ASYMPTOTIC STABILITY OF A PROBLEM WITH KELVIN-VOIGT TERM AND BALAKRISHNAN-TAYLOR DAMPING.

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Abstract. The purpose of this work is to study the energy Decay of solutions for a nonlinear equation of the kelving voigt type with balakrishnan taylor damping and acoustic boundary in a bounded domain in $\mathbb{R}^n$.

1. Introduction

we investigate for the following problem

$$\left|u'\right|^m u'' = \left(a^2 + b\int_\Omega |\nabla u|^2\, dx + \sigma \int_\Omega \nabla u.\nabla u'\, dx\right)\Delta u + 2\lambda\Delta u' \quad \text{in } \Omega\times\mathbb{R}^+, \tag{1}$$

$$u = 0 \quad \text{in } \Gamma_0\times\mathbb{R}^+, \tag{2}$$

$$\left(a^2 + b\int_\Omega |\nabla u|^2\, dx + \sigma \int_\Omega \nabla u.\nabla u'\, dx\right)\frac{\partial u}{\partial v} + 2\lambda\frac{\partial u'}{\partial v} = y' \quad \text{in } \Gamma_1\times\mathbb{R}^+, \tag{3}$$

$$u' + p(x)y' + q(x)y = 0 \quad \text{in } \Gamma_1\times\mathbb{R}^+, \tag{4}$$

$$u(0) = v_0,\ u'(0) = u_1 \quad \text{in } \Omega, \tag{5}$$

$$y(0) = y_0 \ \text{ in } \Gamma_1. \tag{6}$$

where Ω a bounded, connected set in $\mathbb{R}^n (n \geq 1)$ having a smooth boundary $\Gamma = \partial\Omega$ consisting of two parts Γ_0 and Γ_1 such that $\overline{\Gamma_0}\cup\overline{\Gamma_1} = \Gamma$. Primes denote the time derivative, Δ the Laplacian in $\mathbb{R}^n$ taken in space variables, v the unit

normal of Γ pointing towards exterior of Ω and $\mathbb{R}^+ = (0,\infty)$ The parameters $\lambda > 0$ is a small internal material damping coefficient, $a > 0$, $b > 0$, $\sigma > 0$. are constant real numbers p and q are functions satisfying $p(x) > 0$, $q(x) > 0$ fo rall $x\in\Gamma_1$.

Theorem 1.1. *For the initial data $(u_0, u_1, y_0) \in \left(V\cap H^2(\Omega)\right)\times V\times L^2(\Gamma_1)$,there exists a unique pair of functions $(u; y)$, which is a solution to the problem (1)-(6) in the class*

$$\begin{array}{c} u \in L^\infty\left(0, T; V\times H^2(\Omega)\right),\ u_t \in L^\infty(0, T; V) \\ u_{tt} \in L^\infty\left(0, T; \times L^2(\Omega)\right), y, y_t \in L^\infty\left(0, \infty; L^2(\Gamma_1)\right). \end{array} \tag{7}$$

We defined the functional energyof system (1-6) by

$$E(t) = \frac{1}{m+2}\left\|u_t(x,t)\right\|_{m+2}^{m+2} + \frac{1}{2}\left(a^2 + \frac{b}{2}\left\|\nabla u(x,t)\right\|_2^2\right)\left\|\nabla u(x,t)\right\|_2^2 + \frac{1}{2}\int_{\Gamma_1} q(x)(y(t))^2 d\Gamma. \tag{8}$$

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Lemma 1.2. *For every solution $u(x;t)$ of the system (1-6), the time derivative of the functional*

$$\Psi(t)=\frac{1}{m+1}\int_{\Omega}\left|u_t\right|^m u_t u dx+\lambda\int_{\Omega}\left|\nabla u\right|^2 dx+\frac{\sigma}{4}\left(\int_{\Omega}\left|\nabla u\right|^2 dx\right)^2+\frac{1}{2}\int_{\Gamma_1}p(x)y^2 d\Gamma+\int_{\Gamma_1}uyd\Gamma, \tag{9}$$

satisfies

$$\Psi^{'}(t)\leq\frac{1}{m+1}\left\|u_t\right\|_{m+2}^{m+2}-2\int_{\Gamma_1}uy_t d\Gamma-\int_{\Gamma_1}q(x)y^2 d\Gamma-a^2\left\|\nabla u\right\|_2^2-b\left\|\nabla u\right\|_2^4+\frac{\sigma}{4}\left(\frac{1}{2}\frac{d}{dt}\left\|\nabla u\right\|_2^2\right)^2. \tag{10}$$

Il existe deux constantes positives M_1, M_2, telles que

$$\left(1-\varepsilon M_1\right)E(t)\leq V(t)\leq\left(1+\varepsilon M_2\right)E(t),\ \forall t\geq 0, \tag{11}$$

où

$$0<\varepsilon<\frac{1}{M_1}.$$

Theorem 1.3. *If $u(x;t)$ is a regular solution of the system (1-6) with initial values $(u_0,u_1,y_0)\in\left(V\cap H^2(\Omega)\right)\times V\times L^2\left(\Gamma_1\right)$ then the energy $E(t)$ of the system de ned by (1-6) satis es*

$$E(t)<Me^{-\mu t}E(0),\ \ pour\ t\in(0,\infty), \tag{12}$$

for some real constants $M>1$ and $\mu>0$.

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