Heat Exchangers

5.0

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Objectives

Complete students' knowledge and teach them new concepts such as transient heat transfer, conduction through fins in the presence of a heat source, as well as heat exchangers, and methods for calculating heat transfer equipment.

Introduction

Heat exchangers are critical components in numerous engineering and industrial applications, playing a pivotal role in efficiently transferring thermal energy between different media. This pedagogical booklet aims to provide a comprehensive and structured introduction to the fundamentals of heat transfer and the design of heat exchangers, emphasizing both theoretical concepts and practical applications to equip students and engineers with a deep understanding of this field.

The booklet begins by revisiting the core principles of heat transfer, including thermal conduction, convection, and radiation, with detailed explanations of Fourier's, Newton's, and Stefan-Boltzmann's laws, complemented by practical examples to illustrate these concepts. It then delves into the heat diffusion equation, analyzing heat transfer under steady-state and transient conditions, with a focus on multidimensional applications. The booklet also explores finned surfaces, their configurations, and efficiency, alongside an in-depth study of thermal convection (natural and forced) and key dimensionless numbers such as Nusselt, Prandtl, and Reynolds.

In its later chapters, the booklet focuses on heat exchangers without phase change, covering their theoretical foundations, various types (such as double-tube, plate, and shell-and-tube heat exchangers), and performance calculation methods, including the overall heat transfer coefficient, the log mean temperature difference method, and the effectiveness-NTU method. This booklet aims to equip learners with analytical and practical tools to confidently design and evaluate heat exchanger performance.

We hope this booklet serves as a valuable educational guide, enhancing your understanding of heat transfer principles and their application in the design and analysis of heat exchangers across diverse engineering fields.

Chapter 1: Reminders of the Laws of Heat Transfer

I

In this chapter we lay the groundwork for much of the material covered in the text. We do this by raising several questions:

What is heat transfer? How is heat transferred? Why is it important? One goal is to develop an appreciation of the fundamental concepts and principles underlying heat transfer processes.

A second goal is to illustrate how knowledge of heat transfer can be used with the first law of thermodynamics (**conservation of energy**) to solve problems in technology and society [1]*.

1. Heat transfer

1.1. What is heat transfer?

Definition: Heat transfer?

Heat transfer (or heat) is thermal energy in transit due to a spatial **temperature difference**.

Definition: Driving force?

Temperature difference as voltage difference in electric current as pressure difference in fluid flow. The rate depends on the **gradient ΔT**.

Whenever a temperature difference exists in a medium or between media, **heat transfer must occur**.

1.2. How is heat transferred?

As shown in **Figure 1.1**, we refer to different types of heat transfer processes as modes. When a temperature gradient exists in a stationary medium, which may be a solid or a fluid, we use the term **conduction** to refer to the heat transfer that will occur across the medium.

In contrast, the term **convection** refers to heat transfer that will occur between a surface and a moving fluid when they are at different temperatures.

The third mode of heat transfer is termed **thermal radiation**. All surfaces of finite temperature emit energy in the form of electromagnetic waves. Hence, in the absence of an intervening medium, there is net heat transfer by radiation between two surfaces at different temperatures [2]*.

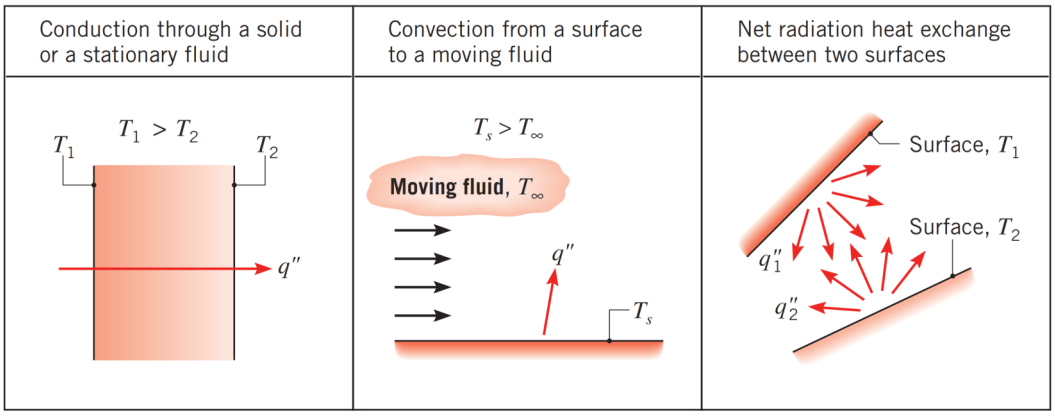


Figure 1.1: Conduction, convection, and radiation heat transfer modes.

2. Thermal conduction

2.1. Definition

Definition: Conduction

Heat (energy) transfer without **macroscopic** transport of matter.

Example:Seen as

Higher temperatures are associated with higher molecular energies. When neighboring molecules collide, as they are constantly doing, a transfer of energy from the more energetic to the less energetic molecules must occur. In the presence of a temperature gradient, energy transfer by conduction must then occur in the direction of decreasing temperature. This would be true even in the absence of collisions, as is evident from **Figure 1.2**.

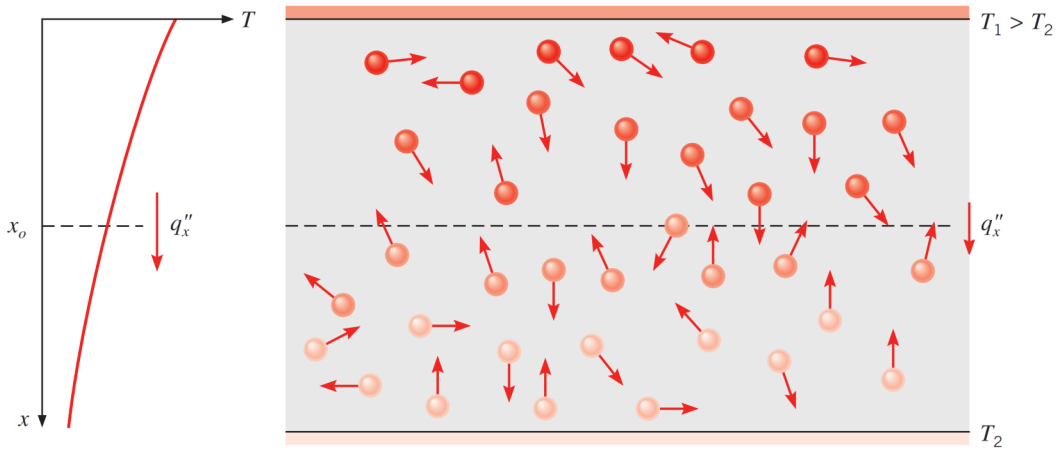


Figure 1.2: Association of conduction heat transfer with diffusion of energy due to molecular

2.2. Fourier's law (1822)

Heat transfer processes can be quantified in terms of appropriate **rate equations.** These equations may be used to compute the amount of energy being transferred per unit time.

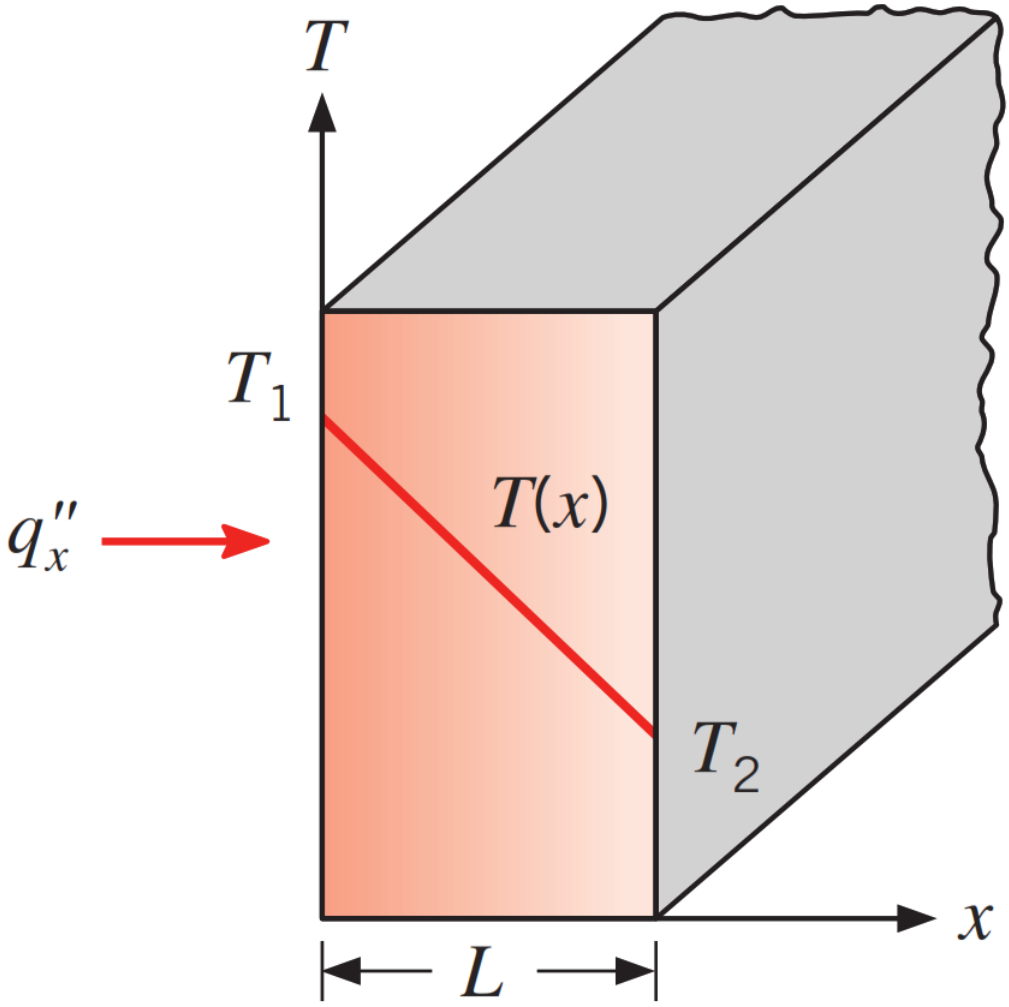


Figure 1.3: One-dimensional heat transfer by conduction (diffusion of energy).

For heat **conduction**, the rate equation is known as **Fourier's law**. For the one-dimensional plane wall shown in Figure 1.3, having a temperature distribution T(x), the rate equation is expressed as:

$$q''_x = -k\frac{dT}{dx} \quad (1.1)$$

-The **heat flux** q''_x(W/m^2) is the heat transfer rate in the x-direction per unit area perpendicular to the direction of transfer, and it is proportional to the temperature gradient, dT/dx, in this direction.

-The parameter k is a transport property known as the **thermal conductivity** (W/m. K) and is a characteristic of the wall material.

-The minus sign is a consequence of the fact that heat is transferred in the direction of decreasing temperature.

- Under the steady-state conditions shown in **Figure 1.3**, where the temperature distribution is linear, the temperature gradient may be expressed as:

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

and the heat flux is then:

$$q''_x = -k\frac{T_2 - T_1}{L}$$

or

$$q''_x = k\frac{T_1 - T_2}{L} = k\frac{\Delta T}{L} \quad (1.2)$$

Note

Note that this equation provides a **heat flux** , that is, the rate of heat transfer per unit area.

The **heat rate** by conduction, q_x (W), through a plane wall of **area A** is then the product of the **flux** and the **area**, $q_x = q''_x.A$.

$$q_{\text{cond}} = -kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{dT}{dx}$$

2.3. EXAMPLE 1.1

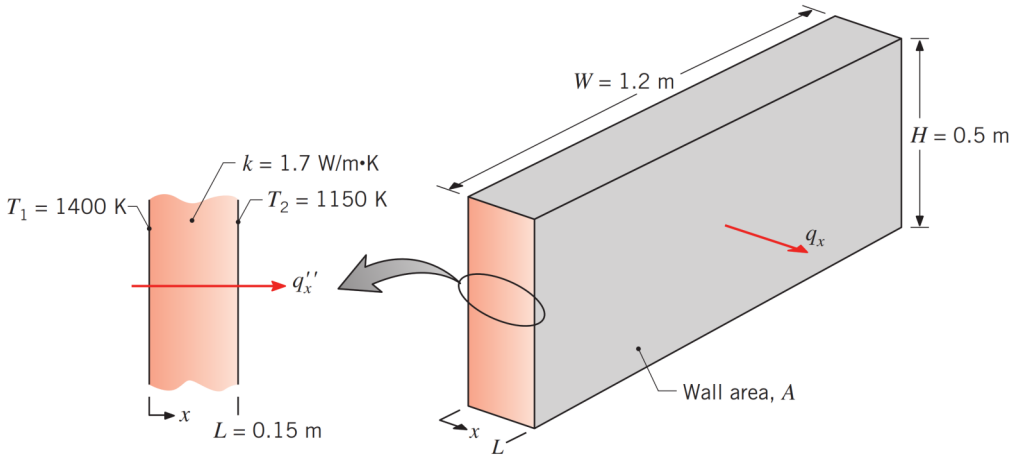
The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m . K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m x 1.2 m on a side?

SOLUTION

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity, and surface temperatures.

Find: Wall heat loss.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction through the wall.
3. Constant thermal conductivity.

Analysis:

Since heat transfer through the wall is by conduction, the heat flux may be determined from Fourier's law. Using Equation 1.2, we have

$$q_x'' = k\frac{\Delta T}{L} = 1.7\ \text{W/m} \cdot \text{K} \times \frac{250\ \text{K}}{0.15\ \text{m}} = 2833\ \text{W/m}^2$$

The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area is then A= H x W.

$$q_x = (HW)q_x'' = (0.5\ \text{m} \times 1.2\ \text{m})2833\ \text{W/m}^2 = 1700\ \text{W}$$

Comments: Note the direction of heat flow and the distinction between heat flux and heat rate.

3. Heat convection

3.1. Definition

Definition: Convection

Heat (energy) transfer with **macroscopic** transport of matter.

Warning

Composed of two mechanisms:

Energy transfer due to random molecular motion - **diffusion**.

Energy transfer by the mass movement of the fluid - **advection**.

Example:Seen as

We are especially interested in convection heat transfer, which occurs between a fluid in motion and a bounding surface when the two are at different temperatures. Consider fluid flow over the heated surface of **Figure 1.5**.

A consequence of the fluid-surface interaction is the development of a region in the fluid through which the velocity varies from zero at the surface to a finite value u_∞ associated with the flow. This region of the fluid is known as the **hydrodynamic**, or **velocity**, **boundary layer**. Moreover, if the surface and flow temperatures differ, there will be a region of the fluid through which the temperature varies from T_s at y=0 toT_∞ in the outer flow. This region, called the **thermal boundary layer**, may be smaller, larger, or the same size as that through which the velocity varies. In any case, if $T_s > T_\infty$, convection heat transfer will occur from the surface to the outer flow.

The convection heat transfer mode is sustained both by random molecular motion and by the bulk motion of the fluid within the boundary layer. The contribution due to random molecular motion (diffusion) dominates near the surface where the fluid velocity is low. In fact, at the interface between the surface and the fluid (y =0), the fluid velocity is zero, and heat is transferred by this mechanism only. The contribution due to bulk fluid motion originates from the fact that the boundary layer **grows** as the flow progresses in the x-direction. In effect, the heat that is conducted into this layer is swept downstream and is eventually transferred to the fluid outside the boundary layer. Appreciation of boundary layer phenomena is essential to understanding convection heat transfer. For this reason, the discipline of fluid mechanics will play a vital role in our later analysis of convection.

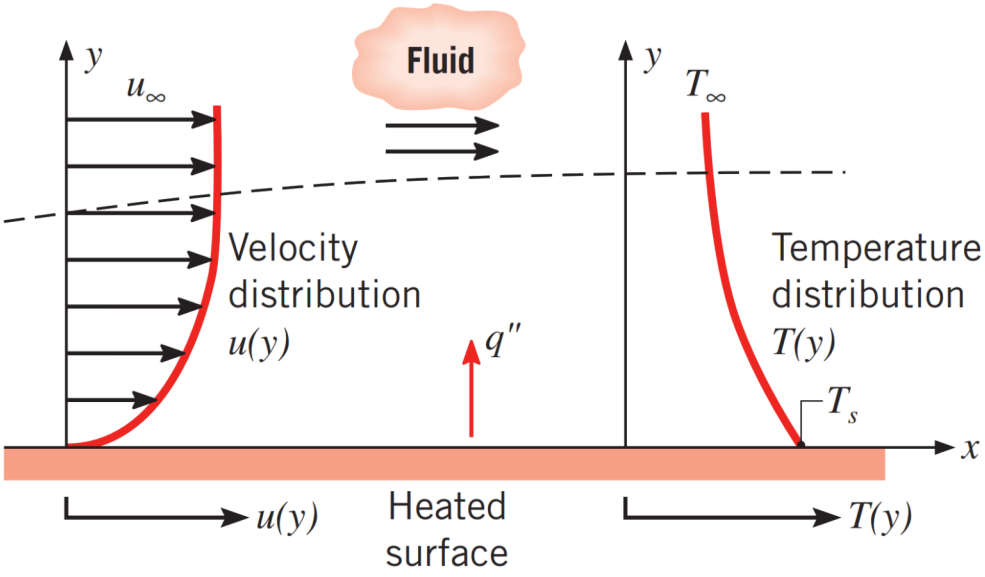


Figure 1.5: Boundary layer development in convection heat transfer.

3.2. Classification

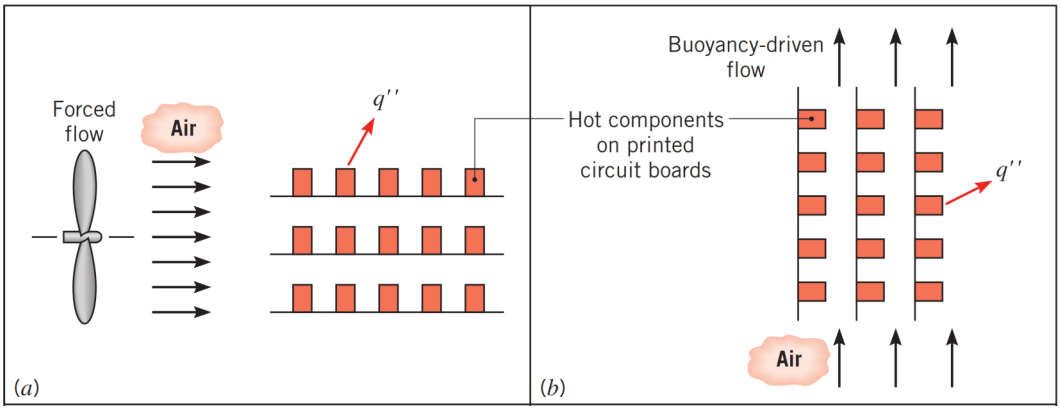


Figure 1.6: Convection heat transfer processes. (a) Forced convection. (b) Natural convection.

Definition: Forced convection (provoked)

Convection heat transfer may be classified according to the nature of the flow. We speak of **forced convection** when the ¯ow is caused by external means, such as by a fan, a pump, or atmospheric winds.

Example:Forced convection

As an example, consider the use of a fan to provide forced convection air cooling of hot electrical components on a stack of printed circuit boards (**Figure 1.6a**).

Free convection (natural)

Free convection (or natural), the flow is induced by **buoyancy forces**, which are due to density differences caused by temperature variations in the fluid.

Example:Free convection

An example is the free convection heat transfer that occurs from hot components on a vertical array of circuit boards in air (**Figure 1.6b**). Air that makes contact with the components experiences an increase in temperature and hence a reduction in density. Since it is now lighter than the surrounding air, buoyancy forces induce a vertical motion for which warm air ascending from the boards is replaced by an in flow of cooler ambient air.

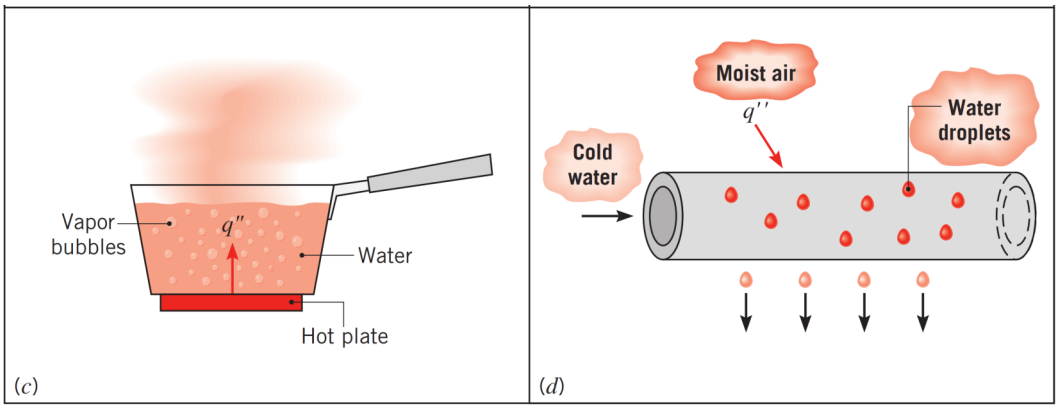


Figure 1.7: Convection heat transfer processes. (c) Boiling. (d) Condensation.

We have described the convection heat transfer mode as energy transfer occurring within a fluid due to the combined effects of conduction and bulk fluid motion. Typically, the energy that is being transferred is the sensible, or internal thermal, energy of the fluid. However, for some convection processes, there is, in addition, latent heat exchange. This latent heat exchange is generally associated with a phase change between the liquid and vapor states of the fluid.

Two special cases of interest in this text are **boiling** and **condensation**.

Example:Boiling and Condensation

For example, convection heat transfer results from fluid motion induced by vapor bubbles generated at the bottom of a pan of boiling water (**Figure 1.7c**) or by the condensation of water vapor on the outer surface of a cold water pipe (**Figure 1.7d**).

3.3. Newton's Law of Cooling

Regardless of the nature of the heat transfer process by convection, the transmitted **heat flux** equation is given by the relationship known as **Newton's law**:

$$q'' = h\,(T_s - T_\infty) \qquad (1.3a)$$

where q", the convective heat flux (W/m^2), is proportional to the difference between the surface and fluid temperatures, T_s and T_∞, respectively. This expression is known as **Newton's law of cooling**, and the parameter h (W/m^2. K) is termed the **convection heat transfer coefficient**. This coefficient depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion, and an assortment of fluid thermodynamic and transport properties.

Any study of convection ultimately reduces to a study of the means by which h may be determined. Although consideration of these means is deferred to Chapter 6, convection heat transfer will frequently appear as a boundary condition in the solution of conduction problems (Chapters 2 through 5). In the solution of such problems we presume h to be known, using typical values given in **Table 1.1**.

Process	h (W/m².K)
Free convection	
Gases	2-25
Liquids	50-1000
Forced convection	
Gases	25-250
Liquids	100-20,000
Convection with phase change	
Boiling or condensation	2500-100,000

Table 1.1: Typical values of the convection heat transfer coefficient

⚠ *Warning*

When **Equation 1.3a** is used, the convection heat flux is presumed to be **positive** if heat is transferred from the surface ($T_s > T_\infty$) and **negative** if heat is transferred to the surface ($T_\infty > T_s$). However, nothing precludes us from expressing Newton's law of cooling as:

$$q'' = h(T_\infty - T_s) \quad (1.3b)$$

in which case heat transfer is positive if it is to the surface.

The **heat rate** by convection, q_x (W), is then the product of the **heat flux** and the **surface area** (section) A.

$$q_{conv} = hA_s(T_s - T_\infty)$$

4. Heat radiation

4.1. Definition

Definition: Radiation

Thermal radiation is energy emitted by matter that is at a nonzero temperature (greater than 0 K) transported by **electromagnetic waves** (or photons).

4.2. Loi de Stefan-Biltzmann

Definition: Emittance (Surface emissive power)

Consider radiation transfer processes for the surface of **Figure 1.8a**. Radiation that is **emitted** by the surface originates from the thermal energy of matter bounded by the surface, and the rate at which energy is released per unit area (W/m^2) is termed the surface **emissive power**, E. There is an upper limit to the emissive power, which is prescribed by the **Stefan Boltzmann law**:

$$E_b = \sigma T_s^4 \quad (1.4)$$

Where Ts is the absolute temperature (K) of the surface and σ is the **Stefan Boltzmann constant** ($\sigma = 5.67 \times 10^{-8}$ $W/m^2.K^4$). Such a surface is called an ideal radiator or blackbody.

Definition: Emissivity

The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by:

$$E = \varepsilon \sigma T_s^4 \quad (1.5)$$

Where ε is a radiative property of the surface termed the **emissivity**. With values in the range $0 \leq \varepsilon \leq 1$, this property provides a measure of how efficiently a surface emits energy relative to a blackbody.

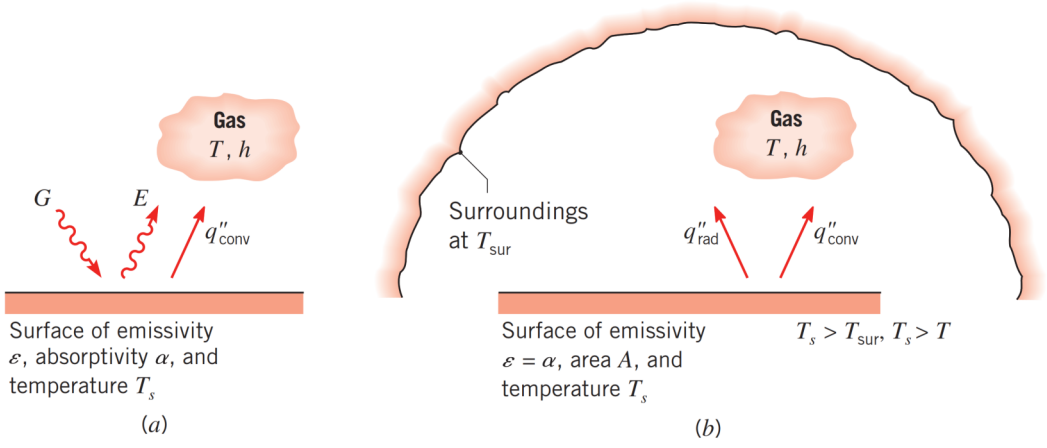


Figure 1.8: Radiation exchange: (a) at a surface and (b) between a surface and large surroundings.

Complement:Irradiation G

Radiation may also be **incident** on a surface from its surroundings. The radiation may originate from a special source, such as the sun, or from other surfaces to which the surface of interest is exposed. Irrespective of the source(s), we designate the rate at which all such radiation is incident on a unit area of the surface as the **irradiation G** (**Figure 1.8a**).

A portion, or all, of the irradiation may be absorbed by the surface, thereby increasing the thermal energy of the material. The rate at which radiant energy is absorbed per unit surface area may be evaluated from knowledge of a surface radiative property termed the absorptivity α. That is:

$$G_{abs} = \alpha G \quad (1.6)$$

Where $0 \leq \alpha \leq 1$. If $\alpha < 1$ and the surface is opaque, portions of the irradiation are reflected. If the surface is **semitransparent**, portions of the irradiation may also be **transmitted**.

However, whereas absorbed and emitted radiation increase and reduce, respectively, the thermal energy of matter, reflected and transmitted radiation have no effect on this energy.

Note

Note that the value of α depends on the nature of the irradiation, as well as on the surface itself. For example, the absorptivity of a surface to solar radiation may differ from its absorptivity to radiation emitted by the walls of a furnace.

Method

A special case that occurs frequently involves the exchange of radiation between a small surface at $\mathbf{T_s}$ and a much larger isothermal surface $\mathbf{T_{sur}}$ which completely surrounds the smaller one (**Figure 1.8b**).

The surroundings could, for example, be the walls of a room or a furnace whose temperature $\mathbf{T_{sur}}$ differs from that of an enclosed surface ($\mathbf{T_{sur} \neq T_s}$).

$$G = \sigma T_{\text{sur}}^4$$

If the surface is assumed to be one for which $\alpha = \varepsilon$ (**a gray surface**), the net rate of radiation heat transfer from the surface, expressed per unit area of the surface, is:

$$q''_{\text{rad}} = \frac{q}{A} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{\text{sur}}^4\right) \quad (1.7)$$

This expression provides the difference between thermal energy that is released due to radiation emission and that gained due to radiation absorption.

For many applications, it is convenient to express the net radiation heat exchange in the form:

$$q_{\text{rad}} = h_r A\,(T_s - T_{\text{sur}}) \quad (1.8)$$

Where, from Equation 1.7, the radiation heat transfer coefficient $\mathbf{h_r}$ is:

$$h_r \equiv \varepsilon\sigma\,(T_s + T_{\text{sur}})\left(T_s^2 + T_{\text{sur}}^2\right) \quad (1.9)$$

Here we have modeled the radiation mode in a manner similar to convection. In this sense we have **linearized** the radiation rate equation, making the heat rate proportional to a temperature difference rather than to the difference between two temperatures to the fourth power.

Note, however, that $\mathbf{h_r}$ depends strongly on temperature, whereas the temperature dependence of the convection heat transfer coefficient **h** is generally weak.

The surfaces of **Figure 1.8** may also simultaneously transfer heat by convection to an adjoining gas. For the conditions of **Figure 1.8b**, the total rate of heat transfer from the surface is then

$$q = q_{\text{conv}} + q_{\text{rad}} = hA\,(T_s - T_\infty) + \varepsilon A\sigma\left(T_s^4 - T_{\text{sur}}^4\right) \quad (1.10)$$

4.3. EXAMPLE 1.2

An uninsulated steam pipe passes through a room in which the air and walls are at 25° C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200 °C and 0.8, respectively. What are the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 W / m^2.K, what is the rate of heat loss from the surface per unit length of pipe?

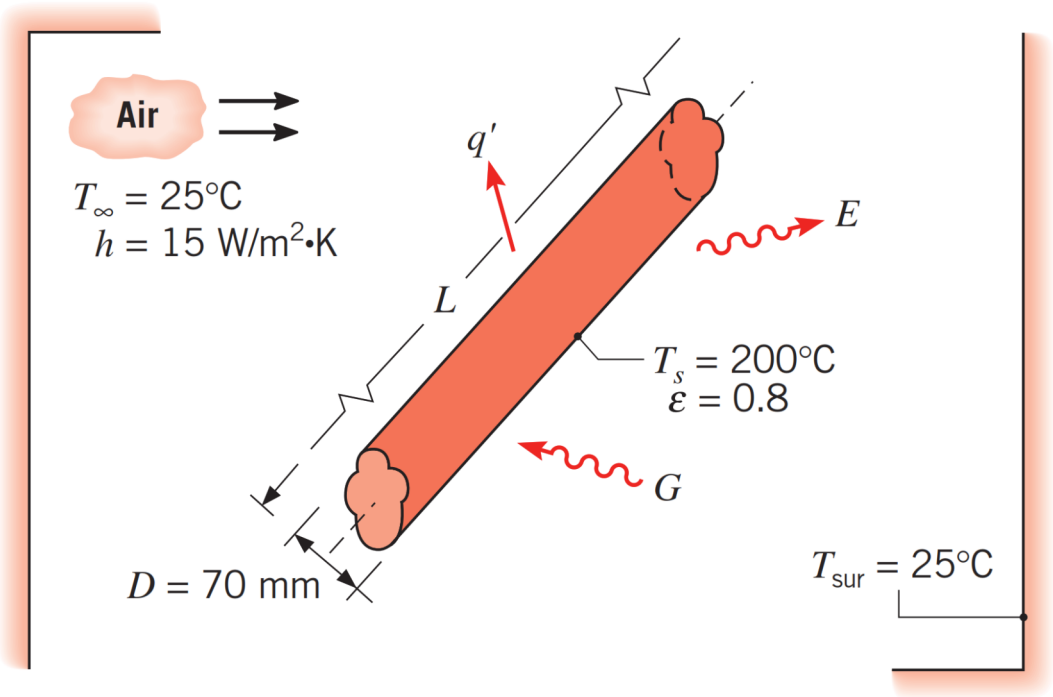
SOLUTION

Known: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures.

Find:

1. Surface emissive power and irradiation.
2. Pipe heat loss per unit length, q'.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Radiation exchange between the pipe and the room is between a small surface and a much larger enclosure.
3. The surface emissivity and absorptivity are equal.

Analysis:

1. The surface emissive power may be evaluated from Equation 1.5, while the irradiation corresponds to $\mathbf{G=\sigma T_{sur}^4}$. Hence

$$E = \varepsilon\sigma T_s^4 = 0.8\left(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\right)(473\ \mathrm{K})^4 = 2270\ \mathrm{W/m^2}$$
$$G = \sigma T_{\mathrm{sur}}^4 = 5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}(298\ \mathrm{K})^4 = 447\ \mathrm{W/m^2}$$

2. Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q= q_{conv} + q_{rad}$ and from Equation 1.10, with $A= \pi DL$,

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma\left(T_s^4 - T_{\mathrm{sur}}^4\right)$$

The heat loss per unit length of pipe is then

$$q' = \frac{q}{L} = 15\ \mathrm{W/m^2\cdot K}(\pi\times0.07\ \mathrm{m})(200-25)^\circ\mathrm{C} + 0.8(\pi\times0.07\ \mathrm{m})5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\left(473^4 - 298^4\right)\mathrm{K}^4$$
$$q' = 577\ \mathrm{W/m} + 421\ \mathrm{W/m} = 998\ \mathrm{W/m}$$

Comments:

Note that temperature may be expressed in units of °C or K when evaluating the temperature difference for a convection (or conduction) heat transfer rate. However, temperature must be expressed in kelvins (K) when evaluating a radiation transfer rate.

5. The concept of thermal resistance

5.1. Thermal resistance

The three modes of heat transfer were introduced in the preceding sections. As is evident from Equations 1.2, 1.3, and 1.8, the heat transfer rate can be expressed in the form

$$q = q''A = \frac{\Delta T}{R_t} \quad (1.11)$$

where **ΔT** is a relevant temperature difference and **A** is the area normal to the direction of heat transfer. The quantity $\mathbf{R_t}$ is called a thermal resistance and takes different forms for the three different modes of heat transfer.

Example

For example, Equation 1.2 may be multiplied by the area **A** and rewritten as:

$$q_x = \frac{\Delta T}{R_{t,c}}$$

Where Rt,c is a thermal resistance associated with conduction, having the units K/W.

$$R_{t,c} = \frac{L}{KA}$$

6. Problem solving: Methodology

6.1. Objectives:

Analyzing different problems will give you a deeper appreciation of the fundamentals of the subject and you will gain confidence in your ability to apply these fundamentals to the solution of engineering problems.

6.2. Methodology

Advice:Be consistent by following these steps:

1. **Known**
2. **Find**
3. **Schematic**
4. **Assumptions**
5. **Analysis**
6. **Comments**

Chapter 2: Heat diffusion equation

II

In this chapter, we will study the heat conduction equation and its solutions. Thermal conduction in a medium is said to be stationary (permanent) when the temperature does not vary with time and transient when it does.

Thermal conduction in a medium is said to be **one-dimensional** when the conduction is significant in only one dimension and negligible in the other two dimensions.

It is said to be **two-dimensional** when the conduction in the **third dimension** is negligible and three-dimensional when the conduction in all dimensions is significant.

In heat transfer analysis, the conversion of electrical, chemical, or nuclear energy into thermal energy is characterized as heat generation [1]*.

1. Problem and application

Determine the temperature distribution in a system resulting from the conditions imposed at its boundaries.

The conduction heat flow at any point in the system or surface can be calculated from **Fourier's law**.

This information could be used to determine thermal stresses, expansions and deformations.

Temperature distribution can also be used to optimize the thickness of an insulating material or to determine the compatibility of special coatings or adhesives used with the material.

2. Heat transfer in Steady-State and transient conditions

2.1. Steady state vs transient

Heat transfer problems are often classified as being steady (also called steady-state) or transient (also called unsteady).

The term steady implies no change with time at any point within the medium, while transient implies variation with time or time dependence. Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location, although both quantities may vary from one location to another (**Figure 2.1**).

For example, heat transfer through the walls of a house will be steady when the conditions inside the house and the outdoors remain constant for several hours. But even in this case, the temperatures on the inner and outer surfaces of the wall will be different unless the temperatures inside and outside the house are the same.

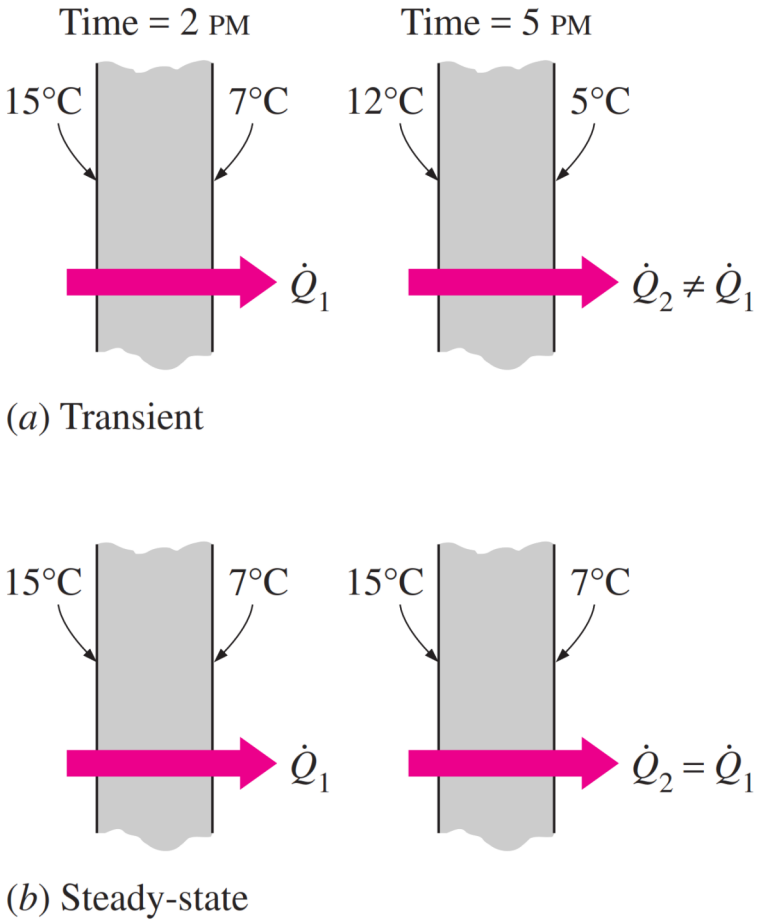


Figure 2.1: Steady and transient heat conduction in a plane wall.

3. Heat generation and energy storage

3.1. Heat generation (Source)

A medium through which heat is conducted may involve the conversion of electrical, nuclear, or chemical energy into heat (or thermal) energy. In heat conduction analysis, such conversion processes are characterized as **heat generation**.

For example, the temperature of a resistance wire rises rapidly when electric current passes through it as a result of the electrical energy being converted to heat at a rate of $\mathbf{I^2 R}$, where I is the current and R is the electrical resistance of the wire. The safe and effective removal of this heat from the sites of heat generation (the electronic circuits) is the subject of electronics cooling, which is one of the modern application areas of heat transfer.

Likewise, a large amount of heat is generated in the fuel elements of nuclear reactors as a result of nuclear fission that serves as the heat source for the nuclear power plants. The natural disintegration of radioactive elements in nuclear waste or other radioactive material also results in the generation of heat throughout the body [2]*.

The **heat generated** in the sun as a result of the fusion of hydrogen into helium makes the sun a large nuclear reactor that supplies heat to the earth.

Another source of heat generation in a medium is exothermic chemical reactions that may occur throughout the medium. The chemical reaction in this case serves as a heat source for the medium. In the case of endothermic reactions, however, heat is absorbed instead of being released during reaction, and thus the chemical reaction serves as a heat sink. The heat generation term becomes a negative quantity in this case.

Note that heat generation is a **volumetric phenomenon**. That is, it occurs throughout the body of a medium. Therefore, the rate of heat generation in a medium is usually specified per **unit volume** and is denoted by $\dot{E}_g$, whose unit is $\mathbf{W/m^3}$ or $\mathbf{Btu/h \cdot ft^3}$.

The rate of **heat generation** in a medium may vary with time as well as position within the medium. When the variation of heat generation with position is known, the total rate of heat generation in a medium of volume V can be determined from:

$$\dot{E}_g = \dot{q} dx dy dz \quad (2.1)$$

3.2. Energy Storage

In addition, changes may occur in the amount of the internal thermal energy stored by the material in the control volume. If the material is not experiencing a change in phase, latent energy effects are not pertinent, and the **energy storage** term may be expressed as:

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dx dy dz \quad (2.2)$$

where $\rho c_p \partial T / \partial t$ is the time rate of change of the sensible (thermal) energy of the medium per unit volume.

Once again it is important to note that the terms $\dot{E}_g$ and $\dot{E}_{st}$ represent different physical processes. The energy generation term $\dot{E}_g$ is a manifestation of some energy conversion process involving thermal energy on one hand and some other form of energy, such as chemical, electrical, or nuclear, on the other. **The term is**

positive (a source) if thermal energy is being generated in the material at the expense of some other energy form; it is **negative** (a sink) if thermal energy is being consumed. In contrast, the energy storage term $\dot{E}_{st}$ refers to the rate of change of thermal energy stored by the matter.

4. Multidimensional Heat Transfer

4.1. Definition

Heat transfer problems are also classified as being **one-dimensional**, **twodimensional**, or **three-dimensional**, depending on the relative magnitudes of heat transfer rates in different directions and the level of accuracy desired. In the most general case, heat transfer through a medium is **three-dimensional.**

That is, the temperature varies along all three primary directions within the medium during the heat transfer process. The temperature distribution throughout the medium at a specified time as well as the heat transfer rate at any location in this general case can be described by a set of three coordinates such as the x, y, and z in the rectangular (or Cartesian) coordinate system; the r, Φ, and z in the cylindrical coordinate system; and the r, Φ, and θ in the spherical (or polar) coordinate system. The temperature distribution in this case is expressed as T(*x, y, z, t*), T(*r, Φ, z, t*), and T(*r, Φ, θ, t*) in the respective coordinate systems.

The general form of the **energy conservation equation** (Overall Energy Balance) is:

$$\dot{E}_{\text{in}} + \dot{E}_g - \dot{E}_{\text{out}} = \dot{E}_{\text{st}} \quad (2.3)$$

Hence, recognizing that the conduction rates constitute the energy inflow $\dot{E}_{\text{in}}$ and outflow $\dot{E}_{\text{out}}$

4.2. Rectangular coordinates (or Cartesian)

Consider a homogeneous medium within which there is no bulk motion (advection) and the temperature distribution T(x, y, z) is expressed in Cartesian coordinates. Following the methodology of applying conservation of energy (**2.3**), we first define an infinitesimally small (differential) control volume, $dx \cdot dy \cdot dz$, as shown in **Figure 2.2**.

Choosing to formulate the first law at an instant of time, the second step is to consider the energy processes that are relevant to this control volume.

In the absence of motion (or with uniform motion), there are no changes in mechanical energy and no work being done on the system. Only thermal forms of energy need be considered. Specifically, if there are temperature gradients, conduction heat transfer will occur across each of the control surfaces.

The conduction heat rates perpendicular to each of the control surfaces at the $x-, y-, and z-$coordinate locations are indicated by the terms $q_x, q_y,$ and q_z, respectively.

The conduction heat rates at the opposite surfaces can then be expressed as a **Taylor** series expansion where, neglecting higher-order terms,

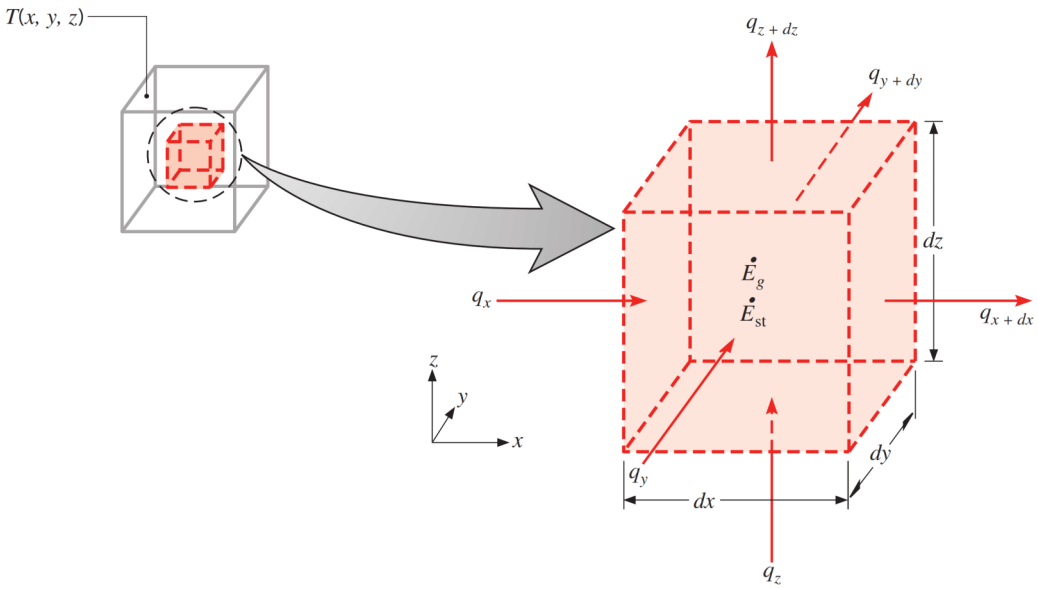


Figure 2.2: Differential control volume, dx dy dz, for conduction analysis in Cartesian coordinates.

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx \quad (2.4a)$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy \quad (2.4b)$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz \quad (2.4c)$$

In words, **Equation 2.4a** simply states that the x-component of the heat transfer rate at $x + dx$ is equal to the value of this component at x plus the amount by which it changes with respect to x times dx.

Substituting Equations 2.1 and 2.2 in Equations 2.3, we obtain:

$$q_x + q_y + q_z + \dot{q}dxdydz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho c_p \frac{\partial T}{\partial t}dxdydz \quad (2.5)$$

Substituting from **Equations 2.4**, it follows that

$$-\frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz + \dot{q}dxdydz = \rho c_p \frac{\partial T}{\partial t}dxdydz \quad (2.6)$$

The conduction heat rates in an **isotropic material** may be evaluated from **Fourier's law**,

$$q_x = -kdydz\frac{\partial T}{\partial x} \quad (2.7a)$$

$$q_y = -kdxdz\frac{\partial T}{\partial y} \quad (2.7b)$$

$$q_z = -kdxdy\frac{\partial T}{\partial z} \quad (2.7c)$$

Where each heat flux component has been multiplied by the appropriate control surface (differential) area to obtain the heat transfer rate. Substituting **Equations 2.7** into **Equation 2.6** and dividing out the dimensions of the control volume ($dxdydz$), we obtain

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.8)$$

It is often possible to work with simplified versions of **Equation 2.8**. For example, if the thermal conductivity is constant, the heat equation is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (2.9)$$

Where $\alpha = k/\rho c_p$ is the **thermal diffusivity**.

4.3. Cylindrical Coordinates

The general heat conduction equation in cylindrical coordinates can be obtained from an energy balance on a volume element in cylindrical coordinates, shown in **Figure 2.3**, by following the steps described above. It can also be obtained directly from **Equation 2.8** by coordinate transformation using the following relationships between the coordinates of a point in the rectangular and cylindrical coordinate systems:

$$x = r\cos\Phi, y = r\sin\Phi \text{ et } z = z$$

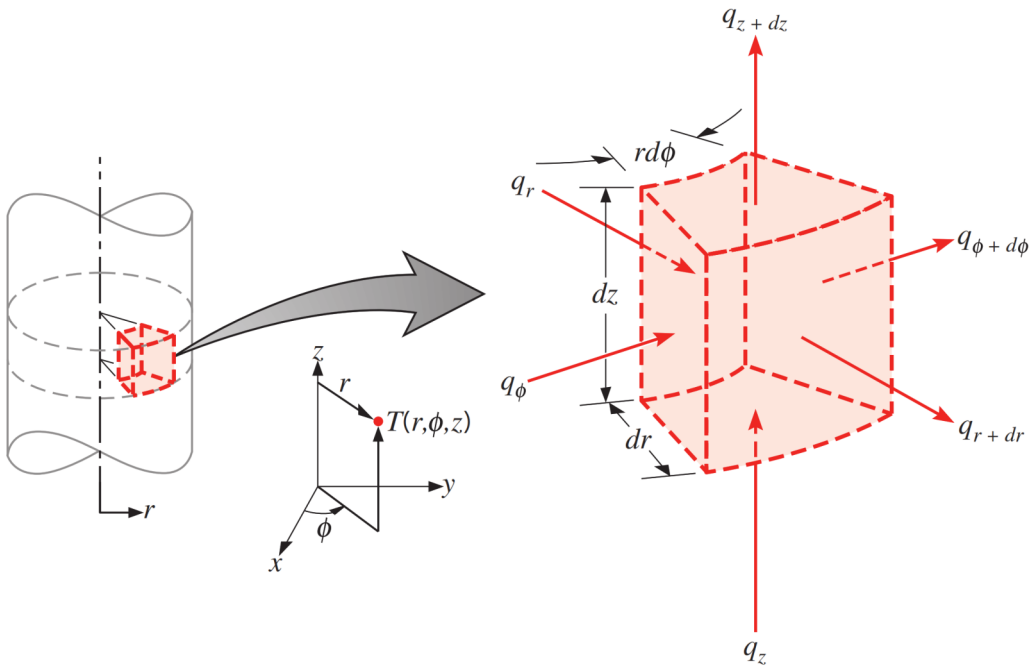


Figure 2.4: Differential control volume, dr. r dΦ. dz, for conduction analysis in cylindrical coordinates (r, Φ , z).

The following general form of the heat equation is obtained:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.11)$$

4.4. Spherical Coordinates

The general heat conduction equations in spherical coordinates can be obtained from an energy balance on a volume element in spherical coordinates, shown in **Figure 2.4**, by following the steps described above. It can also be obtained directly from **Equation 2.8** by coordinate transformation using the following relationships between the coordinates of a point in the rectangular and spherical coordinate systems:

$$x = r\cos\Phi\sin\theta, y = r\sin\Phi\sin\theta \text{ et } z = \cos\theta$$

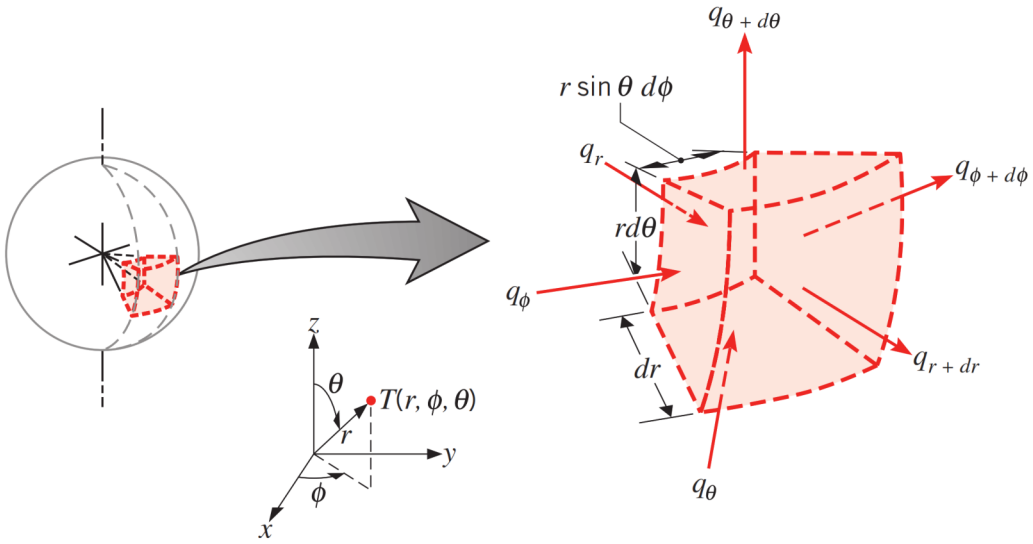


Figure 2.5: Differential control volume, dr. rsinθ dΦ. r dθ, for conduction analysis in spherical coordinates (r,Φ, θ).

The following general form of the heat equation is obtained:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t} \qquad (2.12)$$

5. EXAMPLE 2.1

5.1. EXAMPLE 2.1

The temperature distribution across a wall 1 m thick at a certain instant of time is given as

$$T(x) = a + bx + cx^2$$

where T is in degrees Celsius and x is in meters, while $a = 900°\text{C}, b = -300°\text{C/m}$, and $c = -50°\text{C/m}^2$. A uniform heat generation, $\dot{q} = 1000\ \text{W/m}^3$, is present in the wall of area $10\ \text{m}^2$ having the properties $\rho = 1600\ \text{kg/m}^3, k = 40\ \text{W/m} \cdot \text{K}$, and $c_p = 4\ \text{kJ/kg} \cdot \text{K}$.

1. Determine the rate of heat transfer entering the wall (x = 0) and leaving the wall (x = 1 m).
2. Determine the rate of change of energy storage in the wall.
3. Determine the time rate of temperature change at x = 0, 0.25, and 0.5 m.

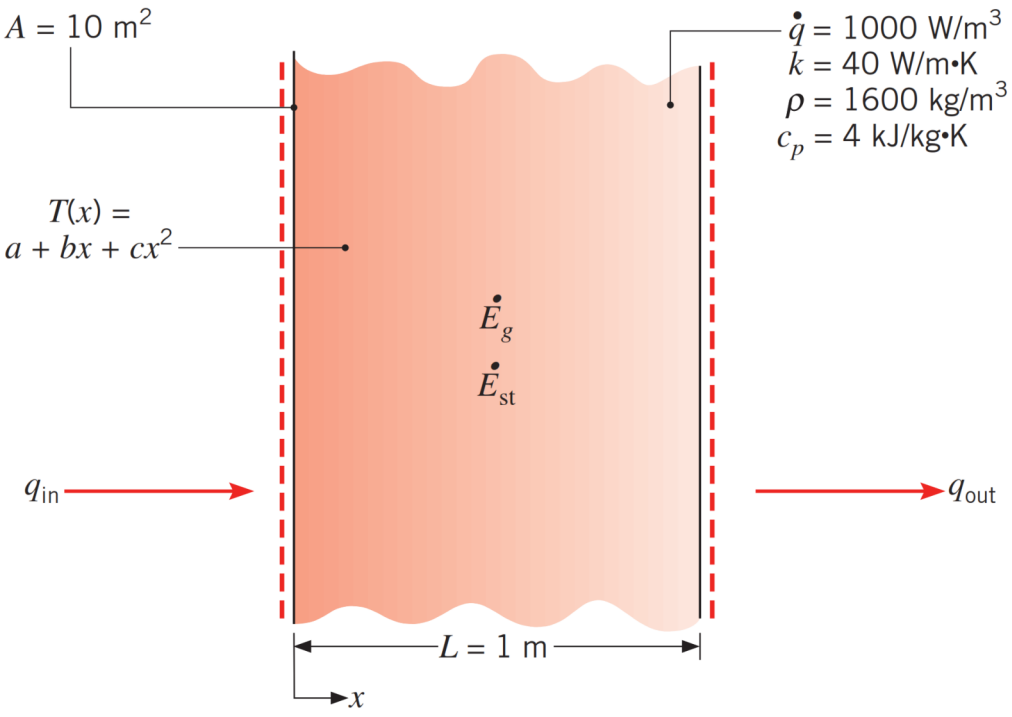
SOLUTION

Known: Temperature distribution T(x) at an instant of time t in a one-dimensional wall with uniform heat generation.

Find:

1. Heat rates entering, $q_{\text{in}}\ (x = 0)$, and leaving, $q_{\text{out}}\ (x = 1\ \text{m})$, the wall.
2. Rate of change of energy storage in the wall, $\dot{E}_{\text{st}}$.
3. Time rate of temperature change at $x = 0, 0.25$, and $0.5\ \text{m}$.

Schematic:



Assumptions:

1. One-dimensional conduction in the x-direction.
2. Isotropic medium with constant properties.
3. Uniform internal heat generation, $\dot{q}\left(\mathrm{W/m^3}\right)$.

Analysis:

1. Recall that once the temperature distribution is known for a medium, it is a simple matter to determine the conduction heat transfer rate at any point in the medium or at its surfaces by using Fourier's law. Hence the desired heat rates may be determined by using the prescribed temperature distribution with Equation 2.1. Accordingly,

$$q_{\text{in}} = q_x(0) = -kA\frac{\partial T}{\partial x}\bigg|_{x=0} = -kA(b + 2cx)_{x=0}$$

$$q_{\text{in}} = -bkA = 300^\circ\text{C/m} \times 40\ \text{W/m} \cdot \text{K} \times 10\ \text{m}^2 = 120\ \text{kW}$$

Similarly,

$$q_{\text{out}} = q_x(L) = -kA\frac{\partial T}{\partial x}\bigg|_{x=L} = -kA(b + 2cx)_{x=L}$$

$$q_{\text{out}} = -(b + 2cL)kA = -\left[-300^\circ\text{C/m} + 2\left(-50^\circ\text{C/m}^2\right) \times 1\ \text{m}\right] \times 40\ \text{W/m} \cdot \text{K} \times 10\ \text{m}^2 = 160\ \text{kW}$$

2. The rate of change of energy storage in the wall $\dot{E}_{\text{st}}$ may be determined by applying an overall energy balance to the wall. Using Equation 2.3 for a control volume about the wall,

$$\dot{E}_{\text{in}} + \dot{E}_g - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$$

where $\dot{E}_g = \dot{q}AL$, it follows that

$$\dot{E}_{\text{st}} = \dot{E}_{\text{in}} + \dot{E}_g - \dot{E}_{\text{out}} = q_{\text{in}} + \dot{q}AL - q_{\text{out}}$$

$$\dot{E}_{\text{st}} = 120\ \text{kW} + 1000\ \text{W/m}^3 \times 10\ \text{m}^2 \times 1\ \text{m} - 160\ \text{kW}$$

$$\dot{E}_{\text{st}} = -30\ \text{kW}$$

3. The time rate of change of the temperature at any point in the medium may be determined from the heat equation, Equation 2.9, rewritten as

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho c_p}$$

From the prescribed temperature distribution, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}(b + 2cx) = 2c = 2\left(-50^\circ\text{C/m}^2\right) = -100^\circ\text{C/m}^2$$

Note that this derivative is independent of position in the medium. Hence the time rate of temperature change is also independent of position and is given by

$$\frac{\partial T}{\partial t} = \frac{40\ \mathrm{W/m \cdot K}}{1600\ \mathrm{kg/m^3} \times 4\ \mathrm{kJ/kg \cdot K}} \times \left(-100^\circ \mathrm{C/m^2}\right) + \frac{1000\ \mathrm{W/m^3}}{1600\ \mathrm{kg/m^3} \times 4\ \mathrm{kJ/kg \cdot K}}$$

$$\frac{\partial T}{\partial t} = -6.25 \times 10^{-4\circ}\mathrm{C/s} + 1.56 \times 10^{-4\circ}\mathrm{C/s}$$

$$= -4.69 \times 10^{-4\circ}\mathrm{C/s}$$

Comments:

1. From this result, it is evident that the temperature at every point within the wall is decreasing with time.
2. Fourier's law can always be used to compute the conduction heat rate from knowledge of the temperature distribution, even for unsteady conditions with internal heat generation.

Chapter 3: HEAT TRANSFER FROM FINNED SURFACES

III

The chapter "**Heat Transfer from Finned Surfaces**" explores the principles and optimization of heat transfer using finned surfaces, which are critical in enhancing thermal performance by increasing the surface area for heat dissipation. It begins with the Fin Equation, which describes the heat transfer rate from finned surfaces by considering factors like geometry, material properties, and boundary conditions, providing a foundation for predicting fin performance. The concept of Fin Efficiency is then introduced, measuring how effectively a fin conducts heat relative to its maximum potential, influenced by parameters such as geometry and thermal conductivity. Next, the chapter covers Fin Effectiveness, which compares the heat transfer rate of finned surfaces to those without fins, helping determine the practical benefits of adding fins. Finally, the importance of selecting the Proper Length of a Fin is discussed, highlighting the point at which extending the fin length yields diminishing returns on heat transfer, balancing performance with cost and space considerations. Together, these topics provide a comprehensive overview of optimizing finned surfaces for improved heat transfer in various engineering applications.

1. Definitions

The term **finned surfaces** is commonly used to depict an important special case involving heat transfer by conduction within a solid and heat transfer by convection (and/or radiation) from the boundaries of the solid. Until now, we have considered heat transfer from the boundaries of a solid to be in the same direction as heat transfer by conduction in the solid. In contrast, for an extended surface, the direction of heat transfer from the boundaries is perpendicular to the principal direction of heat transfer in the solid.

Consider a strut that connects two walls at different temperatures and across which there is fluid flow (**Figure 3.1**). With $T_1 > T_2$, temperature gradients in the x-direction sustain heat transfer by conduction in the strut. However, with $T_1 > T_2 > T_\infty$, there is concurrent heat transfer by convection to the fluid, causing q_x, and hence the magnitude of the temperature gradient, $|dT/dx|$, to decrease with increasing x.

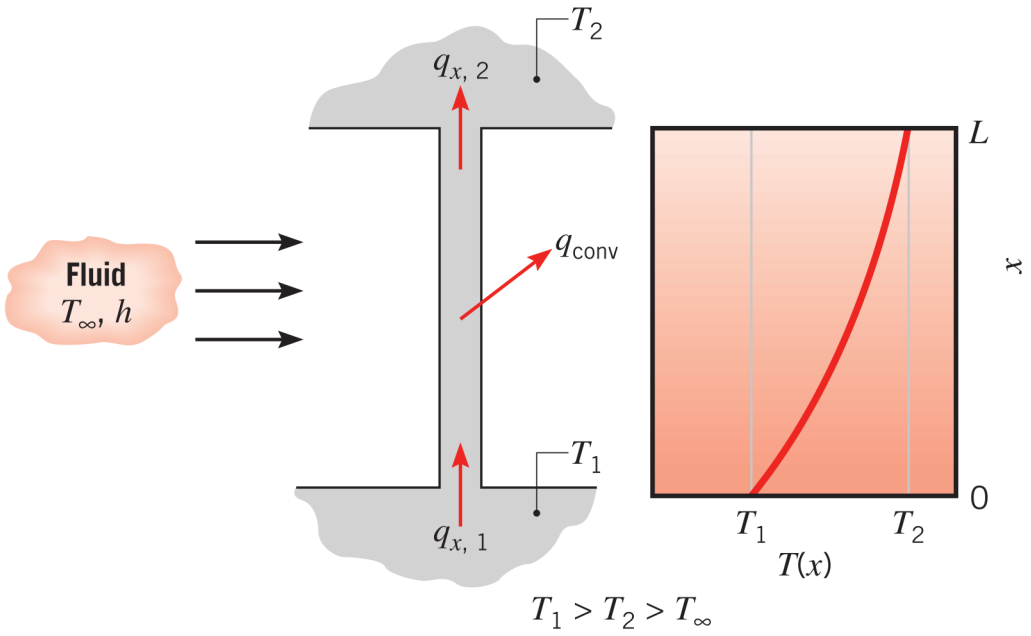


Figure 3.1: Combined conduction and convection in a structural element.

Although there are many different situations that involve such combined **conduction- convection** effects, the most frequent application is one in which an extended surface is used specifically to *enhance* heat transfer between a solid and an adjoining fluid. Such an extended surface is termed a fin.

2. Different Fin configurations

Consider the plane wall of **Figure 3.2a** . If Ts is fixed, there are two ways in which the heat transfer rate may be increased. The convection coefficient h could be increased by increasing the fluid velocity, and/or the fluid temperature T_∞ could be reduced.

However, there are many situations for which increasing h to the maximum possible value is either insufficient to obtain the desired heat transfer rate or the associated costs are prohibitive. Such costs are related to the blower or pump power requirements needed to increase h through increased fluid motion. Moreover, the second option of reducing T_∞ is often impractical.

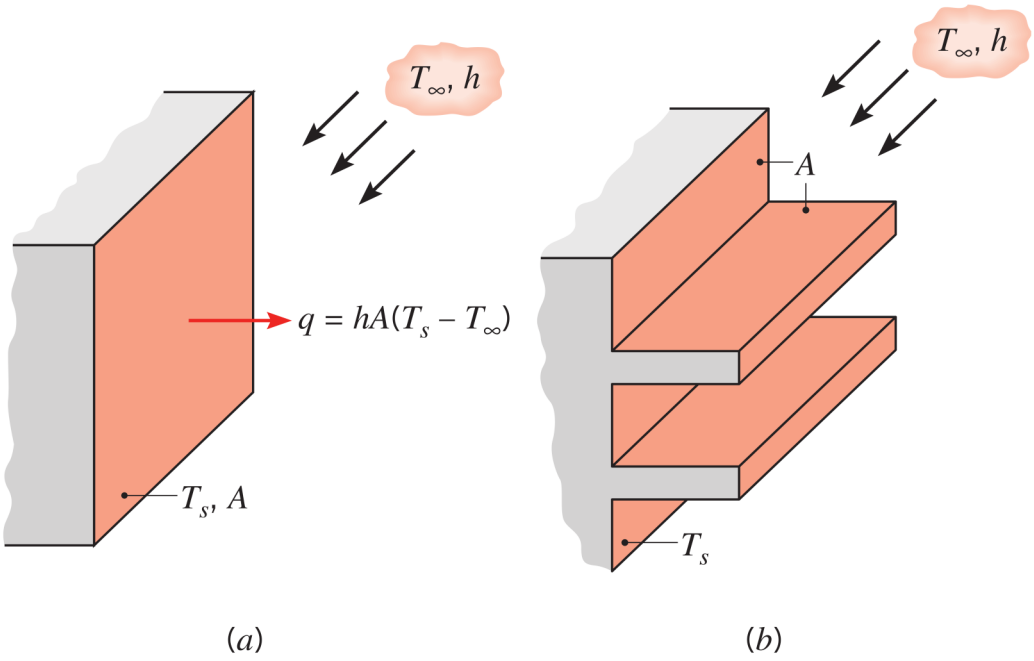


Figure 3.2: Use of fins to enhance heat transfer from a plane wall. (a) Bare surface. (b) Finned surface.

Examining **Figure 3.2b** , however, we see that there exists a third option. That is, the heat transfer rate may be increased by increasing the surface area across which the convection occurs. This may be done by employing fins that extend from the wall into the surrounding fluid.

Consider also the tubes with attached fins used to promote heat exchange between air and the working fluid of an air conditioner. Two common finned-tube arrangements are shown in **Figure 3.3**.

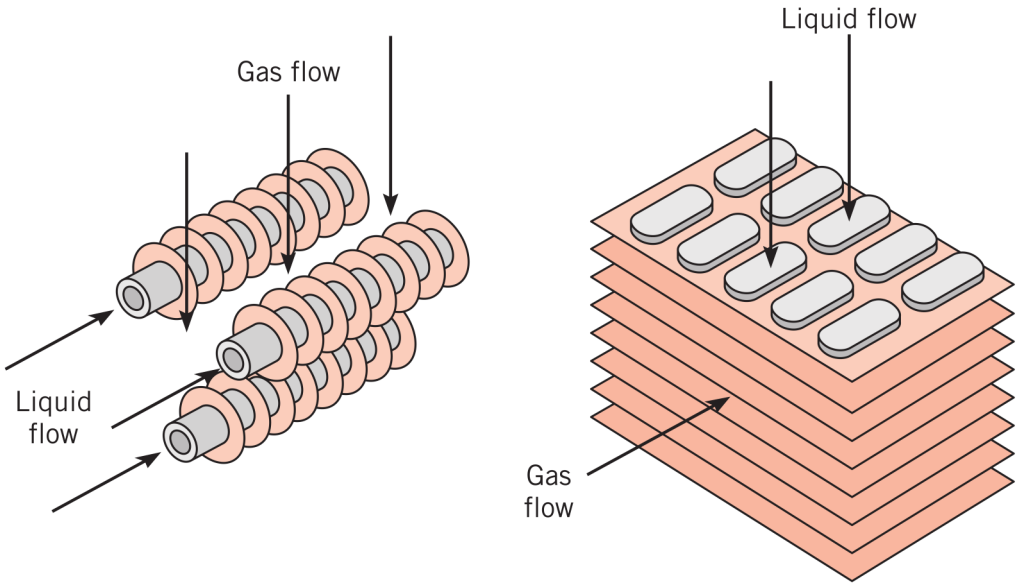


Figure 3.3: Schematic of typical finned-tube heat exchangers.

A General Conduction Analysis

To determine the heat transfer rate associated with a fin, we must first obtain the temperature distribution along the fin. As we have done for previous systems, we begin by performing an energy balance on an appropriate differential element. Consider the extended surface of **Figure 3.4**.

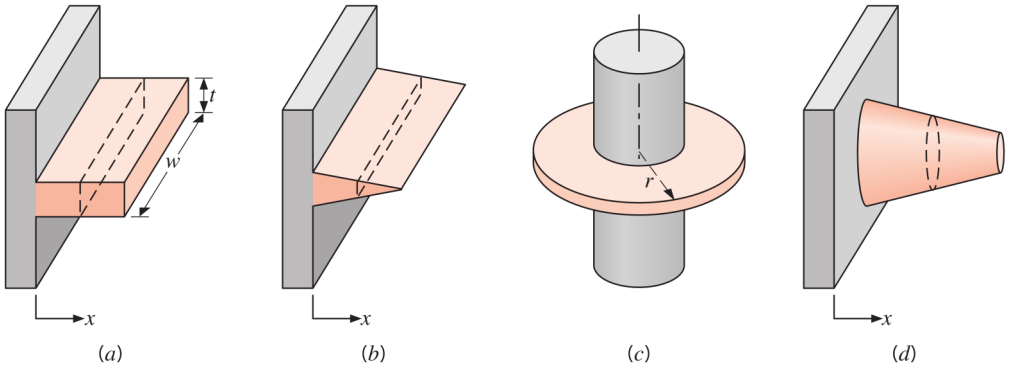


Figure 3.4: Fin configurations. (a) Straight fin of uniform cross section. (b) Straight fin of nonuniform cross section. (c) Annular fin. (d) Pin fin.

3. Fin Equation

Consider a volume element of a fin at location x having a length of dx, crosssectional area of A_c, and a perimeter of p, as shown in **Figure 3.5**. Under steady conditions, the energy balance on this volume element can be expressed as:

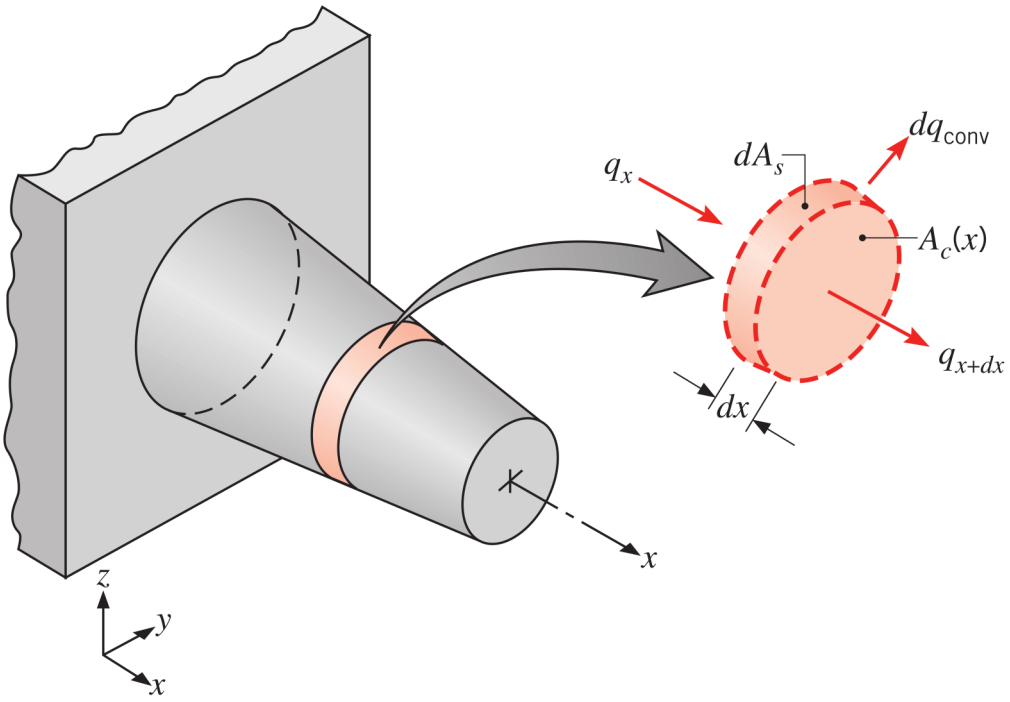


Figure 3.5: Energy balance for an extended surface.

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$

$$\dot{E}_{\text{st}} \equiv \frac{dE_{\text{st}}}{dt} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g \qquad (3.1)$$

Applying the conservation of energy requirement, **Equation 3.1**, to the differential element of **Figure 3.5**, we obtain:

$$q_x = q_{x+dx} + dq_{\text{conv}} \qquad (3.2)$$

where:

$$q_{\text{conv}} = h(p\,dx)\,(T - T_\infty) \qquad (3.3)$$

Substituting and dividing by dx, we obtain:

$$\frac{q_{\text{cond},x+dx} - q_{\text{cond},x}}{dx} + hp(T - T_\infty) = 0 \qquad (3.4)$$

Taking the limit as $dx \to 0$ gives:

$$\frac{dq_{\text{cond}}}{dx} + hp(T - T_\infty) = 0 \qquad (3.5)$$

From Fourier's law of heat conduction we have:

$$q_{cond} = -kA_c \frac{dT}{dx} \qquad (3.6)$$

where A_c is the cross-sectional area of the fin at location x. Substitution of this relation into Equation 3.5 gives the differential equation governing heat transfer in fins,

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0 \qquad (3.7)$$

4. Fins of Uniform Cross-Sectional Area

4.1. Equation Solving

To solve **Equation 3.7** it is necessary to be more specific about the geometry. We begin with the simplest case of straight rectangular and pin fins of uniform cross section (**Figure 3.6**).

In general, the cross-sectional area A_c and the perimeter p of a fin vary with x, which makes this differential equation difficult to solve. In the special case of *constant cross section* and *constant thermal conductivity*, the differential **Equation 3.7** reduces to:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad (3.8)$$

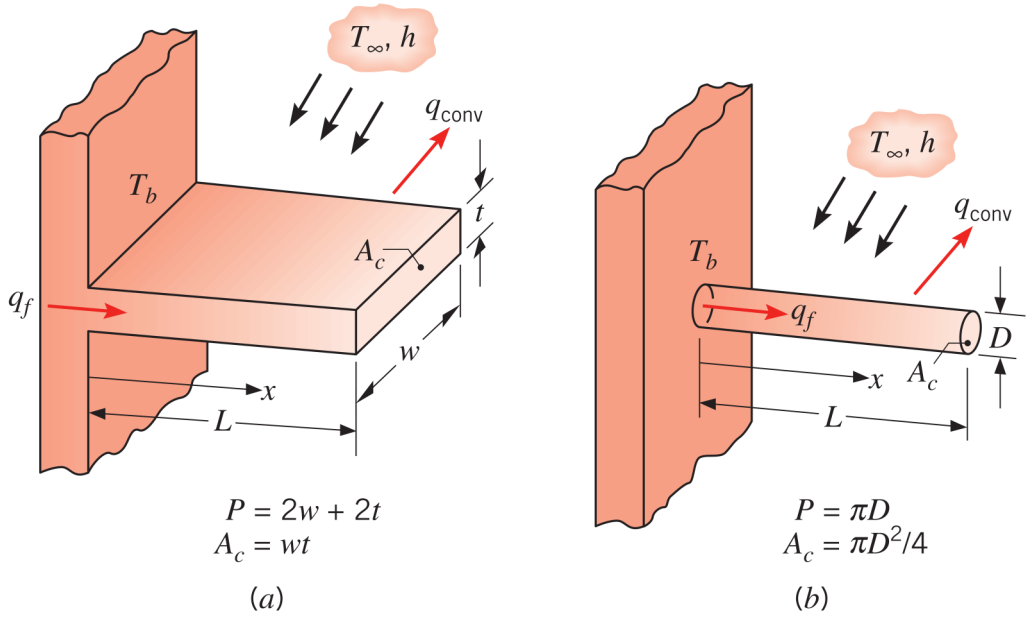


Figure 3.6: Straight fins of uniform cross section. (a) Rectangular fin. (b) Pin fin.

where:

$$m^2 \equiv \frac{hP}{kA_c} \qquad (3.9)$$

and $\theta = T - T_\infty$ is the ***temperature excess.*** At the fin base we have $\theta_b = T_b - T_\infty$.

Equation 3.8 is a linear, homogeneous, second-order differential equation with constant coefficients. Its general solution is of the form:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad (3.10)$$

By substitution it may readily be verified that **Equation 3.10** is indeed a solution to **Equation 3.8**.

To evaluate the constants C_1 and C_2 of **Equation 3.10**, it is necessary to specify appropriate ***boundary conditions***. One such condition may be specified in terms of the temperature at the base of the fin $(x = 0)$.

Boundary condition at fin base: $\theta(0) = \theta_b = T_b - T_x \qquad (3.11)$

The ***second condition***, specified at the fin tip $(x = L)$, *may correspond to one of four different physical situations.*

4.2. Case A: Convection from Fin Tip

The first condition, **Case A**, considers convection heat transfer from the fin tip. Applying an energy balance to a control surface about this tip (**Figure 3.7**), we obtain:

Boundary condition at fin tip:

$$hA_c[T(L) - T_\infty] = -kA_c \left.\frac{dT}{dx}\right|_{x=L} \qquad (3.12)$$

or:

$$h\theta(L) = -k \left.\frac{d\theta}{dx}\right|_{x=L}$$

That is, the rate at which energy is transferred to the fluid by convection from the tip must equal the rate at which energy reaches the tip by conduction through the fin. Substituting **Equation 3.10** into **Equations 3.11** and **3.12**, we obtain, respectively,

$$\theta_b = C_1 + C_2 \qquad (3.13)$$

and

$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

Solving for C_1 and C_2, it may be shown, after some manipulation, that:

Convection from fin tip:

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL} \qquad (3.14)$$

The form of this temperature distribution is shown schematically in **Figure 3.7**. Note that the magnitude of the temperature gradient decreases with increasing x. This trend is a consequence of the reduction in the conduction heat transfer $q_x(x)$ with increasing x due to continuous convection losses from the fin surface.

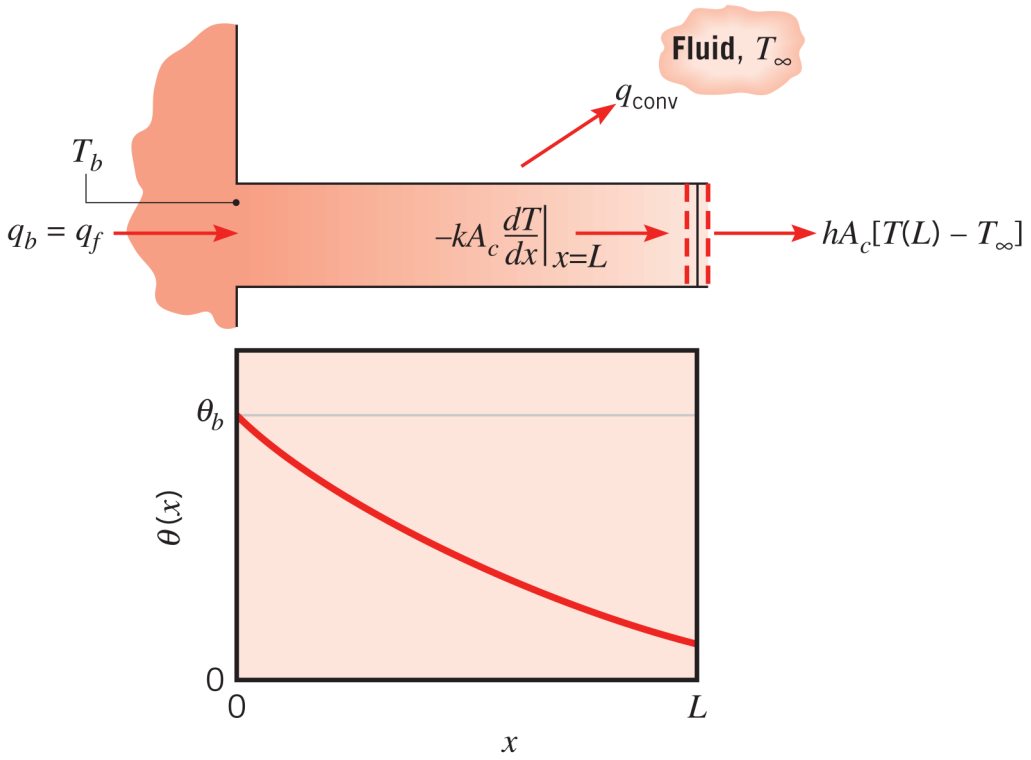


Figure 3.7: Conduction and convection in a fin of uniform cross section.

The ***rate of heat transfer*** from the fin can be found by substituting the temperature gradient at the base of the fin, obtained from **Equation 3.14**, into the Fourier's law of heat conduction. The result is:

$$q_f = q_b = -kA_c \frac{dT}{dx}\bigg|_{x=0} = -kA_c \frac{d\theta}{dx}\bigg|_{x=0} \qquad (3.15)$$

Hence, knowing the temperature distribution,$\theta(x)$, q_f may be evaluated, giving:

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \qquad (3.16)$$

4.3. Case B: Negligible Heat Loss from the Fin Tip (Adiabatic fin tip, q(fin tip)=0)

The second tip condition, **Case B**, corresponds to the assumption that the convective heat loss from the fin tip is negligible, in which case the tip may be treated as adiabatic.

Boundary condition at fin tip:

$$\frac{d\theta}{dx}\bigg|_{x=L} = 0 \qquad (3.17)$$

Substituting from **Equation 3.10** and dividing by m, we then obtain:

$$C_1 e^{mL} - C_2 e^{-mL} = 0$$

Using this expression with **Equation 3.13** to solve for C_1 and C_2 and substituting the results into **Equation 3.10**, we obtain the desired relation for the temperature distribution:

Adiabatic fin tip:

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL} \qquad (3.18)$$

Using this temperature distribution with **Equation 3.15**, the *fin heat transfer rate* is then:

$$q_f = \sqrt{hPkA_c}\theta_b \tanh mL \qquad (3.19)$$

4.4. Case C: Specified Temperature T(fin, tip) = TL

In this case the temperature at the end of the fin (**the fin tip**) is fixed at a specified temperature T_L. This case could be considered as a generalization of the case of ***Infinitely Long Fin*** where the fin tip temperature was fixed at T_∞ . The condition at the fin tip for this case is:

Boundary condition at fin tip:

$$\theta(L) = \theta_L = T_L - T_\infty \qquad (3.20)$$

In the same manner, we can obtain the fin temperature distribution and heat transfer rate for **Case C**, where the temperature is prescribed at the fin tip. That is, the second boundary condition is $\theta(L) = \theta_L$, and the resulting expressions are of the form

Specified fin tip temperature:

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL} \qquad (3.21)$$

$$q_f = \sqrt{hPkA_c}\theta_b \frac{\cosh mL - \theta_L/\theta_b}{\sinh mL} \qquad (3.22)$$

Note

Note that **Equations 3.21** and **3.22** reduce to **Equations 3.24** and **3.25** for the case of *infinitely long fin* $(L \to \infty)$.

4.5. Case D: Infinitely Long Fin T(fin tip)=T∞

The very long fin, **Case D**, is an interesting extension of these results. In particular, as $L \to \infty, \theta_L \to 0$ and it is easily verified that:

Boundary condition at fin tip:

$$\theta(L) = T(L) - T_\infty = 0 \quad \text{as} \quad L \to \infty \qquad (3.23)$$

The **temperature distribution** along the fin in this case can be expressed as:

Very long fin:

$$\frac{\theta}{\theta_b} = e^{-mx} \qquad (3.24)$$

The steady **rate of heat transfer** from the entire fin can be determined from Fourier's law of heat conduction:

$$q_f = \sqrt{hPkA_c}\theta_b \qquad (3.25)$$

The foregoing results are summarized in **Table 3.1**:

TABLE 3.1 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic: $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\theta_b$

5. PROBLEM 3,1

5.1. PROBLEM 3,1

A very long rod 5 mm in diameter has one end maintained at $100°C$. The surface of the rod is exposed to ambient air at $25°C$ with a convection heat transfer coefficient of $100 W/m^2 \cdot K$.

1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss.

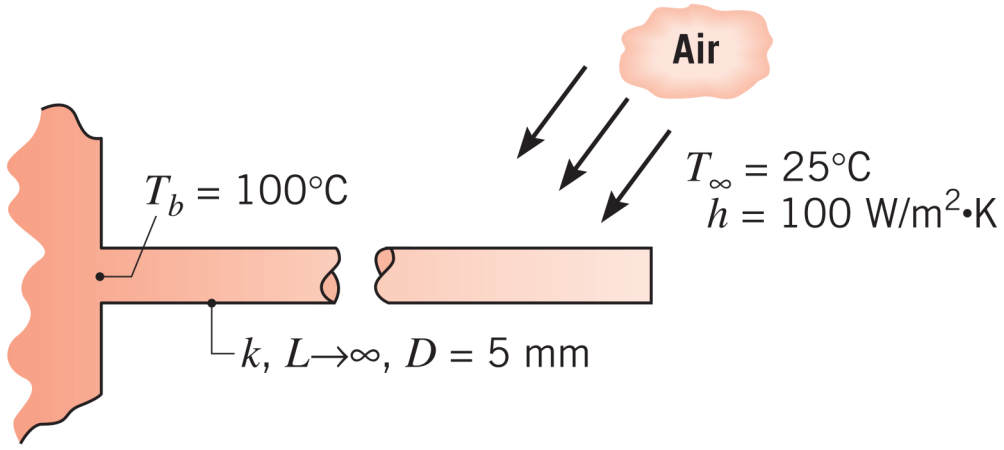
SOLUTION

Known: A long circular rod exposed to ambient air.

Find:

1. Temperature distribution and heat loss when rod is fabricated from copper, an aluminum alloy, or stainless steel.
2. How long rods must be to assume infinite length.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction along the rod.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Uniform heat transfer coefficient.
6. Infinitely long rod.

Properties: Table A.1, copper $[T = (T_b + T_\infty)/2 = 62.5°C \approx 335K]$: $k = 398 W/m \cdot K$. Table A. 1, 2024 aluminum$(335K)$: $k = 180 W/mK$. *Table A.1, stainless steel, AISI 316* (335K): $k = 14 W/m \cdot K$.

Analysis:

1. Subject to the assumption of an infinitely long fin, the temperature distributions are determined from **Equation 3.24**, which may be expressed as:

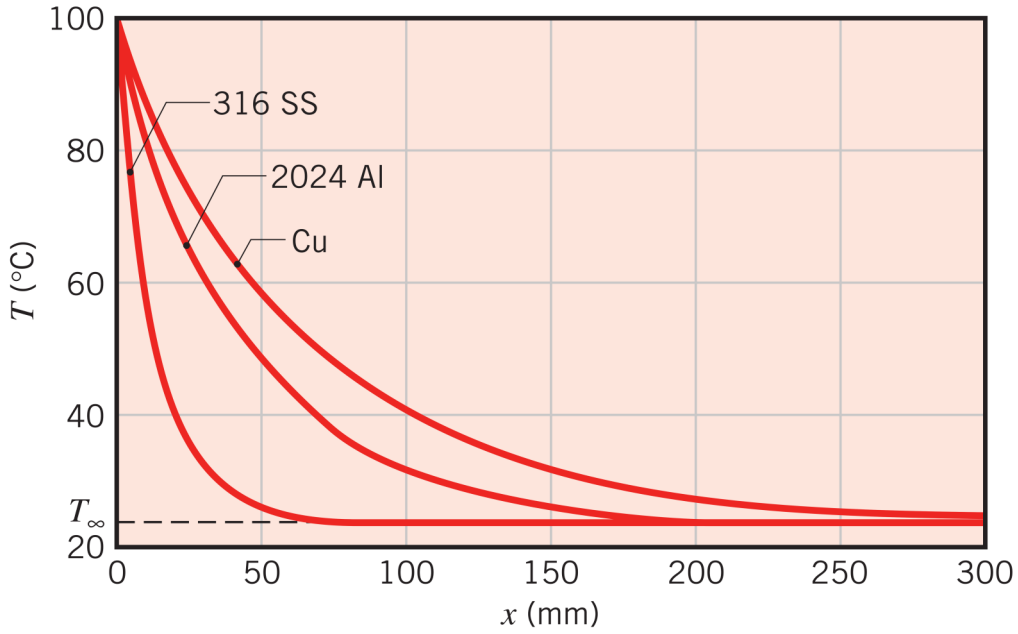
$$T = T_\infty + (T_b - T_\infty)e^{-mx}$$

where

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{4h}{kD}}$$

Substituting for h and D, as well as for the thermal conductivities of copper, the aluminum alloy, and the stainless steel, respectively, the

values of m are $14.2, 21.2, \text{and } 75.6\text{m}^{-1}$. The temperature distributions may then be computed and plotted as follows:



From these distributions, it is evident that there is little additional heat transfer associated with extending the length of the rod much beyond 50, 200, and 300 mm, respectively, for the stainless steel, the aluminum alloy, and the copper.

From **Equation 3.25**, the heat loss is:

$$q_f = \sqrt{hPkA_c}\theta_b$$

Hence for copper,

$$\begin{aligned} \text{q} &= \left[100\text{W/m}^2\cdot\text{K} \times \pi \times 0.005\text{m}\right] \\ &\quad \times 398\text{W/m}\cdot\text{K} \times \frac{\pi}{4}(0.005\text{m})^2\Big]^{1/2} (100 - 25)^\circ\text{C} \\ &= 8.3\ \text{W} \end{aligned}$$

Similarly, for the aluminum alloy and stainless steel, respectively, the heat rates are $q_f = 5.6\text{W and } 1.6\text{ W}$.

2. Since there is no heat loss from the tip of an infinitely long rod, an estimate of the validity of this approximation may be made by comparing **Equations 3.19** and **3.25**. To a satisfactory approximation, the expressions provide equivalent results if $\tanh \text{mL} \geq 0.99$ or $mL \geq 2.65$. Hence a rod may be assumed to be infinitely long if:

$$L \geq L_\infty \equiv \frac{2.65}{m} = 2.65\left(\frac{kA_c}{hP}\right)^{1/2}$$

For copper,

$$L_\infty = 2.65\left[\frac{398\text{W/m} \cdot \text{K} \times (\pi/4)(0.005\text{m})^2}{100\text{W/m}^2 \cdot \text{K} \times \pi(0.005\text{m})}\right]^{1/2} = 0.19\text{m}$$

Results for the aluminum alloy and stainless steel are $L_\infty = 0.13\text{ m and } L_\infty = 0.04\text{ m}$, respectively.

Comments:

1,The foregoing results suggest that the fin heat transfer rate may accurately be predicted

from the infinite fin approximation if $mL \geq 2.65$. However, if the infinite fin approximation is to accurately predict the temperature distribution $T(x)$, a larger value of mL would be required. This value may be inferred from Equation 3.84 and the requirement that the tip temperature be very close to the fluid temperature. Hence, if we require that $\theta(L)/\theta_b = \exp(-mL) < 0.01$, it follows that $mL > 4.6$, in which case $L_\infty \approx 0.33, 0.23$, and 0.07 m for the copper, aluminum alloy, and stainless steel, respectively. These results are consistent with the distributions plotted in part 1.

2. This example is solved in the Advanced section of IHT.

6. Fin Efficiency

In the limiting case of zero thermal resistance or infinite thermal conductivity $(k \to \infty)$, the temperature of the fin will be uniform at the base value of T_b. The heat transfer from the fin will be maximum in this case and can be expressed as:

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}(T_b - T_\infty) \qquad (3.26)$$

We define a ***fin efficiency*** as:

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} \qquad (3.27)$$

or

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}hA_{\text{fin}}(T_b - T_\infty) \qquad (3.28)$$

Where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of ***very long*** fins and ***fins with adiabatic tips***, the fin efficiency can be expressed as:

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL} \qquad (3.29)$$

and

$$\eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)\tanh mL}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{\tanh mL}{mL} \qquad (3.30)$$

Since $A_{\text{fin}} = pL$ for fins with constant cross section.

7. Fin Effectiveness

The performance of fins expressed in terms of the ***fin effectiveness*** ε_{fin} is defined as **Figure 3.8:**

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} \qquad (3.31)$$

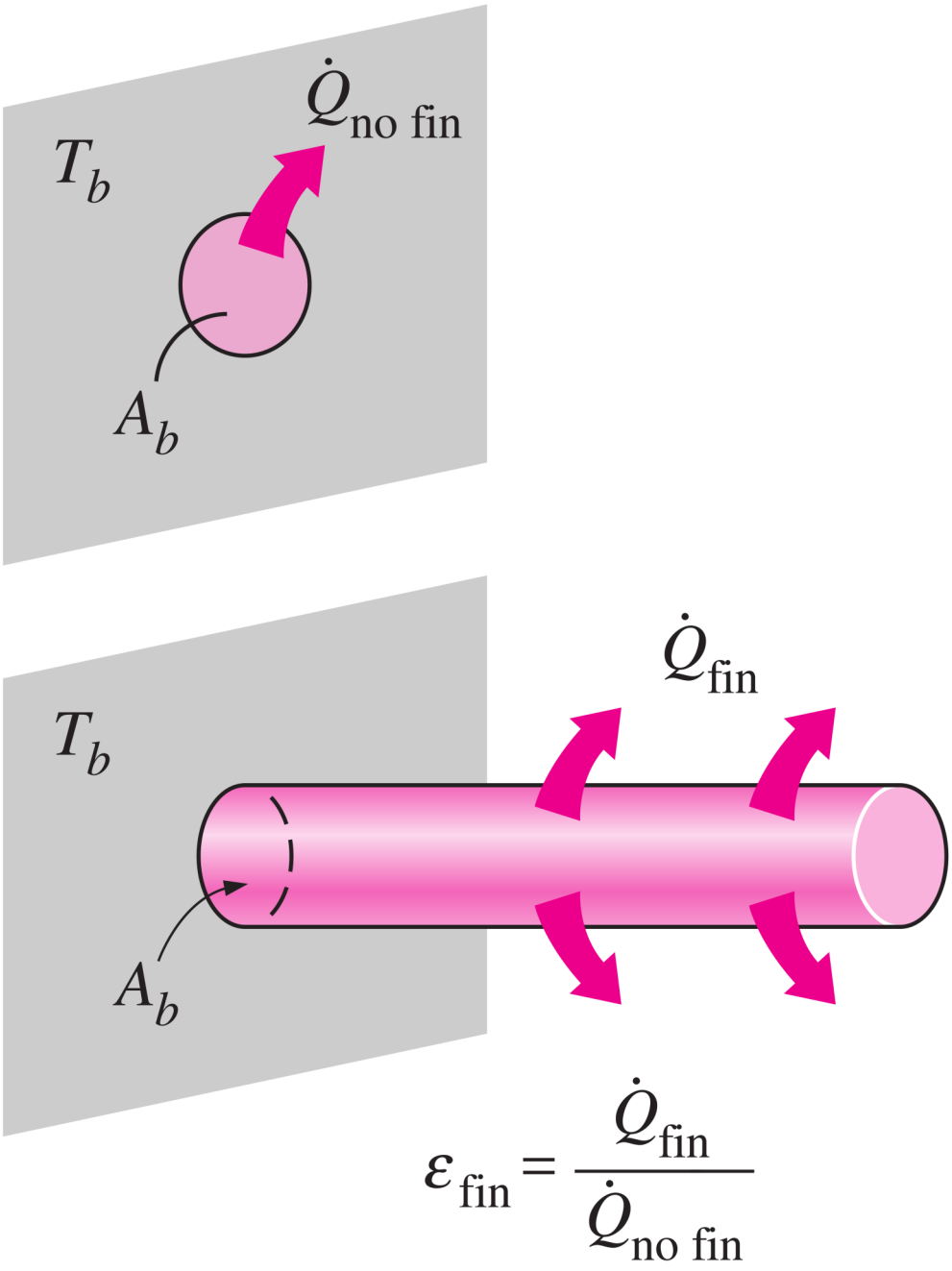


Figure 3.8: The effectiveness of a fin.

Note

Note that both the ***fin efficiency*** and ***fin effectiveness*** are related to the performance of the fin, but they are different quantities. However, they are related to each other by:

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}} \qquad (3.32)$$

The rate of heat transfer from a sufficiently long fin of uniform cross section under steady conditions is given by **Equation 3.25**. Substituting this relation into **Equation 3.31**, the effectiveness of such a long fin is determined to be:

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}} \qquad (3.33)$$

Since $A_c = A_b$ in this case.

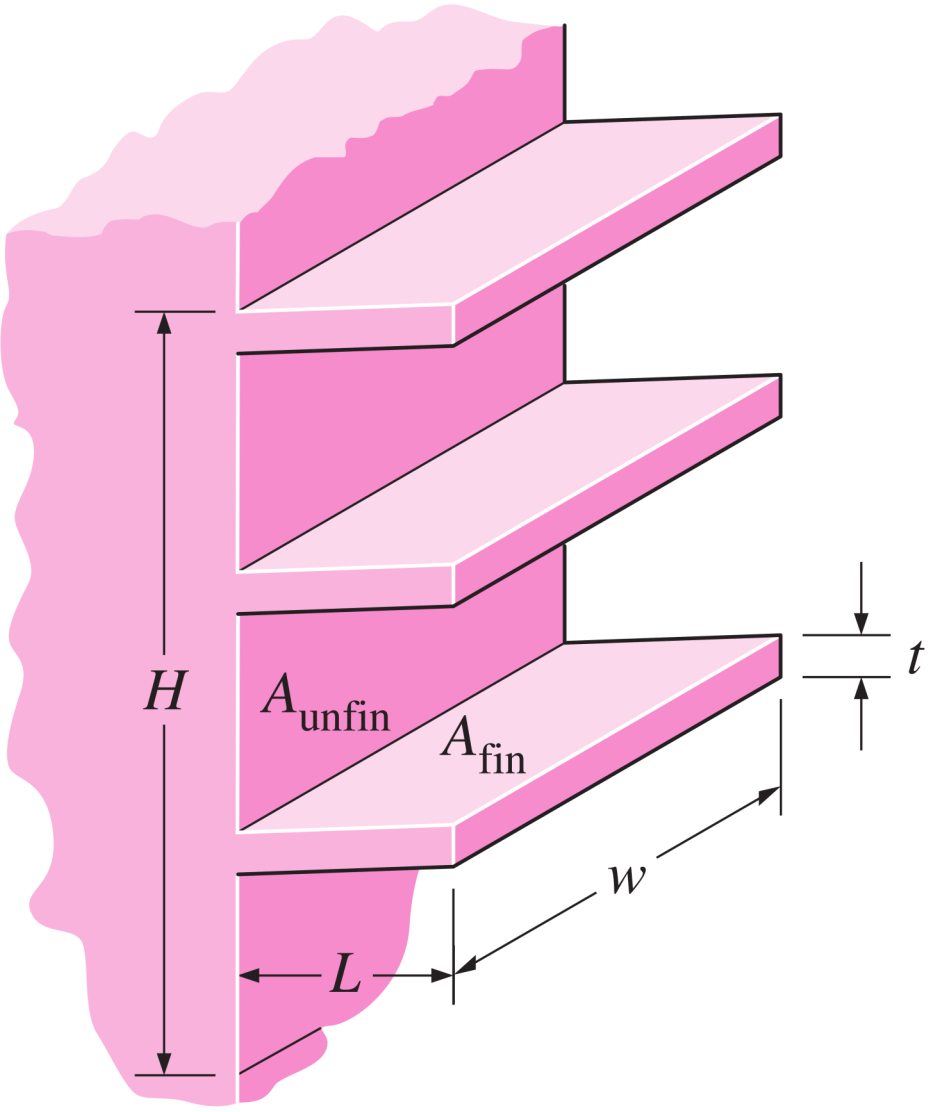
When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins. Therefore, *the rate of heat transfer for a surface containing n fins can be expressed as*:

$$\begin{aligned} \dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\ &= hA_{\text{unfin}}\,(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty) \end{aligned} \qquad (3.34)$$

We can also define an ***overall effectiveness*** for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins:

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)} \qquad (3.35)$$

Where $A_{\text{no fin}}$ is the area of the surface when there are no fins, A_{fin} is the total surface area of all the fins on the surface, and A_{unfin} is the area of the unfinned portion of the surface (***Figure 3.9***).



$$A_{no\ fin} = w \times H$$
$$A_{unfin} = w \times H - 3 \times (t \times w)$$
$$A_{fin} = 2 \times L \times w + t \times w \text{ (one fin)}$$
$$\approx 2 \times L \times w$$

Figure 3.9: Various surface areas associated with a rectangular surface with three fins.

Chapter 4: Thermal Convection

IV

1. General information

Thermal convection is a fundamental phenomenon in fluid dynamics, where heat is transferred within a fluid (liquid or gas) through the combined effects of conduction and fluid motion. This process plays a crucial role in a wide range of natural and industrial applications, including atmospheric dynamics, ocean circulation, cooling of electronic devices, and various engineering systems. The study of thermal convection involves understanding how heat transfer occurs due to the movement of fluid particles and how this movement is influenced by different forces, such as buoyancy and external flow.

Convection can be classified into two primary types: natural (or free) convection and forced convection. In natural convection, the fluid motion is generated by buoyancy forces that arise from temperature differences within the fluid. In contrast, forced convection occurs when an external source, such as a fan or a pump, drives the fluid motion. Each type of convection presents unique challenges and requires different approaches for analysis and modeling.

To analyze thermal convection, several dimensionless numbers are employed to characterize the flow and heat transfer behavior. The Nusselt number (Nu) represents the ratio of convective to conductive heat transfer, while the Prandtl number (Pr) relates the fluid's viscosity to its thermal diffusivity. The Reynolds number (Re) describes the ratio of inertial forces to viscous forces in the flow, and the Grashof number (Gr) indicates the significance of buoyancy forces compared to viscous forces. These dimensionless numbers provide insights into the convective heat transfer mechanisms and help predict the heat transfer rate under various conditions.

Empirical correlations have been developed to estimate the heat transfer coefficients in both free and forced convection scenarios. These correlations, based on experimental data, provide practical tools for engineers and scientists to design and optimize thermal systems. The following sections will explore the dimensionless numbers in detail, discuss their significance in different convection regimes, and present empirical correlations used in engineering applications to predict heat transfer rates in both natural and forced convection.

1.1. Natural and forced convection

Convection is called ***forced convection*** if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. In contrast, convection is called ***natural (or free) convection*** if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid (**Figure 4.1**). For example, in the absence of a fan, heat transfer from the surface of the hot block is by

natural convection since any motion in the air in this case is due to the rise of the warmer (and thus lighter) air near the surface and the fall of the cooler (and thus heavier) air to fill its place. Heat transfer between the block and the surrounding air is by conduction if the temperature difference between the air and the block is not large enough to overcome the resistance of air to movement and thus to initiate natural convection currents.

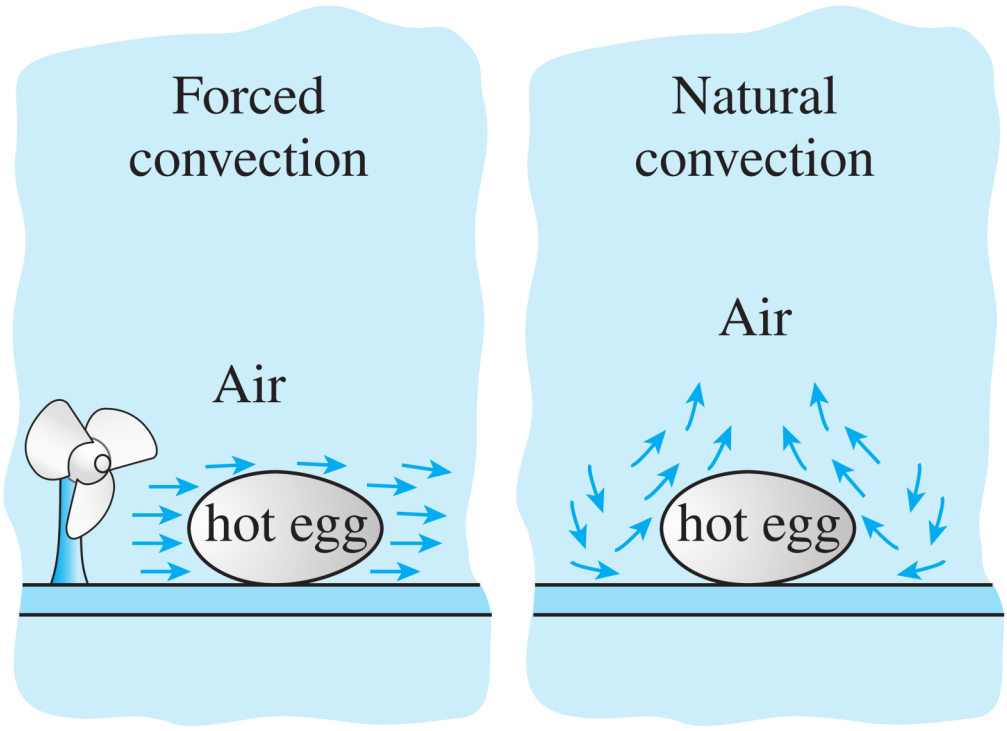


Figure 4.1: The cooling of a boiled egg by forced and natural convection.

2. Dimensionless numbers

2.1. Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient (h) with the Nusselt number (Nu), defined as:

$$\mathrm{Nu} = \frac{hL_c}{k} \qquad (4.1)$$

Where k is the thermal conductivity of the fluid and L_c is the characteristic length.

2.2. Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless parameter **Prandtl number**, defined as:

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \qquad (4.2)$$

Where $\alpha = k/\left(\rho c_p\right)$ is the thermal diffusivity and μ is the dynamic viscosity.

2.3. Reynolds number

For flow in a circular tube, the ***Reynolds number*** is defined as:

$$\mathrm{Re} = \frac{V_{\mathrm{avg}} D}{\nu} = \frac{\rho V_{\mathrm{avg}} D}{\mu} = \frac{\rho D}{\mu}\left(\frac{\dot{m}}{\rho \pi D^2/4}\right) = \frac{4\dot{m}}{\mu \pi D} \qquad (4.3)$$

where V_{avg} is the average flow velocity, D is the diameter of the tube, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

And The average velocity V_{avg}:

$$V = \frac{\dot{m}}{\rho A_c}$$

For flow through noncircular tubes, the *Reynolds number* as well as the *Nusselt number*, and the *friction factor* are based on the **hydraulic diameter** D_h defined as:

$$D_h = \frac{4A_c}{p} \qquad (4.4)$$

where A_c is the *cross sectional area* of the tube and p is its wetted *perimeter*.

The *hydraulic diameter* for ***circular tubes*** sinc:

$$D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D \qquad (4.5)$$

Consider a ***concentric annulus*** of inner diameter Di and outer diameter Do. The *hydraulic diameter* of the annulus is:

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i \qquad (4.6)$$

The *Reynolds number* at a distance x from the leading edge of a ***flat plate*** is expressed as:

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu} \qquad (4.7)$$

Note

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\mathrm{Re}_L = VL/\nu$ at the end of the plate.

2.4. The Grashof Number

The dimensionless parameter in the brackets represents the natural convection effects, and is called the ***Grashof number*** Gr_L (**Figure 4.2**):

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \qquad (4.8)$$

where:

g = gravitational acceleration, $\mathrm{m/s^2}$

β = coefficient of volume expansion, $1/\mathrm{K}$ ($\beta = 1/T$ for ideal gases)

T_s = temperature of the surface, $^\circ\mathrm{C}$

T_∞ = temperature of the fluid sufficiently far from the surface, $^\circ\mathrm{C}$

L_c = characteristic length of the geometry, m

ν = kinematic viscosity of the fluid, $\mathrm{m^2/s}$

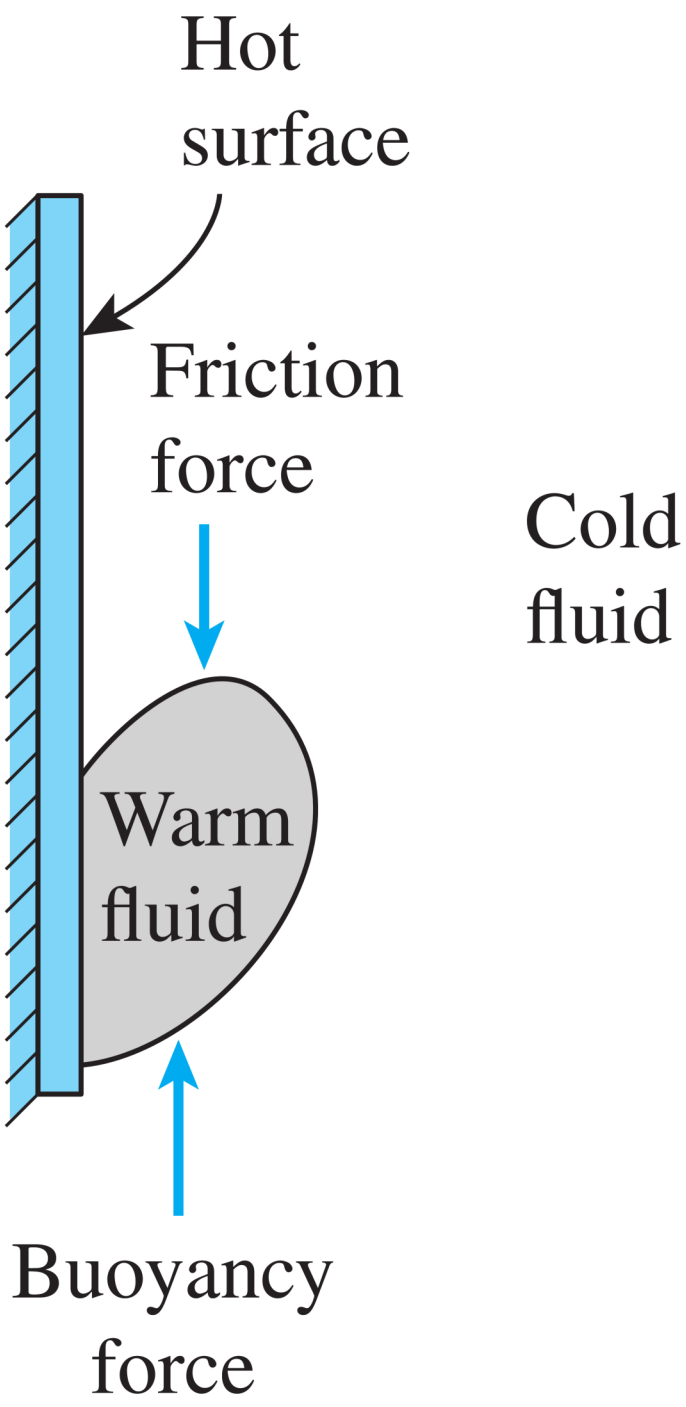


Figure 4.2: The Grashof number Gr is a measure of the relative magnitudes of the buoyancy force and the opposing viscous force acting on the fluid.

Note that the best accuracy is achieved by redefining the ***characteristic length*** as:

$$L = \frac{A_s}{P}$$

where A_s is the surface area and P is the surface perimeter.

2.5. Rayleigh number

The **Rayleigh number** , which is the product of the ***Grashof*** and ***Prandtl numbers***:

$$\mathrm{Ra}_L = \mathrm{Gr}_L \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr} \qquad (4.9)$$

3. Empirical correlations

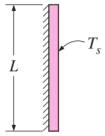
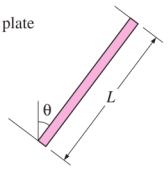
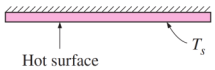
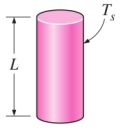
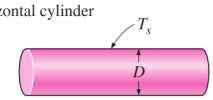
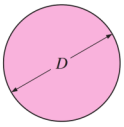
3.1. Empirical correlations for natural convection

The following ***correlations*** have been derived for purely buoyancy driven flows (i.e. ***natural or free convection***) and for such circumstances the **Nusselt number** is typically expressed as a function of ***Rayleigh number*** and **PrandtI number**,i.e.:

$$Nu = f\,(Ra, Pr)$$

TABLE 4.1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ (9-19) $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ (9-20) $\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\mathrm{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ (9-22) $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$ (9-23) $\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\mathrm{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\mathrm{Ra}_D \leq 10^{12}$	$\mathrm{Nu} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$\mathrm{Ra}_D \leq 10^{11}$ ($\mathrm{Pr} \geq 0.7$)	$\mathrm{Nu} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{[1 + (0.469/\mathrm{Pr})^{9/16}]^{4/9}}$ (9-26)

3.2. Empirical correlations for forced convection

The following ***correlations*** have been developed for cases where ***forced convection*** is the dominant factor for heat transfer and boundary layer behavior. For such circumstances the ***Nusselt number*** is typically described as a function of ***Reynolds number*** and ***Prandtl number***, ie.:

$$Nu = f\,(Re, Pr)$$

3.2.1. Forced convection on a flat plate in parallel flow

Figure 4.3 shows a typical boundary layer formation from the leading edge of a flat horizontal plate in parallel flow.

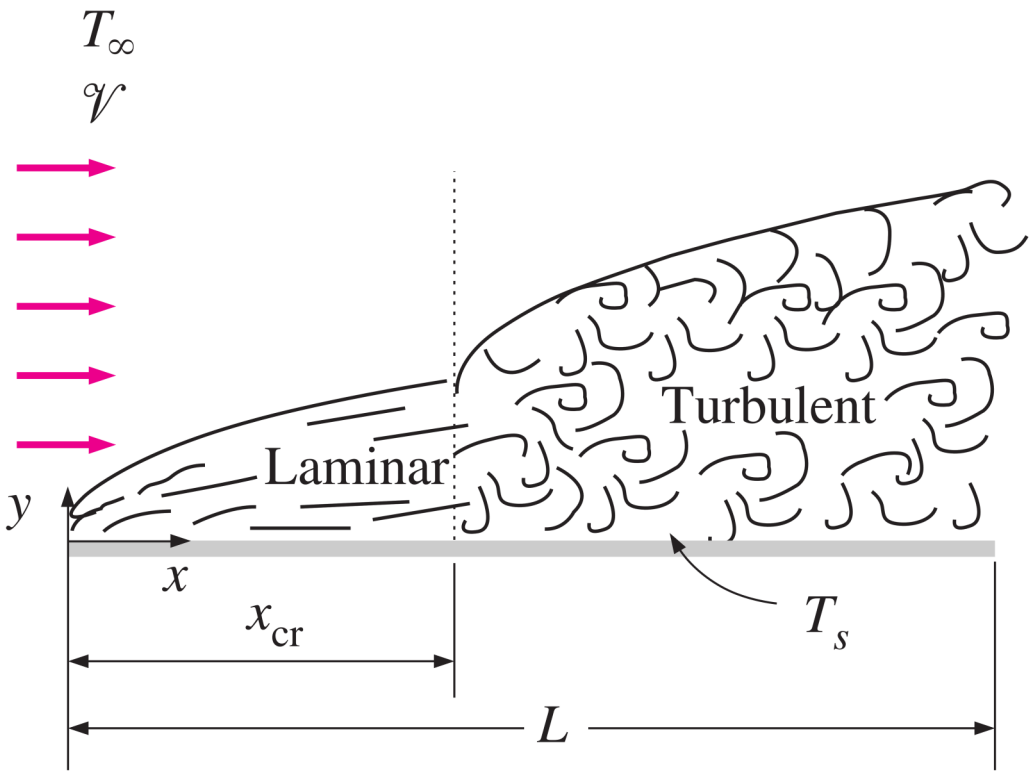


Figure 4.3: Laminar and turbulent regions of the boundary layer during flow over a flat plate.

a) Correlations for an isothermal plate:

For an **isothermal plate** and a steady, incompressible, laminar flow with constant fluid properties, neglecting viscous dissipation and assuming that $dp/dx = 0$ it may be shown that the local *Nusselt number* at a point $x \leq x_c$ can be expressed as:

Laminar:

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\ \mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3} \quad \mathrm{Pr} > 0.60 \qquad (4.10)$$

Turbulent:

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60$$
$$5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \qquad (4.11)$$

The ***average Nusselt number*** over the entire plate:

Laminar:

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\ \mathrm{Re}_L^{0.5}\ \mathrm{Pr}^{1/3} \quad \mathrm{Re}_L < 5 \times 10^5 \qquad (4.12)$$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\ \mathrm{Re}_L^{0.8}\ \mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60$$
$$5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \qquad (4.13)$$

b) Correlations for a plate with uniform heat flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature,

the *local Nusselt number* is given by:

Laminar:

$$Nu_x = 0.453\ Re_x^{0.5} Pr^{1/3} \quad Pr \gtrsim 0.6 \qquad (4.14)$$

Turbulent:

$$Nu_x = 0.0308\ Re_x^{0.8} Pr^{1/3} \quad 0.6 \lesssim Pr \lesssim 60 \qquad (4.15)$$

3.2.2. Forced convection on a cylinder in cross flow

Another common *forced convection* application is the single cylinder in cross flow (see **Figure 4.4** below).

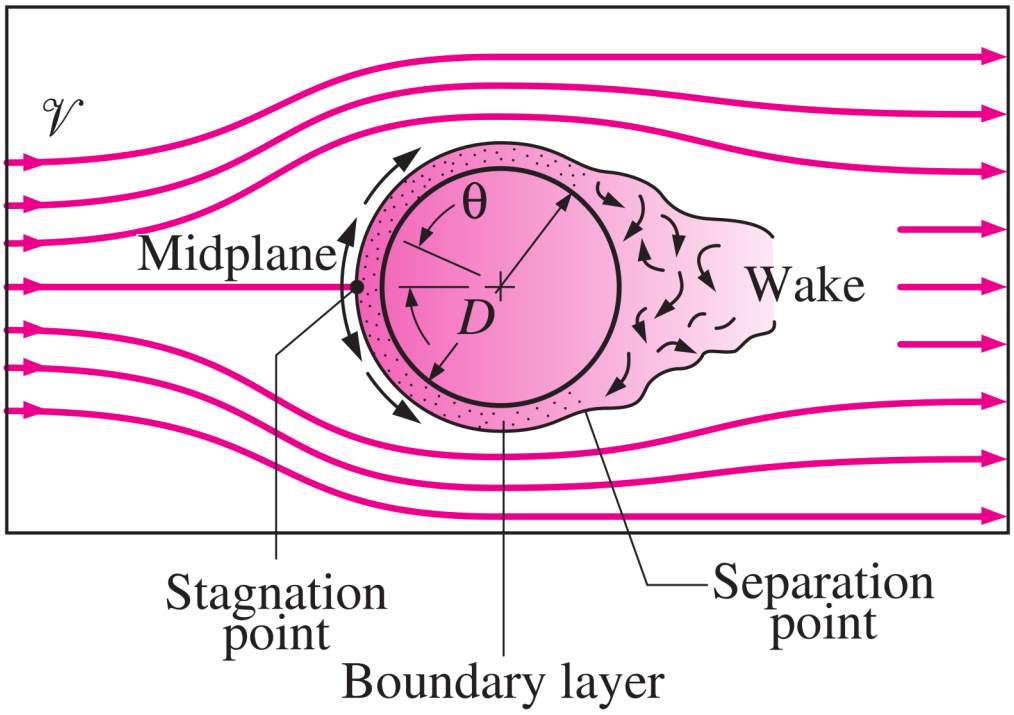


Figure 4.4: Typical flow patterns in cross flow over a cylinder.

Churchill and Bernstein have proposed a correlation for the *average Nusselt number* claimed to cover a wide range of $\mathrm{Nu}_{\mathrm{cyl}}$ and a wide range of P_r. This correlation is deemed valid for $Re_D Pr > 0.2$ and reads

$$\mathrm{Nu}_{\mathrm{cyl}} = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5} \qquad (4.16)$$

The **average Nusselt number** for flow across cylinders can be expressed compactly as. One such widely used correlation has been suggested by ***Hilpert***:

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = C\, Re_D^m\, Pr^{1/3} \qquad (4.17)$$

Suggested values for C and m are found in **Table 4.2** below:

Re_D	C	m
0.4 - 4	0.989	0.330
4 - 40	0.911	0.385
40 - 4000	0.683	0.466
4000 - 40000	0.193	0.618
40000 - 400000	0.027	0.805

For flow over a ***sphere***, *Whitaker* recommends the following comprehensive *correlation*:

$$\mathrm{Nu_{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \qquad (4.18)$$

which is valid for $3.5 \leq \mathrm{Re} \leq 80{,}000$ and $0.7 \leq \mathrm{Pr} \leq 380$.

3.2.3. Forced convection in a circular tube

a) Laminar Flow in Tubes

Constant Surface Temperature

Circular tube, laminar (T_s = constant):

$$\mathrm{Nu} = \frac{hD}{k} = 3.66 \qquad (4.19)$$

Constant Surface Heat Flux

Circular tube, laminar ($\dot{q}_x$ = constant):

$$\mathrm{Nu} = \frac{hD}{k} = 4.36 \qquad (4.20)$$

b) Turbulent Flow in Tubes

For ***turbulent tube flow*** and assuming small to moderate temperature differences ($T_s - T_m$) the *DIttus-Boelter equation* is a common approximation of the **Nusselt number**. This correlation reads:

$$Nu_D = 0.023\,Re_D^{0.8}\,Pr^n \qquad (4.21)$$

where $n = 0.4$ for ***heating*** and $n = 0.3$ for ***cooling*** of the fluid flowing through the tube.

Chapter 5: Description of Heat Exchangers without Phase Change

V

The process of exchanging heat between two fluids at different temperatures, separated by a solid wall, occurs in many engineering applications. The device used to implement this exchange is called a ***heat exchanger***. Heat exchangers facilitate the transfer of heat while keeping the two fluids from mixing with each other. They are commonly used in a wide range of applications, such as space heating and cooling, power generation, waste heat recovery, and chemical processing. For example, in a car radiator, heat is transferred from the hot water flowing through the radiator tubes to the air flowing through the closely spaced thin plates attached to the tubes outside. Unlike mixing chambers, heat exchangers do not allow the two fluids involved to mix.

1. Heat Exchanger Theory

General Principle

A heat exchanger operates by circulating two fluids through channels that allow them to come into thermal contact, separated by a metal wall that facilitates heat transfer. Typically, heat is transferred from the hotter fluid to the cooler one.

The primary challenge is determining an adequate surface area for heat exchange to ensure the required amount of heat is transferred under specific conditions. The heat transfer rate depends not only on the surface area available between the two fluids but also on several other factors.

Key factors influencing the rate of heat transfer include:

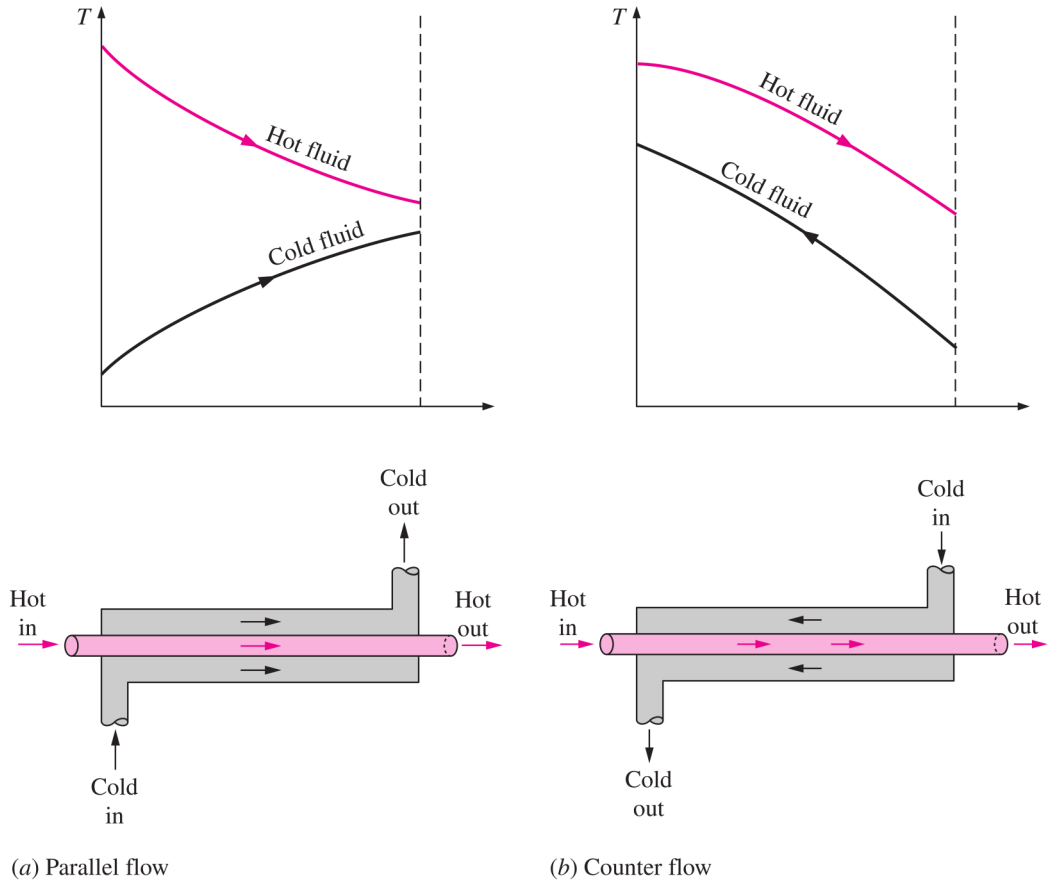
- The inlet temperatures of the fluids ($T_{\text{in,hot}}$ and $T_{\text{in,cold}}$),
- The thermal properties of the fluids, such as specific heat capacity (C_p) and thermal conductivity (k),
- The convection heat transfer coefficients (h).

2. TYPES OF HEAT EXCHANGERS

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment. The attempt to match the heat transfer hardware to the heat transfer requirements within the specified constraints has resulted in numerous types of innovative heat exchanger designs.

2.1. Double tube heat exchanger

The simplest type of *heat exchanger* consists of two concentric pipes of different diameters, as shown in **Figure 5.1**, called the double-pipe heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a ***double-pipe*** heat exchanger: in ***parallel flow***, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction. In ***counter flow***, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions.



(*a*) Parallel flow (*b*) Counter flow

Figure 5.1: Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.

2.2. Compact heat exchanger (plate)

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the ***compact heat exchanger***.

The ratio of the heat transfer surface area of a heat exchanger to its volume is called the ***area density*** . A heat exchanger with $\beta > 700\ \mathrm{m^2/m^3} (\text{or } 200\ \mathrm{ft^2/ft^3})$ is classified as being compact. Examples of compact heat exchangers are car radiators $(\beta \approx 1000\ \mathrm{m^2/m^3})$, glass ceramic gas turbine heat exchangers $(\beta \approx 6000\ m^2/m^3)$, the regenerator of a Stirling engine $(\beta \approx 15,000\ \mathrm{m^2/m^3})$, and the human lung $(\beta \approx 20{,}000 \mathrm{m^2/m^3})$.

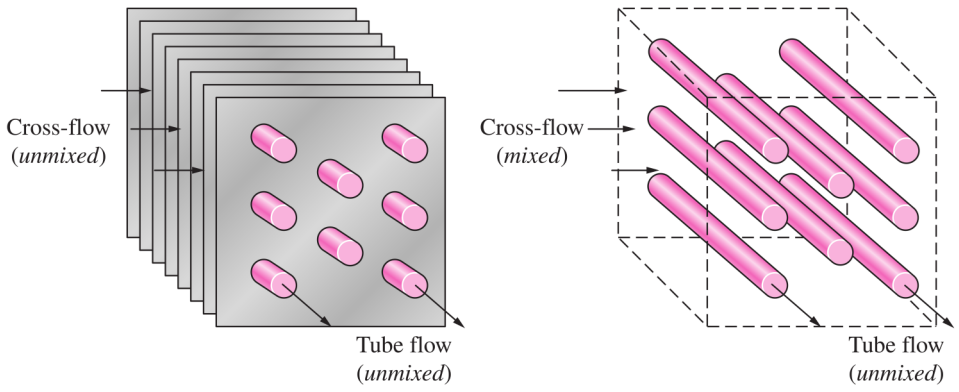
Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers (**Figure 5.2**).

The large surface area in compact heat exchangers is obtained by attaching closely spaced ***thin plate or corrugated fins*** to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas and gas-toliquid (or liquid-to-gas) heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area. In a car radiator, which is a water-to-air compact heat exchanger, for example, it is no surprise that fins are attached to the air side of the tube surface.

In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called ***cross-flow***. The cross-flow is further classified as ***unmixed*** and ***mixed flow***, depending on the flow configuration, as shown in **Figure 5.2**. In (a) the cross-flow is said to be unmixed since the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction (i.e., parallel to the tubes). The cross-flow in (b) is said to be mixed since the fluid now is free to move in the transverse direction. Both fluids are unmixed in a car radiator.

Note

The presence of mixing in the fluid can have a significant effect on the heat transfer characteristics of the heat exchanger.



(*a*) Both fluids unmixed (*b*) One fluid mixed, one fluid unmixed

2.3. Shell-and-Tube Heat Exchanger (With Complex Bundles)

Perhaps the most common type of heat exchanger in industrial applications is the ***shell-and-tube*** heat exchanger, shown in **Figure 5.3**. Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell.

Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the

tubes through the shell.

Baffles are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes.

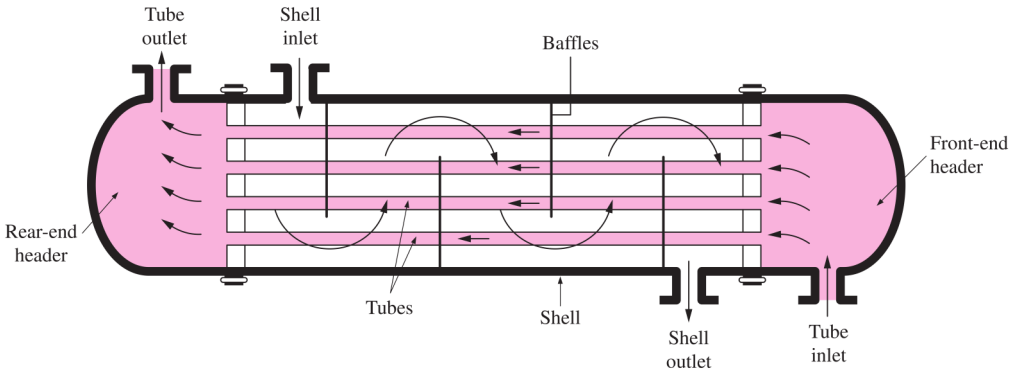
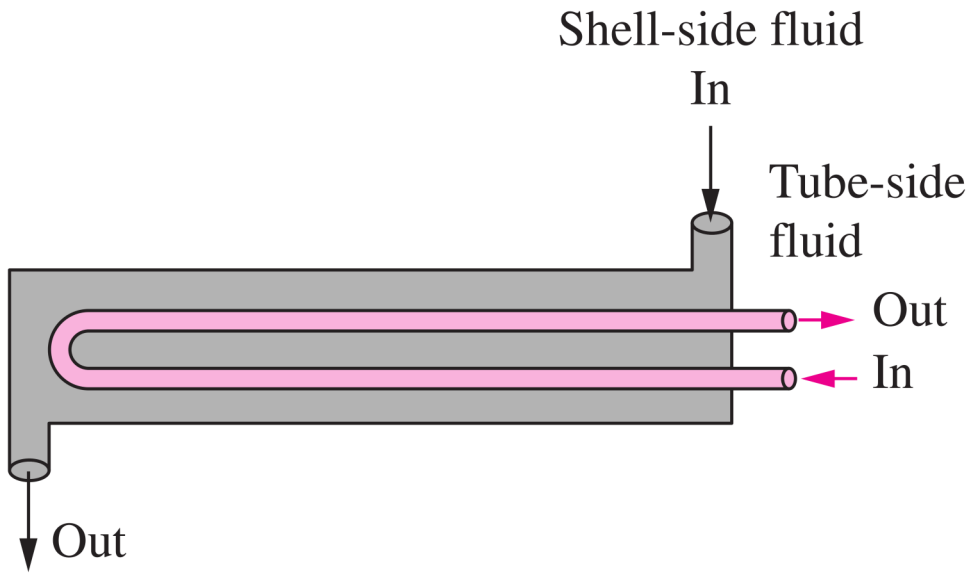


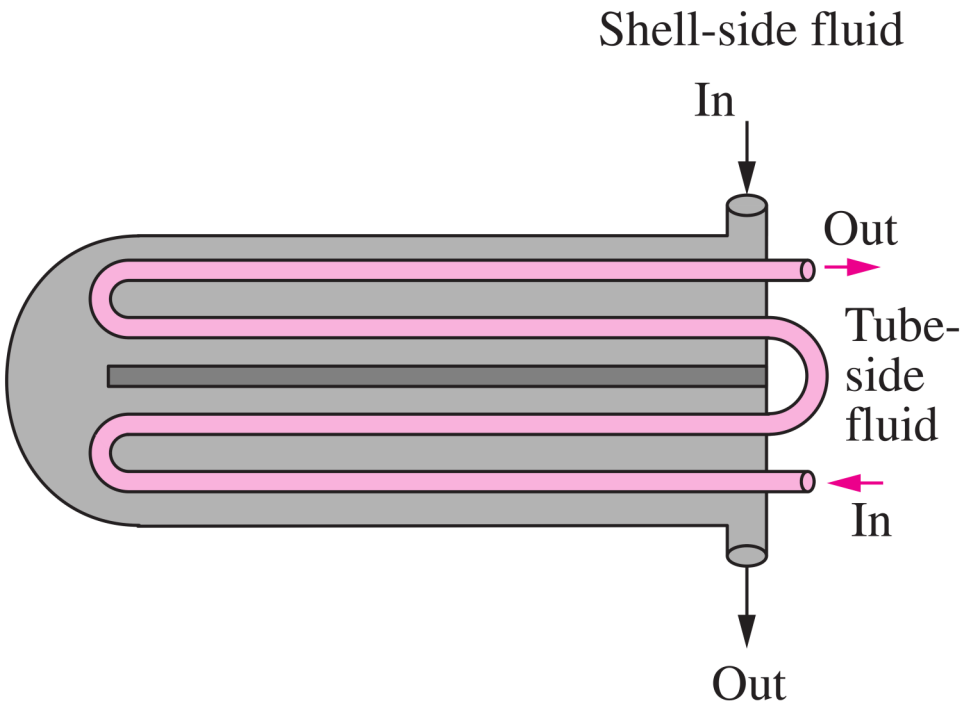
Figure 5.3: The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass).

Despite their widespread use, shelland-tube heat exchangers are not suitable for use in automotive and aircraft applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called ***headers*** at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them.

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called ***one-shell-pass and twotube-passes*** heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a ***two-shell-passes and four-tube-passes*** heat exchanger (**Figure 5.4**).



(*a*) One-shell pass and two-tube passes



(*b*) Two-shell passes and four-tube passes

Figure 5.4: Multipass flow arrangements in shell-and-tube heat exchangers.

Extra

Heat exchangers are often given specific names to reflect the specific application for which they are used. **For example**, a ***condenser*** is a heat exchanger in which one of the fluids is cooled and condenses as it flows through the heat exchanger. A ***boiler*** is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A ***space radiator*** is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

Chapter 6: Heat Exchanger Calculations

VI

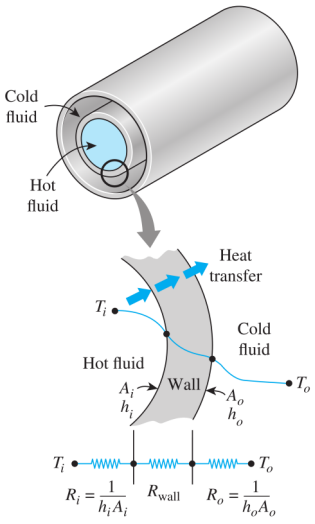
A **heat exchanger** typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects are usually included in the convection heat transfer coefficients.

1. THE OVERALL HEAT TRANSFER COEFFICIENT

A **heat exchanger** typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects are usually included in the convection heat transfer coefficients.

1.1. The thermal resistance network

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in **Figure 6.1**.



Here the subscripts ***i*** and ***o*** represent the inner and outer surfaces of the inner tube. For a double-pipe heat exchanger, **the thermal resistance** of the tube wall is:

Figure 6.1: Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

$$R_{\text{wall}} = \frac{\ln(D_o/D_i)}{2\pi kL} \qquad (6.1)$$

where ***k*** is the thermal conductivity of the wall material and **L** is the length of the tube. Then **the total thermal resistance** becomes:

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o} \qquad (6.2)$$

The $\boldsymbol{A_i}$ is the area of the inner surface of the wall that separates the two fluids, and $\boldsymbol{A_o}$ is the area of the outer surface of the wall. In other words, $\boldsymbol{A_i}$ and $\boldsymbol{A_o}$ are surface areas of the separating wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$ (**Figure 6.2**).

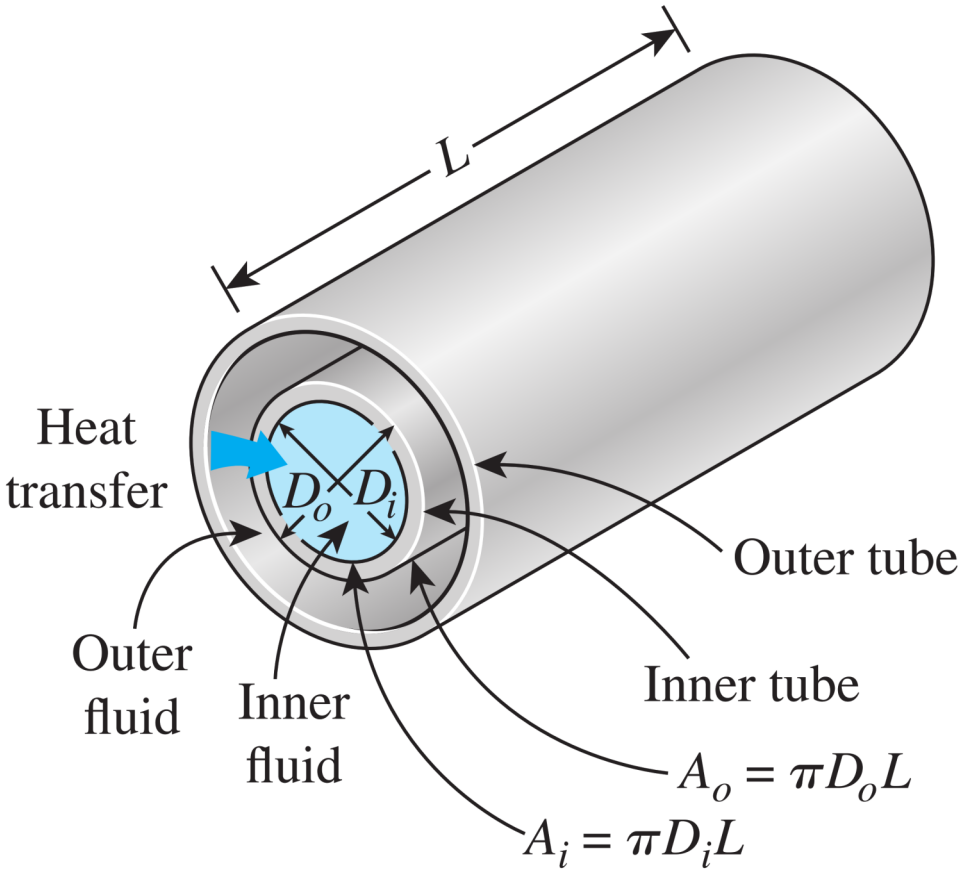


Figure 6.2: The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes, Di ≈ Do and thus Ai ≈ Ao).

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance ***R***, and to express the rate of heat transfer between the two fluids as:

$$\dot{Q} = \frac{\Delta T}{R} = UA_s\Delta T = U_iA_i\Delta T = U_oA_o\Delta T \qquad (6.3)$$

where A_s is the surface area and **U** is **the overall heat transfer coefficient**, whose unit is $\mathrm{W/m^2 \cdot K}$, which is identical to the unit of the ordinary convection coefficient **h**. Canceling ΔT, **Eq. (6.3)** reduces to:

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o} \qquad (6.4)$$

Note

Note that $U_iA_i = U_oA_o$, but $U_i \neq U_o$ unless $A_i = A_c$. Therefore, the overall heat transfer coefficient **U** of a heat exchanger is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible $(R_{\text{wall}} \approx 0)$ and the inner and outer surfaces of the tube are almost identical $(A_i \approx A_o \approx A_s)$. Then **Eq. (6.4)** for the overall heat transfer coefficient simplifies to:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \qquad (6.5)$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, hi and ho, are determined using the convection relations for internal and external forced convection flows, respectively.

The overall heat transfer coefficient **U** in **Eq. (6.5)** is dominated by the smaller convection coefficient, since the inverse of a large number is small.

When one of the convection coefficients is *much smaller* than the other (say, $h_i \leqslant h_o$), we have $1/h_i \geqslant 1/h_o$, and thus $U \approx h_i$.

Therefore, the smaller heat transfer coefficient creates a *bottleneck* on the path of heat transfer and seriously impedes heat transfer. This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product ***UA*** and thus the heat transfer on that side.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes:

$$A_s = A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}} \qquad (6.6)$$

where A_{fin} is the surface area of the fins and A_{unfinned} is the area of the unfinned portion of the tube surface. For short fins of high thermal conductivity, we can use this total area in the convection resistance relation $R_{\text{conv}} = 1/hA_s$ since the fins in this case will be very nearly isothermal. Otherwise, we should determine the effective surface area from:

$$A_s = A_{\text{unfinned}} + \eta_{\text{fin}} A_{\text{fin}} \qquad (6.7)$$

where η_{fin} is the fin efficiency which can be obtained from the relationships discussed in **Chapter 3**. This way, the temperature drop along the fins is accounted for. Note that $\eta_{\text{fin}} = 1$ for isothermal fins, and thus **Eq. (6.7)** reduces to **Eq. (6.6)** in that case. Note that for the finned surface, the relevant area $(A_i \text{ or } A_o)$ in **Eq. (6.4)** should be calculated from **Eq. (6.7)**.

1.2. Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of *deposits* represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor** R_f, which is a measure of the *thermal resistance* introduced by fouling.

The most common type of fouling is the *precipitation* of solid deposits in a fluid on the heat transfer surfaces. You can observe this type of fouling even in your house. If you check the inner surfaces of your teapot after prolonged use, you will probably notice a layer of calcium-based deposits on the surfaces at which boiling occurs. This is especially the case in areas where the water is hard. The scales of such deposits come off by scratching, and the surfaces can be cleaned of such deposits by chemical treatment. Now imagine those mineral deposits forming on the inner surfaces of fine tubes in a heat exchanger (**Figure 6.3**) and the detrimental effect it may have on the flow passage area and the heat transfer. To avoid this potential problem, water in power and process plants is extensively treated and its solid contents are removed before it is allowed to circulate through the system. The solid ash particles in the flue gases accumulating on the surfaces of air preheaters create similar problems.

Figure 6.3: Precipitation fouling of ash particles on superheater tubes.

Another form of fouling, which is common in the chemical process industry, is *corrosion* and other *chemical fouling*. In this case, the surfaces are fouled by the accumulation of the products of chemical reactions on the surfaces. This form of fouling can be avoided by coating metal pipes with glass or using plastic pipes instead of metal ones. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called *biological fouling* and can be prevented by chemical treatment.

In applications where it is likely to occur, fouling should be considered in the design and selection of heat exchangers. In such applications, it may be necessary to select a larger and thus more expensive heat exchanger to ensure that it meets the design heat transfer requirements even after fouling occurs. The periodic cleaning of heat exchangers and the resulting down time are additional penalties associated with fouling.

Note

The fouling factor is obviously zero for a new heat exchanger and increases with time as the solid deposits build up on the heat exchanger surface. The fouling factor depends on the *operating temperature* and the *velocity* of the fluids, as well as the length of service. Fouling increases with *increasing temperature and decreasing velocity* .

The overall heat transfer coefficient relation **Eq. (6.4)** or **(6.5)**, for an unfinned double-pipe heat exchanger is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For an unfinned double-pipe heat exchanger, it can be expressed as

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o} \qquad (6.8)$$

where $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

2. ANALYSIS OF HEAT EXCHANGERS

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to *select a heat exchanger* that will achieve a *specified temperature change* in a fluid stream of known mass flow rate, or to *predict the outlet temperatures* of the hot and cold fluid streams in a *specified heat exchanger*.

In upcoming sections, we discuss the two methods used in the analysis of heat exchangers. Of these, the ***log mean temperature difference (or LMTD)*** method is best suited for the first task and the ***effectiveness–NTU*** (the number of transfer units) method for the second task. But first we present some general considerations.

Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modeled as *steady-flow* devices. As such, the mass flow rate of each fluid remains constant, and the fluid properties such as temperature and velocity at any inlet or outlet remain the same. Also, the fluid streams experience little or no change in their velocities and elevations, and thus the kinetic and potential energy changes are negligible. The specific heat of a fluid, in general, changes with temperature. But, in a specified temperature range, it can be treated as a constant at some average value with little loss in accuracy. Axial heat conduction along the tube is usually insignificant and can be considered negligible. Finally, the outer surface of the heat exchanger is assumed to be *perfectly insulated*, so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only.

The idealizations stated above are closely approximated in practice, and they greatly simplify the analysis of a heat exchanger with little sacrifice from accuracy. Therefore, they are commonly used. Under these assumptions, the *first law of thermodynamics* requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}c_{pc}(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \qquad (6.9)$$

and

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\,\text{in}} - T_{h,\,\text{out}}) \qquad (6.10)$$

where the subscripts ***c*** and ***h*** stand for ***cold*** and ***hot*** fluids, respectively, and

$\dot{m}_c, \dot{m}_h$ = mass flow rates

c_{pc}, c_{ph} = specific heats

$T_{c,\,\text{out}}, T_{h,\,\text{out}}$ = outlet temperatures

$T_{c,\,\text{in}}, T_{h,\,\text{in}}$ = inlet temperatures

Note that the heat transfer rate $\dot{Q}$ is taken to be a positive quantity, and its direction is understood to be from the hot fluid to the cold one in accordance with the second law of thermodynamics.

In heat exchanger analysis, it is often convenient to combine the product of the mass flow rate and the specific heat of a fluid into a single quantity. This quantity is called the **heat capacity rate** and is defined for the hot and cold fluid streams as:

$$C_h = \dot{m}_h c_{ph} \text{ and } C_c = \dot{m}_c c_{pc} \qquad (6.11)$$

The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1°C as it flows through a heat exchanger.

Note

Note that in a heat exchanger, the fluid with a *large* heat capacity rate experiences a *small* temperature change, and the fluid with a *small* heat capacity rate experiences a *large* temperature change. Therefore, *doubling* the mass flow rate of a fluid while leaving everything else unchanged will halve the temperature change of that fluid.

With the definition of the heat capacity rate above, **Eqs. (6.9)** and **(6.10)** can also be expressed as:

$$\dot{Q} = C_c(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \qquad (6.12)$$

and

$$\dot{Q} = C_h(T_{h,\,\text{in}} - T_{h,\,\text{out}}) \qquad (6.13)$$

That is, the heat transfer rate in a heat exchanger is equal to the heat capacity rate of either fluid multiplied by the temperature change of that fluid.

Note

Note that the only time the temperature rise of a cold fluid is equal to the temperature drop of the hot fluid is when the heat capacity rates of the two fluids are equal to each other **Figure 6.4**.

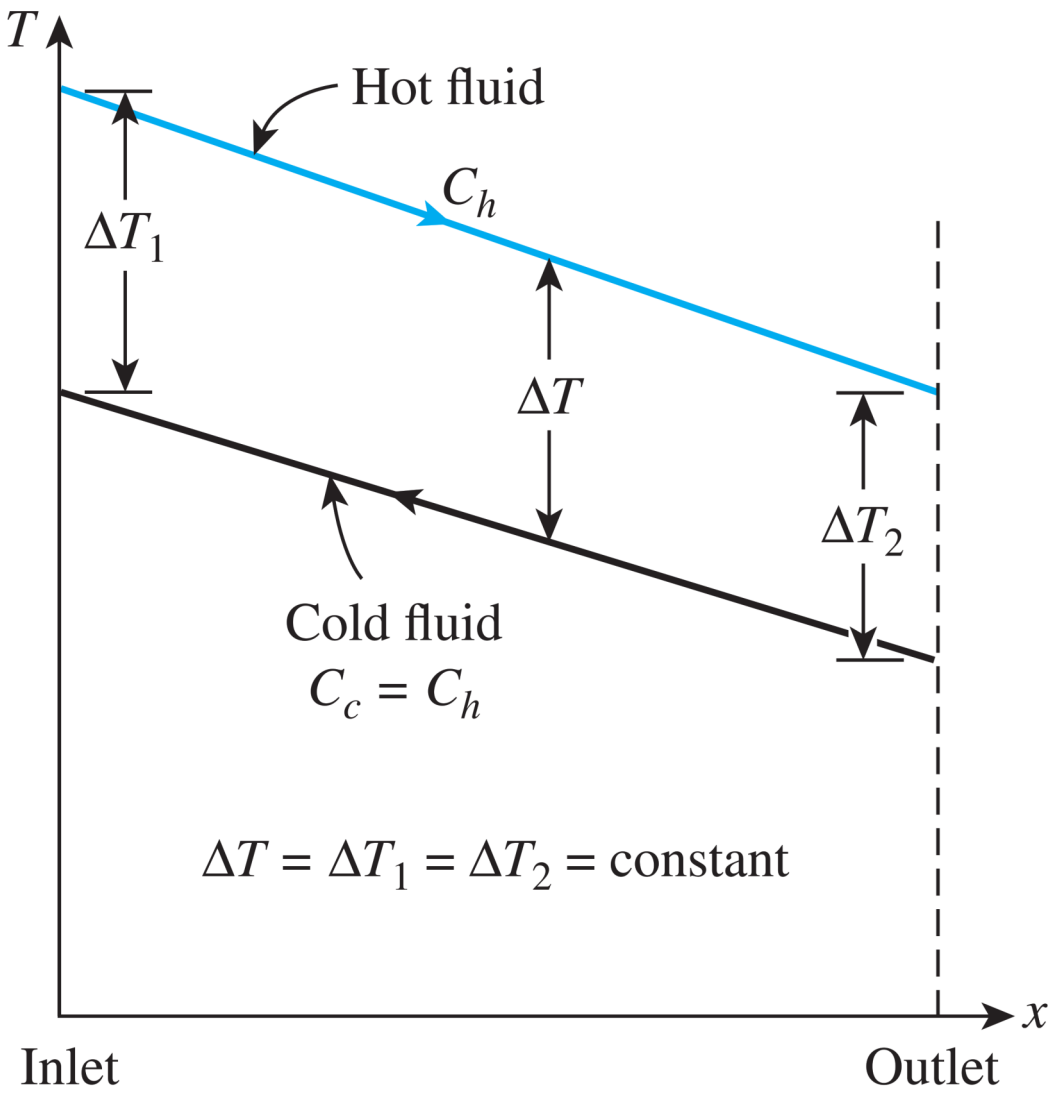


Figure 6.4: Two fluid streams that have the same capacity rates experience the same temperature change in a well-insulated heat exchanger.

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers*. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as:

$$\dot{Q} = \dot{m} h_{fg} \qquad (6.14)$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid, and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in **Figure 6.5**.

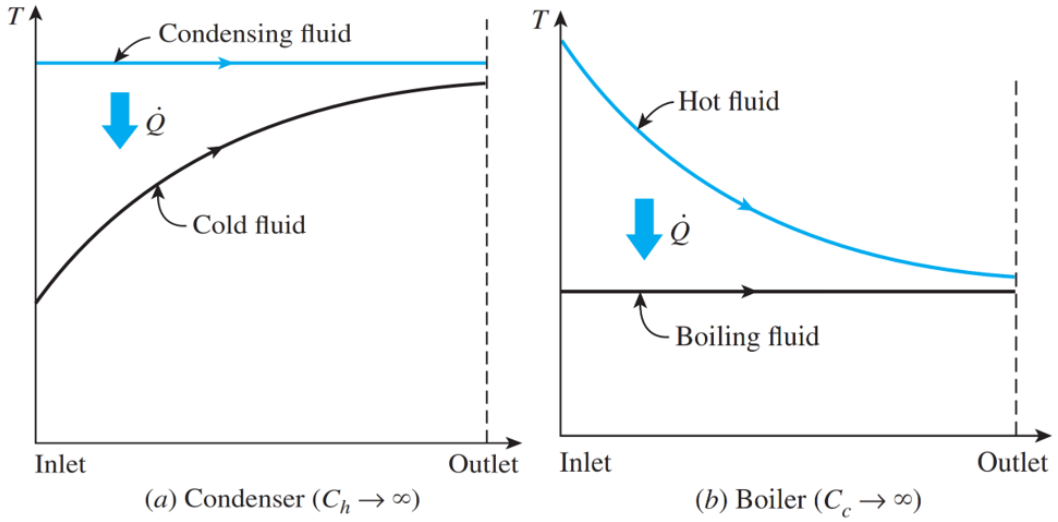


Figure 6.5: Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is, $C = \dot{m}c_p \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m}c_p\Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity*.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as:

$$\dot{Q} = UA_s\Delta T_m \qquad (6.15)$$

where U is the overall heat transfer coefficient, A_s is the heat transfer surface area, and ΔT_m is an appropriate mean temperature difference between the two fluids. Here, the surface area A_s can be determined precisely using the dimensions of the heat exchanger. However, the overall heat transfer coefficient U and the temperature difference ΔT between the hot and cold fluids, in general, may vary along the heat exchanger.

The average value of the overall heat transfer coefficient can be determined as described in the preceding section by using the average convection coefficients for each fluid. It turns out that the appropriate form of the average temperature difference between the two fluids is *logarithmic* in nature, and its determination is presented in **Section 6.4**.

Note

It should be noted that the average temperature difference ΔT_m is dependent on the heat exchanger flow arrangement and its type of construction.

3. THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a

mean temperature difference ΔT_m for use in the relation $\dot{Q} = UA_s\Delta T_m$.

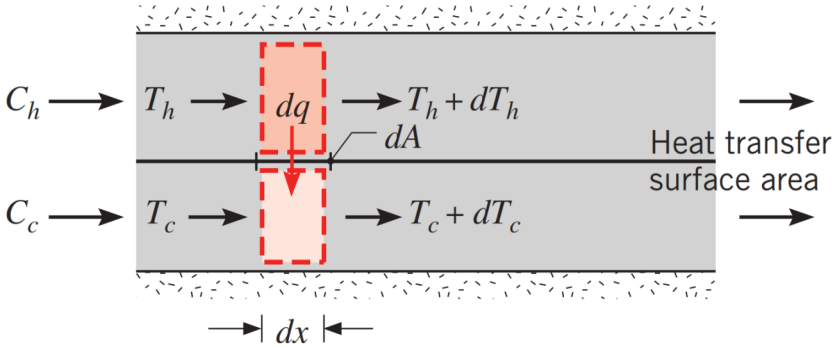
3.1. The Parallel-Flow Heat Exchanger

The hot and cold mean fluid temperature distributions associated with a parallel-flow heat exchanger are shown in **Figure 6.6**

The temperature difference ΔT is initially large but decays with increasing , approaching zero asymptotically. It is important to note that, for such an exchanger, the outlet temperature of the cold fluid never exceeds that of the hot fluid. In ΔT , the subscripts 1 and 2 designate opposite ends of the heat exchanger. This convention is used for all types of heat exchangers considered. For parallel flow, it follows that $T_{h,i} = T_{h,1}, T_{h,o} = T_{h,2}, T_{c,i} = T_{c,1}$, and $T_{c,o} = T_{c,2}$.

The form of ΔT_m may be determined by applying an energy balance to differential elements in the hot and cold fluids. Each element is of length dx and heat transfer surface area dA, as shown in **Figure 6.6**. The energy balances and the subsequent analysis are subject to the following assumptions:

1. The heat exchanger is insulated from its surroundings, in which case the only heat exchange is between the hot and cold fluids.
2. Axial conduction along the tubes is negligible.
3. Potential and kinetic energy changes are negligible.
4. The fluid specific heats are constant.
5. The overall heat transfer coefficient is constant.



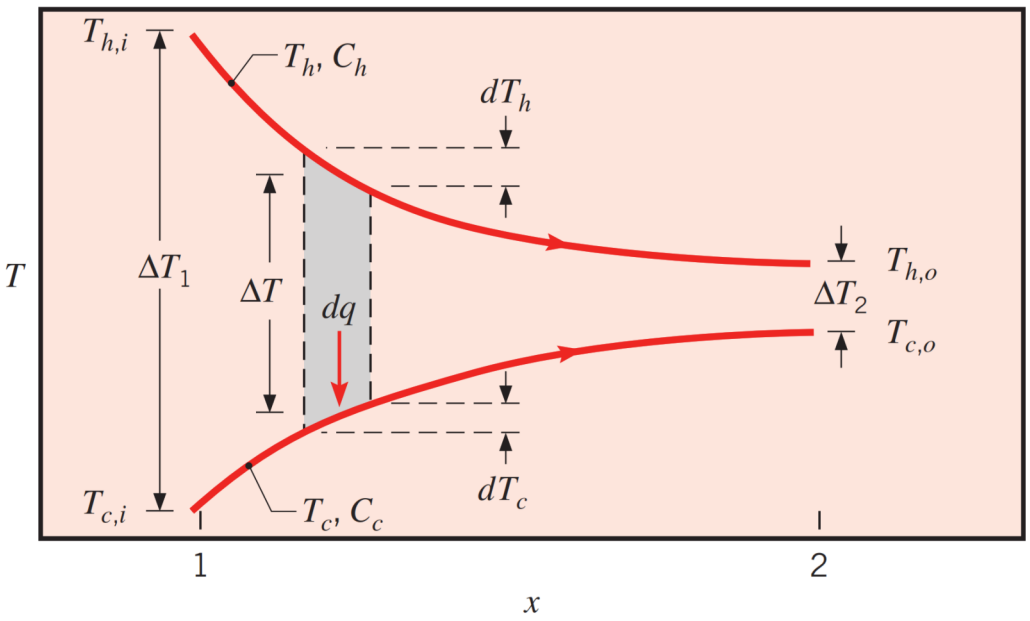


Figure 6.6: Temperature distributions for a parallel-flow heat exchanger.

Comparing the above expression with **Equation (6.15)**, we conclude that the appropriate average temperature difference is a *log mean temperature difference*, ΔT_m. Accordingly, we may write:

$$\dot{Q} = UA_s\Delta T_{\text{lm}} \qquad (6.16)$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \qquad (6.17)$$

is the **log mean temperature difference**, which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids at the two ends (inlet and outlet) of the heat exchanger. It makes no difference which end of the heat exchanger is designated as the inlet or the outlet (**Figure 6.7**).

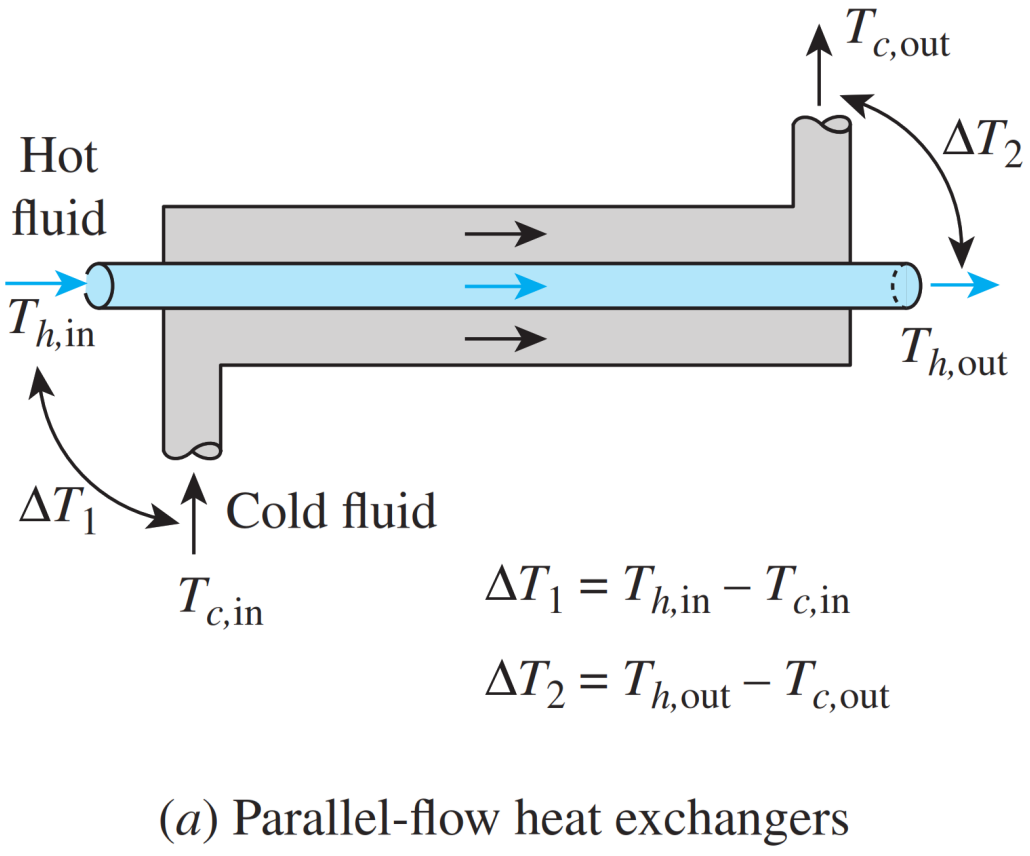


Figure 6.7: The ΔT1 and ΔT2 expressions in parallel-flow heat exchangers.

3.2. The Counterflow Heat Exchanger

The hot and cold fluid temperature distributions associated with a counter-flow heat exchanger are shown in **Figure 6.8**. In contrast to the parallel-flow exchanger, this configuration provides for heat transfer between the hotter portions of the two fluids at one end, as well as between the colder portions at the other. For this reason, the change in the temperature difference, $\Delta T = T_h - T_c$, with respect to x is nowhere as large as it is for the inlet region of the parallel-flow exchanger. Note that the outlet temperature of the cold fluid may now exceed the outlet temperature of the hot fluid.

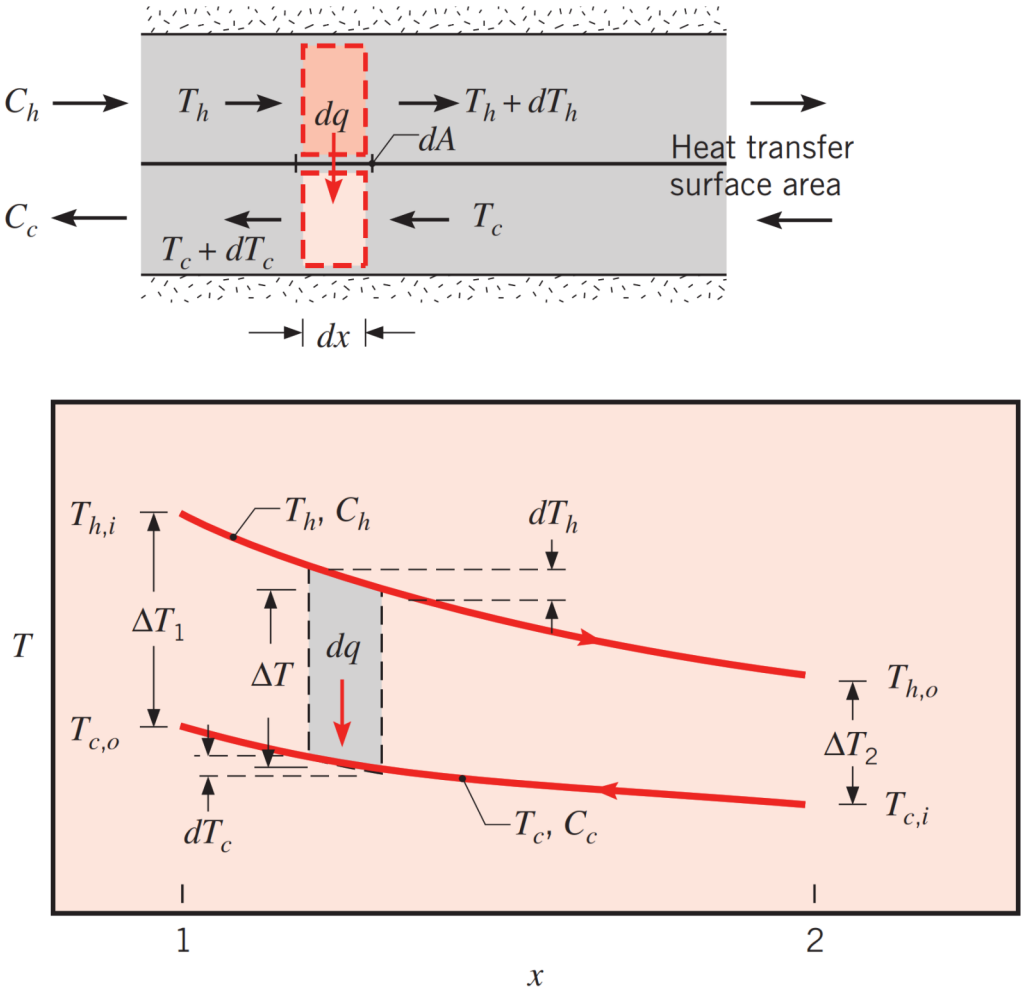
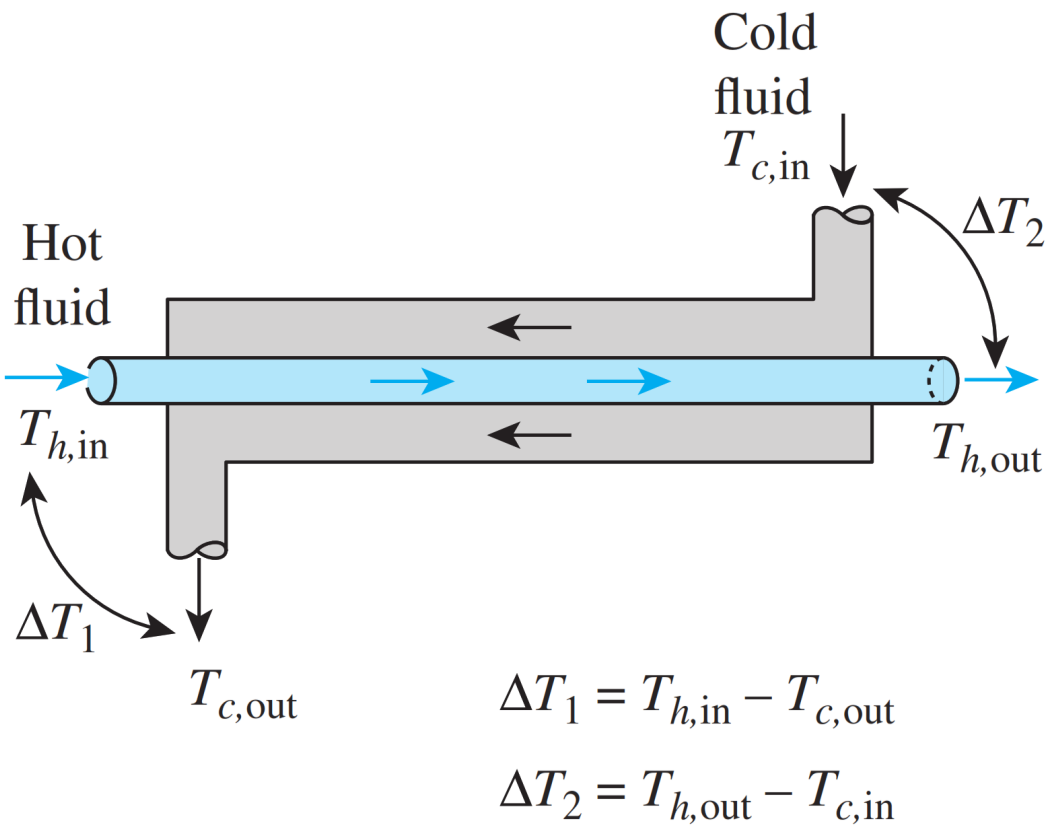


Figure 6.8: Temperature distributions for a counterflow heat exchanger.

The relation already given for the ***log mean temperature difference*** is developed using a parallel-flow heat exchanger, but we can show by repeating the analysis for a counter-flow heat exchanger that it is also applicable to counter-flow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in **Figure 6.9**.



(*b*) Counter-flow heat exchangers

Figure 6.9: The ΔT1 and ΔT2 expressions in counter-flow heat exchangers.

Note

For specified inlet and outlet temperatures, the log mean temperature difference for a counter-flow heat exchanger is always greater than that for a parallel-flow heat exchanger. That is, $\Delta T_{\text{lm, CF}} > \Delta T_{\text{lm, PF}}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger, assuming the same value of the overall heat transfer coefficient. Therefore, it is common practice to use counter-flow arrangements in heat exchangers.

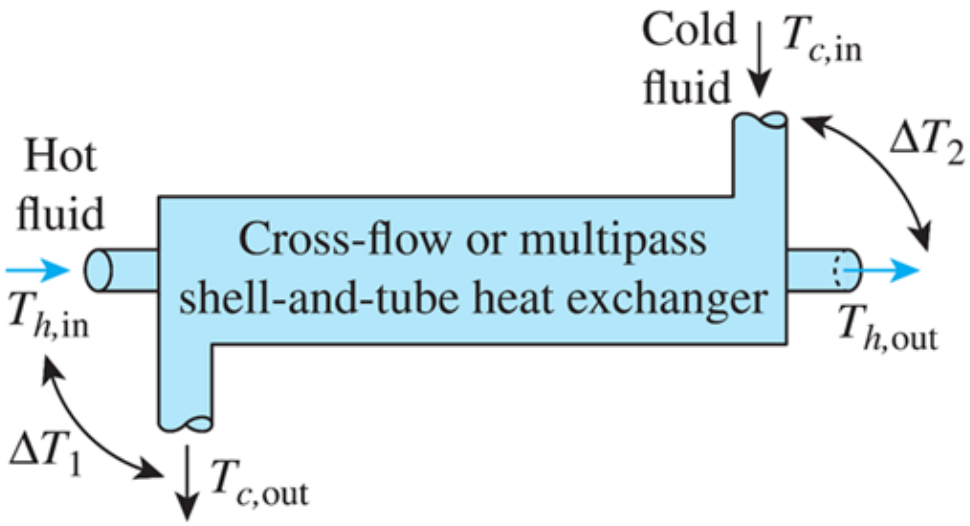
3.3. Multipass and Cross-Flow Heat Exchangers:

The log mean temperature difference ΔT_{lm} relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. Similar relations are also developed for *cross-flow* and *multipass shell-and-tube* heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.

In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as:

$$\Delta T_{\text{lm}} = F \Delta T_{\text{lm, CF}} \qquad (6.18)$$

where F is the ***correction factor***, which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{\text{lm, CF}}$ is the log mean temperature difference for the case of a *counter-flow* heat exchanger with the same inlet and outlet temperatures and is determined from **Eq. (6.17)** by taking $\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$ and $\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ in}}$ (**Figure 6.10**).



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{\text{lm,CF}}$$

where

$$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$$

$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$$

and

$F = \ldots$ (Figure 6.11)

Figure 6.10: The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

The correction factor is less than unity for a cross-flow and multipass shell-and-tube heat exchanger. That is, $F \leq 1$. The limiting value of $F = 1$ corresponds to the counter-flow heat exchanger. Thus, the correction factor F for a heat exchanger *is a measure of deviation of the* ΔT_{lm} *from the corresponding values for the counter-flow case.*

The correction factor F for common cross-flow and shell-and-tube heat exchanger configurations is given in **Figure 6.11** versus two temperature ratios and R defined as:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \qquad (6.19)$$

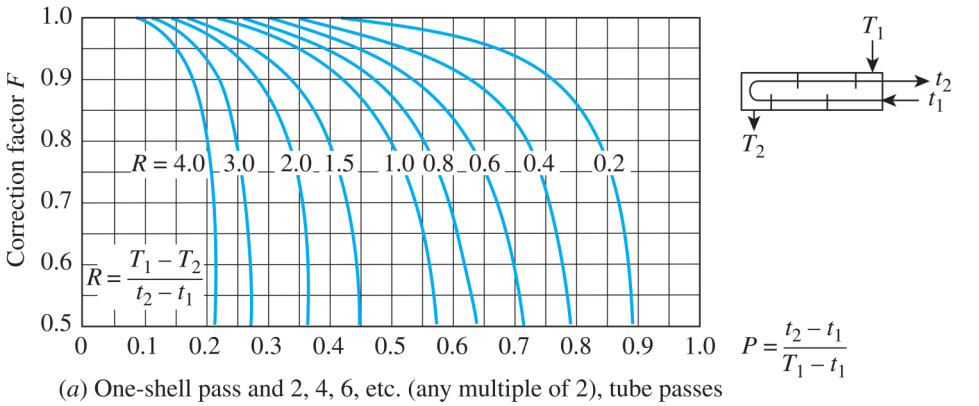
and

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{\text{tube side}}}{(\dot{m}c_p)_{\text{shell side}}} \qquad (6.20)$$

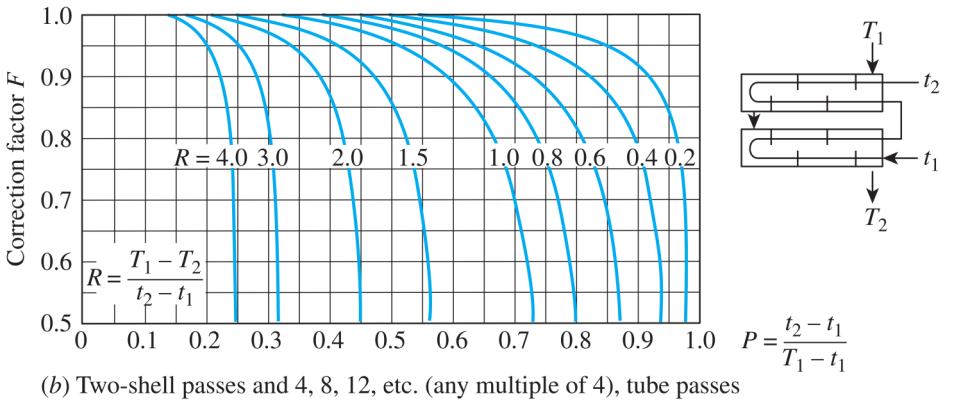
where the subscripts **1** and **2** represent the **inlet** and **outlet**, respectively.

Note

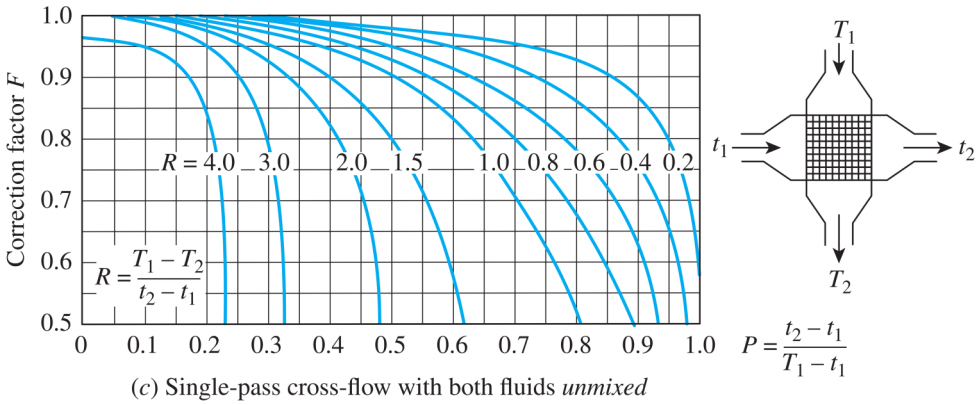
Note that for a shell-and-tube heat exchanger, and represent the *shell-* and *tube-side* temperatures, respectively, as shown in the correction factor charts.



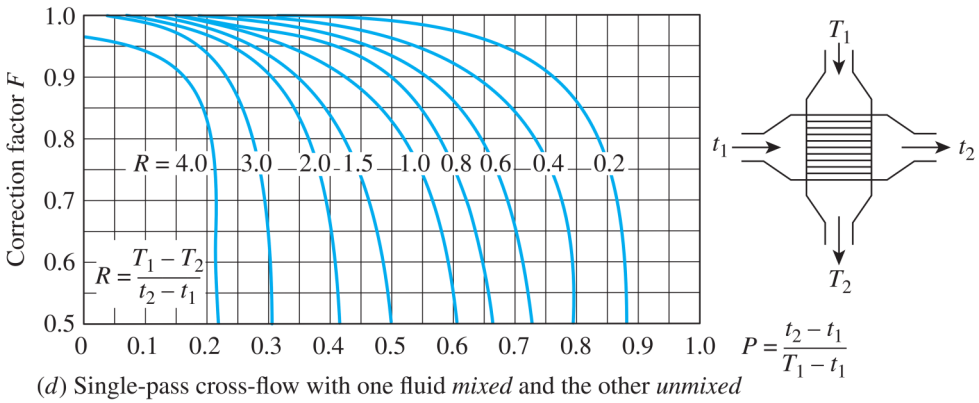
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

Figure 6.11: Correction factor F charts for common shell-and-tube and cross-flow heat exchangers.

Warning

It makes no difference whether the hot or the cold fluid flows through the shell or the tube. The determination of the correction factor requires the availability of the inlet and the outlet temperatures for both the cold and hot fluids.

Note

Note that the value of ranges from 0 to 1. The value of , on the other hand, ranges from 0 to infinity, with corresponding to the phase-change (condensation or boiling) on the shell-side and $R \to \infty$ to phase-change on the tube side. The correction factor is for both of these limiting cases. Therefore, the correction factor for a condenser or boiler is , regardless of the configuration of the heat exchanger.

4. THE EFFECTIVENESS–NTU METHOD

It is a simple matter to use the log mean temperature difference (***LMTD***) method of heat exchanger analysis when the fluid inlet temperatures are known and the outlet temperatures are specified or readily determined from the energy balance expressions, **Equations 6.9 and 6.10** The value of ΔT_{lm} for the exchanger may then be determined. However, if only the inlet temperatures are known, use of the LMTD method requires a cumbersome iterative procedure. It is therefore preferable to employ an alternative approach termed the ***effectiveness–NTU (or NTU) method***.

4.1. Definitions

To define the *effectiveness of a heat exchanger*, we must first determine the *maximum possible heat transfer rate*, $\dot{Q}_{\max}$, for the exchanger. This heat transfer rate could, in principle, be achieved in a counterflow heat exchanger (**Figure 6.8**) of infinite length. In such an exchanger, one of the fluids would experience the maximum possible temperature difference, $T_{h,i} - T_{c,i}$.

$$\Delta T_{\max} = T_{h,\text{ in}} - T_{c,\text{ in}} \qquad (6.21)$$

The ***actual heat transfer rate*** in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as:

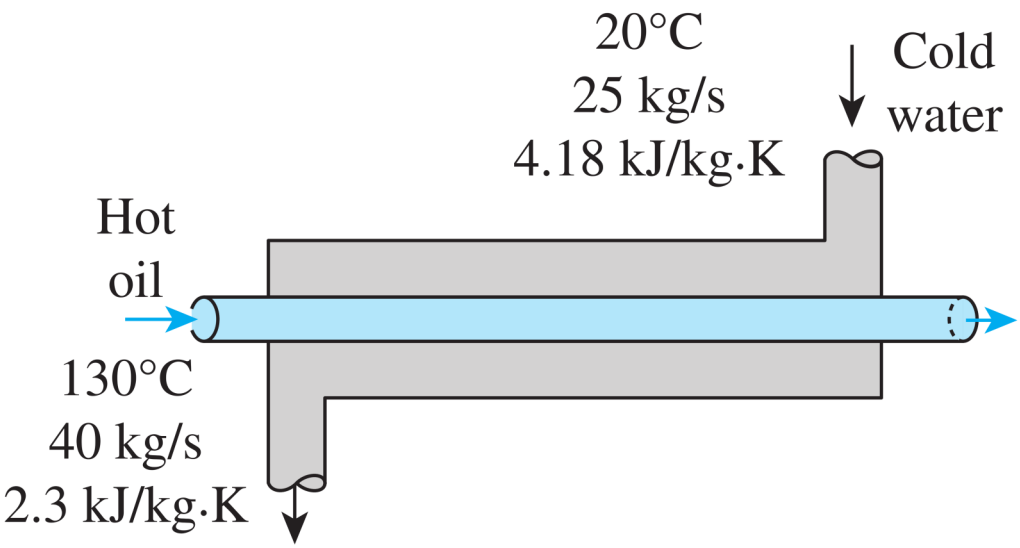
$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \qquad (6.22)$$

where $C_c = \dot{m}_c c_{pc}$ and $C_h = \dot{m}_c c_{ph}$ are the ***heat capacity rates*** of the cold and hot fluids, respectively.

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical $(\text{i.e.}, C_c = C_h)$. When $C_c \neq C_h$ which is usually the case, the fluid with the smaller heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (**Figure 6.12**)

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \qquad (6.23)$$

where $C_{\min}$ is the smaller of C_h and C_c.



$$C_c = \dot{m}_c c_{pc} = 104.5 \text{ kW/K}$$

$$C_h = \dot{m}_c c_{ph} = 92 \text{ kW/K}$$

$$C_{\min} = 92 \text{ kW/K}$$

$$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = 110°\text{C}$$

$$\dot{Q}_{\max} = C_{\min} \Delta T_{\max} = 10{,}120 \text{ kW}$$

Figure 6.12: The determination of the maximum rate of heat transfer in a heat exchanger.

This method is based on a dimensionless parameter called the heat transfer effectiveness ε, defined as:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \qquad (6.24)$$

The determination of $\dot{Q}_{\max}$ requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates*, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from:

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\text{ in}} - T_{\text{c, in}}) \qquad (6.25)$$

where

$$\text{if } C_c = C_{\min}: \qquad \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_c(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{\text{h,in}} - T_{c,\text{in}}}$$

$$\text{if } C_h = C_{\min}: \qquad \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,\text{in}} - T_{h,\text{out}})}{C_h(T_{h,\text{in}} - T_{\text{c,in}})} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{\text{c,in}}}$$

For any heat exchanger it can be shown that:

$$\varepsilon = f\left(\text{NTU}, \frac{C_{\min}}{C_{\max}}\right)$$

where $C_{\min}/C_{\max}$ is equal to C_c/C_h or C_h/C_c, depending on the relative magnitudes of the hot and cold fluid heat capacity rates. ***The number of transfer units (NTU)*** is a dimensionless parameter that is widely used for heat exchanger analysis and is defined as:

$$\text{NTU} = \frac{UA_s}{C_{\min}} \qquad (6.26)$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that ***NTU*** is proportional to A_s. Therefore, for specified values of U and $C_{\min}$, the value of NTU is a measure of the heat transfer surface area A_s. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the is ***heat capacity ratio*** C_r as:

$$C_r = \frac{C_{\min}}{C_{\max}} \qquad (6.27)$$

and from **Equations (6.9) and (6.10)** it follows that:

$$C_r = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

Similar expressions have been developed for a variety of heat exchangers, and representative results are summarized in **Table 6.1**

TABLE 11.3 Heat Exchanger Effectiveness Relations [5]

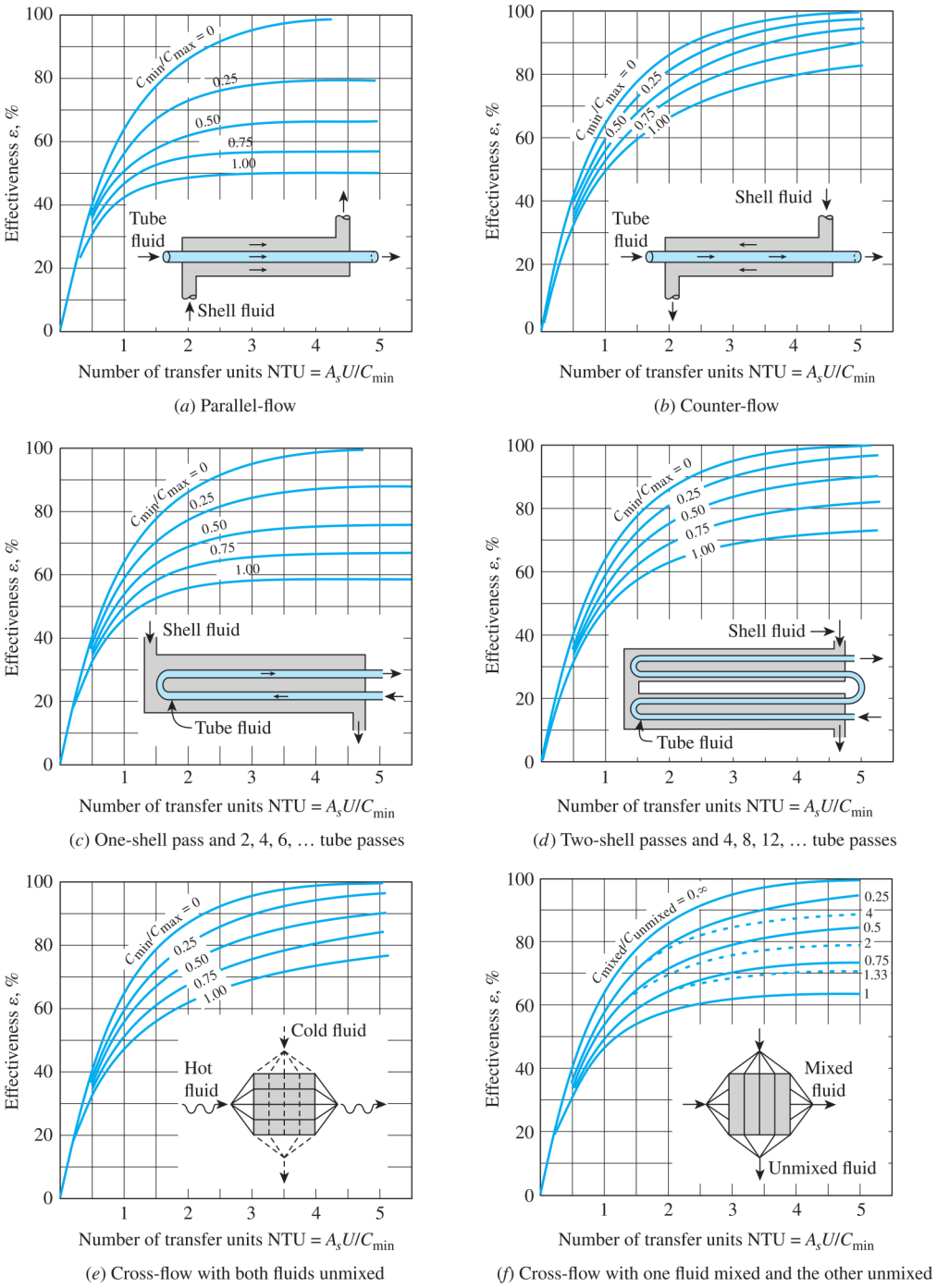
Flow Arrangement	Relation		
Parallel flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterflow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - C_r)]}{1 - C_r \exp[-\text{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\text{NTU}}{1 + \text{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-flow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\text{NTU})^{0.22}\{\exp[-C_r(\text{NTU})^{0.78}] - 1\}\right]$		(11.32)
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\text{NTU})]\})$		(11.33a)
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\text{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\text{NTU})$		(11.35a)

Explicit relations for **NTU** as a function of ε and C_r are provided in **Table 6.1**.

TABLE 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation		
Parallel flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$		(11.28b)
Counterflow	$\text{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$	$(C_r < 1)$	
	$\text{NTU} = \dfrac{\varepsilon}{1 - \varepsilon}$	$(C_r = 1)$	(11.29b)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$(\text{NTU})_1 = -(1 + C_r^2)^{-1/2}\ln\left(\dfrac{E - 1}{E + 1}\right)$		(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$		(11.30c)
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F - 1}{F - C_r}$ $F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n}$ $\text{NTU} = n(\text{NTU})_1$		(11.31b, c, d)
Cross-flow (single pass)			
$C_{\max}$ (mixed), $C_{\min}$ (unmixed)	$\text{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1 - \varepsilon C_r)\right]$		(11.33b)
$C_{\min}$ (mixed), $C_{\max}$ (unmixed)	$\text{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1 - \varepsilon) + 1]$		(11.34b)
All exchangers ($C_r = 0$)	$\text{NTU} = -\ln(1 - \varepsilon)$		(11.35b)

The effectivenesses of some common types of heat exchangers are also plotted in **Figure 6.13**.



(*a*) Parallel-flow

(*b*) Counter-flow

(*c*) One-shell pass and 2, 4, 6, … tube passes

(*d*) Two-shell passes and 4, 8, 12, … tube passes

(*e*) Cross-flow with both fluids unmixed

(*f*) Cross-flow with one fluid mixed and the other unmixed

Figure 6.13: Effectiveness for heat exchangers.

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