New Approach for Multi-Valued Mathematical Morphology Computation

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Abstract. Mathematical Morphology (MM) is a useful tool for spatial image processing. It is based on an infimum operator (min) and a supremum operator (max) applied in local neighborhoods to detect pixel extremes. The MM was initially defined for mono-band images in which each pixel image is a scalar value and it is easy to find pixels extremes by the infimum and the supremum operators. However, in the case of multi-band images, where each pixel image is represented by a vector, establishing an order between image pixels in local neighborhoods by the infimum and supremum operators is not trivial. Many works discussed the feasibility to extend the MM to multi-band images but they did not lead to any consensual definition of the multi-valued mathematical morphology. Nevertheless, these existing works agree that the definition of the MM for multi-band images is based on the notion of vector ordering. In this paper, we propose a multi-valued MM operators computing by introducing a new vector ordering algorithm that allows extending the scalar MM to multi-band images. The proposed multi-valued morphological operations were tested in the experimental phase for the morphological descriptors computation. The obtained results based on use of the proposed vector ordering algorithm for the multi-valued MM computing improve the classification rates.

Keywords: Multi-Band Images, Vector Ordering Algorithm, Multi-Valued Mathematical Morphology, Multi-Valued Morphological Profile.

1 Introduction

Mathematical Morphology (MM) is a useful tool for image processing. It allows the geometrical description of the structures present in the scene. The MM technique was originally developed by Matheron and Serra [1] and was formulated in terms of infimum (finding the min, noted $\wedge$) and supremum (finding the max, noted $\vee$) operators. Even if the MM is well defined for mono-band images, where each pixel is represented by scalar value and it is easy to designate the infimum and supremum pixels for compared pixels in a local neighborhood, there is no consensual definition of the MM for multi-band images in which each pixel is represented by a vector of p component (p is the number of bands, and each pixel have scalar value for each image band). A basic and frequently used idea to extend the morphological operation to the multi-dimensional data is to decompose the initial multi-band image into grayscale images (i.e. mono-band images), on which identical scalar morphological treatments are applied independently. The final result of the MM image processing is obtained by grouping the different isolated results (i.e. single results are assembled into a unique data set). The application of this so-called marginal strategy sometimes results in the creation of new pixel vectors that are not present in the original multi-band image (then the applied morphological treatment is not vectors preserving). Furthermore, this way of adapting the scalar MM to the multi-band images loses the correlations between bands [2]. To avoid these drawbacks, the definition of MM for a multi-band image requires non-scalar morphological approaches that consider the multi-band image as a single data block processed simultaneously. For this purpose, it is necessary to have an appropriate vector ordering strategy allowing vector space manipulation and selecting vectors extremes by the infimum and the supremum operators. The study of the appropriate vector ordering scheme to define the multi-valued MM has been studied for many years by several works. Barnett [2] has classified the existing vector ordering strategies for the MM extension into four groups: the marginal ordering strategy (M-ordering strategy), the conditional ordering strategy (C-ordering strategy), the partial ordering strategy (P-ordering strategy), and the reduced ordering strategy (R-ordering strategy).

The marginal ordering strategy (M-ordering strategy), as mentioned previously, consists of processing each band image separately and independently from the others by the same scalar morphological transformations. Despite its easy implementation, the marginal ordering strategy loses the bands correlation (relation between bands) and can lead to the appearance of some pixel vectors in the output result that are not present in the original image. This second disadvantage can be illustrated by taking the example of 3-dimensional vectors $X = (3, 8, 5)$ and $Y = (4, 7, 2)$. The $\wedge (X, Y) = (3, 7, 2)$ and $\vee(X, Y) = (4, 8, 5)$ (where $\wedge$ indicates the min operator and $\vee$ indicates the max operator). In this illustrative example, the vectors (3, 7, 2) and (4, 8, 5) are two new vectors that do not belong in the initial vectors set. Thus, for this example, the used marginal ordering is not vectors preserving. The reducing dimensionality transformation (with Principal Component Analysis PCA [3, 4, 5] or any other dimensionality reduction method) of the original multi-band image is also applied before a marginal treatment to ensure that the bands of the image are decorrelated and avoid the first problem of losing the bands correlation.

The conditional ordering strategy (C-ordering strategy) gives priority to some particular bands image in the vector ordering process. It uses a prioritization function to calculate the priority of each band (i.e. computing band weight) in the original image. The C-ordering approach is recommended when considering some image bands' weights are greater than others [6]. Thus, with a conditional ordering strategy, two vectors are ordered according to their scalar values for the most prioritized band. In the case of equality, the band with the next priority is considered, and so on. Two vectors are identical in a conditional approach if they are equal component by component (i.e. they have the same scalar values for all image bands). The main limit of the conditional ordering is linked to the difficulty to find the coherent band prioritization function. The lexicographic ordering strategy (L-ordering strategy) is considered as the most known derivative version of the C-ordering strategy. Its principle is taken from the classification of words by alphabetical order, where the first words sorting is based on the first letter, the undecided cases of the previous words classification are resolved by the second letter and so on. For the adaptation of the L-ordering to the vector ordering problem, the first image band is used for the first vectors ordering, the next one is used to solve the unresolved ex-aequo problems of the previous band and so on (1). In the L-ordering strategy, the ordering succession can be reversed by starting with the last band image and gradually advance to the first band image each time there is an indecisive case (2).

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} < \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \Leftrightarrow \begin{matrix} v_1 < \mu_1 \ or \\ v_1 = \mu_1 \ and \ v_2 < \mu_2 \ or \\ \cdots \\ v_1 = \mu_1, \ldots\ldots, \ and \ v_{n-1} = \mu_{n-1} \ and \ v_n < \mu_n \end{matrix} \tag{1}$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} < \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \Leftrightarrow \begin{matrix} v_n < \mu_n \ or \\ v_n = \mu_n \ and \ v_{n-1} < \mu_{n-1} \ or \\ \cdots \\ v_n = \mu_n, v_{n-1} = \mu_{n-1} \ldots\ldots, \ and \ v_2 = \mu_2 \ and \ v_1 < \mu_1 \end{matrix} \tag{2}$$

The Lexicographic order strategy remains the most used vector ordering strategy in the multi-valued MM definition. This can be explained by the fact that the L-ordering is vector preserving and allows to obtain a total order relation between compared vectors (i.e. there is no incomparability situation between two vectors and the unique case of equality between two vectors is the equality component by component between the compared vectors). In practice, the L-ordering strategy frequently corresponds to the exclusive use of the first prioritized bands to make vectors ordering decision. The remaining bands rarely participate in the comparison process [7, 8]. Thus, the lexicographic order is well suited for situations where the first image bands are those including the most relevant information. This case cannot be naturally present in the multi-band images but can be obtained after concentrated image information in the first bands image by projection techniques (such as a principal component analysis PCA or any other projection techniques). The lexicographic ordering and its variants were reported in various works like [9, 10, 11, 12, 13, 14].

The partial ordering strategy (P-ordering strategy) clusters vectors into groups of equivalence according to a given criterion. With the P-ordering strategy, it is possible to compare vectors from two different groups, but not within the same group. Because of the impossibility to make an ordering relation between vectors of the same groups, this vector ordering scheme cannot guarantee the uniqueness of the extremes vectors (i.e. do not guarantee unique infimum vector and/or unique supremum vector). Thus, with partial ordering approaches, there is no total relation between vectors (i.e. some vectors are not comparable). The approaches presented in [15, 16, 17, 18, 19, 20] are examples of using a P-ordering strategy to extend the MM to the multi-band images.

The reduced ordering strategy (R-ordering strategy) reduces vectors to scalar values easily comparable. The passage of an N-dimensional space to one-dimensional space can be obtained by a projection system or by a distance measurement from predefined reference. Once each pixel vector of the multi-band image is replaced by its associated scalar value, the created grayscale image (i.e. mono-band image) can be directly processed by any mono-band morphological transformations. The R-ordering approach using projection techniques, such as the PCA, to transform the multi-band image in one-band image causes losing too much information unlike exploiting distance measurement. Plaza et al. [21] proposed a vectors ordering algorithm using cumulative distances of each pixel vector from all the others. The presented reduced ordering strategy avoids using any predefined reference with which the vectors are compared. The proposed cumulative distances use two new metrics (Spectral Angle Distance “SAD” and Spectral Information Divergence “SID”). Other distances measurements from the predefined reference are also used in the R-ordering strategy like Euclidean distance, the Mahalanobis distance [22, 23], or other distance measurements [24, 25, 26]:

$$\mathrm{v} \leq \mathrm{v}' \Leftrightarrow \mathrm{Distance}(\mathrm{v}, \mathrm{v_{ref}}) \leq \mathrm{Distance}(\mathrm{v}', \mathrm{v_{ref}}) \tag{3}$$

Sometimes, the reduced ordering strategy leads to indecision cases when two different vectors have the same scalar values. In this situation, using an additional condition in the R-ordering approach, such as lexicographic ordering, is necessary to obtain a total order relation (i.e. any vectors are comparable) and decide between vectors having the same scalar estimation. After briefly introducing the four families of vector ordering strategies, we propose, in this work, a new multi-valued MM computation algorithm that extends the morphological operations to the multi-band image by processing all image bands simultaneously. For this purpose, a vector ordering algorithm is introduced for the multi-valued MM computation. This proposed algorithm allows finding the supremum and the infimum of pixel vectors in a local neighborhood by applying the infimum and the supremum operators on pixels that do not have a unidimensional structure. The proposed vectors comparison algorithm is based on outranking relations and operates the vectors ordering process by considering all image bands without excluding any of them. The presented vector ordering algorithm preserves also the original vectors (i.e. is vectors preserving) and does not omit the correlation between bands in the ordering process.

The rest of this paper is organized as follows: the proposed vector ordering algorithm is presented in Section 2. Experimental results obtained on the ROSIS sensor data sets from urban areas are presented in Section 3. Finally, conclusions are drawn in Section 4.

2 Vectors Ordering Based on Outranking Relations

By considering *m* compared vectors with N-dimensional structure (i.e. pixel vectors which constitute the N-bands image), the first step of the proposed vector ordering algorithm is binary comparisons of each pixel vector from all the others included in the vectors comparison set.

From two taken N-dimensional vectors X ($x_1, x_2, \ldots, x_N$) and Y ($y_1, y_2, \ldots, y_N$) belonging in the compared vectors set, two numerical values “α” and “β” are computed for the (X,Y) vectors pair.

The first numerical value “α” expresses the number of the vector components X which are greater than their equivalent components in the vector Y:

$$\alpha = \sum_{i=1}^{N} \Delta(\mathrm{x_i} - \mathrm{y_i}) \tag{4}$$

Where $\Delta(A)$=1 if A is greater than 0 and $\Delta(A)$= 0 otherwise.

The second numerical value "β" represents the number of the vector components Y which are greater than their correspondent components in vector X:

$$\beta = \sum_{i=1}^{N} \Delta(\mathrm{y_i} - \mathrm{x_i}) \tag{5}$$

Note that "$\alpha = m$-β" and "$\beta = m$-α".

It is considered that the vector X and the vector Y are equal (component by component) if "α = β = 0".

When the "α" value is greater than "β", the vector X outrank the vector Y. Otherwise, the vector X is outranked by the vector Y.

This adopted "binary outranking relation" can lead to indecision cases (i.e. incomparability situation between the two compared vectors) between the two compared vectors when "α" and "β" are equal but not null. To resolve the undecided cases an additional ordering must be used. In our approach, the adopted additional ordering strategy is conditional ordering which is recommended when it is possible to find band prioritization function and give priority value for each band of the N-dimensional image. The associated priority function with the C-ordering process is the Improved Score Function (noted IFS function) introduced in [27, 28]. This IFS function was employed in [28] to estimate the discrimination power of the generated morphological descriptors. In this paper, the score function IFS is considered as a tool to give vector components prioritization (i.e. attribute the priority value of each image band) for the conditional ordering scheme. The use of the IFS function as a prioritization function in the conditional vectors ordering algorithm constitutes another contribution of this work.

To estimate the priority value of each band image, the IFS value is computed for each component of the multi-band image by the following formula:

$$IFS_i = \frac{\sum_{j=1}^{l} [avg(x_i^j) - avg(x_i)]^2}{\sum_{j=1}^{l} \frac{1}{n_j - 1} \sum_{k=1}^{n_j} [x_{k,i}^j - avg(x_i^j)]^2} \tag{6}$$

Where:

- IFS_i is the priority value (i.e. the weight) of the i^{th} band image;
- "l" is the obtained objects classes number after a classification of the original multi-band image;
- n_j is the number of pixels belonging to the j^{th} class;
- $avg(x_i)$ is the radiometric average value of the pixels in the i^{th} band image;
- $avg(x_i^j)$ is the radiometric average value of the pixels in the i^{th} band image belonging to the j^{th} class;
- $x_{k,i}^j$ is the k^{th} pixel of the i^{th} band image belonging to the j^{th} class.

Bands with a high priority value (i.e. with high IFS value) are more prioritized bands (i.e. the highest weight bands) in the conditional ordering process. Thus, incomparable pixel vectors with the binary outranking relation are initially ordered according to the scalar value of their first prioritized component. Vectors with the same value for the first prioritized component are ordered according to the scalar value of the next prioritized component, and so on. Therefore, the conditional ordering completes the ordering structure when the binary outranking relation between two compared vectors leads to an indecision situation (i.e. incomparability situation).

The second step of the proposed vectors ranking algorithm is the synthesis of the binary outranking comparisons to give the final ordering of the compared vectors and designate the two pixel-vectors extremes (i.e. designate the infimum pixel vector and the supremum pixel vector).

Note that the most outranked vector (i.e. lower ranked vector) is the infimum vector, while the outranking one (i.e. the higher ranked vector) is a supremum.

The evaluation of the proposed vector ordering algorithm is performed by generating the multi-valued Morphological Profile (MP) computed on all bands of the original image simultaneously. The MP is an association of morphological transformations that allows extracting structures of various sizes and shapes present in the original image (more details for the MP are given in [30, 4]). The MP was originally defined for the mono-band images. In the experiment section, the spatial characterization by MP is operated by the multi-valued MP and is built on all image bands simultaneously. For this purpose, various vector ordering algorithms (two classical algorithms and our proposed algorithm) are used to detect and extract objects of interest by the multi-valued MP.

The obtained results of the vector ordering algorithms used in the multi-valued computation are illustrated and discussed in the next section. The performance measurement of the exploited vector ordering algorithms is determined by SVM classification accuracy.

3 Experimental Results

In this section, performance measurements like classification Overall Accuracy (OA) and Kappa rate are used with the support vector machines (SVM) classifier [31] to measure the impact of the generated morphological descriptors by multi-valued MP on the classification improvement. Note that the three considered scenarios of vector ordering are:

- The lexicographic vector ordering strategy with decreasing priorities of the image bands;
- The lexicographic ordering strategy with increasing priorities of the image bands;
- The proposed vector ordering algorithm based on outranking relations and additional conditional ordering.

The three used vector ordering algorithms for the multi-valued MP computation are applied on all image bands simultaneously.

The experimental analysis was carried out on a multi-band image of Pavia university (northern Italy) acquired by the ROSIS sensor. This ROSIS image is composed of 610*610 pixels with 1.3 m of spatial resolution and 103 image bands. The image was proposed with a ground truth image that differentiates nine object classes (asphalt, meadows, gravel, trees, painted metal sheets, bare sol, bitumen, bricks, and shadows). The original image and its associated ground truths are shown in Fig. 1.

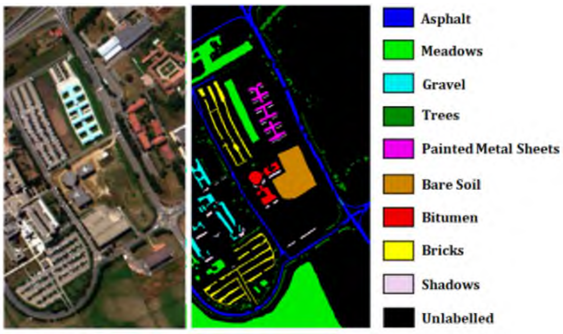


Fig. 1. Pavia University scene with the associated ground truth image.

The multi-band image includes many highly correlated bands resulting in spectral redundancy. This spectral redundancy increases computational complexity and degrades classification accuracy [32]. For this reason, dimensionality reduction was achieved by the PCA projection technique and only the first decorrelated principal components of the original image are considered for the multi-valued MP computing.

The "spectral" classification (without considering spatial descriptors generated by multi-valued MP) of the reduced image gives 82.82% for the OA rate and 77.47% for the Kappa rate.

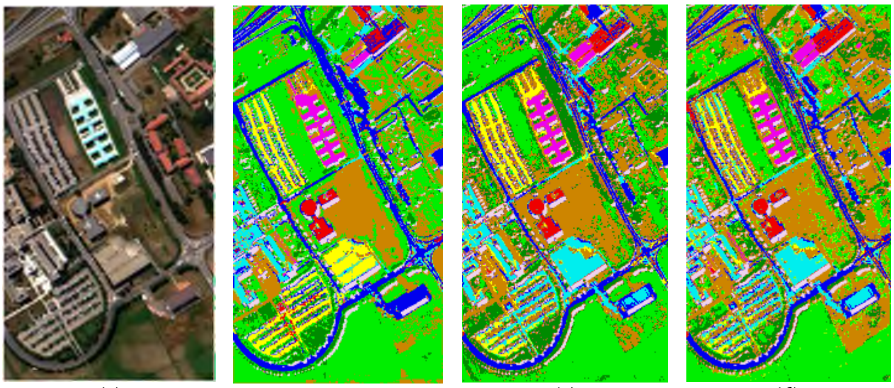
The use of the three vector ordering algorithms on the multi-valued MP computation produces different outcomes. A summary of results in terms of classification accuracies (OA and Kappa rate) is given in Table 1. The use of the vector ordering scheme improves the classification accuracies independently of the used vector ordering algorithm in the multi-valued MP computation.

As shown in Table 1, the classification accuracies obtained by the presented vector ordering algorithm is close to "the lexicographic ordering strategy with increasing priorities" with a small improvement in favor of the proposed vector ordering approach. The obtained results showed also a higher precision for "the lexicographic ordering strategy with decreasing priorities" in comparison to the other compared vector ordering approaches. This is probably due to the dimensionality reduction effect which concentrates the most information present in the image bands on the first bands that are the mainly considered bands in the lexicographic ordering.

Table 1. Classification accuracy using different vector ordering algorithms for the multi-valued MP computation.

Vector ordering algorithm	OA %	Kappa %
Lexicographic ordering with decreasing priorities	90.52	87.58
Lexicographic ordering with increasing priorities	85.35	81.05
Vector ordering algorithm based on outranking relations and conditional ordering	85.79	81.29

The resulting classification maps based on SVM classification of the multi-valued MP computed with the three different vector ordering algorithms is shown in Fig. 2.



(a) (b) (c) (d)

Fig. 2. Classification maps obtained with the SVM classifier. (a) Original multi-band image; (b) classification using the multi-valued MP computed by lexicographic ordering with decreasing priorities; (c) classification using the multi-valued MP computed by lexicographic ordering with increasing priorities; (d) classification using the multi-valued MP computed by the proposed outranking relations and conditional ordering.

4 Conclusion

Mathematical Morphology (MM) is an efficient tool for patterns and object recognition in digital image processing. The basic transformations of mathematical morphology are based on the search for the local minimum and local maximum in the predefined neighborhood. The MM was originally defined for mono-band images where each pixel is associated with a numerical value and the order relation between scalars pixel is natural. The application of the MM logic on multi-bands images is less trivial since there is no predefined order between vector values. In this paper, we have proposed a new vector ordering algorithm based on an idea of outranking relations between vectors. The presented vector ordering algorithm is completed by conditional ordering strategy to have a total order relation between compared vectors (i.e. all vectors are comparable). The experiments are interested in the characterization of the objects present in multi-band images by the multi-valued MP. Even if the classification results of the presented vector ordering algorithm get closer results to the widely used lexicographic approaches, it can be validated by the fact that the proposed algorithm is vector preserving, take into account the band correlation, and provides a total relation order between compared vectors. These proposed algorithms can be also used with other multi-valued morphological operators and is available for any type of multi-band images like color images.

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