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Frictional contact problem between thermo-electro-elastic bodies with long-term memory

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Abstract

We study of a quasistatic frictional contact problem between two thermo-electroelastic bodies with adhesion. The temperature of the materials caused by elastic deformations. The contact is modelled with a version of normal compliance condition and the associated Coulomb's law of friction in which the adhesion of contact surfaces is taken into account. We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem. The proof is based on a classical existence and uniqueness result on parabolic equalities, differential equations and fixed point arguments.

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Keywords: thermo-electroelastic materials, adhesion, Coulomb's law of friction, normal compliance, fixed point.

1 Introduction

The adhesive contact between deformable bodies, when a glue is added to prevent relative motion of the surfaces, has received recently increased attention in the mathematical literature. Analysis of models for adhesive contact can be found in [1, 4, 9, 12] and recently in the monographs [7, 8]. The novelty in all these papers is the introduction of a surface internal variable, the bonding field, denoted in this paper by β, it describes the point wise fractional density of adhesion of active bonds on the contact surface, and some times referred to as the intensity of adhesion. Following [2], the bonding field satisfies the restriction $0 \leq \beta \leq 1$, when $\beta = 1$ at a point of the contact surface, the adhesion is complete and all the bonds are active, when $\beta = 0$ all the bonds are inactive, severed, and there is no adhesion, when $0 < \beta < 1$ the adhesion is partial and only a fraction β of the bonds is active. The aim of this paper is to study the quasistatic contact in thermo-electroelastic materials. For this, we use an thermo-electroelastic constitutive law with long-term memory given by

$$\boldsymbol{\sigma}^\ell = \mathcal{A}^\ell\big(\boldsymbol{\varepsilon}(\boldsymbol{u}^\ell), \theta^\ell\big) + \int_0^t \mathcal{Q}^\ell\big(t-s, \varepsilon(\boldsymbol{u}^\ell(s)), \theta^\ell(s)\big)\, ds - (\mathcal{E}^\ell)^* E^\ell(\varphi^\ell), \tag{1.1}$$

where $\boldsymbol{u}^\ell$ the displacement field, $\boldsymbol{\sigma}^\ell$ and $\boldsymbol{\varepsilon}(\boldsymbol{u}^\ell)$ represent the stress and the linearized strain tensor, respectively, θ^ℓ represents the absolute temperature and α^ℓ represents the damage field. Here $\mathcal{Q}^\ell$ is the relaxation operator, and $\mathcal{A}^\ell$ represents the thermo-elasticity operator with damage. $E(\varphi^\ell) = -\nabla\varphi^\ell$ is the electric field, $\mathcal{E}^\ell$ represents the third order piezoelectric tensor, $(\mathcal{E}^\ell)^*$ is its transposition. In this paper we study a quasistatic Coulomb's frictional contact problem between two thermo-electroelastic bodies with long-term memory. The contact is modelled with normal compliance where the adhesion of the contact surfaces is taken into account and is modelled with a surface variable, the bonding field. We derive a variational formulation of the problem and prove the existence of a unique weak solution. The paper is organized as follows. In section 2 we describe the mathematical models for the frictional contact problem between two thermo-electroelastic bodies with long-term memory. The contact is modelled with normal compliance and adhesion. We introduce some notation, list the assumptions on the problem's data, and derive the variational formulation of the model. We prove in section **??** the existence and uniqueness of the solution, where it is carried out in several steps and is based on a classical existence and uniqueness result on parabolic equalities, differential equations and fixed point arguments.

2 Problem statement

Let us consider two thermo-electro-elasto-viscoplastic bodies, occupying two bounded domains Ω^1, Ω^2 of the space $\mathbb{R}^d (d = 2, 3)$. For each domain Ω^ℓ, the boundary Γ^ℓ is assumed to be Lipschitz continuous, and is partitioned into three disjoint measurable parts Γ_1^ℓ, Γ_2^ℓ and Γ_3^ℓ, on one hand, and on two measurable parts Γ_a^ℓ and Γ_b^ℓ, on the other hand, such that $meas\Gamma_1^\ell > 0$, $meas\Gamma_a^\ell > 0$. Let $T > 0$ and let $[0, T]$ be the time interval of interest. The Ω^ℓ body is submitted to $\boldsymbol{f}_0^\ell$ forces and volume electric charges of density q_0^ℓ. The bodies are assumed to be clamped on $\Gamma_1^\ell \times (0, T)$. The surface tractions $\boldsymbol{f}_2^\ell$ act on $\Gamma_2^\ell \times (0, T)$. We also assume that the electrical potential vanishes on $\Gamma_a^\ell \times (0, T)$ and a surface electric charge of density q_2^ℓ is prescribed on $\Gamma_b^\ell \times (0, T)$. The two bodies can enter in contact along the common part $\Gamma_3^1 = \Gamma_3^2 = \Gamma_3$. The bodies is in adhesive contact over the surface Γ_3. The mechanical problem may be formulated as follows.

Problem P. For $\ell = 1, 2$, find a displacement field $\boldsymbol{u}^\ell : \Omega^\ell \times (0, T) \longrightarrow \mathbb{R}^d$, a stress field $\boldsymbol{\sigma}^\ell : \Omega^\ell \times (0, T) \longrightarrow \mathbb{S}^d$, an electric potential field $\varphi^\ell : \Omega^\ell \times (0, T) \longrightarrow \mathbb{R}$, a temperature $\theta^\ell : \Omega^\ell \times (0, T) \longrightarrow \mathbb{R}$, a bonding field $\beta : \Gamma_3 \times (0, T) \longrightarrow \mathbb{R}$ and a electric displacement field

$\boldsymbol{D}^\ell : \Omega^\ell \times (0,T) \longrightarrow \mathbb{R}^d$ such that

$$\boldsymbol{\sigma}^\ell = \mathcal{A}^\ell\big(\boldsymbol{\varepsilon}(\boldsymbol{u}^\ell), \theta^\ell\big) + \int_0^t \mathcal{Q}^\ell\big(t-s, \varepsilon(\boldsymbol{u}^\ell(s)), \theta^\ell(s)\big)\, ds - (\mathcal{E}^\ell)^* E^\ell(\varphi^\ell) \quad \text{in } \Omega^\ell \times (0,T), \tag{2.1}$$

$$\boldsymbol{D}^\ell = \mathcal{E}^\ell \varepsilon(\boldsymbol{u}^\ell) + \mathcal{G}^\ell E^\ell(\varphi^\ell) \quad \text{in } \Omega^\ell \times (0,T), \tag{2.2}$$

$$\dot{\theta}^\ell - \kappa_0^\ell \Delta\theta^\ell = \Theta^\ell\big(\boldsymbol{\sigma}^\ell, \boldsymbol{\varepsilon}(\boldsymbol{u}^\ell), \theta^\ell\big) + \rho^\ell \quad \text{in } \Omega^\ell \times (0,T), \tag{2.3}$$

$$\operatorname{Div} \boldsymbol{\sigma}^\ell + \boldsymbol{f}_0^\ell = 0 \quad \text{in } \Omega^\ell \times (0,T), \tag{2.4}$$

$$\operatorname{div} \boldsymbol{D}^\ell - q_0^\ell = 0 \quad \text{in } \Omega^\ell \times (0,T), \tag{2.5}$$

$$\boldsymbol{u}^\ell = 0 \quad \text{on } \Gamma_1^\ell \times (0,T), \tag{2.6}$$

$$\boldsymbol{\sigma}^\ell \boldsymbol{\nu}^\ell = \boldsymbol{f}_2^\ell \quad \text{on } \Gamma_2^\ell \times (0,T), \tag{2.7}$$

$$\sigma_\nu^1 = \sigma_\nu^2 \equiv \sigma_\nu, \text{ where } \sigma_\nu = -p_\nu([u_\nu]) + \gamma_\nu \beta^2 R_\nu([u_\nu]) \quad \text{on } \Gamma_3 \times (0,T), \tag{2.8}$$

$$\begin{cases} \boldsymbol{\sigma}_\tau^1 = -\boldsymbol{\sigma}_\tau^2 \equiv \boldsymbol{\sigma}_\tau, \\ \big\|\boldsymbol{\sigma}_\tau + \gamma_\tau \beta^2 \boldsymbol{R}_\tau([\boldsymbol{u}_\tau])\big\| \le \mu p_\nu([u_\nu]), \\ \big\|\boldsymbol{\sigma}_\tau + \gamma_\tau \beta^2 \boldsymbol{R}_\tau([\boldsymbol{u}_\tau])\big\| < \mu p_\nu([u_\nu]) \Rightarrow [\boldsymbol{u}_\tau] = 0, \\ \big\|\boldsymbol{\sigma}_\tau + \gamma_\tau \beta^2 \boldsymbol{R}_\tau([\boldsymbol{u}_\tau])\big\| = \mu p_\nu([u_\nu]) \Rightarrow \exists \lambda \ge 0 \\ \qquad \text{such that } \ \boldsymbol{\sigma}_\tau + \gamma_\tau \beta^2 \boldsymbol{R}_\tau([\boldsymbol{u}_\tau]) = -\lambda[\boldsymbol{u}_\tau] \end{cases} \quad \text{on } \Gamma_3 \times (0,T), \tag{2.9}$$

$$\dot{\beta} = -\Big(\beta\big(\gamma_\nu (R_\nu([u_\nu]))^2 + \gamma_\tau \left|\boldsymbol{R}_\tau([\boldsymbol{u}_\tau])\right|^2\big) - \varepsilon_a\Big)_+ \quad \text{on } \Gamma_3 \times (0,T), \tag{2.10}$$

$$\varphi^\ell = 0 \quad \text{on } \Gamma_a^\ell \times (0,T), \tag{2.11}$$

$$\boldsymbol{D}^\ell . \boldsymbol{\nu}^\ell = q_2^\ell \quad \text{on } \Gamma_b^\ell \times (0,T), \tag{2.12}$$

$$\kappa_0^\ell \frac{\partial^\ell \theta^\ell}{\partial \nu^\ell} + \lambda_0^\ell \theta^\ell = 0 \quad \text{on } \Gamma^\ell \times (0,T), \tag{2.13}$$

$$\boldsymbol{u}^\ell(0) = \boldsymbol{u}_0^\ell, \ \theta^\ell(0) = \theta_0^\ell \quad \text{in } \Omega^\ell, \tag{2.14}$$

$$\beta(0) = \beta_0 \quad \text{on } \Gamma_3. \tag{2.15}$$

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