\Omega$

P: the number of pole pairs.

So :

$$T_e - T_{st} = \frac{J}{P}\frac{d\omega_r}{dt} + \frac{f\omega_r}{P} \qquad \text{(V.10)}$$

V.4- Biphasic Model

The condition for transitioning from a three-phase system to a biphasic system is the creation of a rotating electromagnetic field with equal magnetomotive forces. The three-phase to biphasic transformation results in a family of models for the asynchronous machine, where stator and rotor quantities are projected onto two quadrature axes. The concept of this transformation is based on the idea that a rotating field created by a balanced three-phase system can also be replicated by a two-phase system with two coils spaced by π/2 in space, powered by currents phased by π/2 in time. The objective is to ensure the conservation of magnetomotive forces and instantaneous power.

In our study, for the sake of simplicity, we initially establish a model where the quantities are in the stator-fixed reference frame. Thus, the equivalent winding for the three stator phases consists of two windings in the direct axis α_s and in the quadrature axis β_s. The direct axis α_s coincides with the axis of the first stator phase. Similarly, at the rotor, two windings α_s and β_s are substituted for the equivalent three-phase windings.

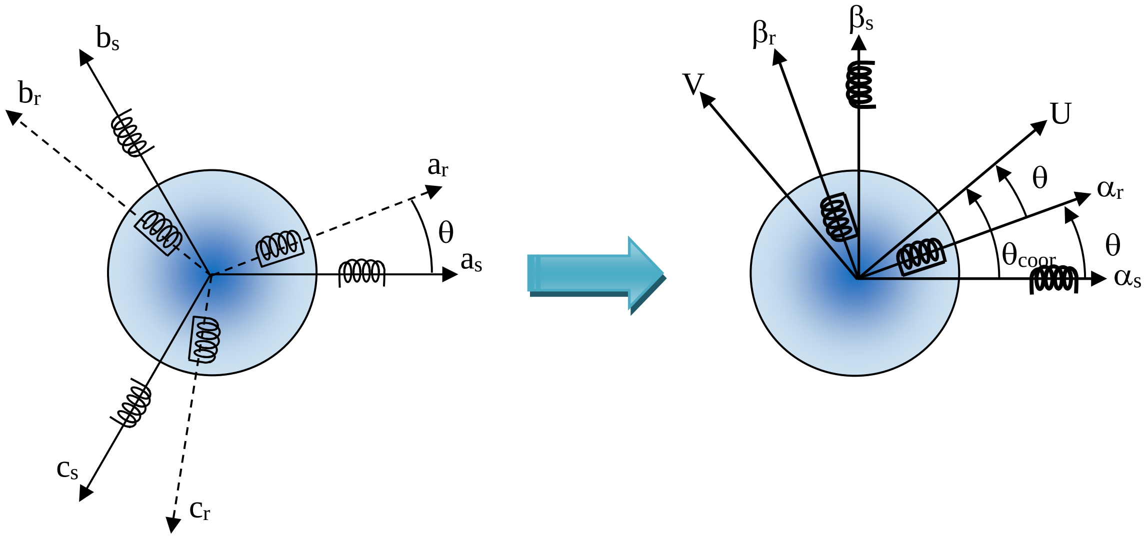


Figure V.3: Representation of the Three-Phase to Biphasic Transformation

V.4.1 Park Transformation:

The Park transformation consists of a three-phase to two-phase transformation followed by a rotation. It facilitates the transition from the (abc) reference frame to the (αβ) frame and subsequently to the (dq) frame. Additionally, it enables a direct transformation from the (abc) frame to the (dq) frame. The (αβ) frame remains fixed relative to the stator's (abc) frame, while the (dq) frame is mobile.

This transformation is a crucial tool in the analysis and control of asynchronous machines. By transforming variables into the (dq) frame, which rotates at the synchronous speed, the dynamics of the system become simpler, aiding in the design and implementation of control strategies. The Park transformation thus plays a significant role in facilitating the understanding and control of asynchronous machines in electrical systems.

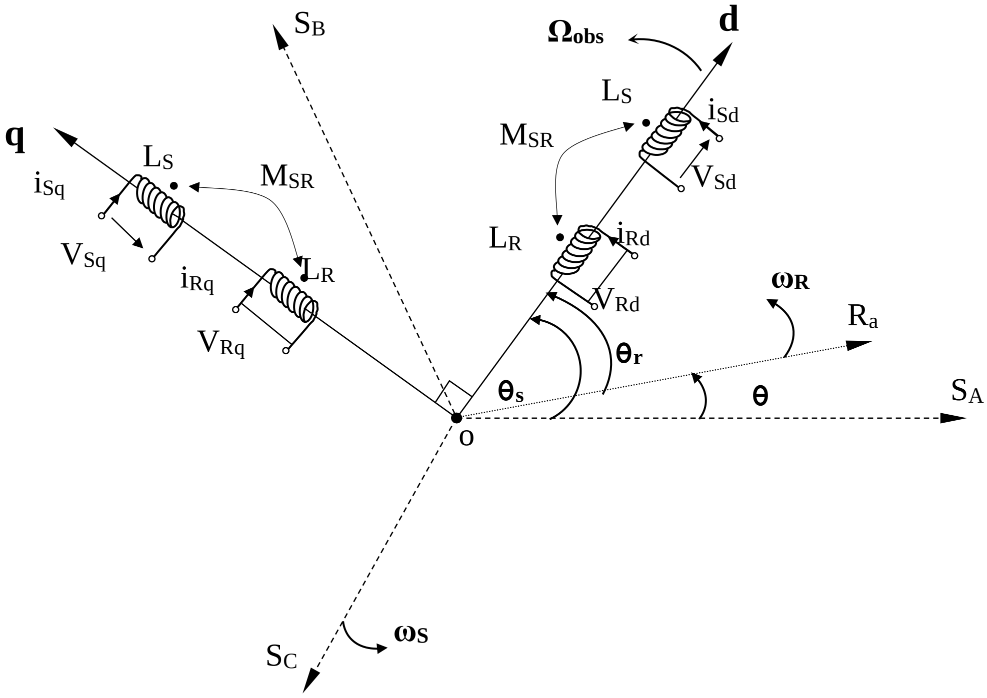


Figure V.4: Angular Positioning of Axis Systems in Electrical Space

In Figure V.4, the angular positioning of the axis systems in electrical space is depicted. It illustrates the necessity for the alignment of the Park transformation reference frames for stator and rotor quantities to simplify their equations. This alignment is achieved by connecting the angles θ_s and θ_r through the relationship:

$$\theta_s = \theta + \theta_r$$

So in this case, the normalized Park transformation is obtained using the Passage Matrix:

$$[P]^{-1}\left[V_{dq0}\right] = [R][P]^{-1}\left[I_{dq0}\right] + \frac{d}{dt}([P]^{-1}\left[\phi_{dq0}\right] \qquad \text{(V.11)}$$

With :

$$[P] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad \text{(V.12)}$$

This matrix is orthogonal, meaning dire $[P(\theta)]^{t} = [P(\theta)]^{-1}$. The Park transformation can be applied to voltages, currents, and fluxes.

The change of variable related to currents, voltages, and fluxes is defined by the Transformation:

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = P(\theta) \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \qquad \text{(V.13)}$$

With x representing voltage, current, or flux, and the following indices representing:

« o » Homopolar axis index.

« d » : Direct axis index.

« q » : Quadrature axis index.

The inverse matrix of the normalized Park transformation is expressed as:

$$[P(\theta)]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \sin\theta & \frac{1}{\sqrt{2}} \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\theta - \frac{4\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad \text{(V.14)}$$

$$\left[V_{dq0}\right] = [R]\left[I_{dqo}\right] + \frac{d}{dt}\left[\emptyset_{dqo}\right] + [P](\frac{d}{dt}[P]^{-1})\left[\emptyset_{dqo}\right] \qquad \text{(V. 15)}$$

We demonstrate that:

$$[P](\frac{d}{dt}[P]^{-1}) = \frac{d\theta}{dt} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{(V. 16)}$$

We finally obtain the system of PARK equations. Which thus constitutes a dynamic electrical model for the equivalent two-phase winding:

$$V_d = RI_d + \frac{d\emptyset_d}{dt} - \left(\frac{d\theta}{dt}\right)\emptyset_q \qquad \text{(V. 17)}$$

$$V_q = RI_q + \frac{d\emptyset_q}{dt} + \left(\frac{d\theta}{dt}\right)\emptyset_d \qquad \text{(V. 18)}$$

$$V_o = RI_o + \frac{d\emptyset_o}{dt} \qquad \text{(V. 19)}$$

For the reduction of the inductance matrix, the proposed transformations establish relationships between the fluxes of the d, q, and o axes and the fluxes of the a, b, c axes::

$$[\emptyset_{sdqo}] = [P(\theta_s)][\emptyset_{sabc}] \qquad \text{(V.20)}$$

$$[\emptyset_{rdqo}] = [P(\theta_r)][\emptyset_{rabc}] \qquad \text{(V.21)}$$

After the calculation, we obtain:

$$\begin{bmatrix} \emptyset_{ds} \\ \emptyset_{qs} \\ \emptyset_{os} \\ \emptyset_{dr} \\ \emptyset_{qr} \\ \emptyset_{or} \end{bmatrix} = \begin{bmatrix} I_s - M_s & 0 & 0 & \frac{3}{2}M_{sr} & 0 & 0 \\ 0 & I_s - M_s & 0 & 0 & \frac{3}{2}M_{sr} & 0 \\ 0 & 0 & I_s + 2M_s & 0 & 0 & 0 \\ \frac{3}{2}M_{sr} & 0 & 0 & I_r - M_r & 0 & 0 \\ 0 & \frac{3}{2}M_{sr} & 0 & 0 & I_r - M_r & 0 \\ 0 & 0 & 0 & 0 & 0 & I_r + 2M_r \end{bmatrix} \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_{os} \\ I_{dr} \\ I_{qr} \\ I_{or} \end{bmatrix} \quad (V.22)$$

$L_s = I_s - M_s$: Stator cyclic inductance.

$L_r = I_r - M_r$: Rotor cyclic inductance.

$M = \frac{3}{2}M_{sr}$: Cyclic mutual inductance between stator and rotor..

The usual mode of powering the stator and the structure of the rotor windings conferring zero to the sums of the stator currents and rotor currents, the components of index (0) are zero.

Under these operating conditions in non-degraded mode, the *d* and *q* axis flows are simply defined by the three constant parameters *Ls, Lr, M*, and linked to the currents by the relation:

$$\begin{bmatrix} \emptyset_{ds} \\ \emptyset_{qs} \\ \emptyset_{dr} \\ \emptyset_{qs} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_{dr} \\ I_{qs} \end{bmatrix} \quad (V.23)$$

V.4.1.1. Electrical equation

The Park equations of the voltages, stator and rotor are written:

$$\begin{cases} V_{ds} = R_s I_{ds} + \dfrac{d\emptyset_{ds}}{dt} - \dfrac{d\theta_s}{dt}\emptyset_{qs} \\ V_{qs} = R_s I_{qs} + \dfrac{d\emptyset_{qs}}{dt} + \dfrac{d\theta_s}{dt}\emptyset_{ds} \\ V_{dr} = R_r I_{dr} + \dfrac{d\emptyset_{dr}}{dt} + \dfrac{d\theta_r}{dt}\emptyset_{qr} = 0 \\ V_{qr} = R_r I_{qr} + \dfrac{d\emptyset_{qr}}{dt} + \dfrac{d\theta_r}{dt}\emptyset_{dr} = 0 \end{cases} \quad (V.24)$$

In the PARK frame (d q) rotating at the angular speed $\omega_s = \frac{d\theta_s}{dt}$ the following equation:

$$\begin{cases} V_{ds} = R_s I_{ds} + \dfrac{d\emptyset_{ds}}{dt} - \omega_s \emptyset_{qs} \\ V_{qs} = R_s I_{qs} + \dfrac{d\emptyset_{qs}}{dt} + \omega_s \emptyset_{ds} \end{cases} \quad (V.25)$$

$$\begin{cases} R_r I_{dr} + \dfrac{d\emptyset_{dr}}{dt} - (\omega_s - \omega)\emptyset_{qr} = 0 \\ R_r I_{qr} + \dfrac{d\emptyset_{qr}}{dt} + (\omega_s - \omega)\emptyset_{dr} = 0 \end{cases} \quad (V.26)$$

V.4.1.2. Magnetic equations

With the flux :

$$\begin{cases} \emptyset_{ds} = L_s I_{ds} + M I_{dr} \\ \emptyset_{qs} = L_s I_{qs} + M I_{qr} \end{cases} \quad (V.27)$$

$$\begin{cases} \emptyset_{dr} = L_r I_{ds} + M I_{ds} \\ \emptyset_{qr} = L_r I_{qr} + M I_{qs} \end{cases} \qquad (V.28)$$

V.4.1.3. Mechanical equations

The equation of torque and that of movement is written:

$$T_e = PM\left[I_{qs} I_{dr} - I_{ds} I_{qr}\right] \qquad (V.29)$$

And :

$$\frac{Jd\omega}{Pdt} = T_e - T_L - \frac{f}{P}\omega \qquad (V.30)$$

J: Moment of inertia of the rotor.

f: Viscous friction coefficient.

P: Number of pole pairs.

T_e: Electromagnetic torque.

T_L: Resistant torque.

V.4.2 Choice of dq Frame

Up to this point, we have expressed the equations and quantities of the machine in a dq frame that makes an electrical angle θ_s with the stator and also makes an electrical angle θ_r with the rotor but is otherwise undefined, meaning it is free.

There are three important choices. We can fix the dq frame to the stator, the rotor, or the rotating field. It is worth noting that the dq frame is the mobile frame, meaning it is our responsibility to calculate the angles of the Park transformations θ_s and θ_r to perform the rotations.

V.4.2.1. Fixed Reference Frame with Respect to the Stator

This choice is characterized by the conditions:

$$\theta_s = 0 \qquad , \qquad \theta_r = -\theta$$

$$\frac{d\theta_s}{dt} = 0 \qquad , \qquad \frac{d\theta_r}{dt} = -\frac{d\theta}{dt}$$

$$\omega_s = 0 \qquad , \qquad \omega_r = -\omega$$

The electrical equations take the form:

$$\begin{cases} V_{ds} = R_s I_{ds} + \dfrac{d\emptyset_{ds}}{dt} \\ V_{qs} = R_s I_{qs} + \dfrac{d\emptyset_{qs}}{dt} \\ 0 = R_r I_{ds} + \dfrac{d\emptyset_{dr}}{dt} + \omega\emptyset_{qr} \\ 0 = R_r I_{qr} + \dfrac{d\emptyset_{qr}}{dt} - \omega\emptyset_{dr} \end{cases} \qquad (V.31)$$

After arranging the equations with (I_{ds} , I_{qs} , I_{dr} , I_{qr}) we end up with:

$$\begin{cases} \frac{dI_{ds}}{dt} = -\frac{1}{T_s\sigma} I_{ds} + \frac{M^2}{L_sL_r\sigma}\omega I_{qs} + \frac{M}{L_sT_r\sigma} I_{dr} + \frac{M}{L_s\sigma}\omega I_{qr} + \frac{V_{ds}}{L_s\sigma} \\ \frac{dI_{qs}}{dt} = -\frac{M^2}{L_sL_r\sigma}\omega I_{ds} - \frac{1}{T_s\sigma} I_{qs} - \frac{M}{L_s\sigma}\omega I_{dr} + \frac{M}{L_sT_r\sigma} I_{qr} + \frac{V_{qs}}{L_s\sigma} \\ \frac{dI_{dr}}{dt} = \frac{M}{L_rT_s\sigma} I_{ds} - \frac{M}{L_r\sigma}\omega I_{qs} - \frac{1}{T_r\sigma} I_{dr} - \frac{1}{\sigma}\omega I_{qr} - \frac{M}{L_sL_r\sigma} V_{ds} \\ \frac{dI_{qr}}{dt} = \frac{M}{L_r\sigma}\omega I_{ds} + \frac{M}{L_rT_{s\sigma}} I_{qs} + \frac{1}{\sigma}\omega I_{dr} - \frac{1}{T_r\sigma} I_{qr} - \frac{M}{L_sL_r\sigma} V_{qs} \end{cases} \quad (V.32)$$

With:

$\sigma = 1 - \frac{M^2}{I_sI_r}$: The coefficient of the total leakage.

$T_s = \frac{I_s}{r_s}$: Stator time constant.

$T_r = \frac{I_r}{r_r}$: Rotor time constant.

V.4.2.2. Fixed Reference Frame with Respect to the Rotor

This reference frame is preferably chosen to study variations in stator quantities. It is characterized by the conditions:

$\theta_s = 0 \quad , \quad \theta_r = 0$

$\frac{d\theta_s}{dt} = \frac{d\theta}{dt} \quad , \quad \frac{d\theta_r}{dt} = 0$

$\omega_r = 0 \quad , \quad \omega_s = \omega$

The electrical equations take the form

$$\begin{cases} V_{ds} = R_sI_{ds} + \frac{d\emptyset_{ds}}{dt} - \omega_s\emptyset_{qs} \\ V_{qs} = R_sI_{qs} + \frac{d\emptyset_{qs}}{dt} + \omega_s\emptyset_{ds} \\ 0 = R_rI_{dr} + \frac{d\emptyset_{dr}}{dt} \\ 0 = R_rI_{qr} + \frac{d\emptyset_{qr}}{dt} \end{cases} \quad (V.33)$$

After arranging the equations, about this:

$$\begin{cases} \frac{dI_{ds}}{dt} = -\frac{1}{T_s\sigma} I_{ds} + \frac{1}{\sigma}\omega I_{qs} + \frac{M}{L_sT_r\sigma} I_{dr} + \frac{M}{L_s\sigma}\omega I_{qr} + \frac{V_{ds}}{L_s\sigma} \\ \frac{dI_{qs}}{dt} = -\frac{1}{\sigma}\omega I_{ds} - \frac{1}{T_s\sigma} I_{qs} - \frac{M}{L_s\sigma}\omega I_{dr} + \frac{M}{L_sT_r\sigma} I_{qr} + \frac{V_{qs}}{L_s\sigma} \\ \frac{dI_{dr}}{dt} = \frac{M}{L_rT_s\sigma} I_{ds} - \frac{M}{L_r\sigma}\omega I_{qs} - \frac{1}{T_r\sigma} I_{dr} - \frac{M^2}{L_sL_r\sigma}\omega I_{qr} - \frac{M}{L_sL_r\sigma} V_{ds} \\ \frac{dI_{dr}}{dt} = \frac{M}{L_r\sigma}\omega I_{ds} + \frac{M}{L_rT_s\sigma} I_{qs} + \frac{M^2}{L_sL_r\sigma}\omega I_{dr} - \frac{1}{T_r\sigma} I_{qr} - \frac{M}{L_sL_r\sigma} V_{qs} \end{cases} \quad (V.34)$$

V.4.2.3. Fixed Reference Frame with Respect to the Rotating Field

This reference frame is the only one that does not introduce simplifications into the formulation of equations. It matches continuous quantities with sinusoidal quantities in steady-state conditions, which is why this reference frame is used in control.

It is characterized by the conditions:

$$\frac{d\theta_s}{dt} = \omega_s$$

$$\frac{d\theta_r}{dt} = \omega_s - \omega = \omega_r$$

The electrical equations take the form :

$$\begin{cases} V_{ds} = R_s I_{ds} + \dfrac{d\emptyset_{ds}}{dt} - \omega_s \emptyset_{qs} \\ V_{qs} = R_s I_{qs} + \dfrac{d\emptyset_{qs}}{dt} + \omega_s \emptyset_{ds} \\ 0 = R_r I_{dr} + \dfrac{d\emptyset_{dr}}{dt} - \omega_r \emptyset_{qr} \\ 0 = R_r I_{qr} + \dfrac{d\emptyset_{qr}}{dt} + \omega_r \emptyset_{dr} \end{cases} \qquad (V.35)$$

After arranging the equations, we arrive at:

$$\begin{cases} \dfrac{dI_{ds}}{dt} = \dfrac{1}{T_s\sigma} I_{ds} + \left(\omega_r + \dfrac{1}{\sigma}\omega\right) I_{qs} + \dfrac{M}{L_s T_r \sigma} I_{dr} + \dfrac{M}{L_s\sigma}\omega I_{qr} + \dfrac{1}{L_s\sigma} V_{ds} \\ \dfrac{dI_{dr}}{dt} = -\left(\omega_r + \dfrac{1}{\sigma}\omega\right) I_{ds} - \dfrac{1}{T_s\sigma} I_{qs} - \dfrac{M}{L_s\sigma}\omega I_{dr} + \dfrac{M}{L_s T_r\sigma} I_{qr} + \dfrac{1}{L_s\sigma} V_{qs} \\ \dfrac{dI_{dr}}{dt} = \dfrac{M}{L_r T_s\sigma} I_{ds} - \dfrac{M}{L_r\sigma}\omega I_{qs} - \dfrac{1}{T_r\sigma} I_{dr} + \left(\omega_r - \dfrac{M^2}{L_s L_r\sigma}\omega\right) I_{qr} - \dfrac{M}{L_s L_r\sigma} V_{ds} \\ \dfrac{dI_{qr}}{dt} = \dfrac{M}{L_r\sigma}\omega I_{ds} + \dfrac{M}{L_r T_s\sigma} I_{qs} + \left(-\omega_r + \dfrac{M^2}{L_s L_r\sigma}\omega\right) I_{dr} - \dfrac{1}{T_r\sigma} I_{qr} - \dfrac{M}{L_s L_r\sigma} V_{qs} \end{cases} \qquad (V.36)$$

V.5- State Equation Formulation for Control:

The generalized form of the system equations in state-space form is:

$$\begin{cases} \dot{X}(t) = A\,X(t) + B\,U(t) \\ Y(t) = C\,X(t) + D\,U(t) \end{cases} \qquad \text{...(V.37)}$$

– X(t) : State vector of the system with dimension n.
– U(t) : Input vector or control vector of the system with dimensionl.
– Y(t) : Output vector of the system with dimension m.
– A : State matrix of the system with dimension n × n.

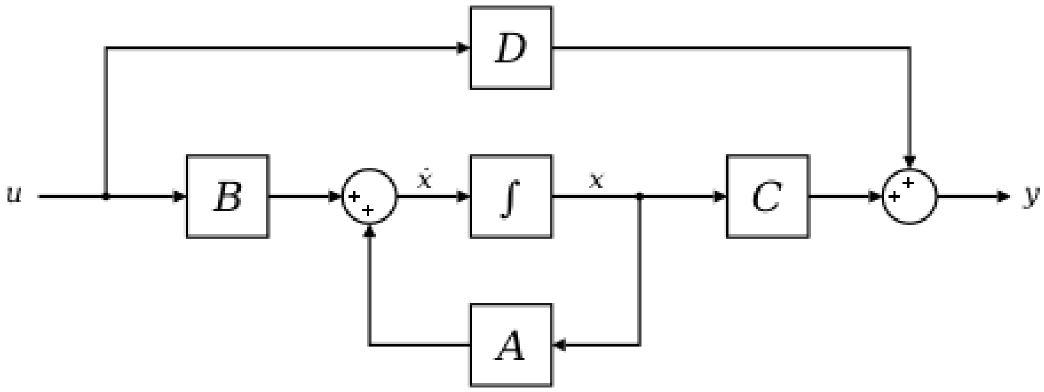


Figure.V.5: Representation of the dynamic state model under functional diagram

– B : input or control matrix of the system of dimension n × l..
– C : output matrix of the system of dimension m × n.
– D : direct transmission matrix of the system of dimension m × l.

The state vector, input vector or output vector will be chosen for control or modeling purposes, then we have the models as follows:

Let us consider the tensions Vds, Vqs, Vdr and Vqr as control quantities, the stator and rotor currents Ids, Iqs, Idr and Iqr as state variables. From the equations for the fixed reference case with respect to the rotating field (36), we can write the system of equations after we pose that:

$T_s = \frac{L_s}{R_s}$: Stator time constant.

$T_r = \frac{L_r}{R_r}$: Rotor time constant.

$\sigma = 1 - \frac{M^2}{L_s.L_r}$: the blondel dispersion coefficient.

The state vector:

$[x] = \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_{dr} \\ I_{qr} \end{bmatrix}$ and

The state matrix A is written as follows::

$$[A] = \begin{bmatrix} -\frac{1}{T_s\sigma} & \left(\omega_r + \frac{1}{\sigma}\omega\right) & \frac{M}{L_sT_r\sigma} & \frac{M}{L_s\sigma}\omega \\ -\left(\omega_r + \frac{1}{\sigma}\omega\right) & -\frac{1}{T_s\sigma} & -\frac{M}{L_s\sigma}\omega & \frac{M}{L_sT_r\sigma} \\ \frac{M}{L_rT_s\sigma} & -\frac{M}{L_r\sigma}\omega & -\frac{1}{T_r\sigma} & \left(\omega_r - \frac{M^2}{L_sL_r\sigma}\omega\right) \\ \frac{M}{L_r\sigma}\omega & \frac{M}{L_rT_s\sigma} & \left(-\omega_r + \frac{M^2}{L_sL_r\sigma}\omega\right) & -\frac{1}{T_r\sigma} \end{bmatrix}$$

We can write the matrix [A] as follows:

$[A] = [A_1] + \omega[A_2] + \omega_r[A_3]$ ...(V.38)

So:

$$[A]=\begin{bmatrix} -\frac{1}{T_s\sigma} & 0 & \frac{M}{L_sT_r\sigma} & 0 \\ 0 & -\frac{1}{T_s\sigma} & 0 & \frac{M}{L_sT_r\sigma} \\ \frac{M}{L_rT_s\sigma} & 0 & -\frac{1}{T_r\sigma} & 0 \\ 0 & \frac{M}{L_rT_s\sigma} & 0 & -\frac{1}{T_r\sigma} \end{bmatrix}+\omega\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}+$$

$$\omega_r\begin{bmatrix} 0 & \frac{1}{\sigma} & 0 & \frac{M}{L_s\sigma} \\ \frac{-1}{\sigma} & 0 & -\frac{M}{L_s\sigma} & 0 \\ 0 & -\frac{M}{L_r\sigma} & 0 & -\frac{M^2}{L_sL_r\sigma} \\ \frac{M}{L_r\sigma} & 0 & \frac{M^2}{L_sL_r\sigma} & 0 \end{bmatrix}..(V.38)$$

The system control vector

$$[U]=[V]=\begin{bmatrix} V_{ds} \\ V_{qs} \\ 0 \\ 0 \end{bmatrix}...(V.39)$$

$$[B]=\begin{bmatrix} \frac{1}{L_s\sigma} & 0 \\ 0 & \frac{1}{L_s\sigma} \\ -\frac{M}{L_sL_r\sigma} & 0 \\ 0 & -\frac{M}{L_sL_r\sigma} \end{bmatrix}...(V.40)$$

When we consider the state vector as the output vector of the system Y,

we must add [C] [D] : $C=\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ et D=0

V.6- Power and torque equations

In the general case, the instantaneous electrical power P_e supplied to the stator and rotor windings is expressed as a function of the axis quantities d, q:

$$P_e = V_{sd}I_{sd} + V_{sq}I_{sq} + V_{rd}I_{rd} + V_{rq}I_{rq} \qquad (V.41)$$

It is broken down into three series of terms which correspond respectively to the three columns of the tension equations:

- Power dissipated in joule losses:

$$P_j = R_s\left(I_{sd}^2 + I_{sq}^2\right) + R_r\left(I_{rd}^2 + I_{rq}^2\right) \qquad (V.42)$$

- Power representing the exchanges of electromagnetic energy with the sources:

$$P_{élm} = I_{sd}\left(\frac{d\emptyset_{sd}}{dt}\right) + I_{sq}\left(\frac{d\emptyset_{sq}}{dt}\right) + I_{rd}\left(\frac{d\emptyset_{rd}}{dt}\right) + I_{rq}\left(\frac{d\emptyset_{rq}}{dt}\right)$$

➢ Mechanical power Pm bringing together all the terms linked to the derivatives of angular positions :

$$P_m = \left(\emptyset_{sd} I_{sq} - \emptyset_{sq} I_{sd}\right)\left(\frac{d\theta_s}{dt}\right) + \left(\emptyset_{rd} I_{rq} - \emptyset_{rq} I_{rd}\right)\left(\frac{d\theta_r}{dt}\right) \qquad (V.43)$$

Taking into account the flux equations we can therefore write that:

$$P_m = \left(\emptyset_{sd} I_{sq} - \emptyset_{sq} I_{sd}\right)\left(\frac{d(\theta_s - \theta_r)}{dt}\right) \qquad (V.44)$$

The mechanical power is also equal to Te. Ω or Te.ω/p, from which the expression for the torque is derived:

$$T_e = P\left(\emptyset_{sd} I_{sq} - \emptyset_{sq} I_{sd}\right) \qquad (V.45)$$

Or

$$T_e = \frac{PM}{Lr}\left(\emptyset_{rd} I_{qs} - \emptyset_{rq} I_{ds}\right) \qquad (V.46)$$

V.7- Conclusion

In conclusion, the modeling of both three-phase and biphasic asynchronous machines has been successfully demonstrated through the application of the Park transformation. Subsequently, the formulation of the state equation for control purposes and its representation in the form of a functional diagram have been comprehensively presented.

These mathematical models of the asynchronous machine provide a valuable framework for simulation, enabling the creation of a model that closely approximates the real-world machine. Additionally, they offer the foundation for implementing appropriate control techniques.

By leveraging these models and control techniques, engineers and researchers can gain insights into the dynamic behavior of asynchronous machines, facilitating the design and implementation of effective control strategies for practical applications. This holistic understanding, from modeling to control, contributes to advancements in the field of electrical machines and systems.

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Abstract

The course "Modeling and Simulation of Electrical Machines" provides an extensive framework for developing mathematical models and simulating the behavior of electrical machines under various operating conditions. The course highlights the critical role of modeling and simulation in analyzing, optimizing, and understanding machine dynamics. It begins with General Modeling of Electrical Machines, introducing magnetic circuits, field and flux relationships, magnetomotive force (MMF), inductance, and mutual coupling as fundamental concepts. The module on Modeling Machines for Dynamic State focuses on deriving mathematical models to study transient and steady-state behaviors. The course also explores Modeling and Simulation of Direct Current Machines, emphasizing their electrical, magnetic, and mechanical models. Additionally, the Modeling and Simulation of Synchronous Machines includes Park and Clarke transformations to simplify three-phase systems into d-q models for dynamic and steady-state analysis. Similarly, the Modeling and Simulation of Induction Machines applies Park and Clarke transformations to derive mathematical models that analyze electrical, magnetic, and mechanical interactions. By integrating theoretical concepts with practical simulation techniques, this course equips students with the skills to model, simulate, and understand the behavior of electrical machines, preparing them for advanced research and real-world engineering challenges.

Keywords: Electrical machine modeling; Dynamic simulation; Park and Clarke transformations; Mathematical models; Synchronous and induction machines; Electrical and mechanical dynamics.

Resumé

Le cours "Modélisation et Simulation des Machines Électriques" offre un cadre détaillé pour le développement de modèles mathématiques et la simulation des performances des machines électriques, en mettant l'accent sur leur comportement dans diverses conditions de fonctionnement. Il souligne l'importance de la modélisation et de la simulation comme outils essentiels pour analyser, optimiser et comprendre la dynamique des machines. Le cours commence par la Modélisation Générale des Machines Électriques, introduisant les circuits magnétiques, les relations de champ et de flux, la force magnétomotrice (MMF), l'inductance et le couplage mutuel comme concepts fondamentaux. Dans le module Modélisation des Machines en Régime Dynamique, les étudiants développent des modèles mathématiques pour étudier le comportement transitoire et en régime permanent des machines électriques. Le cours explore également la Modélisation et Simulation des Machines à Courant Continu, mettant l'accent sur le développement de leurs modèles électriques, magnétiques et mécaniques. De plus, le module Modélisation et Simulation des Machines Synchrones couvre l'application des transformations de Park et Clarke pour simplifier les systèmes triphasés en modèles d-q, permettant l'analyse de leur performance dynamique et en régime permanent. De manière similaire, le module Modélisation et Simulation des Machines Asynchrones intègre les transformations de Park et Clarke pour dériver des modèles mathématiques analysant les interactions électriques, magnétiques et mécaniques dans ces machines. En combinant des principes théoriques et des techniques pratiques de simulation, ce cours dote les étudiants des compétences nécessaires pour modéliser, simuler et comprendre le comportement des machines électriques, renforçant ainsi leur expertise pour des applications en recherche et en ingénierie réelle.

Mots clés : Modélisation des machines électriques ; Simulation dynamique ; Transformations de Park et Clarke ; Modèles mathématiques ; Simulation des machines synchrones et asynchrones ; Dynamiques électriques et mécaniques

الملخص:

يقدم مقرر "نمذجة ومحاكاة الآلات الكهربائية" إطارًا شاملاً لتطوير النماذج الرياضية ومحاكاة أداء الآلات الكهربائية، مع التركيز على سلوكها في ظروف التشغيل المختلفة. يؤكد المقرر على أهمية النمذجة والمحاكاة كأدوات أساسية لتحليل وتحسين وفهم ديناميكيات الآلات. يبدأ المقرر بـ النمذجة العامة للآلات الكهربائية، حيث يتم تقديم الدوائر المغناطيسية وعلاقات المجال والتدفق، وقوة المحرك المغناطيسي(MMF) ، والحث، والاقتران المتبادل كمفاهيم أساسية. في وحدة نمذجة الآلات للحالة الديناميكية، يتم اشتقاق النماذج الرياضية لدراسة السلوك العابر والمستقر للآلات الكهربائية. يتعمق المقرر أيضًا في نمذجة ومحاكاة الآلات ذات التيار المستمر مع التركيز على تطوير نماذجها الكهربائية والمغناطيسية والميكانيكية. بالإضافة إلى ذلك، تغطي وحدة نمذجة ومحاكاة الآلات المتزامنة تطبيق تحويلات بارك وكلارك لتبسيط الأنظمة ثلاثية الطور إلى نماذجd-q ، مما يمكن من تحليل أدائها الديناميكي والحالة المستقرة. وبالمثل، تدمج وحدة نمذجة ومحاكاة الآلات الحثية تحويلات بارك وكلارك لاشتقاق النماذج الرياضية لتحليل التفاعلات الكهربائية والمغناطيسية والميكانيكية في هذه الآلات. من خلال الجمع بين المفاهيم النظرية وتقنيات المحاكاة العملية، يزود المقرر الطلاب بالمهارات اللازمة لنمذجة ومحاكاة وفهم سلوك الآلات الكهربائية، مما يعزز خبراتهم لتطبيقها في البحث المتقدم والسيناريوهات الهندسية الواقعية

الكلمات المفتاحية: نمذجة الآلات الكهربائية، المحاكاة الديناميكية، تحويلات بارك وكلارك، النمذجة الرياضية،محاكاة الآلات المتزامنة والحثية،الديناميكيات الكهربائية والميكانيكية.