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By : **Bouzenna Fatma El-Ghenbazia**

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Study of The Generalized Bose-Einstein Condensation in Various Dimensions Based on Fractional Quantum Mechanics

Publicly defended on : 01/10/2020 to the jury composed by:

E H. Guedda	*Professor*	*El-Oued University*	*President*
M. T. Meftah	*Professor*	*Ourgla University*	*Thesis Director*
D. Dou	*Professor*	*El-Oued University*	*Thesis Co-director*
M. Difallah	*MCA*	*El-Oued University*	*Examiner*
T. Boudjedaa	*Professor*	*Jijel University*	*Examiner*
M. Merad	*Professor*	*Oum El Bouaghi University*	*Examiner*
Z. Korichi	*MCB*	*ENS Ourgla*	*Invited*

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