الجمهورية الجزائرية الديمقراطية الشعبية
وزارة التعليم العالي و البحث العلمي
جامعة الشهيد حمه لخضر – الوادي
كلية التكنولوجيا
قسم الهندسة الميكانيكية

Dr : ZINE Bachir — Maitre de Conférences classe : « B »
Email : zine-bachir@univ-eloued.dz
Tel : 0699513851

Polycopie pédagogique du cours : « Electric Machines »

Filière : Electromécanique

Spécialité : Electromécanique

Semestre : Cinque

Année : 3eme

Table of contents

Chapter.1.Magnetic Circuit

Chapter.1.Magnetic circuit

1.1. Chapter Objectives

After the completion of this unit, students/readers will be able to understand:

- ✓ What is magnetic field and its significance?
- ✓ What is a magnetic circuit?
- ✓ What are the important terms related to magnetism and magnetic circuits?
- ✓ What are the similarities and dissimilarities between magnetic and electric circuits?
- ✓ How series and parallel magnetic circuits are treated?
- ✓ What is leakage flux and how it affects magnetic circuits?
- ✓ What is magnetic hysteresis and hysteresis loss?
- ✓ What is electromagnetic induction phenomenon?
- ✓ What are Faraday's laws of electromagnetic induction?
- ✓ What are self- and mutual inductances and what is their significance?
- ✓ What is the effective value of inductances when these are connected in series–parallel combination?
- ✓ What are electromechanical energy conversion devices?
- ✓ How does torque develop by the alignment of two fields?
- ✓ What are the factors on which torque depends?
- ✓ How to determine the direction of torque or induced emf in rotating machines?

1.2. Introduction

The operation of all the electrical machines such as *DC* machines, transformers, synchronous machines, induction motors, etc., rely upon their magnetic circuits. The closed path followed by the magnetic lines of force is called a *magnetic circuit*. The operation of all the electrical devices (e.g., transformers, generators, motors, etc.) depends upon the magnetism produced by their magnetic circuits. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

In this chapter, we shall focus our attention on the basic fundamentals of magnetic circuits and their applications as electromechanical energy conversion devices.

1.3. Magnetic field and its significance

The region around a magnet where its poles exhibit a force of attraction or repulsion is called **magnetic field.**

The existence of the magnetic field at a point around the magnet can also be determined by placing a magnetic needle at that point as shown in Fig. 1.1. Although magnetic lines of force have no real existence and are purely imaginary, yet their concept is very useful to understand various magnetic effects. It is assumed (because of their effects) that the magnetic lines of force possess the following important properties:

- The direction of magnetic lines of force is from N-pole to the S-pole outside the magnet. But inside the magnet their direction is from S-pole to N-pole.
- They form a closed loop.
- Their tendency is to follow the least reluctance path.
- They act like stretched cords, always trying to shorten themselves.
- They never intersect each other.
- They repel each other when they are parallel and are in the same direction.
- They remain unaffected by non-magnetic materials.

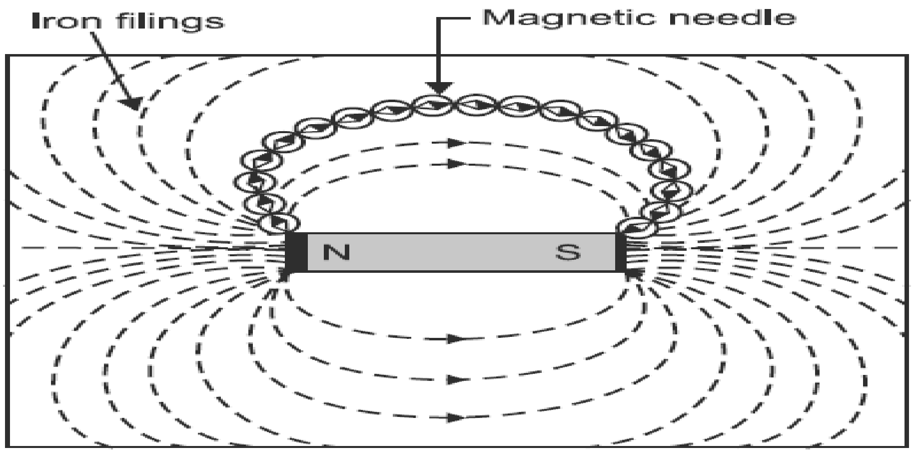


Figure.1.1.Magnetic field around a bar magnet

1.4. Magnetic circuit and its analysis

The closed path followed by magnetic flux is called a magnetic circuit.

A magnetic circuit usually consists of magnetic materials having high permeability (e.g., iron, soft steel, etc.). In this circuit, magnetic flux starts from a point and finishes at the same point after completing its path.

Figure 1.2 shows a solenoid having N turns wound on an iron core (ring).

When current I ampere is passed through the solenoid, magnetic flux φ (Wb) is set-up in the core.

Let:

- ✓ l = mean length of magnetic circuit in m;
- ✓ a = area of cross-section of core in m2;
- ✓ μ_r = relative permeability of core material.

- ✓ Flux density in the core material, B = $\frac{\varphi}{a}$ (Wb/m^2) = T (Tesla)
- ✓ Magnetizing force in the core material:

$$H = \frac{B}{\mu_0 \mu_r} = \frac{\phi}{a \mu_0 \mu_r} \text{ AT/m} \qquad 1.1$$

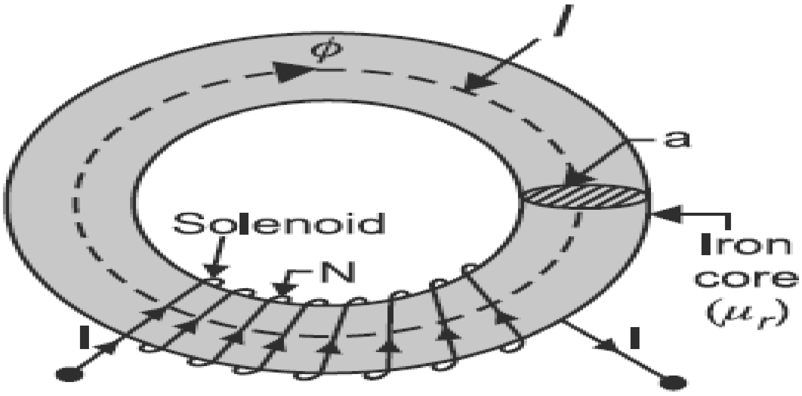


Figure.1.2. Magnetic circuit

According to work law, the work done in moving a unit pole once round the magnetic circuit (or path) is equal to the ampere-turns enclosed by the magnetic circuit.

$$Hl = NI \quad \text{or} \quad \frac{\phi}{a \mu_0 \mu_r} \times l = NI \quad \text{or} \quad \phi = \frac{NI}{(l / a \mu_0 \mu_r)} \text{Wb} \qquad 1.2$$

The above expression reveals that the amount of flux set-up in the core is:

- Directly proportional to N and I i.e., NI, called magnetomotive force (mmf). It shows that the flux increases if either of the two increases and vice-versa.
- Inversely proportional to l/a μ0 μr called reluctance of the magnetic path. In fact, reluctance is the opposition offered to the magnetic flux by the magnetic path. The lower is the reluctance, the higher will be the flux and vice-versa. Thus,

$$\text{Flux} = \frac{\text{m.m.f}}{\text{reluctance}} \qquad 1.3$$

It may be noted that the above expression has a strong resemblance to Ohm's law for electric current (*I* = emf/resistance). The mmf is analogous to emf in electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, the above expression is sometimes referred to as *Ohm's law of magnetic circuits.*

Example.1.1

An iron ring of 400 cm mean circumference is made from round iron of cross-section 20 cm^2. Its permeability is 500. If it is wound with 400 turns, what current would be required to produce a flux of 0·001 Wb?

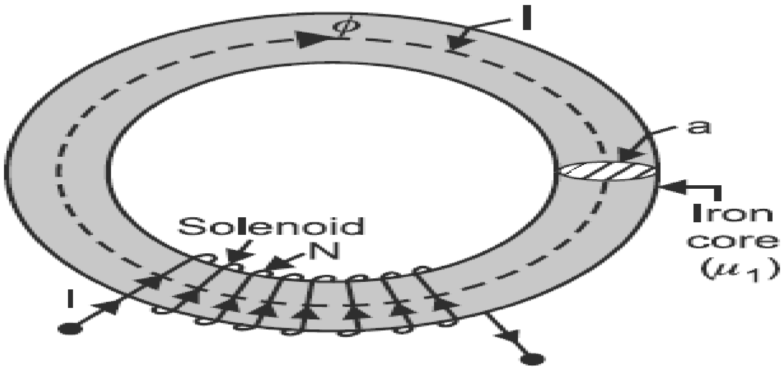


Figure.1.3. Magnetic circuit

Solution:

The magnetic circuit is shown in Fig. 1.3.

Mean length of magnetic path, *lm* = 400 cm = 4 m

Area of cross-section of iron ring, *a* = 20 . 10^{-4} m^2

Absolute permeability, μ0 = 4π . 10^{-7}

Now: mmf = flux . reluctance

$$NI = \phi \times \frac{l_m}{a\mu_0\mu_r}$$

$$400 \times I = 0{\cdot}001 \times \frac{4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500}$$

$$\text{Current, } I = \frac{0.001 \times 4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500 \times 400} = \mathbf{7{\cdot}958}\ (Ans.)$$

1.5. Important Terms

While studying magnetic circuits, generally, we come across the following terms:

1. Magnetic field: The region around a magnet where its poles exhibit a force of attraction or repulsion is called *magnetic field.*

2. Magnetic flux (*φ*): The amount of magnetic lines of force set-up in a magnetic circuit is called *magnetic flux*. Its unit is weber (Wb). It is analogous to *electric current I* in electric circuit.

3. Magnetic flux density (B) *at a point is the flux per unit area at right angles to the flux at that point.*

It is, generally, represented by letter '*B*'. Its unit is Wb/m^2 or Tesla, i.e.,

$$B = \frac{\phi}{A} \text{Wb} / \text{m}^2 \quad \text{or} \quad \text{T} \; (1 \text{ Wb/m}^2 = 1 \times 10^4 \text{ Wb/cm}^2) \qquad 1.4$$

4. Permeability: The ability of a material to conduct magnetic lines of force through it is called the **permeability** of that material.

It is generally represented by μ (mu, a Greek letter). The greater the permeability of a material, the greater is its conductivity for the magnetic lines of force and vice-versa. The permeability of air or vacuum is the poorest and is represented as μ0 (where $\mu 0 = 4.\pi .10^{-7}$ H/m).

5. Relative permeability: The absolute (or actual) permeability μ of a magnetic material is much greater than absolute permeability of air μ0. The relative permeability of a magnetic material is given in comparison with air or vacuum. Hence, the ratio of the permeability of material μ to the permeability of air or vacuum μ0 is called the relative permeability μr of the material.

$$\mu_r = \frac{\mu}{\mu_0} \quad \text{or} \quad \mu = \mu_0 \mu_r \qquad 1.5$$

Obviously, the relative permeability of air would be μ0/μ0 = 1. The value of relative permeability of all the non-magnetic materials is also 1. However, its value is as high as 8000 for soft iron, whereas, its value for mumetal (iron 22% and nickel 78%) is as high as 1,20,000.

5. Magnetic field intensity: The force acting on a unit north pole (1 Wb) when placed at a point in the magnetic field is called the magnetic intensity of the field at that point. It is denoted by H.

In magnetic circuits, it is defined as mmf per unit length of the magnetic path. It is denoted by H, mathematically,

$$H = \frac{\text{m.m.f}}{\text{length of magnetic path}} = \frac{NI}{l} \text{AT} / \text{m} \qquad 1.6$$

6. Magnetomotive force (mmf): The magnetic pressure which sets-up or tends to set-up magnetic flux in a magnetic circuit is called magnetomotive force. As per work law it may be defined as under:

The work done in moving a unit magnetic pole (1 Wb) once round the magnetic circuit is called magnetomotive force. In general mmf = NI ampere-turns (or AT). It is analogous to emf in an electric circuit.

7. Reluctance (S): The opposition offered to the magnetic flux by a magnetic circuit is called its reluctance. It depends upon length (l), area of cross-section (a) and permeability (μ = μ0 μr) of the material that makes up the magnetic circuit. It is measured in AT/Wb.

$$\text{Reluctance, } S = \frac{l}{a\ \mu_0\ \mu_r} \qquad 1.7$$

It is analogous to resistance in an electric circuit.

8. Permeance: It is a measure of the ease with which flux can be set-up in the material. It is just reciprocal of reluctance of the material and is measured in Wb/AT or henry.

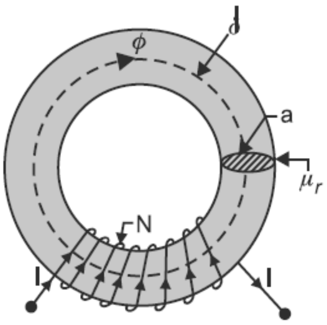
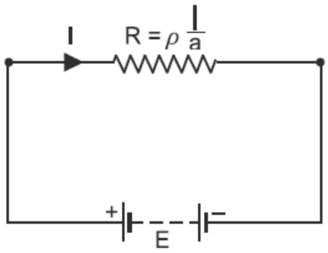
$$\text{Permeance} = \frac{1}{\text{reluctance}} = \frac{a\ \mu_0\ \mu_r}{l}\ \text{Wb/AT} \quad \text{or} \quad \text{H} \qquad 1.8$$

It is analogous to conductance in an electric circuit.

9. Reluctivity: It is specific reluctance and analogous to resistivity in electric circuit.

1.6. Comparison between magnetic and electric circuits

Although magnetic and electric circuits have many points of similarity but still they are not analogous in all respects. A comparison of the two circuits is given below:

Magnetic Circuits	***Electrical Circuits***
 **Fig. 1.3** Magnetic circuit	 **Fig. 1.4** Electric circuit

Similarities	
1. The closed path for magnetic flux is called magnetic circuit.	1. The closed path for electric current is called electric circuit.
2. Flux = mmf/reluctance	2. Current = emf/resistance
3. Flux, ϕ in Wb	3. Current, I in ampere
4. mmf in AT	4. emf in V
5. Reluctance, $S = \frac{l}{a\mu} = \frac{l}{a\mu_0\,\mu_r}$ AT/Wb	5. Resistance, $R = \rho\frac{l}{a}\Omega$ or $R = \frac{1}{\sigma}\frac{l}{a}\Omega$
6. Permeance = 1/reluctance	6. Conductance = 1/resistance
7. Permeability, μ	7. Conductivity, $\sigma = 1/\rho$
8. Reluctivity	8. Resistivity
9. Flux density, $B = \frac{\phi}{a}$ Wb/m^2	9. Current density, $J = \frac{I}{a}$ A/m^2
10. Magnetic intensity, $H = NI/l$	10. Electric intensity, $E = V/d$

1.7. Ampere-turns Calculations

In a magnetic circuit, flux produced,

$$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{l/a\,\mu_0\,\mu_r}$$

$$NI = \frac{\phi l}{a\mu_0\mu_r} = \frac{B}{\mu_0\mu_r}l = Hl$$

1.8. Series magnetic circuits

A magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called a series magnetic circuit. Such as series magnetic circuit (composite circuit) is shown in Fig. 1.5.

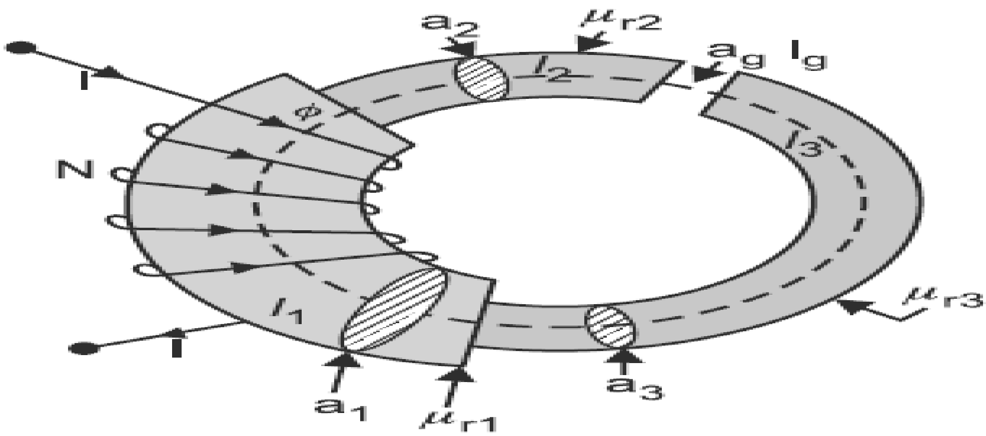


Figure.1.4.Series magnetic circuit

Total reluctance of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g \quad 1.9$$

$$= \frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}$$

Total mmf = ∅.S$_{Tot}$ 1.10

$$= \phi\left(\frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}\right)$$

$$= \frac{B_1 l_1}{\mu_0\,\mu_{r1}} + \frac{B_2 l_2}{\mu_0\,\mu_{r2}} + \frac{B_3 l_3}{\mu_0\,\mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1\,l_1 + H_2\,l_2 + H_3\,l_3 + H_g\,l_g$$

1.9. Parallel magnetic circuits

A magnetic circuit which has two or more than two paths for the magnetic flux is called a *parallel magnetic circuit*. Its behavior can be just compared to a parallel electric circuit.

Figure 1.5 shows a parallel magnetic circuit. A current carrying coil is wound on the central limb *AB*. This coil sets-up a magnetic flux $\varphi 1$ in the central limb which is further divided into two paths i.e., (*1*) path *ADCB* which carries flux $\varphi 2$ and (*2*) path *AFEB* which carries flux $\varphi 3$.

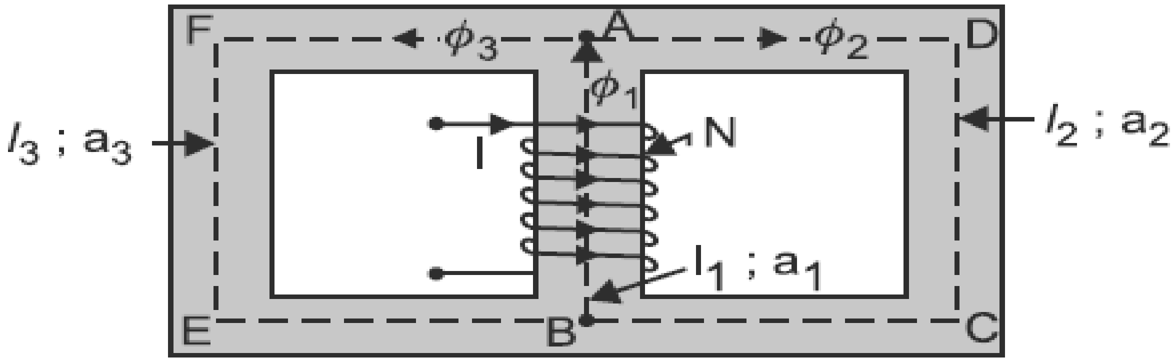


Figure. 1.5. Parallel magnetic circuit

It is clear that:

$$\phi_1 = \phi_2 + \phi_3 \qquad 1.11$$

The two magnetic paths ADCB and AFEB are in parallel. The ATs required for this parallel circuit is equal to the ATs required for any one of the paths.

If S1 = reluctance of path BA i.e., l1/a1 μ0 μr1

S2 = reluctance of path ADCB i.e., l2/a2 μ0 μr2

S3 = reluctance of path AFEB i.e., l3/a3 μ0 μr3

Total mmf required = mmf required for path BA + mmf required path ADCB or path AFEB.

Total mmf or AT = φ1. S1 + φ2. S2 = φ1 .S1 + φ3. S3 1.12

1.10. Leakage flux

The magnetic flux which does not follow the intended path in a magnetic circuit is called leakage flux. When some current is passed through a solenoid, as shown in Fig. 1.6, magnetic flux is produced by it. Most of this flux is set-up in the magnetic core and passes through the air gap (an intended path). This flux is known as useful flux φu. However, some of the flux is just set-up around the coil and is not utilized for any work. This flux is called leakage flux φ l. Total flux produced by the solenoid.

$$\phi = \phi_u + \phi_l \qquad 1.13$$

Leakage co-efficient or leakage factor: The ratio of total flux (φ) produced by the solenoid to the useful flux (φu) set-up in the air gap is known as leakage co-efficient.

It is generally represented by letter 'λ'. Leakage co-efficient :

$$\lambda = \frac{\phi}{\phi_u} \qquad 1.14$$

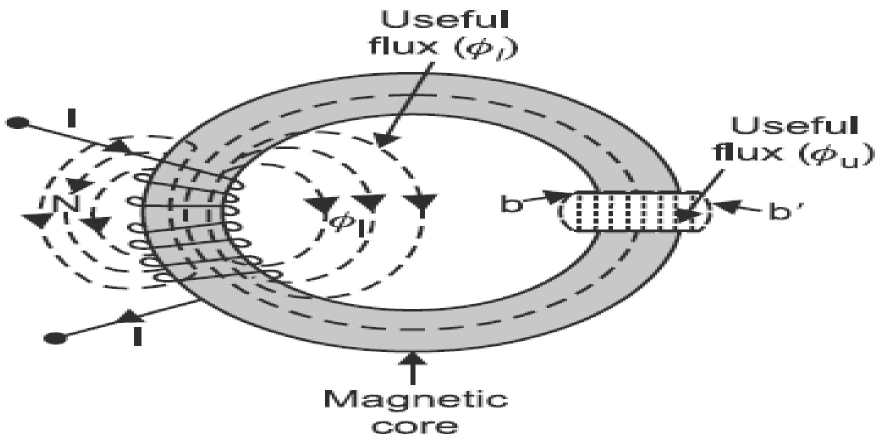


Figure. 1.6. Leakage flux

Fringing: It may be seen in Fig. 1.6 that the useful flux when sets-up in the air gap, it tends to bulge outwards at b and b' since the magnetic lines set-up in the same direction repel each other.

This increases the effective area in the air gap and decreases the flux density. This effect is known as fringing. The fringing is directly proportional to the length of the air gap.

1.11. Magnetization or B-H Curve

The graph plotted between flux density B and magnetizing force H of a material is called the magnetization or B–H Curve of that material. The general shape of the B–H Curve of a magnetic material is shown in Fig. 1.7. The shape of the curve is non-linear this indicates that the relative permeability ($\mu r = B/\mu 0\ H$) of a magnetic material is not constant but it varies. The value of μr largely depends upon the value of flux density. Its shape is shown in Fig. 1.8 (for cast steel).

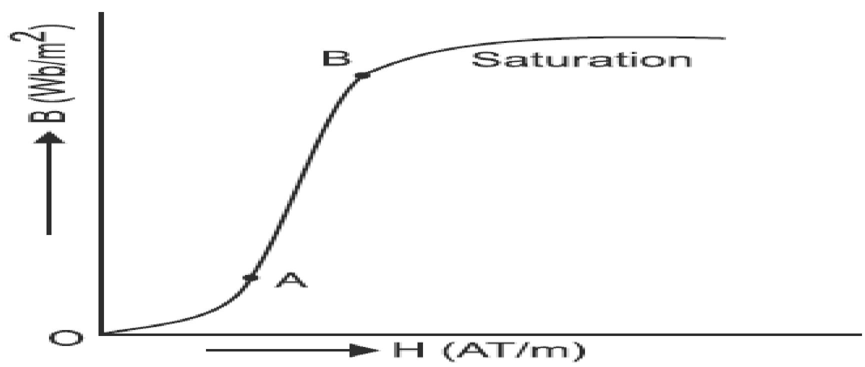


Fig. 1.7. B-H curve of a magnetic material

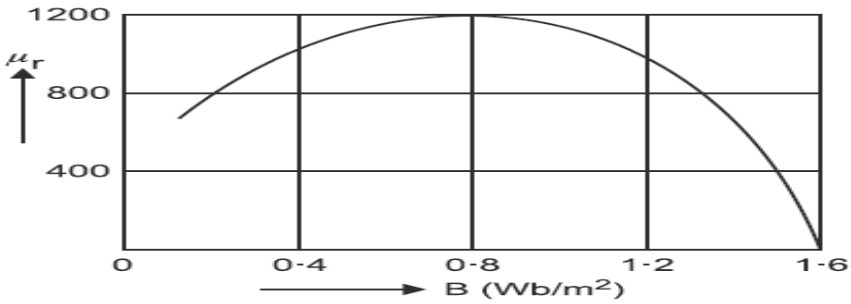


Figure. 1.8. μr - B curve

The B–H curves of some of the common magnetic materials are shown in Fig. 1.9.

The B–H curve for a non-magnetic material is shown in Fig. 1.10. It is a straight line curve since $B = \mu_0 .H$ or $B \propto H$ as the value of μ_0 is constant.

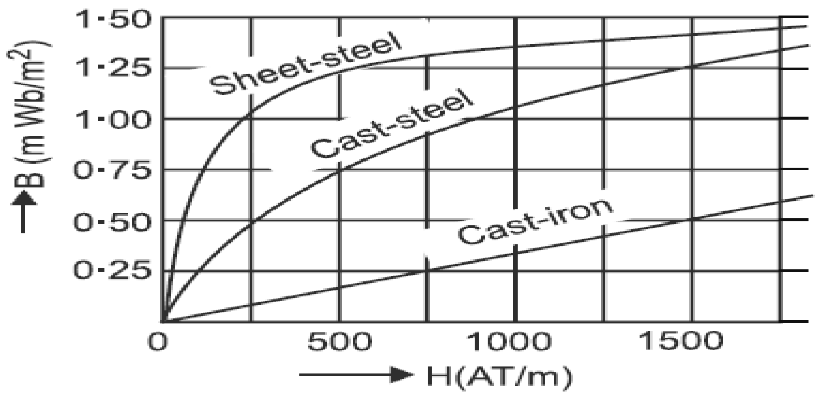


Figure. 1.9. B-H curve for different magnetic materials

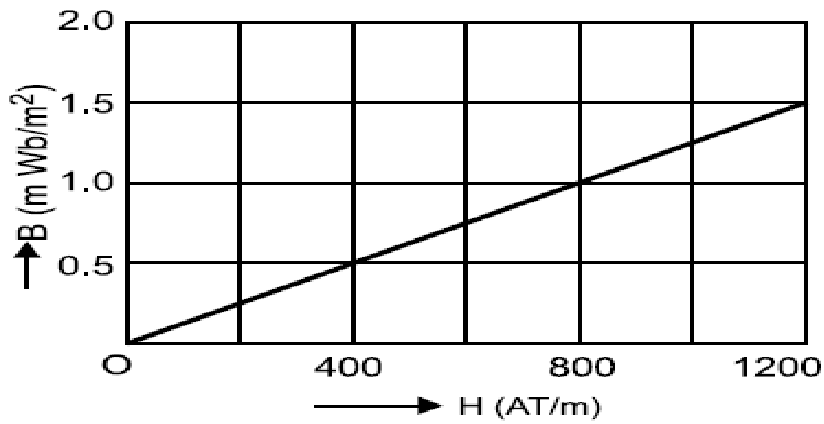


Figure. 1.10. B-H curve for a non-magnetic materials

1.12. Magnetic Hysteresis

When a magnetic material is magnetized first in one direction and then in the other (i.e., one cycle of magnetization), it is found that flux density B in the material lags behind the applied magnetizing force H. This phenomenon is known as magnetic hysteresis.

Hence, the phenomenon of flux density B lagging behind the magnetizing force H in a magnetic material is called **magnetic hysteresis.**

'Hysteresis' is the term derived from the Greek word hysterein meaning to lag behind. To understand the complete phenomenon of magnetic hysteresis, consider a ring of magnetic material on which a solenoid is wound uniformly as shown in Fig. 1.11. The solenoid in connected to a DC source through a double pole double throw reversible switch (position '1').

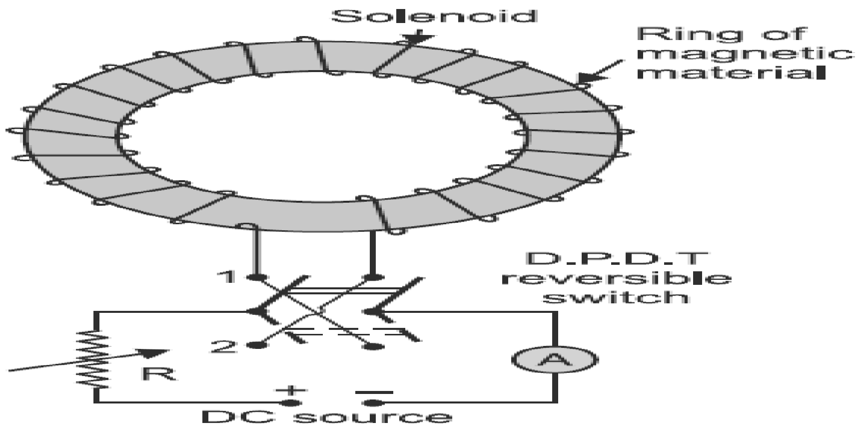


Figure. 1.11. Circuit to trace hysteresis loop

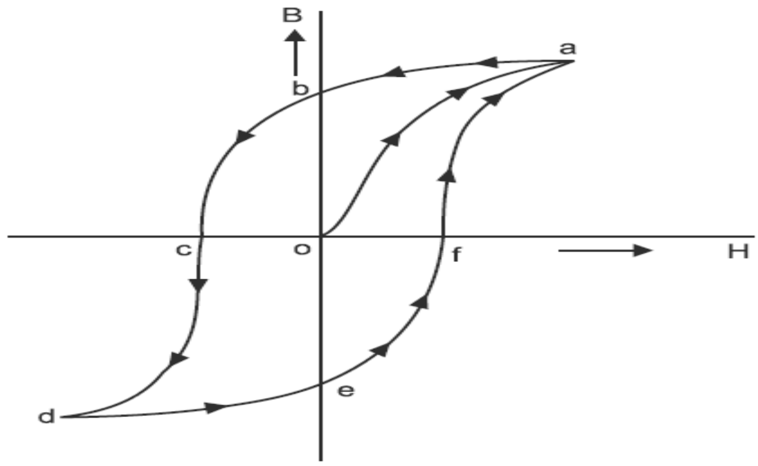


Figure. 1.12. Hysteresis loop

When the field intensity H is increased gradually by increasing current in the solenoid (by decreasing the value of R), the flux density B also increases until saturation point a is reached and curve so obtained is oa. If now the magnetizing force is gradually reduced to zero by decreasing current in the solenoid to zero. The flux density does not become zero and the curve so obtained is ab as shown in Fig. 1.12.

When magnetizing force H is zero, the flux density still has value ob.

Residual Magnetism and Retentivity

This value of flux density 'ob' retained by the magnetic material is called **residual magnetism** and the power of retaining this residual magnetism is called **retentivity** of the material.

To demagnetize the magnetic ring, the magnetizing force H is reversed by reversing the direction of flow of current in the solenoid. This is achieved by changing the position of double pole, double throw switch (i.e., position ‘2’). When H is increased in reverse direction, the flux density starts decreasing and becomes zero and curve follows the path bc. Thus residual magnetism of the magnetic material is wiped off by applying magnetizing force oc in opposite direction.

- **Coercive Force**

This value of magnetizing force oc required to wipe off the residual magnetism is called **coercive force.**
To complete the loop, the magnetizing force H is increased further in reverse direction till saturation reaches (point ‘d’) and the curve follows the path cd. Again H is reduced to zero and the curve follows the path de. Where oe represents the residual magnetism. Then H is increased in the positive direction by changing the position of reversible switch to position ‘1’ and increasing the flow of current in the solenoid. The curve follows the path of efa and the loop is completed. Again of is the magnetizing force utilized to wipe off the residual magnetism oe. Hence, cf is the total coercive force required in one cycle of magnetization to wipe off the residual magnetism.
Since the meaning of hysteresis is lagging behind, and in this case flux density B always lags behind the magnetizing force, H, therefore, loop (abcdefa) so obtained is called hysteresis loop.

1.13. Hysteresis Loss

When a magnetizing force is applied, the magnetic material is magnetized and the molecular magnets are lined up in a particular direction.
However, when the magnetizing force in a magnetic material is reversed, the internal friction of the molecular magnets opposes the reversal of magnetism, resulting in hysteresis.
To overcome this internal friction of the molecular magnets (or to wipe off the residual magnetism), a part of the magnetizing force is used. The work done by the magnetizing force against this internal friction of molecular magnets produces heat. This energy, which is wasted in the form of heat due to hysteresis, is called *hysteresis loss*.
Hysteresis loss occurs in all the magnetic parts of electrical machines where there is reversal of magnetization. This loss results in wastage of energy in the form of heat. Consequently, it increases the temperature of the machine which is undesirable. Therefore, a suitable magnetic material is selected for the construction of such parts, e.g., silicon steel is most suitable in which hysteresis loss is minimum.

1.14. Importance of Hysteresis Loop

The shape and size of hysteresis loop of a magnetic material largely depends upon its nature. For a particular location, the choice of the magnetic material depends upon the shape and size (i.e., area) of its hysteresis loop. The hysteresis loops of some of the common magnetic materials are shown in Fig. 1.13.

- ✓ **Hard steel:** The hysteresis loop for hard steel is shown in Fig. 1.13 (*a*). This loop has larger area which indicates that this material will have more hysteresis loss. Therefore, it is never used for the construction of machine parts. However, its loop shows that the material has high retentivity and coercivity. Therefore, it is more suitable for making permanent magnets.

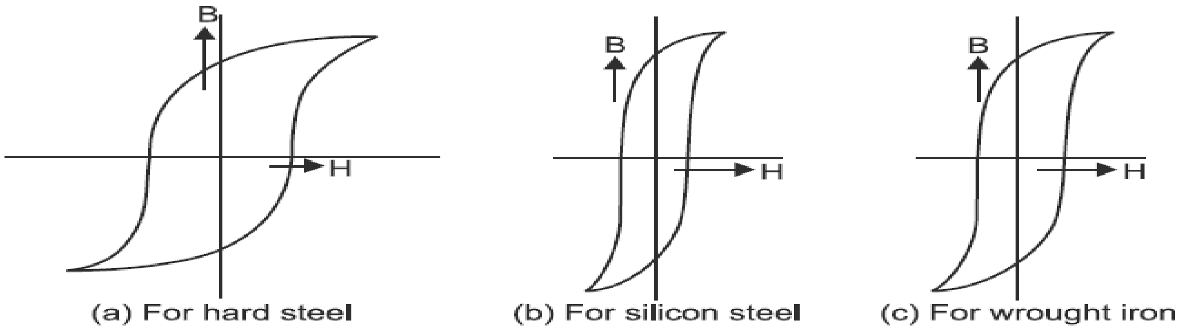


Figure. 1.13 Hysteresis loop for different magnetic materials

- ✓ **Silicon steel:** The hysteresis loop for silicon steel is shown in Fig. 1.13 (b). This loop has smallest area which indicates that this material will have small hysteresis loss. Therefore, it is most suitable for the construction of those parts of electrical machines in which reversal of magnetisation is very quick e.g., armature of DC machines, transformer core, and starter of induction motors etc.
- ✓ **Wrought iron:** Figure 1.13 (c) shows the hysteresis loop for wrought iron. This loop shows that this material has fairly good residual magnetism and coercivity. Therefore, it is best suited for making cores of electromagnets.

Example.1.2

The ring shaped core shown in Fig. 1.14 is made of a material having a relative permeability of 1000. The flux density in the smallest area of cross-section is 2 T. If the current through the coil is not to exceed 1·5 A, compute the number of turns of the coil.

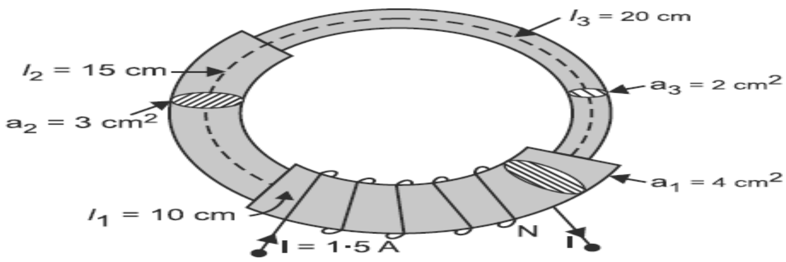


Figure. 1.14 Series magnetic circuit as per data

Solution:

Flux in the core, $\phi = B \times A = 2 \times 2 \times 10^{-4} = 4 \times 10^{-4}$ Wb

Total reluctance of the magnetic path,

$$S = S_1 + S_2 + S_3$$
$$= \frac{l_1}{a_1 \mu_0 \mu_r} + \frac{l_2}{a_2 \mu_0 \mu_r} + \frac{l_3}{a_3 \mu_0 \mu_r}$$
$$= \frac{1}{\mu_0 \mu_r}\left(\frac{l_1}{a_1} + \frac{l_2}{a_2} + \frac{l_3}{a_3}\right)$$
$$= \frac{1}{4\pi \times 10^{-7} \times 1000}\left(\frac{0 \cdot 1}{4 \times 10^{-4}} + \frac{0 \cdot 15}{3 \times 10^{-4}} + \frac{0 \cdot 2}{2 \times 10^{-4}}\right)$$
$$= 13{\cdot}926 \times 10^5 \text{ AT/Wb}$$

Total mmf required, $NI = \phi S$

or $N \times 1{\cdot}5 = 4 \times 10^{-4} \times 13{\cdot}926 \times 10^5$

$\therefore$ No. of turns, $N =$ **371·36** (*Ans.*)

1.15. Electro Magnetic Induction

The phenomenon by which an emf is induced in a circuit (and hence current flows when the circuit is closed) when magnetic flux linking with it changes is called *electro-magnetic induction.*

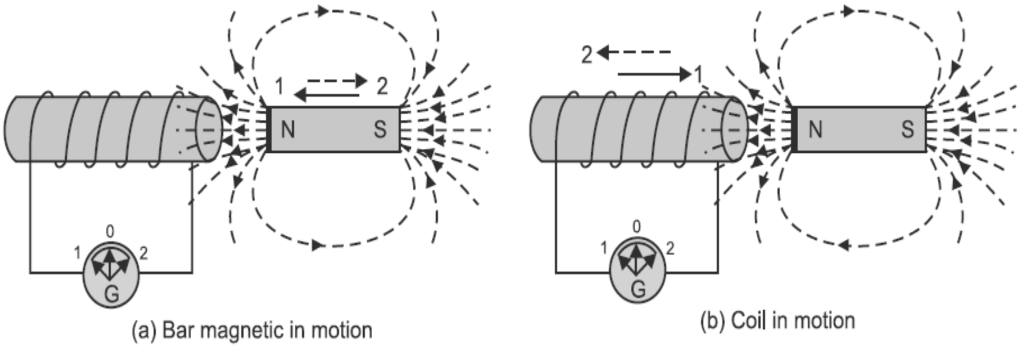


Figure. 1.15 .Electromagnetic induction

However, if the magnet and the coil both are kept stationary, no matter how much flux is linking with the coil, there is no deflection in the galvanometer needle.

The following points are worth noting:

- ✓ The deflection in the galvanometer needle shows that emf is induced in the coil. This condition occurs only when flux linking with the circuit changes i.e., either magnet or coil is in motion.
- ✓ The direction of induced emf in the coil depends upon the direction of magnetic field and the direction of motion of coil.

Faraday’s Laws of Electromagnetic Induction

Michael Faraday summed up conclusions of his experiments regarding electro-magnetic induction into two laws, known as Faraday’s laws of electro-magnetic induction.

First Law: This law states that “Whenever a conductor cuts across the magnetic field, an emf is induced in the conductor.” or “Whenever the magnetic flux linking with any circuit (or coil) changes, an emf is induced in the circuit.”

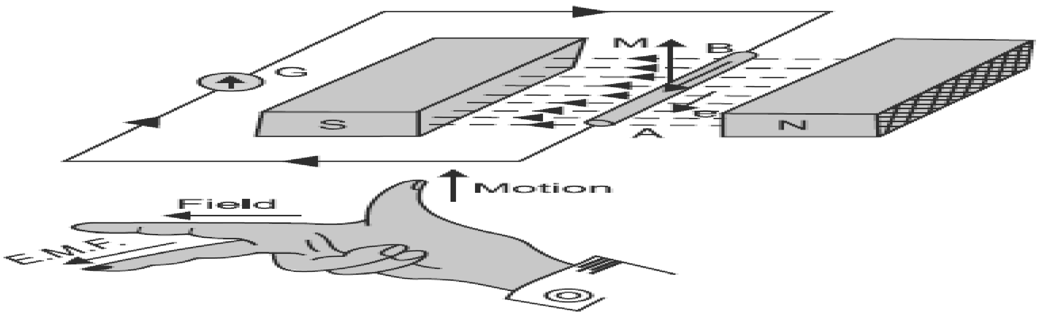


Figure. 1.16 Conductor moving in the field

If the conductor is moved along (parallel) the field, there is no deflection in the needle which indicates that no emf is induced in the conductor.

For the second statement, consider a coil placed near a bar magnet and a galvanometer connected across the coil, as shown in Fig. 1.17.

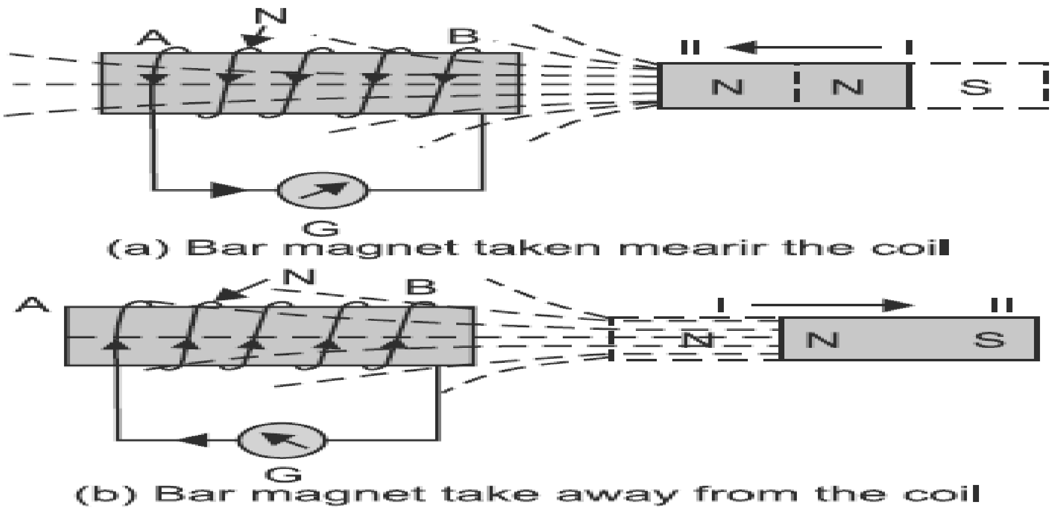


Figure. 1.17. Coil is stationary but bar magnet (field) is moving

Second Law: This law states that “The magnitude of induced emf in a coil is directly proportional to the rate of change of flux linkages.

$$\text{Rate of change of flux linkages} = \frac{N(\phi_2 - \phi_1)}{t} \text{ Wb-turns/s} \qquad 1.15$$

Where, N = No. of turns of the coil; ($\boldsymbol{\varphi}2 - \boldsymbol{\varphi}1$) = change of flux in Wb t = time in second for the change $\boldsymbol{\varphi}$ According to Faraday’s second law of electro-magnetic induction;

Induced emf, $e \propto \frac{N(\phi_2 - \phi_1)}{t}$

$$e = \frac{N(\phi_2 - \phi_1)}{t} \text{ (taking proportionality constant, as unity)}$$

In differential form, $e = N\frac{d\phi}{dt}$ volt

Usually, a minus sign is given to the right-hand side expression which indicates that emf is induced in such a direction which opposes the cause (i.e., change in flux) that produces it (according to Lenz's law).

$$e = -N\frac{d\phi}{dt} \text{ volt} \quad 1.16$$

Direction of Induced emf

The direction of induced emf and hence current in a conductor or coil can be determined by either of the following two methods:

- ✓ Fleming's Right Hand Rule,
- ✓ Lenz's Law: This law is more suitably applied to determine the direction of induced emf in a coil or circuit when flux linking with it changes. It is stated as under: "In effect, electro magnetically induced emf and hence current flows in a coil or circuit in such a direction that the magnetic field set up by it, always opposes the very cause which produces it."

Induced emf

When flux linking with a conductor (or coil) changes, an emf is induced in it. This change in flux linkages can be obtained in the following two ways:

- By either moving the conductor and keeping the magnetic field system stationary or moving the magnetic field system and keeping the conductor stationary, in such a way that conductor cuts across the magnetic field (as in case of DC and AC generators). The emf induced in this way is called dynamically induced emf
- By changing the flux linking with the coil (or conductor) without moving either coil or field system. However, the change of flux produced by the field system linking with the coil is obtained by changing the current in the field system (solenoid), as in transformers. The emf induced in this way is called statically induced emf.

❖ Dynamically Induced emf

By either moving the conductor keeping the magnetic field system stationary or moving the field system keeping the conductor stationary so that flux is cut by the conductor, the emf thus induced in the conductor is called dynamically induced emf.

❖ Mathematical Expression

Considering a conductor of length l metre placed in the magnetic field of flux density B Wb/m2 is moving at right angle to the field at a velocity ***v*** metre/second as shown in Fig. 1.18 (a). Let the conductor be moved through a small distance dx metre in time dt second as shown in Fig. 1.18 (b).

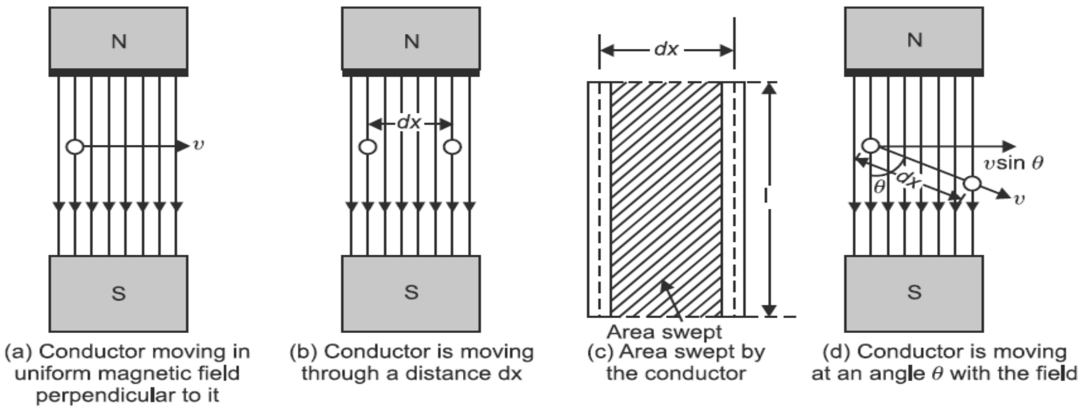


Figure. 1.18. Dynamically induced emf

- ✓ Area swept by the conductor, A = l . Dx
- ✓ Flux cut by the conductor, φ = B . A = B l dx
- ✓ According to Faraday's Law of electro-magnetic induction;

$$\text{Induced emf, } e = \frac{\text{flux cut}}{\text{time}} = \frac{\phi}{dt} = \frac{B\,l\,dx}{dt} = B\,l\,\upsilon \qquad (\text{since } dx/dt = \upsilon \text{ (velocity)})$$

Now, if the conductor is moved at an angle θ with the direction of magnetic field at a velocity ***v*** metre/second as shown in Fig. 1.18 (d). A small distance covered by the conductor in that direction is dx in time dt second. Then the component of distance perpendicular to the magnetic field, which produces emf, is dx sin θ.

$$\text{Area swept by the conductor, } A = l \times dx \sin\theta$$

$$\text{Flux cut by the conductor, } \phi = B \times A = B\,l\,dx \sin\theta$$

$$\text{Induced emf } e = \frac{B l\,dx \sin\theta}{dt} = B\,l\,\upsilon \sin\theta$$

Example.1.3

A coil of 500 turns in linked with a flux of 2 mWb. If this flux is reversed in 4 ms, calculate the average emf induced in the coil.

Solution:

$$\text{Average induced emf, } e = N\frac{d\phi}{dt}$$

where, $N = 500$ turns; $d\phi = 2 - (-2) = 4\ mWb$; $dt = 4 \times 10^3\ s$;

$$\therefore \quad e = 500 \times \frac{4 \times 10^{-3}}{4 \times 10^{-3}} = \mathbf{500\ V}\ (Ans.)$$

Example.1.4

A coil of 250 turns is wound on a magnetic circuit of reluctance 100000 AT/Wb. If a current of 2 A flowing in the coil is reversed in 5 ms, find the average emf induced in the coil.

Solution:

$$\phi = \text{mmf/reluctance i.e., } \phi = NI/S$$

where, $N = 250$; $I = 2\text{A}$ and $S = 100000$ AT/Wb.

$$\therefore \quad \phi = \frac{250 \times 2}{100000} = 5\ \text{m Wb}$$

$$\text{Average induced emf } e = N\frac{d\phi}{dt}$$

where, $d\phi = 5 - (-5) = 10$ m Wb (since current is reversed)

$$e = 250 \times \frac{10 \times 10^{-3}}{5 \times 10^{-3}} = \mathbf{500\ V}\ (Ans.)$$

❖ Statically Induced emf

When the coil and magnetic field system both are stationary but the magnetic field linking with the coil changes (by changing the current producing the field), the emf thus induced in the coil is called statically induced emf.

The statically induced emf may be:

- ✓ Self-induced emf
- ✓ Mutually induced emf

Self-induced emf: The emf induced in a coil due to the change of flux produced by it linking with its own turns is called self-induced emf as shown in Fig. 1.19. The direction of this induced emf is such that it opposes the cause which produces it (Lenz's law) i.e., change of current in the coil.

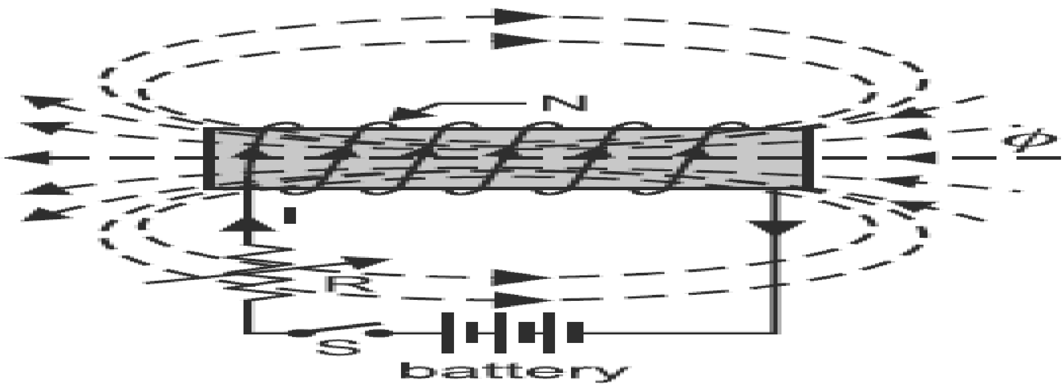


Figure. 1.19. Flux produced by coil linking with its own turns

Since the rate of change of flux linking with the coil depends upon the rate of change of current in the coil. Therefore, the magnitude of self-induced emf is directly proportional to the rate of change of current in the coil:

$$e \propto \frac{dI}{dt} \quad \text{or} \quad e = L\frac{dI}{dt} \qquad 1.17$$

Where L is a constant of proportionality and is called self-inductance or co-efficient of self-inductance or inductance of the coil.

Mutually induced emf: The emf induced in a coil due to the change of flux produced by another (neighbouring) coil, linking with it is called mutually induced emf as shown in Fig. 1.20.

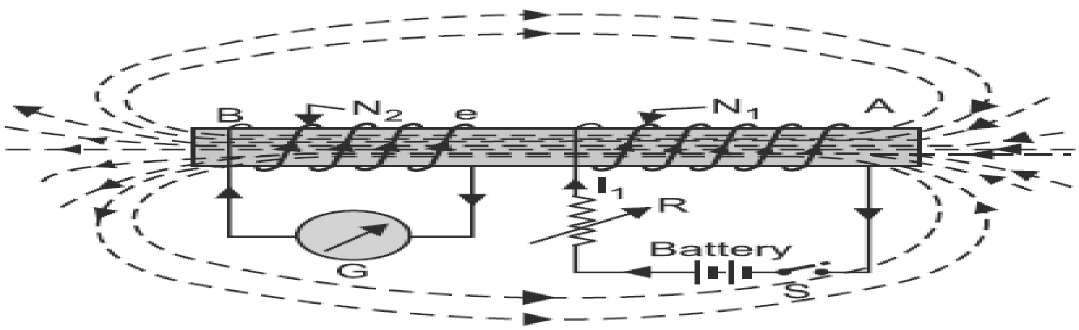


Figure. 1.20. Flux produced by coil-A linking with coil-B

Since the rate of change of flux linking with coil 'B' depends upon the rate of change of current in coil 'A'. Therefore, the magnitude of mutually induced emf is directly proportional to the rate of change of current in coil 'A', i.e.,

$$e_m \propto \frac{dI_1}{dt} \quad \text{or} \quad e_m = M\frac{dI_1}{dt} \qquad 1.18$$

Where M is a constant of proportionality and is called mutual inductance or co-efficient of mutual inductance.

❖ Self-Inductance

The property of a coil due to which it opposes the change of current flowing through itself is called self-inductance or inductance of the coil. This property (i.e., inductance) is attained by a coil due to self-induced emf produced in the coil itself by the changing current flowing through it.

If the current in the coil is increasing (by the change in circuit conditions), the self-induced emf is produced in the coil in such a direction so as to oppose the rise of current i.e., the direction of self-induced emf is opposite to that of the applied voltage.

On the other hand, if the current in the coil is decreasing, the self-induced emf is produced in the coil in such direction so as to oppose the fall of current i.e., the direction of self-induced emf is in the same direction as that of the applied voltage. In fact, self-inductance does not prevent the change of current but it delays the change of current flowing through a coil.

It may be noted that this property of the coil only opposes the changing current (i.e., alternating current). However, it does not affect the steady (i.e., direct) current when flows through it.

In other words, the self-inductance of the coil (by virtue of its geometrical and magnetic properties) will exhibit its presence to the alternating current but it will not exhibit its presence to the direct current.

Expressions for self-inductance:

$$L = \frac{e}{dI / dt}\left(\text{since } e = L\frac{dI}{dt}\right) \quad 1.19$$

$$= \frac{N\phi}{I}\left(\text{since } e = N\frac{d\phi}{dt} = L\frac{dI}{dt}\right) = \frac{N^2}{l / a\mu_0\mu_r}\left(\text{since } \phi = \frac{NI}{l / a\mu_0\mu_r}\right)$$

❖ Mutual Inductance

The property of one coil due to which it opposes the change of current in the other (neighbouring) coil is called mutual-inductance between the two coils. This property (i.e., mutual-inductance) is attained by a coil due to mutually induced emf in the coil while current in the neighbouring coil is changing.

Expression for mutual inductance:

$$M = \frac{e_m}{dI_1 / dt}\left(\text{since } e_m = M\frac{dI_1}{dt}\right) \quad 1.20$$

$$= \frac{N_2\phi_{12}}{I_1}\left(\text{since } e_m = N_2\frac{d\phi_{12}}{dt} = M\frac{dI_1}{dt}\right)$$

$$= \frac{N_1N_2}{l / a\mu_0\mu_r}\left(\text{since } \phi_{12} = \frac{N_1I_1}{l / a\mu_0\mu_r}\right)$$

1.16. AC Excitation in Magnetic Circuits

To magnetise the magnetic circuits of electrical devices such as transformers, AC machines, electromagnetic relays, etc., AC supply is used. The magnetisation of magnetic circuits is called their excitation.

The magnetic circuits are never excited by DC supply, because in case of DC excitation the steadystate current is determined by the impressed voltage and resistance of the circuit. The inductance of the coil comes into picture only during transient period i.e., when the current is building-up or decaying during switching (ON or OFF) instants.

The magnetic flux in the magnetic circuit adjusts itself in accordance with this steady value of current so that the relationship imposed by magnetization (B-H) curve in satisfied.

However, with AC excitation, inductance comes into picture even at steadystate condition.

As a result for most of the magnetic circuits (not for all) the flux is determined by the impressed voltage and frequency. Then the magnetisation current adjusts itself in accordance with this flux so that the relationship imposed by the magnetisation (B-H) curve is satisfied. Usually, for economic reasons, the normal working flux density in a magnetic circuit is kept beyond the linear portion of the magnetisation curve thus accurate analysis cannot be predicted for determining self inductance.

However, for all practical purposes the parameters of the magnetic circuit are considered to be constant.

The reactive effect of the alternatively flux setup by the exciting current can readily be shown as per Faraday's laws i.e.,

$$e = N\frac{d\phi}{dt}$$

Consider a magnetic core which is excited by a coil (winding) having N turns and carrying a current of i ampere as shown in Fig. 1.21.

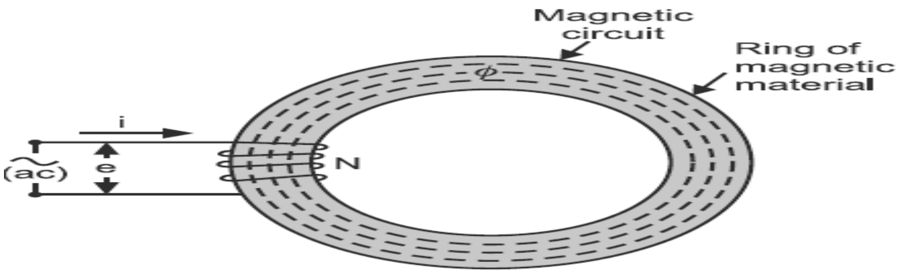


Figure. 1.21. Magnetic core excited by AC

A magnetic flux $\boldsymbol{\varphi}$ is set-up by the exciting current i.

Let the magnetic flux $\boldsymbol{\varphi}$ varies sinusoidally with respect to time t, then its instantaneous value is given by the relation;

$$\phi = \phi_m \sin \omega t = \phi_m \sin 2\pi ft \qquad 1.21$$

where, ϕ_m = maximum value of alternating flux

f = frequency of supply impressed across the coil.

The induced *emf* in the coil

$$e = N\frac{d\phi}{dt} = N\frac{d}{dt}(\phi_m \sin 2\pi ft)$$

$$= 2\pi fN\,\phi_m \cos 2\pi ft = 2\pi fN\,\phi_m \sin\left(2\pi ft + \frac{\pi}{2}\right) \qquad ...(ii)$$

The value of induced *emf* will be maximum when $\cos 2\pi ft = 1$, therefore

$$E_m = 2\pi fN\,\phi_m$$

Its effective or *r.m.s.* value, 1.22

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{2\pi fN\phi_m}{\sqrt{2}} = 4{\cdot}44\,f\,N\,\phi_m$$

Equation (i) and (ii) reveal that the induced emf leads the flux and hence the exciting current by $\boldsymbol{\pi/2}$ radian or 90°.

This induced emf and the coil resistance drop oppose the applied voltage. In case of electrical machines, usually the drop in resistance in only a few percent of applied voltage and therefore, neglected for close approximation. Thus, the induced emf E and applied voltage V may be considered equal in magnitude.

Example.1.5

For the AC excited magnetic circuit shown in Fig. 1.22. Calculate the excitation current and induced emf of the coil to produce a core flux of 0·6 sin 314 t mWb.

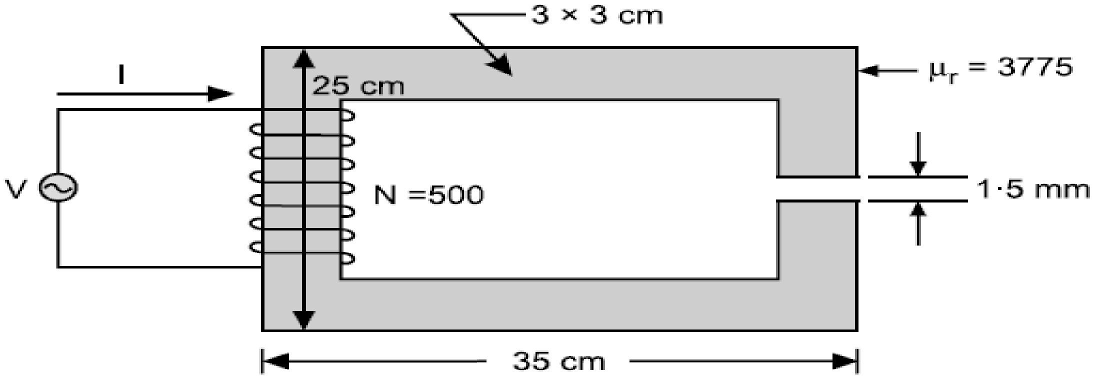


Fig. 1.22. Magnetic core excited by AC

Solution:

Here, $\phi = \phi_m \sin \omega t = 0{\cdot}6 \sin 314\, t$ mWb

Maximum value of flux, $\phi_m = 0{\cdot}6$ mWb $= 6 \times 10^{-4}$ Wb

Area of x-section, $a = 3 \times 3 = 9$ cm^2 $= 9 \times 10^{-4}$ m^2

Flux density, $B_m = \dfrac{\phi_m}{\alpha} = \dfrac{6 \times 10^{-4}}{9 \times 10^{-4}} = 0.667T$

Length of air gap, $l_g = 1{\cdot}5$ mm $= 1{\cdot}5 \times 10^{-3}$ m

Length of iron path, $l_i = \left(25 - 2 \times \frac{3}{2} + 35 - 2 \times \frac{3}{2}\right) \times 2 - 0.15 = 107 \cdot 85$cm

$= 1{\cdot}0785$ m

Total ampere-turns required, $AT_m = \dfrac{B_m}{\mu_0} l_g + \dfrac{B_m}{\mu_0 \mu_r} l_i$

$= \dfrac{0 \cdot 667 \times 1 \cdot 5 \times 10^{-3}}{4\pi \times 10^{-7}} + \dfrac{0 \cdot 667 \times 1 \cdot 0785}{4\pi \times 10^{-7} \times 3775}$

$= 796{\cdot}3 + 151{\cdot}7 = 948$

Maximum value of excitation current required,

$$I_m = \frac{AT_m}{N} = \frac{948}{500} = 1{\cdot}896 \text{ A}$$

RMS value of excitation current,

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{1 \cdot 896}{1 \cdot 414} = \mathbf{1{\cdot}34\ A}\ (Ans.)$$

RMS value of induced emf in the coil, $E_{rms} = 4{\cdot}44 f \text{ N } \phi_m$

$$= 4 \cdot 44 \times \frac{314}{2\pi} \times 500 \times 0 \cdot 6 \times 10^{-3} = \mathbf{66{\cdot}57\ V}\ (Ans.)$$

1.17. Eddy Current Loss

When a magnetic material is subjected to a changing (or alternating) magnetic field, an emf is induced in the magnetic material itself according to Faraday's laws of electro-magnetic induction.

Since the magnetic material is also a conducting material, these emfs. circulate currents within the body of the magnetic material. These circulating currents are known as eddy currents.

As these currents are not used for doing any useful work, therefore, these currents produce a loss (i^2.R loss) in the magnetic material called eddy current loss.

Like hysteresis loss, this loss also increases the temperature of the magnetic material.

The hysteresis and eddy current losses in a magnetic material are called iron losses or core losses or magnetic losses.

When changing flux links with the core itself, an emf is induced in the core which sets-up circulating (eddy) currents (i) in the core as shown in Fig. 1.23 (a).

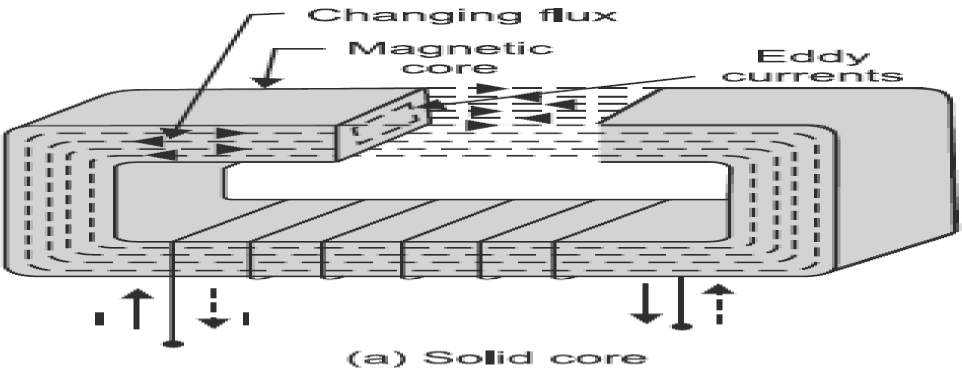


Figure. 1.23. Production of eddy currents

These currents produce eddy current loss (i^2. R), where i is the value of eddy currents and R is resistance of eddy current path. As the core is a continuous iron block of large cross-section, the magnitude of i will be very large and hence greater eddy current loss will result.

To reduce the eddy current loss, the obvious method is to reduce magnitude of eddy currents.

This can be achieved by splitting the solid core into thin sheets (called laminations) in the planes parallel to the magnetic field as shown in Fig. 1.24 (b). Each lamination is insulated from the other by a fine.

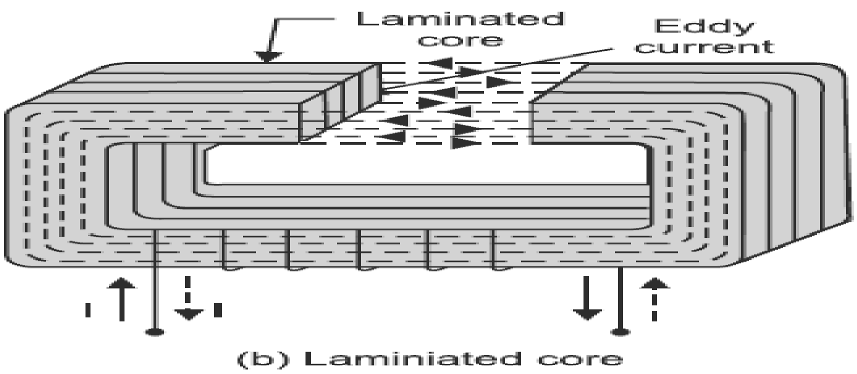


Fig. 1.24. Production of eddy currents

Layer of insulation (varnish or oxide film). This arrangement reduces the area of each section and hence the induced emf It also increases the resistance of eddy currents path since the area through which the currents can pass is smaller.

This loss can further be reduced by using a magnetic material having higher value of resistivity (like silicon steel).

Mathematical Expression for Eddy Current Loss

Although it is difficult to determine the eddy current power loss from the current and resistance values. However, experiments reveal that the eddy current power loss in a magnetic material can be expressed as:

$$P_e = K_e B_m^2 t^2 f^2 \ V \text{ watt} \tag{1.23}$$

Where,

- Ke = co-efficient of eddy current, its value depends upon the nature of magnetic material;
- Bm = maximum value of flux density in Wb/m^2;
- t = thickness of lamination in m;
- f = frequency of reversal of magnetic field in Hz;
- V = volume of magnetic material in m^3.

Chapter.2.Transformer

Chapter .2: Transformer

2.1. CHAPTER OBJECTIVE

After the completion of this unit, students/readers will be able to understand:

- ✓ What is a transformer and its necessity in power system?
- ✓ How a transformer transfers electric power from one circuit to the other i.e., what is basic principle of a transformer.
- ✓ How does a transformer behave? When is it considered to be an ideal one?
- ✓ What are the various factors on which emf induced in a transformer winding depends?
- ✓ How does a transformer behave when it is at no-load?
- ✓ What is exciting current and how is it affected by magnetization?
- ✓ How does a transformer behave when it is loaded?
- ✓ How to draw phases diagrams of a transformer to represent various alternating quantities (neglecting resistance and reactance ampere-turns balance)?
- ✓ How does resistance of transformer windings affect its performance?
- ✓ How to determine equivalent resistance and reactance of transformer windings on its either side?
- ✓ How to draw an equivalent circuit of a transformer and how to simplify it?
- ✓ What is voltage regulation of a transformer?
- ✓ What are the major losses in a transformer?
- ✓ How is efficiency of a transformer calculated at various loads?
- ✓ How to determine the conditions at which a transformer works at maximum efficiency?
- ✓ How to conduct various tests such as, open circuit test, short circuit test on a transformer?

2.2. INTRODUCTION

Transformer is considered to be a backbone of a power system. For generation, transmission and distribution of electric power, AC system is adopted instead of DC system because voltage level can be changed comfortably by using a transformer. For economic reasons, high voltages are required for transmission whereas, for safety reasons, low voltages are required for utilization.

2.3. DEFINITION

A transformer is a static device that transfers AC electrical power from one circuit to the other at the same frequency but the voltage level is usually changed.

The figure.II.1 and 2 show the block diagram of a single-phase transformer and a real transformer.

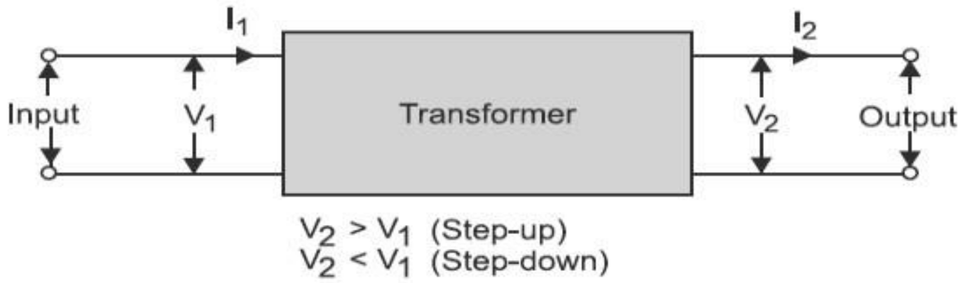


Figure. 2 .1.Block diagram of a single-phase transformer

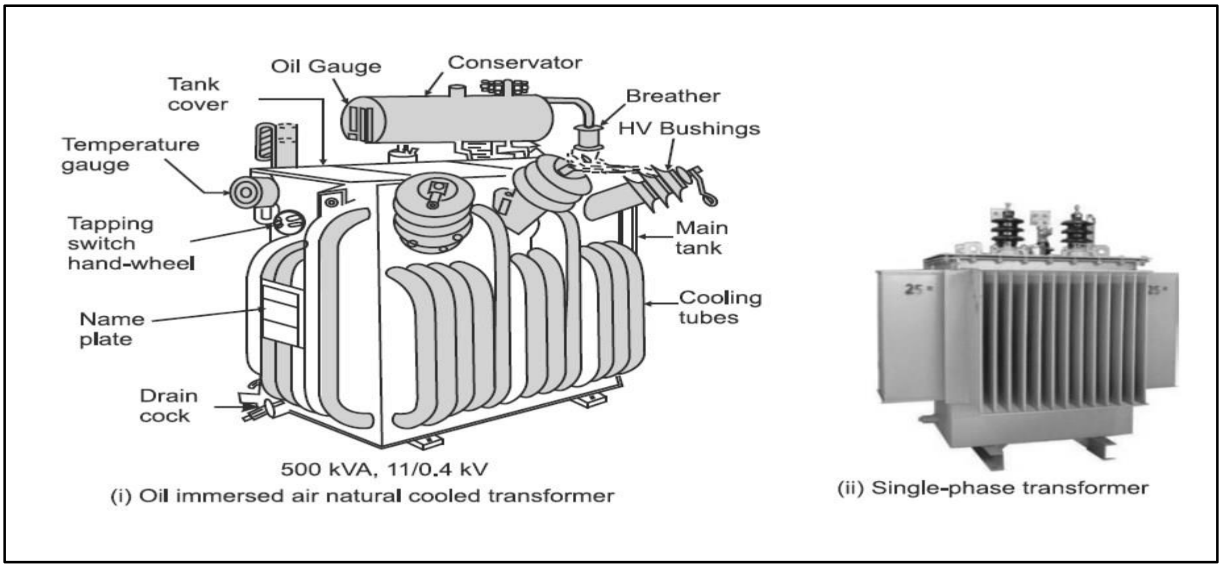


Figure.2.2.Transformer

2.4.WORKING PRINCIPLE OF A TRANSFORMER

The basic principle of a transformer is based in electromagnetic induction law. In addition a single-phase transformer consists of two windings placed over a laminated silicon steel core. The transformer winding having less number of turns is called low-voltage winding and the transformer winding having more number of turns is called high voltage winding.

The Figures.II.3 respectively are presented a single-phase transformer and the flux linking with primary and secondary of transformer. Also, the winding to which AC supply is connected is called a primary winding and the other one is called a secondary winding to which load is connected.

Once AC supply of voltage V1 is given to primary winding, an alternating flux is set-up in the magnetic core which links with the primary and secondary winding.

Consequently, self-induced **emf E1** and mutually-induced **emf E2** are induced in primary and secondary, respectively. These induced **emf's** are developed in phase opposition to V1 as per **Lenz's law.**

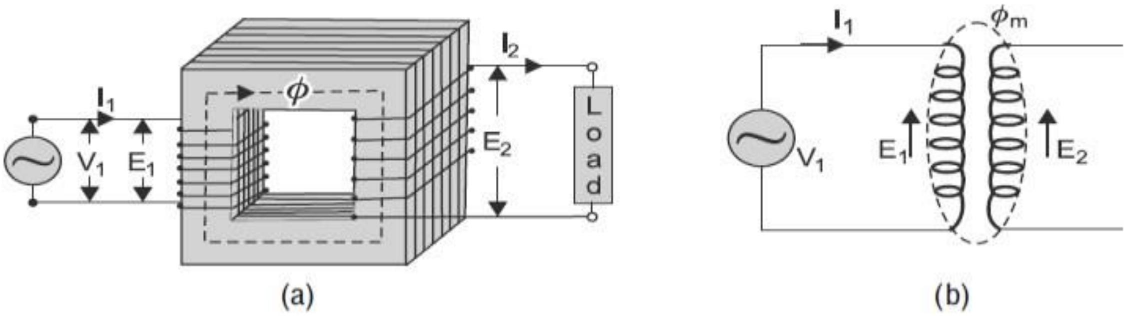


Figure.2.3. (a) Single-phase transformer (core and windings), (b) Flux linking with primary and secondary

Although, there is no electrical connection between primary and secondary winding, still electric power is transferred from one circuit (primary side) to the other circuit (secondary side). It is all because of magnetic coupling, i.e., the alternating flux which is set-up in the core linking with both the windings. When ***N*2 > *N*1**, the transformer is called a step-up transformer, on the other hand, when ***N*2< *N*1** the transformer is called step-down transformer.

- ❖ **Turn ratio:** The ratio of primary to secondary turns is called turn ratio, i.e**., turn ratio = *N*1/ *N*2**.
- ❖ **Transformation ratio:** The ratio of secondary voltage to primary voltage is called voltage transformation ratio of the transformer. It is represented by *K*.

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} \qquad (\text{since } E_2 \propto N_2 \text{ and } E_1 \propto N_1) \qquad 2.1$$

2.5. CONSTRUCTION OF TRANSFORMER

The following are the major elements of a transformer:

- Magnetic circuit mainly comprises of transformer core having limbs and yokes.
- Electric circuits mainly comprises of windings, insulation and bushings.
- Tank mainly comprises of cooling devices, conservator and ancillary apparatus.

The construction of a transformer depends largely on its size and the duty which it is to perform.

2.6. AN IDEAL TRANSFORMER

To understand the theory, operation and applications of a transformer, it is better to view a transformer first as an ideal device. For this, the following assumptions are made:

- ✓ Its coefficient of coupling (k) is unity.
- ✓ Its primary and secondary windings are pure inductors having infinitely large value.
- ✓ Its leakage flux and leakage inductances are zero.
- ✓ Its self and mutual inductances are zero having no reactance or resistance.
- ✓ Its efficiency is 100 percent having no loss due to resistance, hysteresis or eddy current.
- ✓ Its transformation ratio (or turn ratio) is equal to the ratio of its secondary to primary terminal voltage and also as the ratio of its primary to secondary current.

- ✓ Its core has permeability (μ_r) of infinite value.

Thus, a transformer is said to be an ideal one, if it has no ohmic resistances, moreover whole of the flux set-up in the core is considered to be linking with its primary and secondary turns, i.e., it carries no leakage flux. In other words, a transformer is said to be ideal one, when it has no losses, i.e., copper or iron losses. In actual practice, such a transformer cannot exist, but to begin with it may be considered.

For an ideal transformer, the output must be equal to input (since it has no losses), therefore.

i.e., $$E_2\, I_2 \cos\phi = E_1\, I_1 \cos\phi \quad \text{or} \quad E_2\, I_2 = E_1\, I_1 \quad \text{or} \quad \frac{E_2}{E_1} = \frac{I_1}{I_2}$$

since $$E_2 \propto N_2;\ E_1 \propto N_1 \text{ and } E_1 \cong V_1;\ E_2 \cong V_2$$

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K \qquad 2.2$$

2.6.1. BEHAVIOUR AND PHASOR DIAGRAM

Consider an ideal transformer whose secondary is open, as shown in figure.II.4 (a)

When its primary winding is connected to sinusoidal alternating voltage V1, a current Imag flows through it. Since the primary coil is pure inductive, the current Imag lags behind the applied voltage V1 by 90°. This current sets up alternating flux (or mutual flux φm) in the core and magnetizes it.

Hence it is called magnetizing current. Flux is in phase with Imag as shown in the phasor diagram and wave diagram in figures.II.4 (b), and (c), respectively.

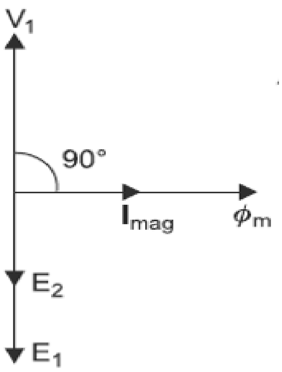


Figure.a.Phasor Diagram

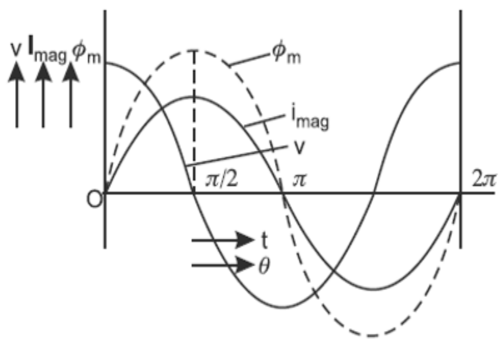


Figure.b.Wave Diagram

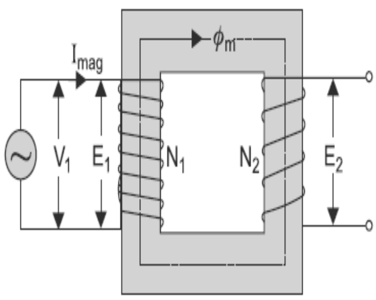


Figure.c.Circuit Diagram

Figure.2.4. Ideal transformer

The alternating flux links with both primary and secondary windings. When it links with primary, it produces self-induced emf E1 in opposite direction to that of applied voltage V2. When it links with secondary winding, it produces mutually-induced emf E2 in opposite direction to that of applied voltage. Both the emfs. E1 and E2 are shown in phasor diagram in figure.II.4 (a).

2.6.2.Emf Equation

When sinusoidal voltage is applied to the primary winding of a transformer, a sinusoidal flux, as shown in figure.II.5, is set up in the iron core which links with primary and secondary winding.

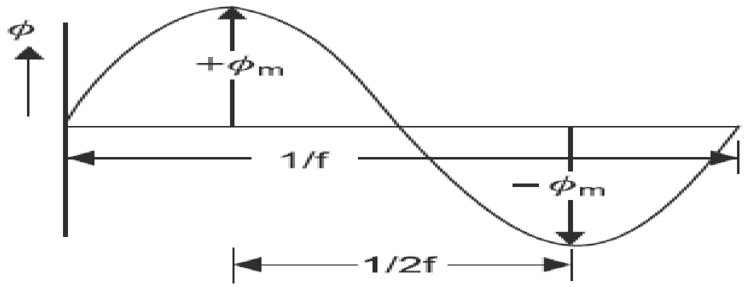


Figure.2.5.Wave diagram of flux

Let, φm = Maximum value of flux in Wb;

f = supply frequency in Hz (or 1/s);

$N1$ = No. of turns in primary;

$N2$ = No. of turns in secondary.

As shown in Figure.II.5, flux changes from + φm to – φm in half a cycle i.e., 1/2 f second,

Average rate of change of flux = $\frac{\emptyset_m - (-\emptyset_m)}{\frac{1}{2} \times f}$ = $4\times f \times \emptyset_m \quad Wb/s$

Now, the rate of change of flux per turn is the average induced emf per turn in volt.

Average emf induced per turn = $4\times f \times \emptyset_m$ volt

For a sinusoidal wave, $\frac{R.M.S.value}{Average\ value}$ = Form factor = 1.11

R.M.S.value of emf induced / turn, E = $1.11\times 4f\emptyset_m = 4.44 \times f \times \emptyset_m$ volt

Since primary and secondary have N_1 and N_2 turns, respectively.

R.M.S.value of emf induced in primary,

$$\mathbf{E_1 = 4.44 \times N_1 \times f \times \emptyset_m \text{ volt}} \quad 2.3$$

Similarly, r.m.s.value of emf induced in secondary,

$$\mathbf{E_2 = 4.44 \times N_2 \times f \times \emptyset_m \text{ volt}} \quad 2.4$$

From eq.(2.3), we get,

$$\mathbf{\frac{E_1}{N_1} = 4.44 \times f \times \emptyset_m \text{ volt / turn}} \quad 2.5$$

From eq.(2.4), we get,

$$\mathbf{\frac{E_2}{N_2} = 4.44 \times f \times \emptyset_m \text{ volt / turn}} \quad 2.6$$

Equations (2.5) and (2.6) are clearly show that emf induced per turn on both the sides i.e, primary and secondary is the same.

Again we can find the voltage ratio, $\frac{E_2}{E_1} = \frac{4.44 \times N_2 \times f \times \emptyset_m}{4.44 \times N_1 \times f \times \emptyset_m} = \frac{N_2}{N_1} = \mathbf{k}$ (transformation ratio)

Equations (2.3) and (2.4) can be written in the form of maximum flux density B_{max} using relation, $\emptyset_m = B_m \times A_i$ (where A_i *is iron area*).

$$\mathrm{E_1} = \mathbf{4.44} \times \boldsymbol{N_1} \times \boldsymbol{f} \times \boldsymbol{B_m} \times \boldsymbol{A_i} \quad \textbf{volt} \qquad 2.7$$

And

$$\mathrm{E_2} = \mathbf{4.44} \times \boldsymbol{N_2} \times \boldsymbol{f} \times \boldsymbol{B_m} \times \boldsymbol{A_i} \quad \textbf{volt} \qquad 2.8$$

- **Example.2.1**

What will be the number of primary and secondary turn of a single-phase 2310/220V, 50 Hz, transformer which has in emf of 13V per turn approximately.

Solution:

$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 13$ V (given); $E_1 = 2310\ V;\ E_2 = 220\ V$

Primary turns, $N_1 = \frac{E_1}{13} = \frac{2310}{13} = 177.69 \cong 178$

Secondary turns, $N_2 = \frac{E_2}{13} = \frac{220}{13} = 16.92 \cong 17$

2.7. Transformer on No-load

A transformer is said to be on no-load when its secondary winding is kept open and no-load is connected across it. As such, no current flows through the secondary i.e., *I2* = 0. Hence, the secondary winding is not causing any effect on the magnetic flux set-up in the core or on the current drawn by the primary. But the losses cannot be ignored.

At no-load, a transformer draws a small current *I0* (usually 2 to 10% of the rated value). This current has to supply the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper loss in the primary (the primary copper losses are so small as compared to core losses that they are generally neglected moreover secondary copper losses are zero as *I*2 is zero).

Therefore, current *Io* lags behind the voltage vector *V*1 by an angle φ0 (called hysteresis angle of advance) which is less than 90, as shown in Fig.2.6 (*b*). The angle of lag depends upon the losses in the transformer. The no-load current *I*0 has two components; One, *Iw* in phase with the applied voltage *V*1, called active or working component. It supplies the iron losses and a small primary copper losses. The other, *Imag* in quadrature with the applied voltage *V*1, called reactive of magnetizing component. It produces flux in the core and does not consume any power. From phasor (vector) diagram shown in Fig. 2.6. (b)

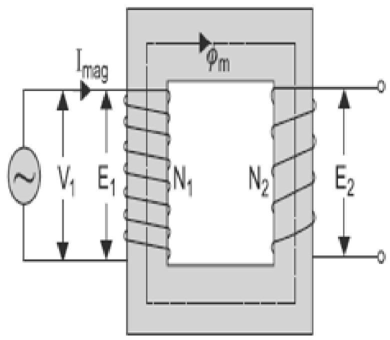


Figure.a.Circuit Diagram

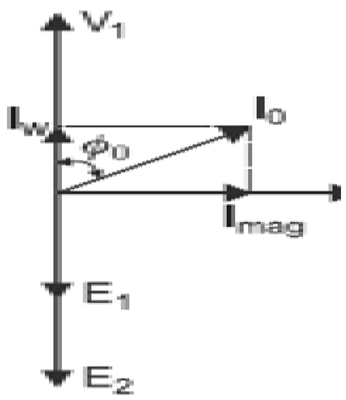


Figure.b.Phasor Diagram

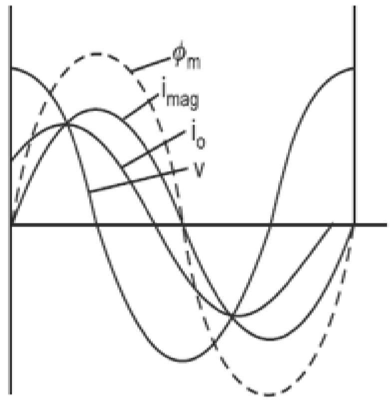


Figure.c.Wave Diagram

Figure.2.6. Transformer on no-load

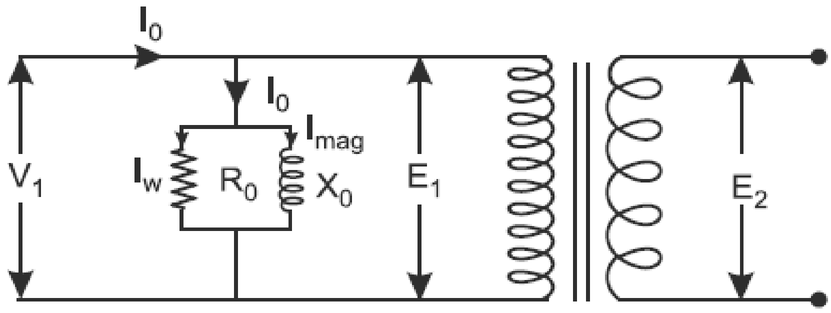


Figure.2.7.Equivalent circuit

Working component, $I_w = I_0 \cos \phi_0$

Magnetising component, $I_{mag} = I_0 \sin \phi_0$

No-load current, $I_0 = \sqrt{I_w^2 + I_{mag}^2}$ 2.9

Primary p.f. at no-load, $\cos \phi_0 = \frac{I_w}{I_0}$ 2.10

No-load power input, $P_0 = V_1 I_0 \cos \phi_0$ 2.11

Exciting resistance, $R_0 = \frac{V_1}{I_w}$ 2.12

Exciting reactance, $X_0 = \frac{V_1}{I_{mag}}$ 2.13

The equivalent circuit of a transformer at no-load is shown in Fig.2.7.

Here, *R*0 represents the exciting resistance of the transformer which carries power loss component of no-load current, i.e., *Iw* used to meet with the no-load losses in the transformer, whereas *X*0 represents the exciting reactance of the transformer which carries wattless component of no-load current, i.e., *I*mag used to set-up magnetic field in the core.

Example.2.2

A 230/110 V single-phase transformer has a core loss of 100 W. If the input under no-load condition is 400 VA, find core loss current, magnetizing current and no-load power factor angle.

Solution:

Here, $V_1 = 230$ V; $V_2 = 110$ V; $P_i = 100$ W

Input at no-load = 400 VA

i.e., $V_1 I_0 = 400$

or No-load current, $I_0 = \frac{400}{230} = 1{\cdot}739$ A

Core loss current, $I_w = \frac{P_i}{V_1} = \frac{100}{230} = \mathbf{0{\cdot}4348\ A}$ (*Ans*)

Magnetising current, $I_{mag} = \sqrt{I_0^2 - I_w^2} = \sqrt{(1{\cdot}739)^2 - (0{\cdot}4348)^2} = \mathbf{1{\cdot}684\ A}$ (*Ans*)

No-load power factor, $\cos\phi_0 = \frac{I_w}{I_0} = \frac{0{\cdot}4348}{1{\cdot}739} = \mathbf{0{\cdot}25}$ ***lag*** (*Ans*)

No-load power factor angle, $\phi_0 = \cos^{-1} 0{\cdot}25 = \mathbf{75{\cdot}52^\circ}$ (*Ans*)

2.8. Transformer on Load

(Neglecting winding resistance and leakage flux)

When a certain load is connected across the secondary, a current I2 flows through it as shown in Fig.2.8 The magnitude of current I2 depends upon terminal voltage V2 and impedance of the load.

The phase angle of secondary current I2 with respect to V2 depends upon the nature of load i.e., whether the load is resistive, inductive or capacitive.

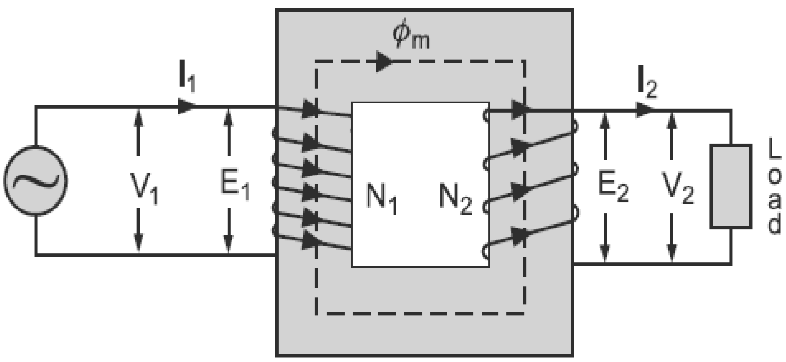


Figure.2.8.Circuit diagram for loaded transformer

The operation of the transformer on load is explained below with the help of number of diagrams; when the transformer is on no-load as shown in Figure.2.9 it draws no-load current *I*0 from the supply mains. The no-load current *I*0 produces an mmf *N*1. *I*0 which sets up flux in the core. When the transformer is loaded, current I2 flows in the secondary winding. This secondary current I2 produces an mmf N2. I2 which sets up flux $\boldsymbol{\varphi}_2$ in the core. As per Lenz's law this flux opposes the main flux $\boldsymbol{\varphi}$ as shown in Figure.2.10.

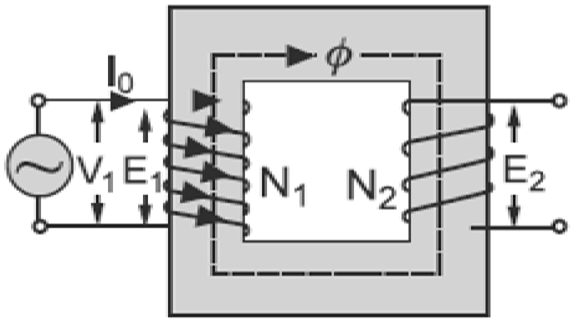


Figure.2.9.transformer on no-load

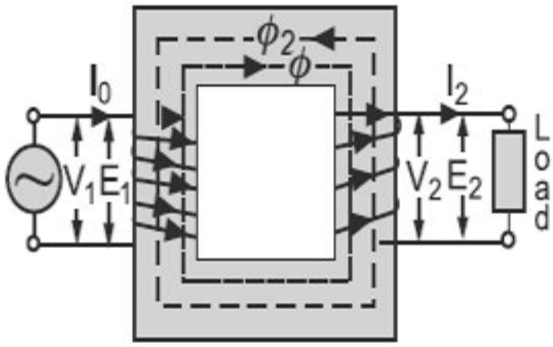


Figure.2.10.transformer on load

As φ_2 is set-up in opposite direction to the main flux, the resultant flux tends to decrease and causes the reduction of self-induced emf E_1 momentarily. Thus, V_1 predominates over E_1 causing additional primary current I'_1 drawn from the supply mains. The amount of this additional current I_1' is such that the original conditions i.e., flux in the core must be restored. so that $V_1 = E_2$. The current I_1 is in phase opposition with I_2 and is called primary counter balancing current. This additional current I_1' produces an mmf N_1. I_1' which sets up flux φ, in the same direction as that of φ as shown in Figure.2.11.a, and cancels the flux φ_2 set up by mmf N_2. I_2.

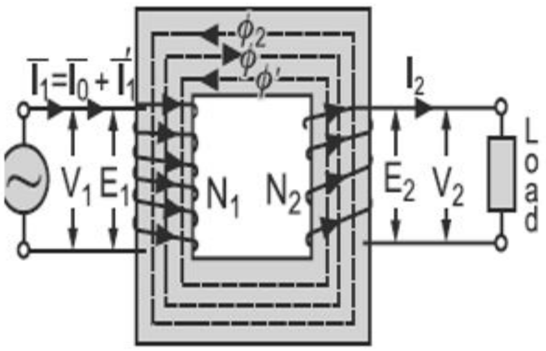


Figure.2.11.a.transformer on no-load

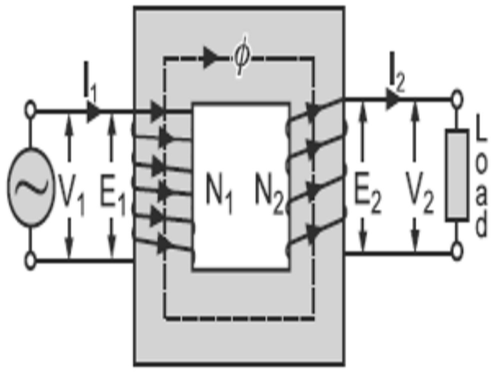


Figure.2.11.b.transformer on load

Figure.2.11.Transformer action when load is applied to it.

Now $\quad N_1 I_1' = N_2 I_2$ (ampere-turns balance)

$$\therefore \quad I_1' = \frac{N_2}{N_1} I_2 = K I_2$$

Thus, the flux is restored to its original value as shown in Figure.2.11.b. The total primary current I_1 is the vector sum of current I_0 and I_1' i.e., $I_1 = I_0 + I_1$'.

This shows that flux in the core of a transformer remains the same from no-load to full-load; this is the reason why iron losses in a transformer remain the same from no-load to full-load.

2.8.1.Phasor Diagram of a Loaded Transformer

Neglecting voltage drops in the windings; ampere-turns balance)

Since the voltage drops in both the windings of the transformer are neglected, therefore, V1 = E1 and E2 = V2, While drawing the phasor diagram the following important points are to be considered:

For simplicity, let the transformation ratio K = l be considered, therefore, E1 = E2.

The secondary current I2 is in phase, lags behind and leads the secondary terminal voltage V2 by an angle $\varphi 2$ for resistive, inductive and capacitive load, respectively.

The counter balancing current I1’ = $\frac{N_2}{N_1}.I_2$ (i.e., I1’= K .I2 here K = 1; I1’= I2) and is 180 out of phase with I2.

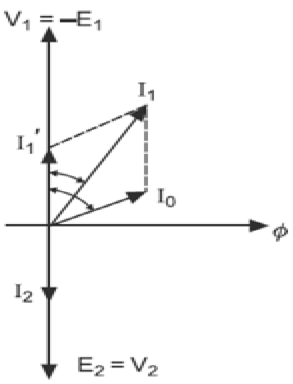


Figure.a. For unity p.f.

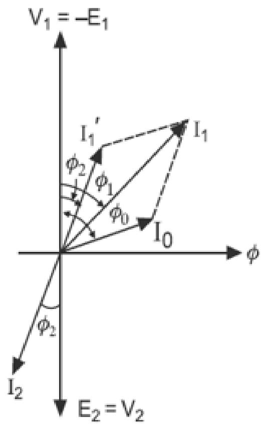


Figure.b. For lagging p.f.

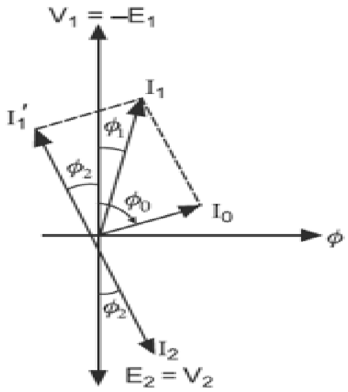


Figure.c. For leading p.f.

Figure.2.12.Phasor diagram on-load (neglecting winding resistance and leakage reactance)

The total primary current I1 is the vector sum of no-load primary current I0 and counter balancing current I1’.

$$\overline{I_1} = \overline{I_0} + \overline{I_1'} \text{ or } \overline{I_1} = \sqrt{(I_0)^2 + (I_1')^2 + 2I_0 I_1' \cos\theta} \qquad 2.14$$

Where θ is the phase angle between $I0$ and $I1$’.

The p.f. on the primary side is cos Ø1 which is less than the load p.f. cos Ø2 on the secondary side. Its value is determined by the relation;

$$\cos\phi_1 = \frac{I_0 \cos\phi_0 + I_1' \cos\phi_2}{I_1} \qquad 2.15$$

The phasor diagrams of the transformer for resistance, inductive and capacitive loads are shown in Figs. 2.12(*a*), (*b*) and (*c*), respectively.

Alternately

The primary current *I*1 can also be determined by resolving the vectors, i.e.,

$$I_v = I_0 \cos\phi_0 + I'_1 \cos\phi_2 \qquad [\text{where } \sin\phi_0 = \sin\cos^{-1}(\cos\phi_0)$$ 2.16

$$I_H = I_0 \sin\phi_0 + I'_1 \sin\phi_2 \qquad \text{and } \sin\phi_2 = \sin\cos^{-1}(\cos\phi_2)]$$ 2.17

$$I_1 = \sqrt{(I_v)^2 + (I_H)^2}$$ 2.18

2.9. Actual Transformer

An actual transformer has (i) Primary and secondary resistances R1 and R2, (ii) primary and secondary leakage reactances X1 and X2 (iii) iron and copper losses and (iv) exciting resistance R0 and exciting reactance X0. The equivalent circuit of an actual transformer is shown in Fig. 2.13.

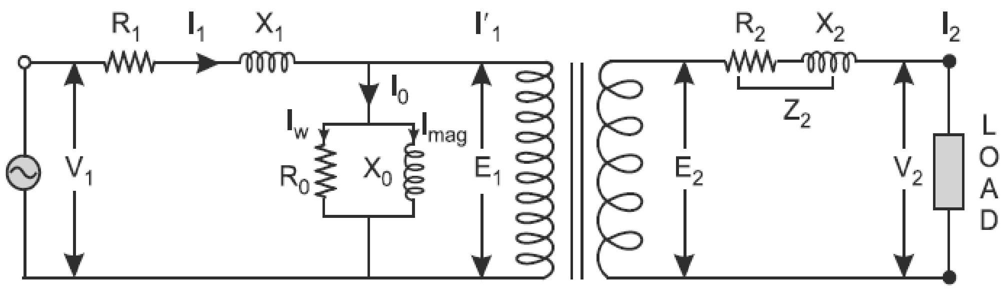


Figure.2.13.Equivalent circuit for loaded transformer

Primary impedance,

$$\overline{Z_1} = R_1 + jX_1$$ 2.19

Supply voltage is V2.

The resistance and leakage reactance of primary winding are responsible for some voltage drop in primary winding.

$$\overline{V_1} = \overline{E_1} + \overline{I_1}(R_1 + jX_1) = \overline{E_1} + \overline{I_1}\,\overline{Z_1}$$ 2.20

where, $\overline{I}_1 = \overline{I'}_1 + \overline{I}_0$

Secondary impedance,

$$\overline{Z_2} = R_2 + jX_2$$ 2.21

Similarly, the resistance and leakage reactance of secondary winding are responsible for some voltage drop in secondary winding. Hence,

$$\overline{V_2} = \overline{E_2} - \overline{I_2}(R_2 + jX_2) = \overline{E_2} - \overline{I_2}\,\overline{Z_2}$$ 2.22

The phasor (vector) diagrams of an actual transformer for resistive, inductive and capacitive loads are shown in Figs.2.14 (*a*), (*b*) and (*c*), respectively. The drops in resistances are drawn in phase with current vectors and drops in reactances are drawn perpendicular to the current vectors.

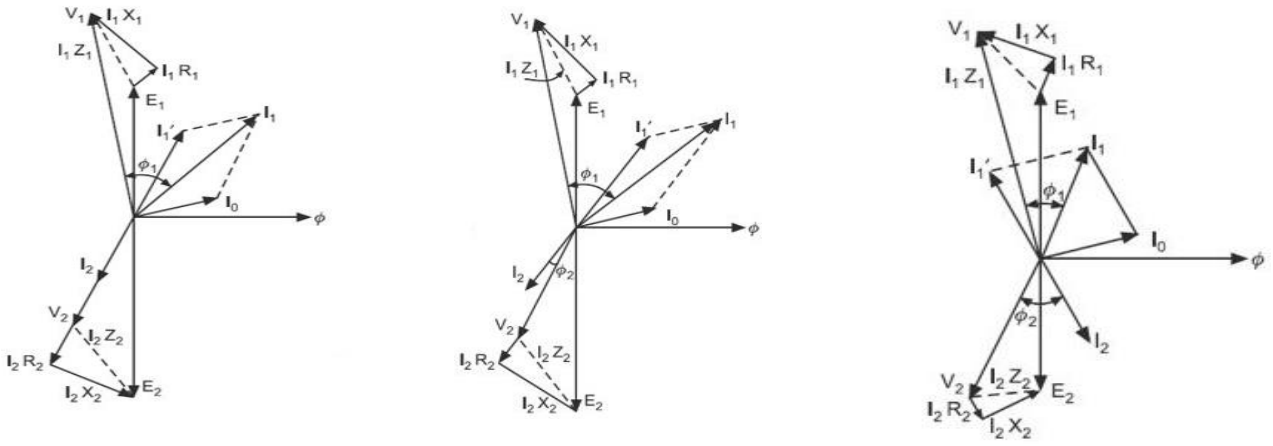


Figure.a. For unity p.f. **Figure.b. For lagging p.f.** **Figure.c. For leading p.f.**

Figure.2.14.Complete phasor diagram for loaded transformer

2.10. Simplified Equivalent Circuit

While drawing simplified circuit of a transformer, the exciting circuit (i.e., exciting resistance and exciting reactance) can be omitted. The simplified equivalent circuit of a transformer is drawn by representing all the parameters of the transformer either on the secondary or on the primary side. The no-load current $I0$ is neglected as its value is very small as compared to full load current, therefore, $I1' = I1$.

- **Equivalent circuit when all the quantities are referred to secondary.**

The primary resistance when referred to secondary side, its value is $R1' = K^2 .R1$ and the total or equivalent resistance of transformer referred to secondary, $Res = R2 + R1'$.

Similarly, the primary reactance when referred to secondary side, its value is $X1' = K^2.X1$ and the total or equivalent reactance of transformer referred to secondary, $Xes = X2 + X1'$.

All the quantities when referred to the secondary side are shown in Fig. 2.15.

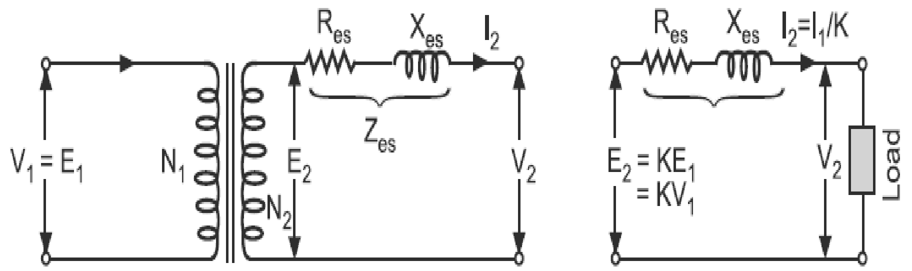


Figure.2.15.Simplified equivalent circuit referred to secondary side

Total or equivalent impedance referred to secondary side,

$$Z_{es} = R_{es} + j\,X_{es} \qquad 2.23$$

There is some voltage drop in resistance and reactance of transformer

$$\overline{V_2} = \overline{E_2} - \overline{I_2}\left(R_{es} + j\,X_{es}\right) = \overline{E_2} - \overline{I_2 Z_{es}} \qquad 2.24$$

Phasor Diagrams: The phasor (vector) diagrams of a loaded transformer when all the quantities are referred to secondary side for resistive, inductive and capacitive loads are shown in Figs.2.16 (*a*), (*b*) and (*c*), respectively.

The voltage drops in resistances (vectors) are taken parallel to the current vector and the voltage drops in reactances (vectors) are taken quadrature to the current vector.

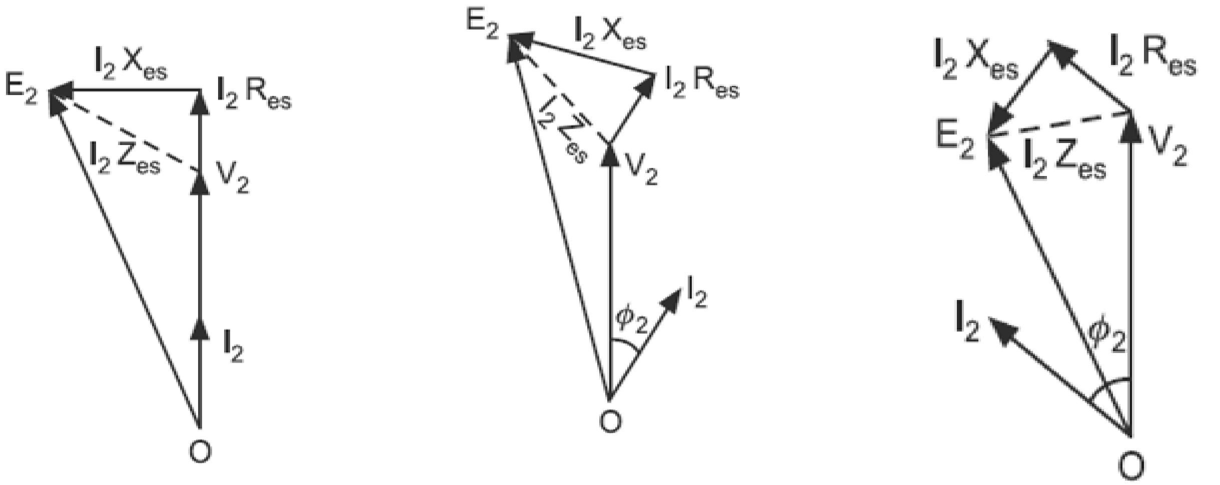


Figure.a. For unity p.f. **Figure.b. For lagging p.f.** **Figure.c. For leading p.f.**

Figure.2.16.Phasor diagram

- **Equivalent circuit when all the quantities are referred to primary.**

In this case, to draw the equivalent circuit all the quantities are to be referred to primary, as shown in Fig. 2.17.

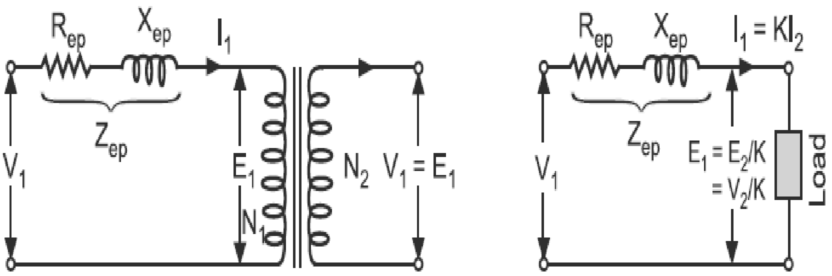


Figure.2.17.Simplified equivalent circuit referred to primary.

Secondary resistance referred to primary, $R_2' = R_2/K^2$

Equivalent resistance referred to primary, $R_{ep} = R_1 + R_2'$

Secondary reactance referred to primary, $X_2' = X_1/K^2$

Equivalent reactance referred to primary, $X_{ep} = X_1 + X_2'$

Total or equivalent impedance referred to primary side,

$$Z_{ep} = R_{ep} + j\,X_{ep} \qquad 2.25$$

There is some voltage drop in resistance and reactance of the transformer referred to primary side.

$$\overline{V_1} = \overline{E_1} + \overline{I_1}\left(R_{ep} + j\,X_{ep}\right) = \overline{E_1} + \overline{I_1}\,\overline{Z_{ep}} \qquad 2.26$$

Phasor diagrams: The phasor diagram to transformer when all the quantities are referred to primary side for different types of loads are shown in Fig. 2.18.

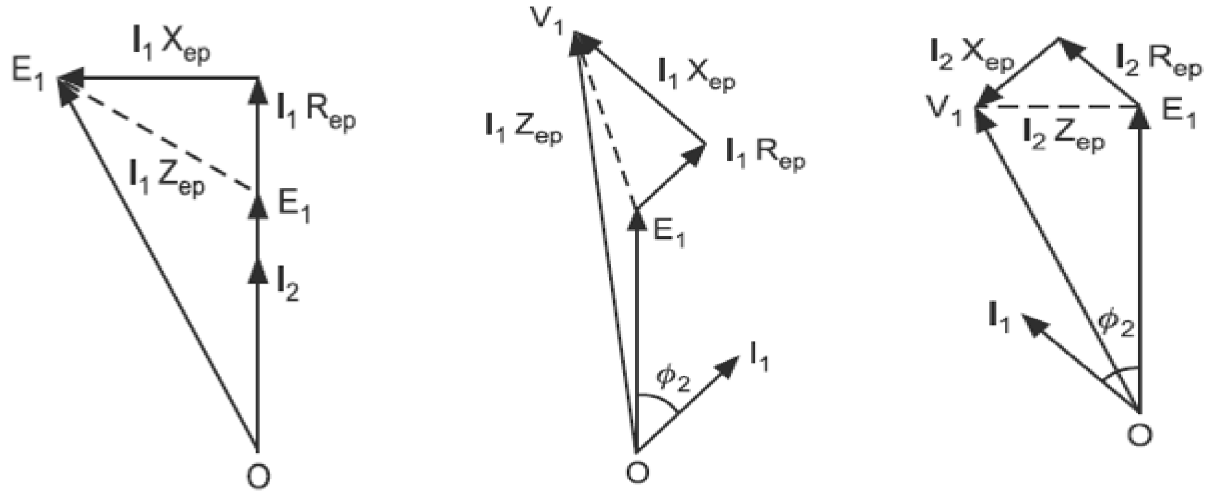


Figure.a. For unity p.f. **Figure.b. For lagging p.f.** **Figure.c. For leading p.f.**

Figure.2.18.Phasor diagram

Example 2.4

The primary winding resistance and reactance of a 10 kVA, 2000/400 V, single phase transformer is 5.5 ohm and 12 ohm, respectively, the corresponding values for secondary are 0.2 ohm and 0.45 ohm. Determine the value of the secondary voltage at full load, 0.8 p.f. lagging, when the primary supply voltage is 2000 V.

Solution:

Transformer rating $= 10$ kVA $= 10 \times 10^3$ VA

Primary induced voltage, $E_1 = 2000$ V ;

Secondary induced voltage, $E_2 = 400$ V

Primary resistance, $R_1 = 5.5$ ohm;

Primary reactance, $X_1 = 12$ ohm

Secondary resistance, $R_2 = 0.2$ ohm; Secondary reactance, $X_2 = 0.45$ ohm

Load p.f., $\cos\phi_2 = 0.8$ lagging

Transformation ratio, $K = \dfrac{E_2}{E_1} = \dfrac{400}{2000} = 0.2$

Primary resistance referred to secondary side,

$$R_1' = K^2 R_1 = (0.2)^2 \times 5.5 = 0.22\ \Omega$$

Equivalent resistance referred to secondary side,

$$R_{es} = R_2 + R_1' = 0.2 + 0.22 = 0.42\ \Omega$$

Primary reactance referred to secondary side,

$$X_1' = K^2 X_1 = (0.2)^2 \times 12 = 0.48\ \Omega$$

Equivalent reactance referred to secondary side,

$$X_{es} = X_2 + X_1' = 0.45 + 0.48 = 0.93\ \Omega$$

Load p.f., $\cos\phi_2 = 0.8$

$\therefore$ $\sin\phi_2 = \sin\cos^{-1} 0.8 = 0.6$

Full load secondary current, $I_2 = \dfrac{10 \times 10^3}{400} = 25\ A$

As the primary supply voltage, $V_1 = E_1 = 2000\ V$

Secondary induced voltage, $E_2 = KE_1 = 0.2 \times 2000 = 400\ V$

Using the expression; $E_2 = V_2 + I_2 R_{es}\cos\phi_2 + I_2 X_{es}\sin\phi_2$

Secondary terminal voltage, $V_2 = E_2 - I_2 R_{es}\cos\phi_2 - I_2 X_{es}\sin\phi_2$

$$= 400 - 25 \times 0.42 \times 0.8 - 25 \times 0.93 \times 0.6$$

$$= 400 - 8.4 - 13.95 = \mathbf{377.65\ V}\ (Ans.)$$

2.11. Approximate Expression for Voltage Regulation

The approximate expression for the no-load secondary voltage is:
For inductive load:

$$E_2 = V_2 + I_2 R_{es}\cos\phi_2 + I_2 X_{es}\sin\phi_2$$

$$E_2 - V_2 = I_2 R_{es}\cos\phi_2 + I_2 X_{es}\sin\phi_2 \qquad 2.27$$

2.12. Losses in a Transformer

The losses which occur in an actual transformer are: Core or iron losses and Copper losses

- ✓ **Core or iron losses**
  - Hysteresis loss: $Ph = Kh \cdot V \cdot f \cdot Bm^{2,6}$

This loss can be minimized by using silicon steel material for the construction of core.

  - Eddy current loss: $Pe = Ke \cdot V \cdot f^2 \cdot t^2 \cdot Bm^2$

This loss can be minimized by making the core of thin laminations.

Hence, iron loss is independent of the load and is known as constant losses.

- ✓ **Copper losses:**

Copper losses occur in both the primary and secondary windings due to their ohmic resistance.

Total copper losses $= R_1 . I_1^2 + R_2 . I_2^2 = I_1^2 . R_{ep} = I_2^2 . R_{es} \qquad 2.28$

The currents in the primary and secondary winding vary according to the load, therefore, these losses vary according to the load and are known as variable loss.

2.13. Efficiency of a Transformer

The efficiency of a transformer is defined as the ratio of output to the input power.

$$\text{Transformer efficiency, } \eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power + losses}}$$

or

$$\eta = \frac{\text{output power}}{\text{output power + iron losses + coper losses}}$$

$$\eta = \frac{V_2\, I_2 \cos\phi_2}{V_2\, I_2 \cos\phi_2 + P_i + P_c} \qquad 2.29$$

where, V_2 = Secondary terminal voltage

I_2 = Full load secondary current

$\cos\phi_2$ = *p.f.* of the load

P_i = Iron losses = Hysteresis losses + eddy current losses (constant losses)

P_c = Full load copper losses = $I^2_2 R_{es}$ (variable losses)

2.13.1. Condition for Maximum Efficiency

Thus, the efficiency of a transformer will be maximum when copper (or variable) losses are equal to iron (or constant) losses.

Copper losses = Iron losses

$$\eta_{max} = \frac{V_2\, I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + 2\, P_i} \qquad [since\ P_c = P_i] \qquad 2.30$$

2.14. Transformer Tests

All the transformers are tested before placing them in the field.

By performing these tests, we can determine the parameters of a transformer to compute its performance characteristics (like voltage regulation and efficiency etc.).

Therefore, to furnish the required information open circuit and short circuit tests are conducted conveniently without actually loading the transformer.

2.14.1. Open-circuit or No-load Test

This test is carried out at rated voltage to determine the no-load loss or core loss or iron loss.

It is also used to determine no-load current $I0$ which is helpful in finding the no-load parameters i.e., exciting resistance $R0$ and exciting reactance $X0$ of the transformer.

Usually, this test is performed on low voltage side of the transformer, i.e., all the measuring instruments such as voltage (*V*), wattmeter (*W*) and ammeter (*A)* are connected in low-voltage side (say primary).

The primary winding is then connected to the normal rated voltage $V1$ and the secondary side is kept open or connected to a voltmeter *V'* as shown in Fig.(*a*).

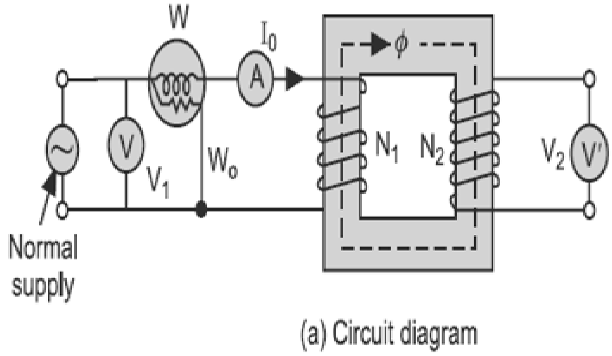


Figure.a.Circuit diagram

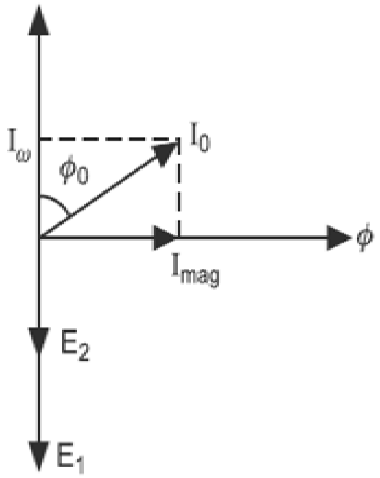


Figure.b.Phasor diagram

Figure.2.19.Open circuit test

The current drawn by the primary is called no-load current $I0$ measured by the ammeter *A*. The value of no-load current $I0$ is very small usually 2 to 10% of the rated full-load current. Thus, the copper loss in the primary is negligibly small and no copper loss occurs in the secondary as it is open.

Therefore, wattmeter reading $W0$ only represents the core or iron losses for all practical purposes. These core losses are constant at all loads. The voltmeter V' if connected on the secondary side measures the secondary induced voltage V2. The ratio of voltmeter readings $\frac{V_2}{V_1}$ gives the transformation ratio of the transformer. The phasor diagram of transformer at no-load is shown in Fig.(*b*).

Let the wattmeter reading = W_0

voltmeter reading = V_1

and ammeter reading = I_0

Then, iron losses of the transformer $P_i = W_0$

i.e., $V_1 I_0 \cos\phi_0 = W_0$

$\therefore$ No-load power factor, $\cos\phi_0 = \dfrac{W_0}{V_1 I_0}$

Working component, $I_w = \dfrac{W_0}{V_1}$ $(\because I_w = I_0 \cos\phi_0)$

Magnetising component $I_{mag} = \sqrt{I_0^2 - I_w^2}$

No-load, parameters, i.e.,

Equivalent exciting resistance, $R_0 = \dfrac{V_1}{I_w}$

Equivalent exciting reactance, $X_0 = \dfrac{V_1}{I_{mag}}$

The Iron losses measured by this test are used to determine transformer efficiency and parameters of exciting circuit of a transformer shown in Fig.2.20.

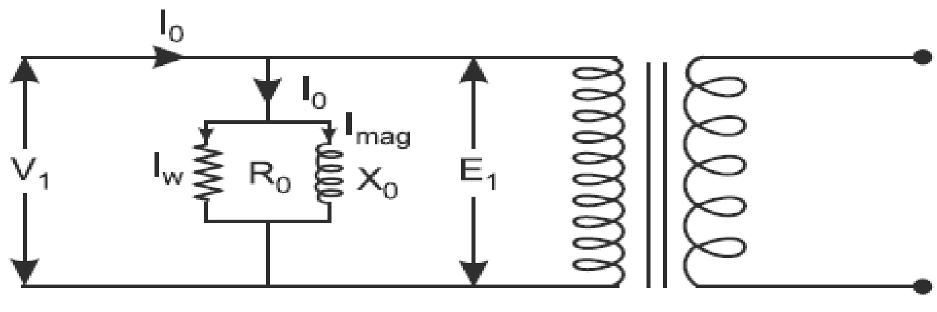


Figure.2.20.Equivalent circuit of a transformer at no-load

2.14.2. Short Circuit Test

This test is carried out to determine the following:

- ✓ Copper losses at full load (or at any desired load). These losses are required for the calculations of efficiency of the transformer.
- ✓ Equivalent impedance (*Zes* or *Zep*), resistance (*Res* or *Rep*) and leakage reactance (*Xes* or *Xep*) of the transformer referred to the winding in which the measuring instruments are connected.
- ✓ Knowing equivalent resistance and reactance, the voltage drop in the transformer can be calculated.

This test is usually carried out on the high-voltage side of the transformer i.e., a wattmeter W, voltmeter V and an ammeter A are connected in high-voltage winding (say secondary). The other winding (primary) is then short circuited by a thick strip or by connecting an ammeter **A'** across the terminals as shown in Fig. 2.21. A low voltage at normal frequency is applied to the high voltage winding with the help of on autotransformer so that full-load current flows in both the windings, measured by ammeters A and A'. Low voltage is essential, failing which an excessive current will flow in both the windings which may damage them.

Since a low voltage (usually 5 to 10% of normal rated voltage) is applied to the transformer winding, therefore, the flux set up in the core is very small about 1/30th to 1/8th of normal flux. The iron losses are negligibly small due to low value of flux as these losses are approximately proportional to the square of the flux. Hence, wattmeter reading *Wc* only represents the copper losses in the transformer windings for all practical purposes. The applied voltage *V2sc* is measured by the voltmeter *V* which circulates the current *I2sc* (usually full load current) in the impedance *Zes* of the transformer to the side in which instruments are connected as shown in Fig. 2.21.

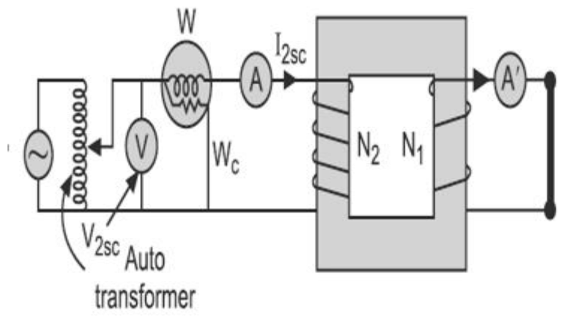


Figure.a.Circuit diagram

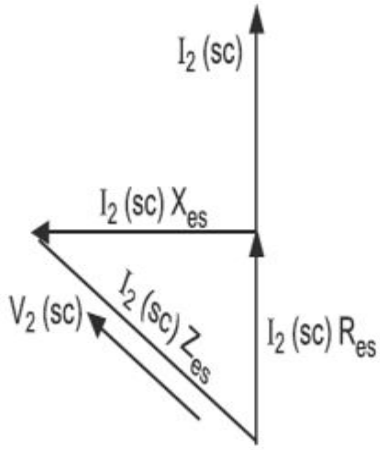


Figure.b.Phasor diagram

Figure.2.21.Schort circuit test

Let the wattmeter reading $= W_c$

voltmeter reading $= V_{2sc}$

and ammeter reading $= I_{2sc}$

Then, full load copper losses of the transformer,

$$P_c = \left(\frac{I_{2fl}}{I_{2sc}}\right)^2 Wc \tag{2.31}$$

$$I_{2sc}^2 R_{es} = W_C \tag{2.32}$$

Equivalent resistance referred to secondary

$$R_{es} = \frac{W_c}{I_{2sc}^2} \tag{2.33}$$

From phasor diagram as shown in fig.2.21.b

$$I_{2sc} Z_{es} = V_{2sc}$$

Equivalent impedance referred to secondary

$$Z_{es} = V_{2sc}/I_{2sc} \tag{2.34}$$

Equivalent reactance referred to secondary

$$X_{es} = \sqrt{(Z_{es})^2 - (R_{es})^2} \tag{2.35}$$

Example.2.5

A 15 kVA, 440/230 V, 50 Hz, single phase transformer gave the following test results:

Open Circuit (LV side) 250 V, 1.8A, 95 W.

Short Circuit Test (HV side) 80 V, 12.0 A, 380 W.

Compute the parameters of the equivalent circuit referred to LV side.

Solution:

Transformer rating = 15 kVA; $E_1 = 440$ V; $E_2 = 230$ V; $f = 50$ Hz

Open circuit test (LV side); $V_2 = 250$ V; $I_0 = 1.8$ A; $W_0 = 95$ W

Short circuit test (HV side); $V_{1(sc)} = 80$ V; $I_{1(sc)} = 12$ A; $W_c = 380$ W

From open circuit test performed on LV side;

$$I_w = \frac{W_0}{V_2} = \frac{95}{250} = 0.38 \text{ A}$$

$$I_{mag} = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.8)^2 - (0.38)^2} = 1.75943 \text{ A}$$

$$\text{Exciting resistance, } R_0 = \frac{V_2}{I_w} = \frac{250}{0.38} = 658\ \Omega$$

$$\text{Exciting reactance, } X_0 = \frac{V_2}{I_{mag}} = \frac{250}{1.75943} = 142\ \Omega$$

From short circuit test performed on HV side;

$$Z_{ep} = \frac{V_{1(sc)}}{I_{1(sc)}} = \frac{80}{12} = 6.667\ \Omega$$

$$R_{ep} = \frac{W_c}{(I_{1(sc)})^2} = \frac{380}{(12)^2} = 2.639\ \Omega$$

$$X_{ep} = \sqrt{Z_{ep}^2 - R_{ep}^2} = \sqrt{(6.667)^2 - (2.639)^2} = 6.122\ \Omega$$

$$\text{Transformation ratio, } K = \frac{E_2}{E_1} = \frac{230}{440} = 0.5227$$

Transformer resistance and reactance referred to LV (secondary) side;

$$R_{es} = R_{ep} \times K^2 = 2.639 \times (0.5227)^2 = 0.7211\ \Omega$$

$$X_{es} = X_{ep} \times K^2 = 6.122 \times (0.5227)^2 = 2.673\ \Omega$$

Chapter.3.DC Machines

Chapter.3. Direct Current Machines

3.1. Chapter Objectives

After the completion of this unit, students will be able to understand:

- What is a DC motor and how does it convert electrical energy into mechanical energy?
- What is back emf and its significance?
- What are the factors on which torque developed in a DC motor depends?
- How DC motors are classified on the basis of their field excitation?
- What are the performance characteristics of DC motors and how are they selected for different applications?
- How is a four point starter for DC shunt motor different than a three point starter?
- Which type of starter is used for starting of DC series motors?
- What are the factors on which the speed of a DC motor depends and how can we control the speed of DC motors as per the requirement?
- What are the various losses of a DC machine, how these are categorised?
- How to determine various losses of a DC machine with the help of different tests?
- How to calculate the efficiency of a DC machine once the losses are determined?

3.2. Introduction

A machine that converts mechanical power into DC electrical power is called a DC generator. The same machine when used to convert DC electrical power into mechanical power, it known as a DC motor. From construction point of view there is no difference between a DC generator and motor. The DC motors are very useful where wide range of speeds and perfect speed regulation is required such as electric traction. An electro-mechanical energy conversion device (electrical machine) that converts DC electrical energy or power (EI) into mechanical energy or power (ω.T) is called a DC motor. The block diagram of energy conversion, when the electro-mechanical device works as a motor, is shown in Fig. 5.1.

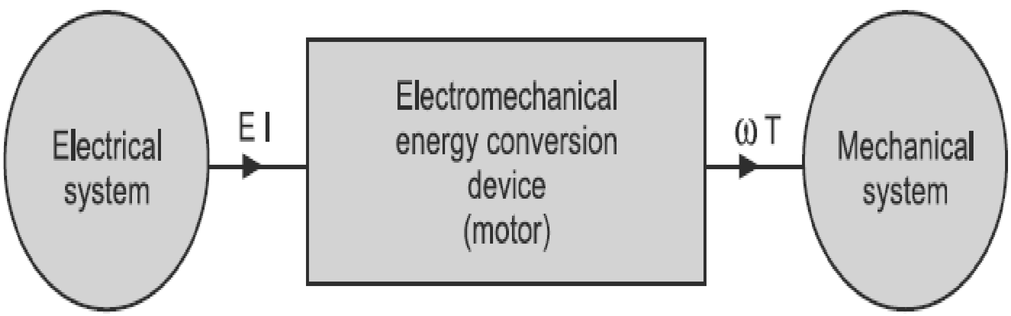


Figure.3.1.Block diagram of electromagnetic energy conversion (motor action)

3.3. Working Principle of DC Motors

The operation of a DC motor is based on the principle that when a current carrying conductor is placed in a magnetic field, a mechanical force is experienced by it. The direction of this force is determined by *Fleming's Left Hand Rule* and its magnitude is given by the relation: ***F = Bil* newton**

For simplicity, consider only one coil of the armature placed in the magnetic field produced by a bipolar machine [see Fig. 5.2(*a*)]. When DC supply is connected to the coil, current flows through it which sets up its own field as shown in Fig. 5.2(*b*). By the interaction of the two fields (i.e., field produced by the main poles and the coil), a resultant field is set up as shown in Fig. 5.2(*c*). The tendency of this is to come to its original position i.e., in straight line due to which force is exerted on the two coil sides and torque develops which rotates the coil.

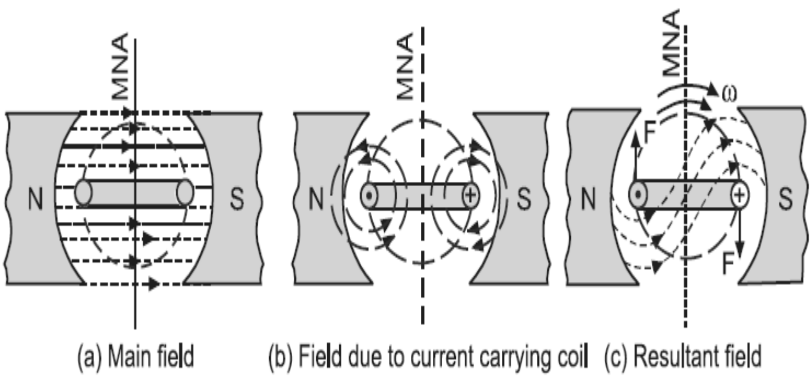


Figure.3.2.Working principle of a motor

Alternately, it can be said that the main poles produce a field *Fm*. Its direction is marked in Fig. 5.3.

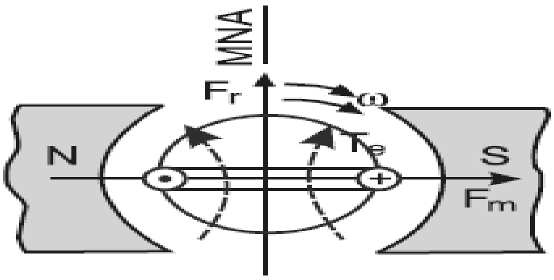


Figure.3.3.Position of main field F_m and rotor field F_r

When current is supplied to the coil (armature conductors), it produces its own field marked as *Fr*. This field tries to come in line with the main field and an electromagnetic torque develops in clockwise direction as marked in Fig. 5.3. In actual machine, a large number of conductors are placed on the armature. All the conductors, placed under the influence of one pole (say, North pole) carry the current in one direction (outward). Whereas, the other conductors placed under the influence of other pole i.e., south pole, carry the current in opposite direction as shown in Fig. 5.4.

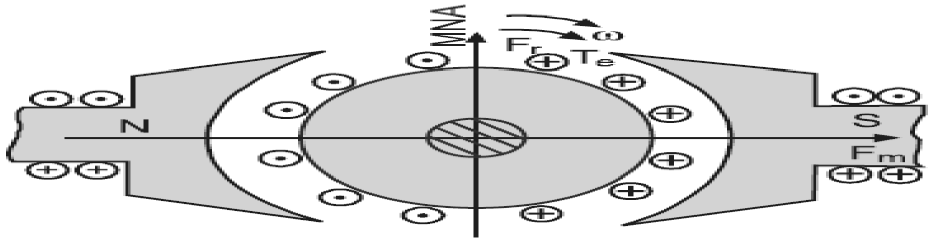


Figure.3.4.Motor action

A resultant rotor field is produced. Its direction is marked by the arrow-head *Fr*. This rotor field *Fr* tries to come in line with the main field *Fm* and torque (*Te*) develops. Thus, rotor rotates. It can be seen that to obtain a continuous torque, the direction of flow of current in each conductor or coil side must be reversed when it passes through the magnetic neutral axis (*MNA*). This is achieved with the help of a commutator. The function of a commutator in DC motors is to reverse the direction of flow of current in each armature conductor when it passes through the M.N.A. to obtain continuous torque.

3.4. Back emf

It has been seen that when current is supplied to the armature conductors, as shown in Fig. 5.5(*a*), placed in the main magnetic field, torque develops and armature rotates.

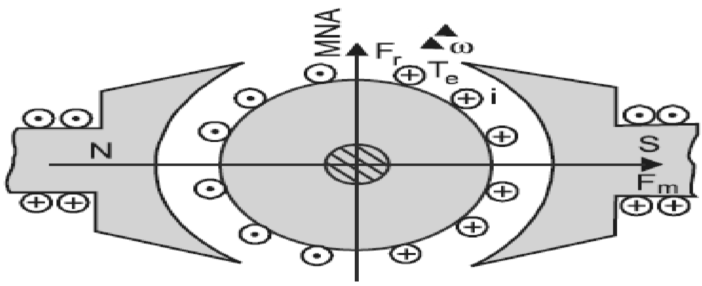
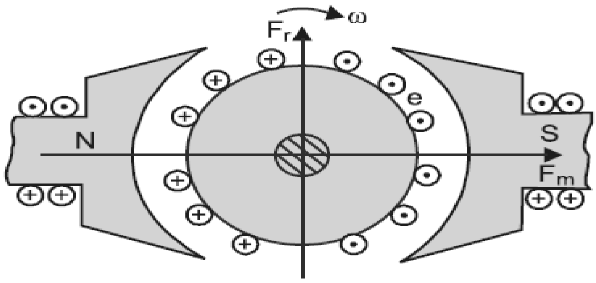


Figure.a.Torque development due to alignment of rotor field with main field



Figures.b. Production of E_b

Figure.3.5.Back emf

Simultaneously, the armature conductors cut across the magnetic field and an emf is induced in these conductors. The direction of this induced emf in the armature conductors is determined by Fleming's Right Hand Rule and is marked in Fig. 5.5(*b*). It can be seen that the direction of this induced emf is opposite to the applied voltage. That is why this induced emf is called back emf (*Eb*). The magnitude of this induced emf is given by the relation;

$$E_b = \frac{PZ\phi N}{60\,A}$$

or $$E_b = \frac{ZP}{60\,A}\,\phi N \text{ or } E_b \propto \phi N \qquad \left(\text{since } \frac{ZP}{60\,A} \text{ are constant}\right)$$

Also, $N \propto \frac{E_b}{\phi}$ shows that speed of motor is inversely proportional to magnetic field or flux

A simple conventional circuit diagram of the machine working as motor, is shown in Fig. 5.6.

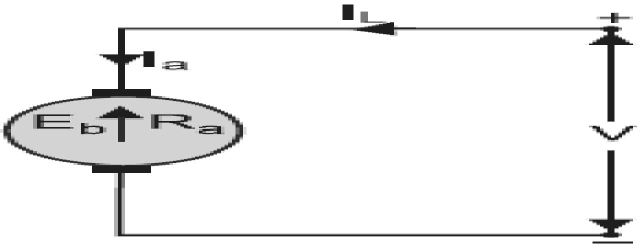


Fig. 5.6 Circuit diagram ($E_b < V$)

Figure.3.6.Circuit diagram ($E_b < V$)

In this case, the supply voltage is always greater than the induced or back emf (i.e., $V > Eb$).

Therefore, current is always supplied to the motor from the mains and the relation among the various quantities will be

$$E_b = V - I_a R_a.$$

3.4.1. Significance of Back emf

The current flowing through the armature is given by the relation:

$$I_a = \frac{V - E_b}{R_a}$$

When mechanical load applied on the motor increases, its speed decreases which reduces the value of *Eb*. As a result the value ($V - Eb$) increases which consequently increases *Ia*. Hence, motor draws extra current from the mains. Thus, the back emf regulates the input power as per the extra load.

3.5. Electro-magnetic Torque Developed in DC Motor

The electrical power which is supplied to a DC motor is converted into mechanical power. The power developed in the armature is given as

$$EI_a = \omega T_e \qquad \text{or} \qquad EI_a = \frac{2\pi N}{60} \times T_e$$

$$\text{or} \qquad \frac{\phi\ ZNP}{60A} \times I_a = \frac{2\pi N}{60} \times T_e \qquad \text{or} \qquad T_e = \frac{PZ\phi N}{2\pi A}\text{Nm}$$

For a particular machine, the number of poles (P), number of conductors per parallel path (Z/A) are constant.

$$T = K\frac{\phi I_a}{2\pi} \quad \text{where } K = \frac{PZ}{A} \quad \text{is a constant}$$

The constant K for a given machine is the same for the emf equation as well as the torque equation.

As well as,

$$T = K_2\ \phi\ I_a \text{ where } K_2 = \frac{PZ}{2\pi A} \text{ is another constant or } T \propto \phi\ I_a$$

Thus, it is concluded that torque produced in the armature of a DC machine is directly proportional to flux per pole and armature current. Moreover, the direction of electromagnetic torque developed in the armature depends upon the direction of flux or magnetic field and the direction of flow of current in the armature conductors. If either of the two is reversed the direction of torque produced is reversed and hence the direction of rotation. But when both are reversed the direction of torque (or rotation) does not change.

Example.3.1

The induced emf in a DC machine is 200 V at a speed of 1200 rpm. Calculate the electromagnetic torque developed at an armature current of 15 A.

Solution:

Here, $E_b = 200$ V; $N = 1200$ rpm; $I_a = 15$ A

Now power developed in the armature,

$$\omega\ T_e = E_b\ I_a$$

or

$$T_e = \frac{E_b\ I_a}{\omega} = \frac{E_b\ I_a}{2\pi N} \times 60 \qquad \left(\because \omega = \frac{2\pi N}{60}\right)$$

$$= \frac{200 \times 15}{2\pi \times 1200} \times 60 = \mathbf{23.87\ Nm}\ (Ans.)$$

Example.3.2

A four-pole DC motor has a wave-wound armature with 594 conductors. The armature current is 40 A and flux per pole is 7.5 m Wb. Calculate H.P. of the motor when running at 1440 rpm.

Solution:

Torque developed, $$T = \frac{PZ\phi I_a}{2\pi A} = \frac{4 \times 594 \times 7.5 \times 10^{-3} \times 40}{2\pi \times 2} = \mathbf{56.72\ Nm}\ (Ans.)$$

Power developed $= \omega\ T$ watt; where $\omega = \frac{2\pi N}{60}$

$$H.P. = \frac{\text{power developed}}{735.5};\ H.P. = \frac{\omega T}{735.5} = \frac{2\pi\text{NT}}{60 \times 735.5}$$

or

$$H.P. = \frac{2 \times 1440 \times 56.72}{60 \times 735.5} = \mathbf{11.63}\ (Ans)$$

3.6. Types of DC Motors

On the basis of the connections of armature and their field winding, DC motors can be classified as;

A. Separately excited DC motors:

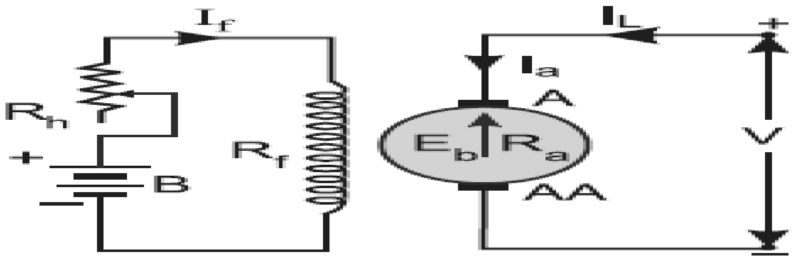


Figure.3.7.Circuit diagram of separately excited DC motor

$$E_b = V - I_a R_a$$

B. Self-excited DC motors:

These motors can be further classified as;

- **Shunt motors:**

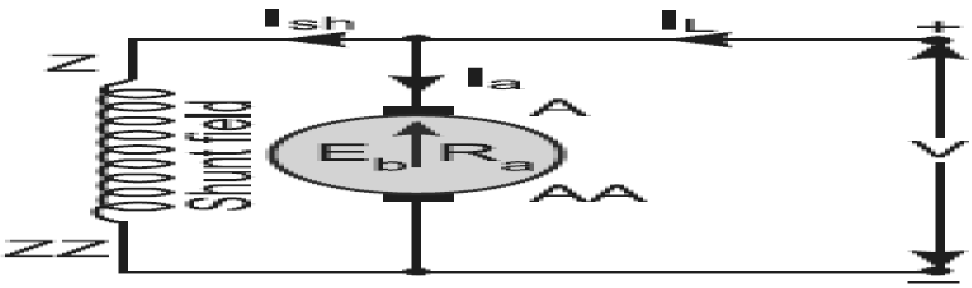


Figure.3.8.Circuit diagram of DC shunt motor

$$I_{sh} = V/R_{sh}$$
$$I_a = I_L - I_{sh}$$
$$E_b = V - I_a R_a$$

- **Series motor**

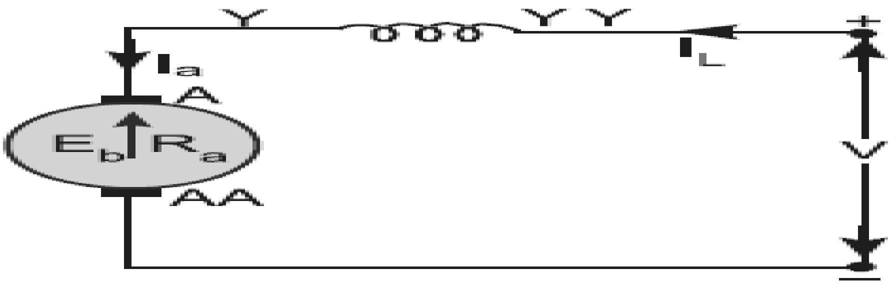


Figure.3.9.Circuit diagram of DC series motor

$$I_L = I_a = I_{se}$$

$$E_b = V - I_a (R_a + R_{se})$$

- **Compound motor:**

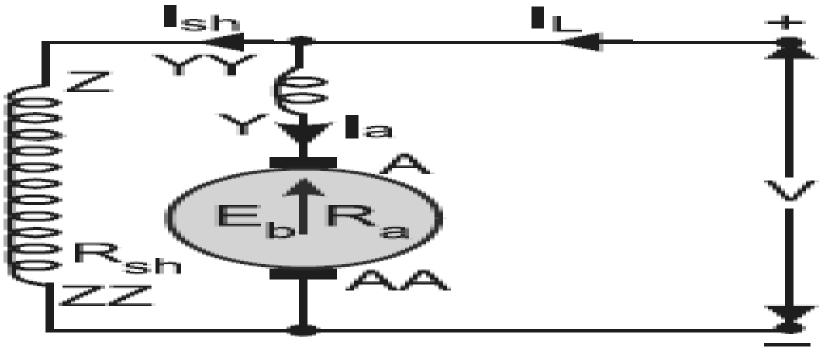


Figure.3.10.Circuit diagram of DC compound motor

$$I_{sh} = \frac{V}{R_{sh}}; I_a = I_L - I_{sh;} E_b = V - I_a (R_a + R_{se})$$

Example.3.3

The armature resistance of a DC shunt motor is 0.5 ohm, it draws 20 A from 220 V mains and is running at a speed of 80 radian per second. Determine (i) Induced emf (ii) Electromagnetic torque (iii) Speed in rpm.

Solution:

Here, $V = 220$ V; $I_a = 20$ A; $R_a = 0.5\ \Omega$; $\omega = 80$ rad/s

Induced emf, $E = V - I_a R_a = 220 - 20 \times 0.5 = \mathbf{210\ V}$ (*Ans.*)

Electromagnetic torque, $T = \frac{EI_a}{\omega} = \frac{210 \times 20}{80} = \mathbf{52.5\ Nm}$ (*Ans.*)

Speed in rpm, $N = \frac{60\omega}{2\pi} = \frac{60 \times 80}{2 \times \pi} = \mathbf{764\ rpm}$ (*Ans.*)

3.7. Characteristics of DC Motors

The performance of a DC motor can be easily judged from its characteristic curves, known as motor characteristics. The characteristics of a motor are those curves which show relation between the two quantities. On the basis of these quantities, the following characteristics can be obtained:

1. Speed and Armature current i.e., N – Ia Characteristics: It is also known as speed characteristics.
2. Torque and Armature current i.e., T–Ia Characteristics:
3. Speed and Torque i.e., N–T characteristics: It is also known as mechanical characteristics.

The following important relations must be kept in mind while discussing the motor characteristics:

$$E_b \propto N\,\phi \quad \text{or} \quad N \propto \frac{E_b}{\phi} \quad \text{and} \quad T \propto \phi\, I_a$$

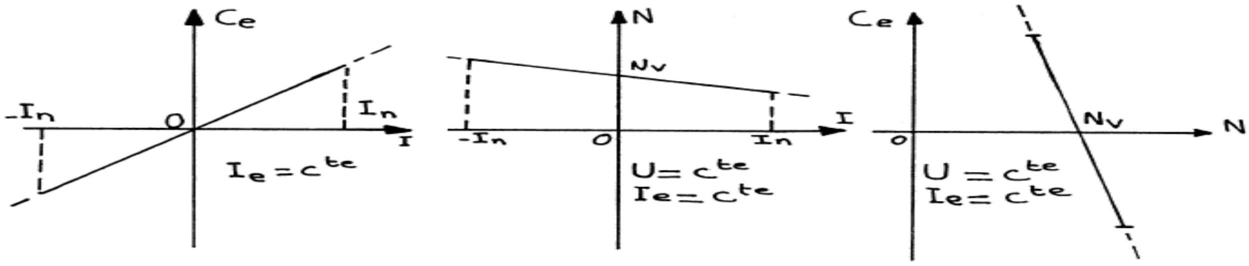


Figure.3.11.Mechanical characteristics of separately excited DC machine

3.8. Applications of DC Motors

As per the characteristics of DC motors, different types of DC motors are applied for different jobs as explained below:

i. **Separately excited motors:**

- ✓ Very accurate speeds can be obtained by these motors.
- ✓ Moreover, these motors are best suited where speed variation is required from very low value to high value.
- ✓ These motors are used in steel rolling mills, paper mills, diesel – electric propulsion of ships, etc.

ii. **Shunt motors:**

From the characteristics of a shunt motor we have seen that it is almost constant speed motor. It is, therefore, used;

- ✓ Where the speed between no-load to full load has to be maintained almost constant.
- ✓ Where it is required to drive the load at various speeds and any one of the speed is required to be maintained almost constant for a relatively long period.
- ✓ As such the shunt motors are most suitable for industrial drives such as lathes, drills, grinders, shapers, spinning and weaving machines, line shafts in the group drive, etc.

iii. **Series motors:**

The characteristics of a series motor reveal that it is variable speed motor i.e., the speed is low at higher torques and vice-versa. Moreover, at light loads or at no-load, the motor attains dangerously high speed. It is, therefore, employed:

- ✓ Where high torque is required at the time of starting to accelerate heavy loads quickly.
- ✓ Where the load is subjected to heavy fluctuations and speed is required to be adjusted automatically.
- ✓ As such the series motors are most suitable for electric traction, cranes, elevators, vacuum cleaners, hair driers, sewing machines, fans and air compressors, etc.

Note:

- ✓ The series motors are always directly coupled with loads or coupled through gears.
- ✓ Belt loads are never applied to series motor, because the belt may slip over the pulley or it may break.

- ✓ Then the motor will operate at light loads or at no-load and will attain dangerously high speed which may damage the motor.

iv. Compound motors:

The important characteristic of this motor is that the speed falls appreciably on heavy loads as in a series motor, but at light loads, the maximum speed is limited to safe value. It is, therefore, used;

- ✓ Where high torque is required at the time of starting and where the load may be thrown off suddenly.
- ✓ Where the load is subjected to heavy fluctuations.
- ✓ As such the cumulative compound, motors are best suited for punching and shearing machines, rolling mills, lifts and mine - hoists, etc.

3.9. Speed Control of DC Motors

The speed of a DC motor is given by the relation $N \propto \frac{E_b}{\phi}$ where $E_b = V - I_a R_a$

$$\therefore \quad N \propto \frac{V - I_a R_a}{\phi}$$

From the above equation it is clear that the speed of DC motors can be controlled;

- o By varying flux per pole $\emptyset$. This is known as flux or field control method.
- o By varying the armature drop, i.e., by varying the resistance of armature circuit. This is known as armature control method.
- o By varying the applied voltage. This is known as voltage control method.

3.10. Losses in a DC Machine

These losses affect the efficiency of the machine. A reduction in these losses leads to higher efficiency. Thus, the major objective in the design of a DC machine is to reduce these losses. The various losses occurring in a DC machine can be sub-divided as:

- ❖ Copper losses.
- ❖ Iron losses.
- ❖ Mechanical losses

a. Copper losses:

The various windings of the DC machine, made of copper, have some resistance.
When current flows through them, there will be power loss proportional to the square of their respective currents. These power losses are called copper losses. In general, the various copper losses in a DC machine are:

- ✓ Armature copper loss = Ia^2 Ra
- ✓ Shunt field copper loss = Ish^2 Rsh
- ✓ Series field copper loss = Ise^2 Rse

b. Iron losses:

The losses which occur in the iron parts of a DC machine are called iron losses or core losses or magnetic losses. These losses consist of the following:

- ✓ Hysteresis loss
- ✓ Eddy current loss

c. Mechanical losses:

As the armature of a DC machine is a rotating part, some power is required to overcome:

- ✓ Air friction of rotating armature (windage loss).
- ✓ Friction at the bearing and friction between brushes and commutator (friction loss).

These losses are known as mechanical losses. To reduce these losses proper lubrication is done at the bearings.

3.11. Power Flow Diagram

Power absorbed by the armature: $P_{ai} = U \cdot I$ (electrical power in watts).

Power absorbed by the field winding: $P_{ae} = U_{exc} \cdot I_{exc} = r_{exc} \cdot I_{exc}^2 = \frac{U_{exc}^2}{r_{exc}}$.

Total absorbed power: $P_a = P_{ai} + P_{ae} = U_a \cdot I_a + U_{exc} \cdot I_{exc}$.

Joule losses in the armature: $P_{ji} = R_a \cdot I_a^2$.

Joule losses in the field winding: $P_{jexc} = U_{exc} \cdot I_{exc}$ (all the power absorbed by the field winding is lost; it only serves to create the magnetic flux).

Electromagnetic Power: $P_{em} = E \cdot I = T_{em} \cdot \Omega$

Useful Power: $P_u = P_a -$ the sum of losses in the motor $= T_u \cdot \Omega$

3.12. Efficiency of a DC Machine

Motor Efficiency: $\eta = \frac{P_u}{P_a}$

Where:

- $P_u = T_u \cdot \Omega$
- $P_a = U_a \cdot I_a + U_{ex} \cdot I_{ex}$

- Condition for Maximum Efficiency

Variable losses = Constant losses

Example.3.4

A 200 V shunt motor has Ra = 0.1 ohm, Rf = 240 ohm and rotational loss = 236 W. On full load the line current is 9.8 A with the motor running at 1450 rpm. Determine

(a) the mechanical power developed

(b) the power output

(c) the load torque

(d) the full load efficiency

Solution:

Here, $V = 200$ V; $R_a = 0.1\ \Omega$; $R_{sh} = 240\ \Omega$; (Mech. + Iron loss) = 236 W

$$I_L = 9.8 \text{ A}; N = 1450 \text{ rpm}$$

$$\text{Shunt field current, } I_{sh} = \frac{V}{R_{sh}} = \frac{200}{240} = 0.833 \text{ A}$$

$$\text{Armature current, } I_a = I_L - I_{sh} = 9.8 - 0.833 = 8.967 \text{ A}$$

$$E_b = V - I_a R_a = 200 - 8.967 \times 0.1 = 199.1 \text{ V}$$

(*a*) $\text{Mechanical power developed} = E_b \times I_a = 199.1 \times 8.967 = 1785.33 \text{ W}$

(*b*) $\text{Power output} = \text{Mech. power developed} - \text{rotational losses}$

$$= 1785.33 - 236 = 1549.33 \text{ W}$$

(*c*) Now $\dfrac{2\pi NT}{60} = \text{Power output}$

or $\text{Load torque, } T = \dfrac{\text{Power output} \times 60}{2\pi N}$

or $\text{Load torque, } T = \dfrac{1549.33 \times 60}{2\pi \times 1450} = \mathbf{10.2\ Nm}$ (*Ans.*)

(*d*) $\text{Input to the motor} = VI_L = 200 \times 9.8 = 1960 \text{ W}$

$$\text{Full-load efficiency, } \eta = \frac{\text{Power output}}{\text{Input}} \times 100 = \frac{1549.3}{1960} \times 100 = \mathbf{79.04\%} \text{ (\textit{Ans.})}$$

Chapter.4.Asynchronous Machines

Chapter.4.Induction Motor

4.1. Chapter objectives

After the completion of this unit, students will be able to understand:

- ✓ What are the various parts of 3-phase induction motor?
- ✓ What is basically a squirrel cage and phase wound induction motor?
- ✓ How a rotating field is developed in the stator core of a 3-phase induction motor?
- ✓ What is the basic principle of operation of this motor and why it is named as induction motor?
- ✓ Why it is also called as asynchronous motor?
- ✓ How the direction of rotation of a 3-phase induction motor is reversed?
- ✓ What are the parameters of the rotor of an induction motor?
- ✓ How to draw an equivalent circuit of rotor circuit?
- ✓ What are stator parameters and how to draw exact equivalent circuit of a 3-phase induction motor?
- ✓ How to draw its phasor diagram?
- ✓ How power flows in an induction motor?
- ✓ What are the factors on which torque produced depends in an induction motor?
- ✓ Under what condition, torque developed in an induction motor is maximum.
- ✓ How the performance of an induction motor is affected by the addition of resistance in its rotor circuit?
- ✓ How to perform various tests on an induction motor to determine its parameters and performance?

4.2. Introduction

Induction machines are also called asynchronous machines i.e., the machines which never run at a synchronous speed. Whenever we say induction machine we mean to say induction motor.
Induction motors may be single-phase or three-phase. The single phase induction motors are usually built in small sizes (upto 3 H.P). Three phase induction motors are the most commonly used AC motors in the industry because they have simple and rugged construction, low cost, high efficiency, reasonably good power factor, self-starting and low maintenance cost. Almost more than 90% of the mechanical power used in industry is provided by three phase induction motors.

4.3. Constructional Features of a Three-phase Induction Motor

A 3-phase induction motor consists of two main parts, namely stator and rotor.

a. Stator:

It is the stationary part of the motor. It has three main parts, namely. (*i*) Outer frame, (*ii*) Stator core and (*iii*) Stator winding. Outer frame: It is the outer body of the motor. Its function is to support the stator core and to protect the inner parts of the machine.

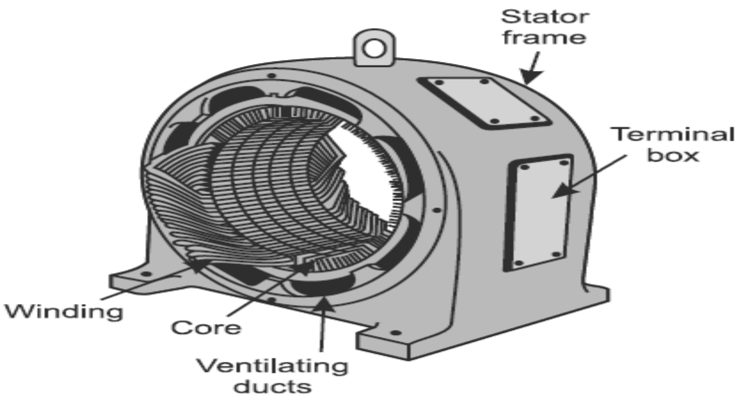


Figure.4.1.Stator

- Stator core:

When AC supply is given to the induction motor, an alternating flux is set -up in the stator core. This alternating field produces hysteresis and eddy current loss. To minimize these losses, the core is made of high grade silicon steel stampings. The stampings are assembled under hydraulic pressure and are keyed to the frame. Each stamping is insulated from the other with a thin varnish layer. The thickness to the stamping usually varies from 0.3 to 0.5 mm.

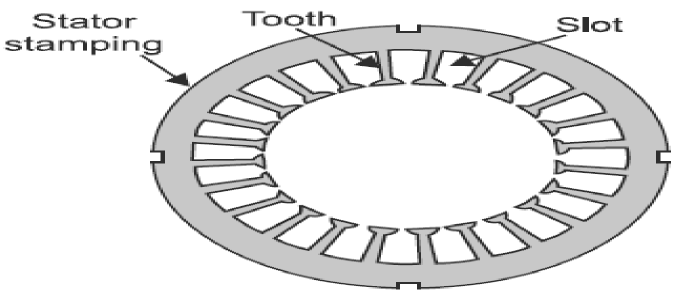


Figure.4.2.Stator stamping

- Stator winding:

The stator core carries a three phase winding which is usually supplied from a three phase supply system. The six terminals of the winding (two of each phase) are connected in the terminal box of the machine. The stator of the motor is wound for definite number of poles, the exact number being determined by the requirement of speed. It will be seen that greater the number of poles, the lower is the speed and vice-versa, since

$$N_S \propto \frac{1}{P}\left(\because N_S = \frac{120f}{P}\right)$$

The three- phase winding may be connected in star or delta externally through a starter.

b. Rotor:

The rotating part of the motor is called rotor. Two types of rotors are used for 3-phase induction motors.

- ✓ Squirrel cage rotor
- ✓ Phase wound rotor

- **Squirrel cage rotor:**

The motors in which these rotors are employed are called Squirrel cage induction motors. Because of simple and rugged construction, the most of the induction motors employed in the industry are of this type. A squirrel cage rotor consists of a laminated cylindrical core having semi-closed circular slots at the outer periphery. Copper or aluminium bar conductors are placed in these slots and short circuited at each end by copper or aluminium rings, called short circuiting rings, as shown in Fig. 9.3.

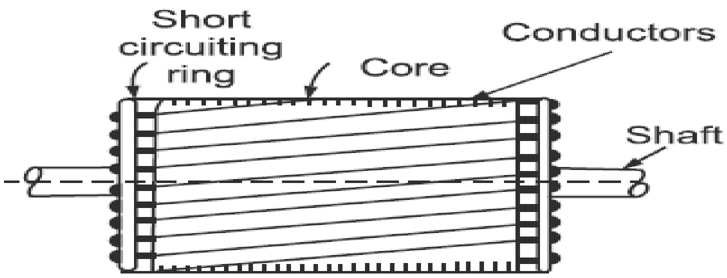


Figure.4.3.Squirrel cage rotor

Thus, in these rotors, the rotor winding is permanently short-circuited and no external resistance can be added in the rotor circuit. Figure 9.3 clearly shows that the slots are not parallel to the shaft but these are skewed. The skewing provides the following advantages:

- ✓ At different positions of the rotor, smooth and sufficient torque is obtained.
- ✓ It reduces the magnetic locking of the stator and rotor,
- ✓ It increases the rotor resistance due to the increased length of the rotor bar conductors.

- **Phase wound rotor**:

It is also known as slip ring rotor and the motors in which these rotors are employed are known as phase wound or slipring induction motors. This rotor is also cylindrical in shape which consists of large number of stampings. A number of semi-closed slots are punched at its outer periphery. A 3-phase insulated winding is placed in these slots. The rotor is wound for the same number of poles as that of stator. The rotor winding is connected in star and its remaining three terminals are connected to the slip rings. The rotor core is keyed to the shaft. Similarly, slip-rings are also keyed to the shaft but these are insulated from the shaft. (see Fig. 9. 4). In this case, depending upon the requirement any external resistance can be added in the rotor circuit. In this case also the rotor is skewed. A mild steel shaft is passed through the centre of the rotor and is fixed to it with key. The purpose of shaft is to transfer mechanical power.

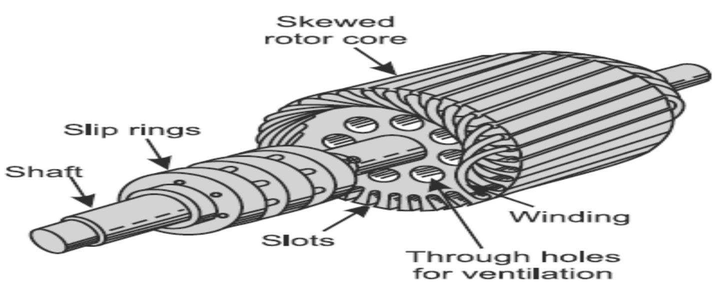


Figure.4.4.Phase wound rotor

4.4. Principle of operation

When 3-phase supply is given to the stator winding of a 3-phase wound induction motor, a revolving field is set up in the stator core. The resultant magnetic field set-up by the stator core, at any instant, is shown in Fig. 9.9.

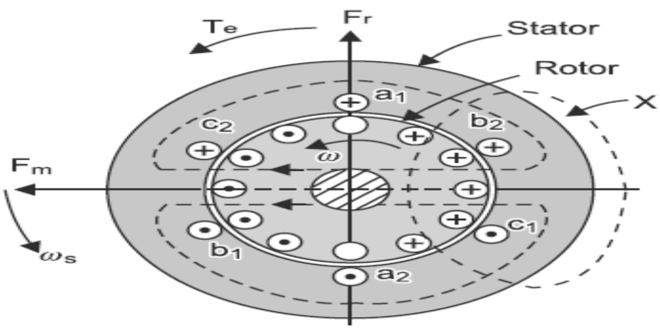


Figure.a.Stator field position at instant t_1 and direction of induced emf in rotor conductors

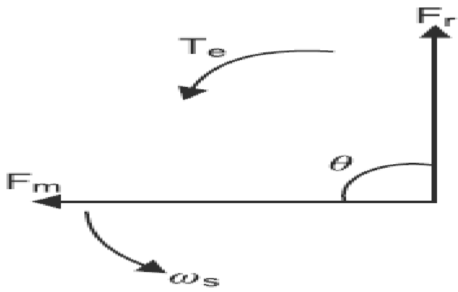


Figure.b.Phasor position of field

Figure.4.5.Phasor position of stator and rotor field

The direction of the resultant field is marked by an arrow head Fm. As per the supply sequence, let this field is rotating in an anti-clockwise direction at synchronous speed Ns radian per second.
The revolving field is cut by the stationary rotor conductors and an emf is induced in the rotor conductors. Since the rotor conductors are short circuited, current flows through them in the direction as marked in Fig. 9.9(a). A resultant field Fr is set-up by the rotor current carrying conductors. This field tries to come in line with the stator revolving field Fm, due to which an electromagnetic torque Te develops and rotor starts rotating in same direction as that of stator revolving field. It is because if the rotor attains the synchronous speed then the relative speed between revolving stator field and rotor will be zero, no emf will be induced in rotor conductors. No emf means no current, no rotor field Fr and hence no torque is produced. Thus, an induction motor can never run at synchronous speed. It always runs at a speed less than synchronous speed. Since, the principle of operation of this motor depends upon electromagnetic induction, hence the name induction motor.

4.5. Slip

In an induction motor, the speed of rotor is always less than synchronous speed. The difference between the speed of revolving field (Ns) and the rotor speed (N) is called slip. The slip is usually expressed as a percentage of synchronous speed (Ns) and is represented by symbol S. Mathematically,

% slip, $$\% S = \frac{N_s - N}{N_s} \times 100$$

or Fractional slip, $$S = \frac{N_s - N}{N_s}$$

Rotor speed, $$N = N_s (l - S)$$

The difference between synchronous speed and rotor speed is called slip speed i.e., Slip speed = Ns – N
The value of slip at full load varies from about 6% small motors to about 2% for large motors.

4.5.1. Importance of Slip

Slip plays an important role in the operation of an induction motor. We have already seen that the difference between the rotor speed and synchronous speed of flux determine the rate at which the flux is cut by rotor conductors and hence the magnitude of induced emf i.e.,

$$e_2 \propto N_S - N$$

Rotor current, $i_2 \propto e_2$ and torque, $T \propto i_2$

$$\therefore \qquad T = K(N_S - N)$$

$$\text{or} \qquad T = KN_S\left(\frac{N_s - N}{N_s}\right)$$

$$\text{or} \qquad T = K_1 S$$

Hence, $T \propto S$

Thus, greater the slip greater will be the induced emf or rotor current and hence larger will be the torque developed. At no-load, induction motor requires small torque to meet with the losses only such as mechanical, iron losses. Therefore rotor speed at no-load is very high and the slip is very small. When the motor is loaded, greater torque is required to drive the load, therefore, the slip increases and rotor speed decreases slightly.

4.6. Frequency of Rotor Currents

The frequency of rotor currents depends upon the relative speed between rotor and stator field. When the rotor is stationary, the relative speed between stator revolving field and stationary rotor conductors is (*Ns*– 0 = *Ns)* the frequency of rotor currents is the same as that of the supply frequency. But once the rotor starts rotating, the frequency of rotor currents decreases depends upon relative speed or slip speed (*Ns* – *N*). Let at any speed *N*, the frequency of rotor currents be *fr*. Then

$$f_r = \frac{(N_s - N)P}{120} = \frac{(N_s - N)}{N_s} \cdot \frac{N_s P}{120} = S \times f$$

Example.4.1

A 3-phase, 4 pole induction motor is connected to 400 V, 50 Hz supply. Determine: (i) Synchronous speed. (ii) Actual speed of the motor when running at 4% slip (iii) Frequency of emf induced in rotor.

Solution:

Synchronous speed, $N_S = \frac{120f}{P} = \frac{120 \times 50}{4}$ = **1500 rpm** (*Ans.*)

Actual speed of motor, $N = N_S(1 - S)$ where, $S = 0{\cdot}04$

$N = 1500\,(1 - 0{\cdot}04)$ = **1440 rpm** (*Ans*)

Frequency of rotor emf, $f_r = Sf = 0.04 \times 50$ = **2 Hz** (*Ans*)

4.7. Rotor emf

The revolving magnetic field set up in the stator by poly phase currents is common to both stator and rotor winding. This field induces emfs. in both the windings. The stator induced emf per phase is given by the relation;

$$E_1 = 4.44\ kw1\ T_1\ f\ f_m$$

Where kw_1 = winding factor i.e., product or coil span factor kc and distribution factor kd.

T_1 = No. of turns/phase of stator winding;

f = stator or supply frequency and

f_m = maximum value of flux.

The rotor induced emf/phase, $E_2 = 4.44\ kw_2\ T_2 f_r f_m$

Where *fr* is the rotor current frequency, and under stationary condition i.e., at the start *fr = f*.

Therefore, rotor induced emf/phase at stand still or start,

E2s = 4.44 *kw*2 T2 *f fm*

Dividing eq. (*ii*) by (*i*), we get,

$$\frac{E_{2s}}{E_1} = \frac{4{\cdot}44\ kw_2 T_2 f \phi_m}{4{\cdot}44\ kw_1 T_1 f \phi_m} = \frac{T_2}{T_1} = K \quad \text{(i.e., transformation ratio)}$$

From eq. (*ii*), induced emf in the rotor under running condition,

$$E_2 = 4{\cdot}44\ kw_2\ T_2\ (Sf)\ \phi_m = SE_{2S}$$

The induced emf in the rotor circuit is maximum at the start and varies according to the value of slip under running condition. Since, the value of normal slip under loaded condition is nearly 5% therefore, the rotor induced emf is nearly 5% of the maximum value.

4.8. Rotor Resistance

Since the rotor winding is made of some conducting material (copper or aluminium), it has a definite resistance ($R = \rho.\ l/a$). Its value remains constant and is denoted by *R*2.

4.9. Rotor Reactance

Whole of the flux produced by the rotor currents does not link with the stator winding. The part of rotor flux which links the rotor conductors but not with the stator winding is called leakages flux and hence develops leakage inductance (*L*2). Depending upon the rotor current frequency, rotor reactance will be developed.

Rotor reactance, $X_2 = 2\pi\ fr\ L_2 = 2\pi\ Sf\ L_2 = S\ (2\pi\ f\ L_2)$

When the rotor is standstill i.e., at the start, when slip, $S = 1$. The value of rotor reactance = $X2s = 2\pi f L2$

Thus, under normal running, rotor reactance, $X2 = SX2s$

4.10. Rotor Impedance

$$\text{Rotor impedance,} = R_2 + j X_2 = R_2 + j SX_2 s$$

$$\text{Magnitude of rotor impedance, } Z_2 = \sqrt{(R_2)^2 + (SX_{2s})^2}$$

4.11. Rotor Current and Power Factor

The rotor circuit diagram of an induction motor is shown in Fig. 9.12.

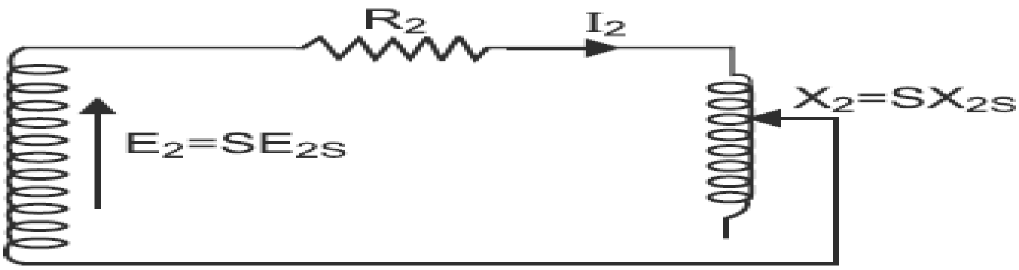


Figure.4.6.Rotor equivalent circuit

Under running condition;

$$\text{Rotor induced emf} = E_2 = SE_2 s$$

$$\text{Rotor impedance, } Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{(R_2)^2 + (SX_{2s})^2}$$

$$\text{Rotor current, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2)^2 + (X_2)^2}}$$

$$= \frac{SE_{2s}}{\sqrt{(R_2)^2 + (SX_{2s})^2}}$$

$$\text{Rotor power factor, } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{(R_2)^2 + (SX_{2s})^2}$$

4.12. Simplified Equivalent Circuit of Rotor

The various parameters and electrical quantities are represented on the circuit diagram, as shown in Fig.4.7.

The rotor current is given by the expression:

$$I_2 = \frac{SE_{2s}}{\sqrt{(R_2)^2 + (SX_{2s})^2}}$$

The other expression for the rotor current is

$$I_2 = \frac{E_{2s}}{\sqrt{(R_2 / S)^2 + (X_{2s})^2}}$$ (dividing the numerator and denominator by S)

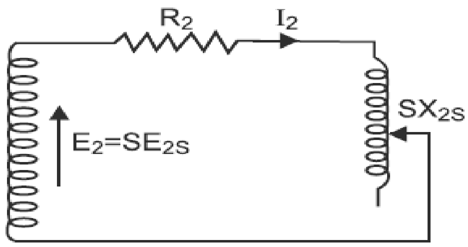


Figure.a

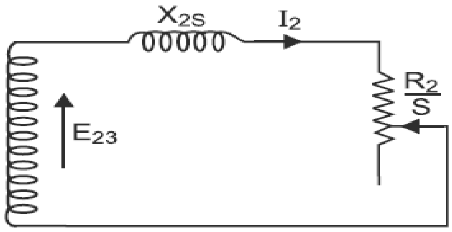


Figure.b.

Figure.4.7.Rotor equivalent circuit

This expression gives a convenient form of equivalent circuit as shown in Fig. 9.14. The resistance is a function of slip and can be split into two parts;

$$\frac{R_2}{S} = R_2 + R_2\left(\frac{1-S}{S}\right)$$

Where $R_2\left(\frac{1-S}{S}\right)$ represents electrical load on the rotor, Say R_L

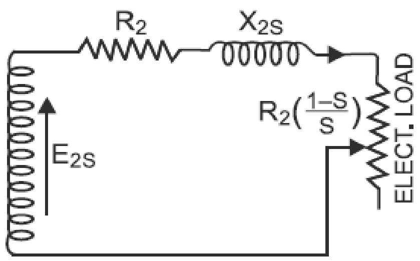


Figure.a. Simplified rotor circuit

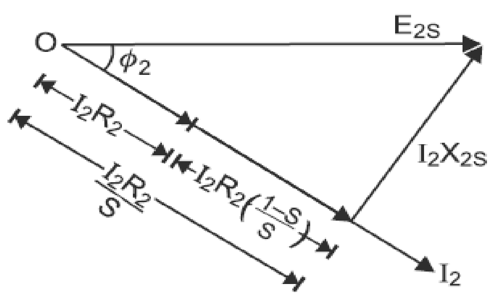


Figure.b. Phasor diagram

Figure.4.8.Simplified rotor equivalent circuit with phasor diagram

Thus, the final simplified equivalent rotor circuit is shown in Fig. 9.15 (*a*). Where R_2 is rotor resistance and X_{2s} is standstill leakage reactance. The resistance $R_2\left(\frac{1-S}{S}\right)$ is fictitious resistance representing load (R_L).

The power consumed by this fictitious resistance i.e., $I_2^2 R_2\left(\frac{1-S}{S}\right)$ is the electrical power which is converted into mechanical power to pick the load. After subtracting the mechanical losses, we get the output power available at the shaft.

Thus, electrical power converted into mechanical power, $= I_2^2 R_2\left(\frac{1-S}{S}\right)$ watt

From the simplified equivalent circuit the phasor diagram of rotor circuit is drawn as shown in Fig. 9.15 (*b*).

Rotor current I_2 lags behind the rotor standstill induced emf E_{2s} by an angle f.

The voltage drop across R_2 i.e., I_2R_2 and across $R_2\left(\frac{1-S}{S}\right)$ i.e., $I_2R_2\left(\frac{1-S}{S}\right)$ are in phase with current I_2, whereas the voltage drop in X_{2s} i.e., I_2X_{2s} leads the current I_2 by 90°.

The vector sum of all the three drops is equal to E_{2s} i.e.,

$$E_{2s} = I_2\sqrt{(R_2/S)^2 + (X_{2s})^2}$$

Power factor of rotor circuit,

$$\cos\phi_2 = \frac{R_2/S}{\sqrt{(R_2/S)^2 + (X_{2s})^2}}$$

4.13. Stator Parameters

Like rotor the stator winding of the motor also has resistance R1. The flux produced by stator winding linking with its own turns only (leakage flux) produces leakage reactance X1. Of the total voltage V applied to the stator, a part of it is consumed by stator resistance (I1Rl) and leakage reactance (I1X1) and the remaining is utilized in establishing mutual flux which links with stator and rotor winding both. When it links with the stator winding it produces self-induced emf E1.

$$\overline{E}_1 = \overline{V}_1 - \overline{I}_1\overline{R}_1 - \overline{I}_1\overline{X}_1$$

4.14. Induction Motor on No-load

In slip ring induction motor rotor circuit can be opened. Under this condition, when stator is connected to 3-phase supply, it draws a very small current called no-load current *I*0. This current has two components i.e., working component *Iw* and magnetising component *Imag*. Working component is in phase with the supply voltage and it supplies the stator iron losses. Whereas, magnetising component lags behind the supply voltage *V* by 90° and produces the mutual flux which links with stator and rotor winding and induces *E*1 and *E*2*s*.

The equivalent circuit and phasor diagram of the motor under this condition is shown in Fig.4.9.

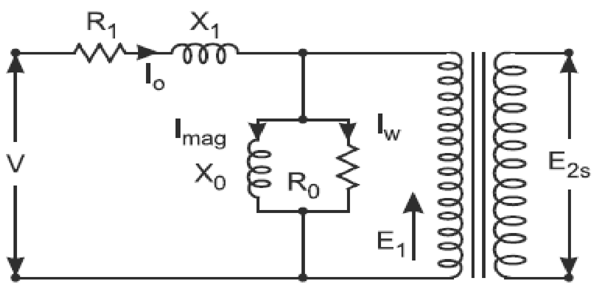


Figure.a. Circuit diagram

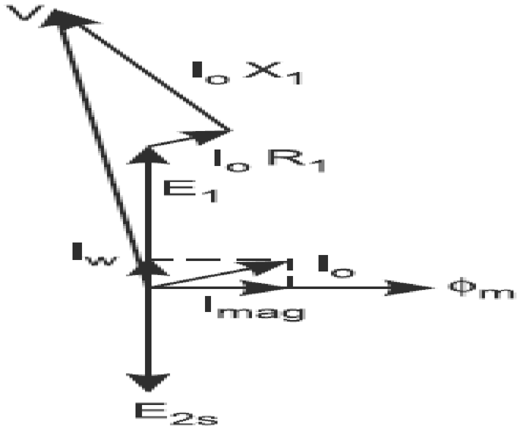


Figure.b. Phasor diagram

Figure.4.9. Induction motor at no-load

4.15. Induction Motor on Load

When load is applied on the induction motor its speed decreases slightly and slip increases. Thus rotor current *I*2 increases. Simultaneously, to meet with this load, motor draws extra current from the supply mains similar to that of a transformer. In fact, power is transferred through magnetic field or flux. The complete circuit diagram and phasor diagram of a loaded induction motor is shown in Fig.4.10.(a) and (b), respectively.

Here, X_0 – exciting reactance $= \frac{V}{I_{mag}}$

R_0 – exciting resistance $= \frac{V}{I_w}$

All other abbreviation have their usual meaning. It may be noted that in the equivalent circuit (per phase) of a 3-phase induction motor, the mechanical load has been replaced by an equivalent electrical resistance *RL* given as:

$$R_L = R_2\left(\frac{1-S}{S}\right)$$

The circuit shown in Fig. 20(a) is similar to the equivalent circuit of a transformer with secondary load equal to *RL*. The rotor emf in the equivalent circuit only depends on the transformation ratio

$$K\ (= E_2 / E_1)$$

Hence, an induction motor can be represented as an equivalent transformer connected to a variable load

$$R_L \left(= R_2 \frac{1-S}{S}\right)$$

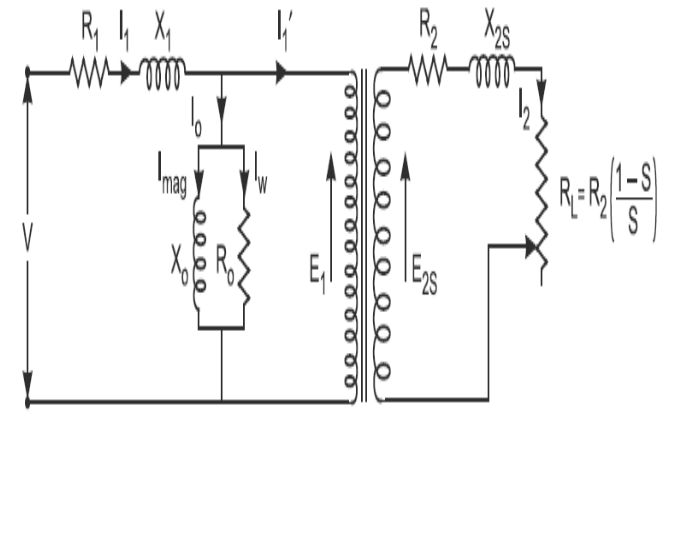


Figure.a. Circuit diagram

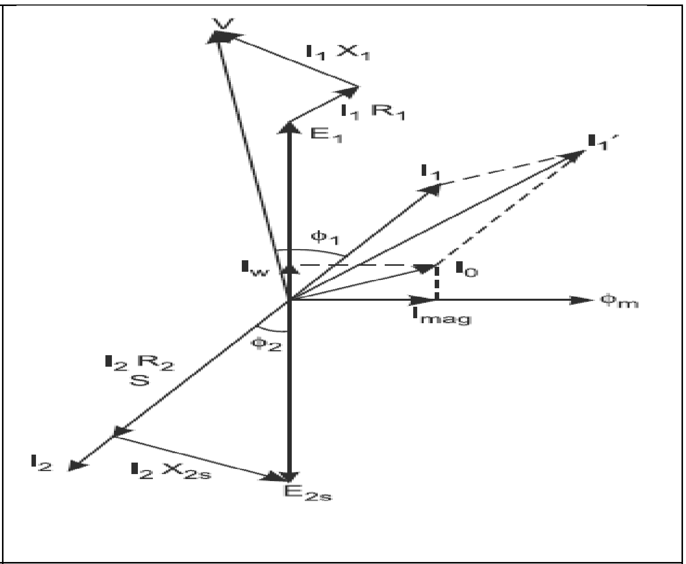


Figure.b. Phasor diagram

Figure.4.10.Circuit diagram and phasor diagram of an induction motor on-load

4.16. Main Losses in an Induction Motor

The major losses in an induction motor are:

1. **Stator losses:** The losses which occur in the stator of an induction motor are called stator losses.
   - ✓ Stator copper losses = $I1^2$.R1 (per phase)
   - ✓ Stator iron losses: These are the hysteresis and eddy current losses.
2. **Rotor losses**: which occur in the rotor of an induction motor are called rotor losses.
   - ✓ Rotor copper losses = $I2^2.R2$ (per phase)
   - ✓ Rotor iron losses: Since under normal running condition rotor frequency is very small, therefore these losses are so small that they are neglected.
3. **Mechanical losses**: The sum of windage and friction losses is called mechanical losses.

4.17. Power Flow Diagram

Electrical power input is given to the stator. There are stator copper and iron losses and the remaining power i.e., stator output is transferred to the rotor through magnetic flux called rotor input. In the rotor there are rotor copper losses and the remaining power is converted into mechanical power called mechanical power developed in the rotor. Then there are mechanical losses and the remaining power is available at the shaft called mechanical power output. The power flow diagram is shown in Fig.4.11.

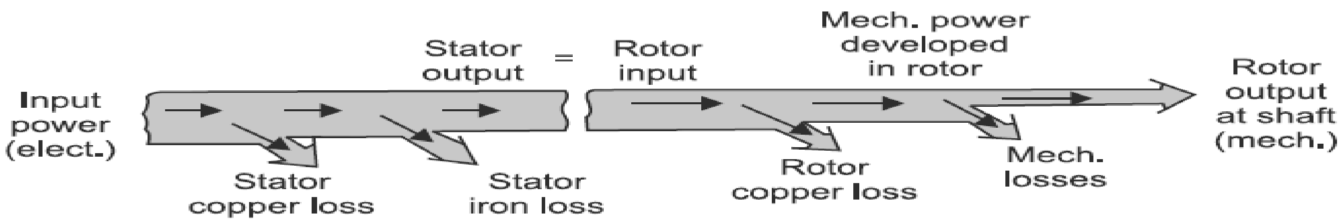


Figure.4.11. power flow diagram

4.18. Relation between Rotor Copper Loss, Slip and Rotor Input

We have seen that the electrical power developed in the rotor is converted into mechanical power which is given by the relation:

$$\text{Mech. power developed in the rotor} = I_2^2 R_2 \left(\frac{I - S}{S}\right) \quad \ldots(i)$$

$$\text{The rotor copper losses} = I_2^2 R_2 \quad \ldots(ii)$$

From power flow diagram:

$$\text{Rotor input} = \text{Mech. power developed} + \text{rotor copper losses}$$

$$= I_2^2 R_2 \left(\frac{I - S}{S}\right) + I_2^2 R_2 = \left(\frac{I_2^2 R_2}{S}\right) \quad \ldots(iii)$$

From eq. (*i*) and (*ii*), we get,

$$\frac{\text{Rotor copper loss}}{\text{Mech. power developed}} = \frac{I_2^2 \ R_2}{I_2^2 \ R_2 \left(\frac{1 - S}{S}\right)}$$

$$\therefore \quad \textbf{Rotor copper loss} = \left(\frac{\mathbf{S}}{\mathbf{1 - S}}\right) \textbf{Mech. power developed}$$

From. eq. (*ii*) and (*iii*), we get,

$$\frac{\text{Rotor copper loss}}{\text{Rotor input}} = \frac{I_2^2 \ R_2}{I_2^2 \ R_2 \ / \ S}$$

$$\therefore \quad \textbf{Rotor copper loss} = \mathbf{S \times \textbf{Rotor input}}$$

4.19. Motor Efficiency

$$\boldsymbol{\eta} = \frac{P_u}{P_{abs}} \times 100$$

$$\text{Rotor efficiency} = \frac{\text{Mech. Power developed}}{\text{Rotor input}} = \frac{I_2^2 \ R_2 \left(\frac{1 - S}{S}\right)}{I_2^2 \ R_2 \ / \ S} = (1 - S)$$

Example.4.2

A 10 H.P., 4 pole, 25 Hz, 3-phase, wound rotor induction motor is taking 9100 watt from the line.

Core loss is 290 watt, stator copper loss is 568 watt, rotor copper loss in 445 watt, friction and windage losses are 100 watt. Determine;

(a) Power transferred across air gap;

(b) Mechanical power in watt developed by rotor;

(c) Mechanical power output in watt;

(d) Efficiency;

(e) Slip.

Solution:

Power input to motor or stator = 9100 watt

Power transferred across air gap = Stator input – Stator core loss – Stator copper loss

= 9100 – 290 – 568 = **8242 W** (*Ans*)

Mechanical power developed in rotor = rotor input – Rotor copper loss = 8242 – 445 = **7797**

Rotor output = Mechanical power developed – Mechanical loss

= 7797 – 100 = **7697 W** (*Ans.*)

$$\text{Motor efficiency} = \frac{\text{Output}}{\text{input}} \times 100 = \frac{7697}{9100} \times 100 = \mathbf{84{\cdot}58\ \%}\ (Ans)$$

$$\text{Slip, } S = \frac{\text{Rotor copper loss}}{\text{Rotor input}} = \frac{445}{8242} = \mathbf{0{\cdot}05399}\ (Ans)$$

4.20. Torque Developed by an Induction Motor

We have already seen that the electrical power of 3-phase induction motor converted into mechanical power is given by the relation;

$$P_0 = 3I_2^2R_2\left(\frac{1-S}{S}\right) \qquad \ldots(i)$$

also,

$$P_0 = \omega T \qquad \ldots(ii)$$

Where, ω = angular speed of the rotor in rad/sec. and

T = torque developed by an induction motor in Nm. Equating eq. (*i*) and (*ii*), we get

$$\omega\, T = 3I_2{}^2R_2\left(\frac{1-S}{S}\right) = 3\frac{I_2^2R_2}{S}\cdot\frac{1-S}{\omega}$$

or $$T = \frac{3}{\omega_2}\cdot\frac{I_2^2\; R_2}{S}$$ [since $\omega = \omega_s\,(1-S)$]

where ω_s = angular synchronous speed in rad/sec.

As $$I_2 = \frac{SE_{2s}}{\sqrt{(R_2)^2 + (SX_{2s})^2}}$$

$\therefore$ $$T = \frac{3}{\omega_2}\left(\frac{SE_{2s}}{\sqrt{(R_2)^2 + (SX_{2s})^2}}\right)^2\cdot\frac{R_2}{S}$$

or $$T = \frac{3}{\omega_s}\,\frac{SE_{2s}^2R_2}{[(R_2)^2 + (SX_{2s})^2]} = \frac{3}{\omega_s}\,\frac{E_{2s}^2R_2\,/\,S}{[(R_2\,/\,S)^2 + (X_{2s})^2]}$$

This is the expression for full load torque.

4.21. Condition for Maximum Torque and Equation for Maximum Torque

The full load torque developed in an induction motor is given by the relation:

$$T = \frac{3}{\omega_2}\,\frac{SE_{2s}^2R_2}{[(R_2)^2 + (SX_{2s})^2]}$$

or $$T \propto \frac{SR_2}{R_2^2 + S^2X_{2x}^2}$$ (since $\frac{3}{\omega_s}E_{2s}^2$ is constant)

The torque developed will be maximum at a particular value of slip. As, slip (S) is a variable quantity, therefore, to obtain the condition for maximum torque, the above expression for torque is differentiated with respect to S and equated to zero.

$$\frac{dT}{dS} = \frac{(R_2^2 + S^2X_{2s}^2)\,R_2 - SR_2(0 + 2SX_{2s}^2)}{(R_2^2 + S^2X_{2s}^2)^2} = 0$$

or $$(R_2{}^2 + S^2\,X_{2s}^2\,)\,R_2 = 2R_2S^2\,X_{2s}{}^2$$

or $$R_2{}^2 = (SX_{2s})^2 \text{ or } R_2 = SX_{2s}$$

or $S = R_2/X_{2s}$ is the slip at which torque is maximum.

To obtain the expression for maximum torque substitute the value of $R2 = SX2s$ in the expression for full load torque, we get,

$$\text{Maximum torque, } T_m = \frac{3}{\omega_s}\,\frac{SE_{2s}^2\;(SX_{2s})}{[(SX_{2s})^2 + (SX_{2s})^2]} = \frac{3E_{2s}^2}{2\omega X_{2s}}$$

Thus, the maximum torque is independent of rotor resistance but it is inversely proportional to rotor reactance at standstill (i.e., X2s). Therefore, to achieve higher value of maximum torque, the leakage reactance of the rotor should be kept minimum. This is achieved (i) by placing the rotor conductors very near to the outer periphery of the rotor and (ii) by reducing the air gap between stator and rotor to smallest possible value.

4.22. Starting Torque

At start rotor is stationary and the value of slip is one i.e., $S = 1$. Thus, to obtain the expression for starting torque, substitute the value of slip, $S = 1$ in the expression of full load torque;

$$\therefore \quad \text{Starting torque, } T_s = \frac{3}{\omega_s} \frac{E_{2s}^2 R_2}{[(R_2)^2 + (X_{2s})^2]}$$

Sometimes maximum torque is required at start. In that case, in the condition for maximum torque substitute the value of $S = 1$.

$$R_2 = SX_{2s} = X_{2s} \qquad \text{(since } S = 1 \text{ at start)}$$

Thus, to obtain maximum torque at start, the value of rotor resistance must be equal to rotor leakage reactance at standstill. Therefore, at start some external resistance is added in the rotor circuit. This is only possible in case of slip ring induction motors. This is the reason, why slip ring induction motors are applied where heavy loads are required to be picked up at start such as in lifts, cranes, elevators etc. Once the motor picks up the load the external resistance is gradually reduced to zero. In case of squirrel cage induction motors, the rotor resistance is fixed and is kept quite low in comparison to rotor reactance, otherwise the rotor copper losses would be high and the efficiency of the motor would fall to low value. However to obtain higher starting torque in case of squirrel cage induction motors another cage is embedded in the rotor and the motor is called a *double cage induction motor.*

Torque-speed Curve and Operating Region. From the curve, it is clear that induction motor develops the same torque at point X and Y.

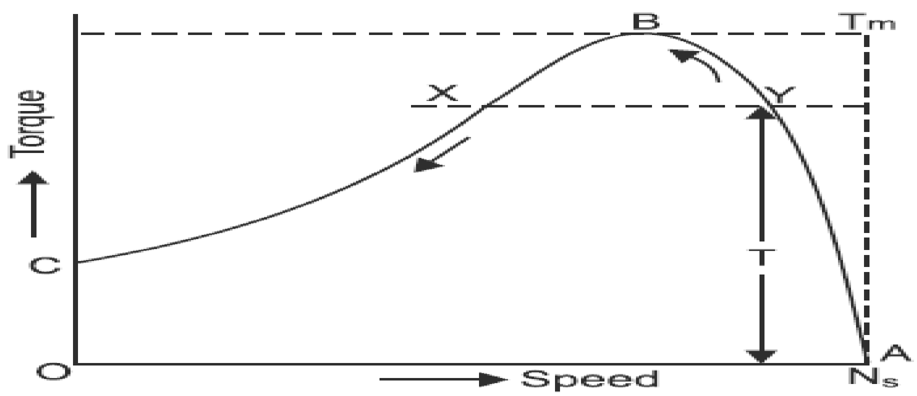


Figure.4.12.Torque – Speed curve

However at point X the motor is unstable because with the increase in load speed decreases and the torque developed by the motor also decreases. Therefore, the motor could not pick up the load and the result is that the motor slows down and eventually stops. The miniature circuit breakers will be tripped open if the circuit has been so protected. At point Y, the motor is stable because in this region with the increase in load speed decreases but the torque developed by the motor increases. Thus the motor will be in position to pick up the extra load effectively. Thus, on the torque-speed curve region BC is the unstable region and region AB is the stable or operating region of the induction motor as shown in Fig.4.12.

4.23. Applications of Three-phase Induction Motors

The applications of squirrel cage induction motors and slip-ring (phase wound) induction motors are given below:

1. **Squirrel cage induction motors:**

These motors are mechanically robust and are operated almost at constant speed. These motors operate at high power factor and have high over load capacity. However, these motors have low starting torque. (i.e., these motors cannot pick-up heavy loads) and draw heavy current at start. On the bases of these characteristics, these motors are best suited for:

- Printing machinery
- Flour mills
- Saw mills
- Shaft drives of small industries
- Pumps
- Prime-movers with small generators etc.

2. **Slip-ring (or phase-wound) induction motors:**

These motors have all the important characteristics (advantage) of squirrel cage induction motors and at the same time have the ability to pick-up heavy loads at start drawing smaller current from the mains. Accordingly these motors are best suited for;

- Rolling mills
- Lifts and hoists
- Big flour mills
- Large pumps
- Line shafts of heavy industries
- Prime-moves with medium and large generators.

4.24. Comparison of Squirrel Cage and Phase Wound Induction Motors

Se. No.	*Particulars*	Squirrel Cage induction motor	*Phase Wound Induction motor*
1.	Construction	It is *simple in construction* and mechanically robust.	Its rotor is phase wound which *needs care.*
2.	Cost	It is *cheaper* in construction cost.	It is more *costly.*
3.	Maintenance	It requires *less maintenance.*	Because of slip rings, brushes etc. it requires *more maintenance.*
4.	Utilisation of slot space	*Space of rotor slots* is better utilised since rotor conductors are in shape of bars.	Whole of the *slots space* is not utilised since winding is placed in the slots.
5.	Coper losses	It has got *small copper losses* because of low rotor resistance.	It has got *more copper losses.*
6.	Efficiency	Its *efficiency* is *high.*	It has *lower efficiency.*
7.	Cooling	The heat can be dissipated more efficiently because the end rings are bare thus there is more space for providing a *cooling fan.*	In this case, overhang of the winding occupies space and less space is left for the provision of a good cooling fan. Thus *cooling* is not quite efficient.
8.	Starting torque	It develops very *poor starting torque* due to low resistance which cannot be increased by ordinary means.	In this case an external resistance can be added in the rotor circuit at the start to *improve its starting torque.*
9.	Starter	Starters are applied on stator side and *starting methods* are quite simple.	Starter is applied on rotor side through *sliprings and brush gears.*

4.25. Comparison between Induction Motor and Synchronous Motor

Se. No.	*Particulars*	*Induction motor*	*Synchronous motor*
1.	Starting	It is a *self-starting* motor.	It is inherently *not a self-starting* motor.
2.	DC Excitation	Its basic principle is mutual induction and *no excitation* is required.	It *requires* DC for *field excitation.*
3.	Speed	Its speed is always *less than synchronous* speed and speed decreases with the increase in load.	It only runs at a *synchronous speed* and its speed is not affected by load. With the increase in load the torque angle δ increases.
4.	Power factor	It runs at *lagging p.f.* which may become very low at light loads.	It can be operated under wide range of power factors *both lagging and leading* by changing its excitation.
5.	Speed control	Its *speed* can be *controlled.*	It runs only at *synchronous speed.* The only way to change its speed is to vary the supply frequency.
6.	Applications	Its *application* is limited to the supply of mechanical load.	It is also used to supply mechanical load and in addition it *improves the p.f.* of the system.
7.	Cost	It is very *cheap to manufacture* and mechanically robust.	It is *costlier* and complicated to manufacture.
8.	Maintenance	It requires *less maintenance.*	It requires *more maintenance.*

Chapter.5.Synchronous Machines

Chapter.5.Synchronous Machines

5.1. Introduction

Synchronous generators, often referred to as alternators, play a pivotal role in modern power generation systems. These machines are responsible for converting mechanical energy, typically derived from prime movers such as turbines or internal combustion engines, into electrical energy in the form of alternating current (AC). Their ability to operate in synchrony with the electrical grid, maintaining a constant frequency and voltage, makes them indispensable for stable and efficient power distribution.

The significance of synchronous generators extends across diverse sectors, including utility-scale power generation, industrial applications, and renewable energy systems. In power plants, they are the cornerstone of electricity production, transforming the kinetic energy of water, steam, or gas into electrical energy. In renewable energy systems, such as hydroelectric and wind power installations, synchronous generators contribute to integrating sustainable energy sources into the grid.

This chapter delves into the fundamental principles, construction, and operational characteristics of synchronous generators. Key topics include their design features, excitation systems, performance under varying loads, and the role of synchronization in grid operations. Furthermore, the chapter highlights the efficiency and reliability of synchronous generators in ensuring a steady and uninterrupted power supply.

5.2. Classification of synchronous machines

A synchronous machine can be classified under three categories:

- ✓ Classification based on the arrangement of field and armature winding.
- ✓ Classification based on the shape of the shape of the field
- ✓ Classification based of the type of prime mover used

5.2.1. Classification based on the Arrangement of Field and Armature Winding

- Rotating armature-type
- Rotating field-type

5.2.2. Classification Based on the Shape of the field

- Non-salient pole machine (Cylindrical rotor)
- Salient pole machine

5.2.3. Classification based on types of prime movers used

- Generator driven by hydraulic turbine called the hydro-generator.

- Generator driven by steam turbine called the turbo-generator.
- Generator driven by IC

5.3. Advantages of rotating field and stationary armature

- It is easier to insulate stationary winding of alternators.
- It is comparatively easier to collect current from stationary parts of the generator.
- Slip ring voltages are less.
- Efficiency is high

5.4. Construction

The rotors of a synchronous motor are of two types, salient pole and cylindrical rotor. The cylindrical pole rotor has the DC field winding embedded in it. The cylindrical rotor provides greater mechanical strength and permits more accurate dynamic balancing. It is particularly used in high-speed turbo generators. The salient pole rotor has projecting poles in it. These projecting poles lessen its mechanical strength. This type of rotor construction is used for low-speed applications such as hydroelectric generators. Large number of poles in the rotor makes the rotor larger in diameter and smaller in length.

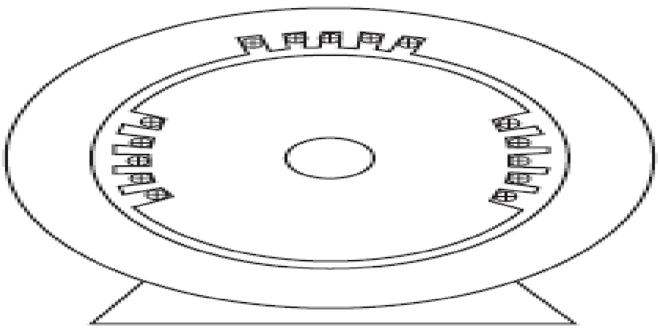


Figure5.1.Generator of Cylindrical rotor

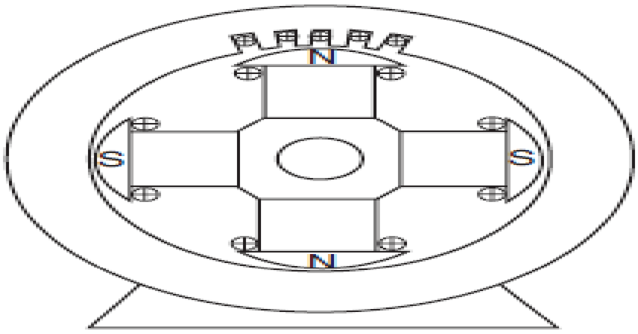


Figure5.2.Generator of Salient pole rotor

5.5. Operation of synchronous generator

The field winding which is present on the rotor of a synchronous machine is supplied with DC current. The DC current will set up a magnetic field on the rotor. The rotor is then rotated at synchronous speed with the help of a prime mover. The magnetic field of the rotor conductor will cut the stationary stator coils and an e.m.f. will be induced in it (Faraday`s law). This e.m.f. will be displaced in phase by 120 degree

If the number of poles on the rotor is P, then in one revolution $P/2$ cycles of e.m.f. is completed. Then the frequency of induced e.m.f. will be

$$f = \frac{P}{2} n$$

Where n is the number of revolution per second of the rotor.

$$f = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120}$$

Where N is the number of revolution per minute of the rotor.

5.6. EMF Equation

Let Z be the number of conductors of coil side in series per phase, φ be the flux per pole in Wb, P be the number of poles in the rotor, N be the speed of the rotor in rpm, f be the frequency in Hz, T be the number of coils or turns per phase.

$$\text{Average induced e.m.f.} = \frac{P\varphi Z}{60} \times \frac{120 f}{P} = 2\varphi Z f$$

$$\text{r.m.s. value of induced e.m.f.} = K_d K_c\, 4.44 \varphi T f$$

5.7. Armature Reaction

In case of an alternator, we supply electricity to pole to produce magnetic field and output power is taken from the armature. According to Faraday's law of electromagnetic induction, there would be an e.m.f. induced in the armature.

5.7.1. EMF Equation

When the current starts flowing through the armature conductor, there is a reverse effect of this current on the main field flux of the alternator (or synchronous generator). This reverse effect is referred as armature reaction in alternator or synchronous generator.

5.8. Synchronous reactance and synchronous impedance

Synchronous reactance is the combined effect of the armature leakage reactance and the armature reaction. When the synchronous reactance is combined vectorally with the armature resistance, the quantity is called the synchronous impedance.

5.9. Equivalent circuit of synchronous generator

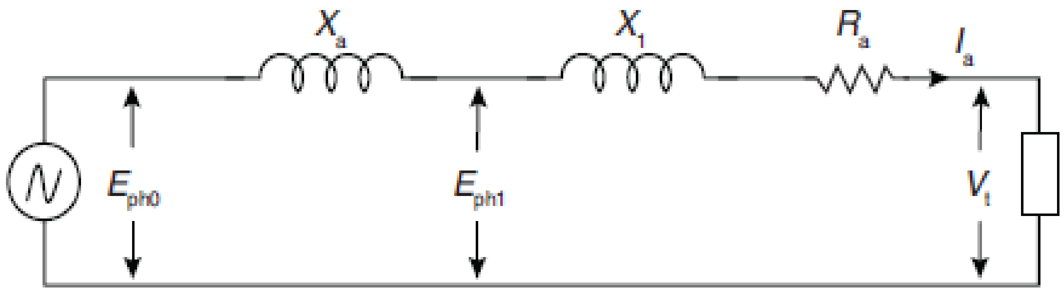


Figure.5.3.Equivalent circuit per phase

5.10. Phasor diagram of an alternator

5.10.1. Phasor diagram of an alternator at lagging load

$$E_{ph} = \sqrt{(V_t \cos\theta + I_a R_a)^2 + (V_t \sin\theta + I_a X_s)^2}$$

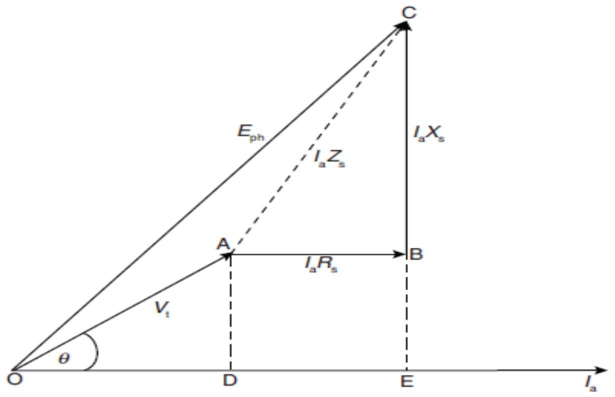


Figure.5.4. Phasor diagram of an alternator at lagging load

5.10.2. Phasor diagram of an alternator at leading load

$$E_{ph} = \sqrt{(V_t \cos\theta + I_a R_a)^2 + (V_t \sin\theta \pm I_a X_s)^2}$$

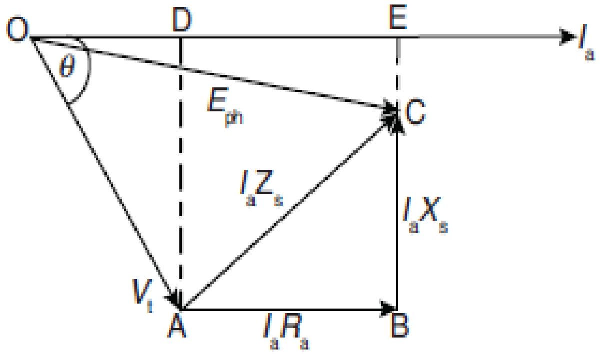


Figure.5.5. Phasor diagram of an alternator at leading load

5.10.3. Phasor diagram of an alternator at unity power factor

$$E_{ph} = \sqrt{(V_t + I_a R_a)^2 + (I_a X_s)^2}$$

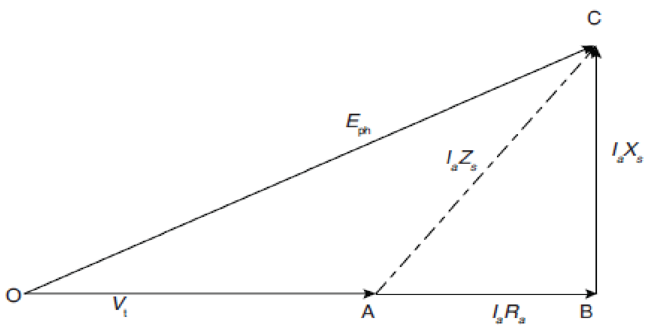


Figure.5.6. Phasor diagram of an alternator at unity power factor

5.11. Voltage regulation of an alternator

As the load on the generator increases, the terminal voltage drops (lagging and unity PF loads cases). But, the terminal voltage, must be maintained constant, and hence the excitation on the machine is varied, or input power to the generator is varied. That means, EA has to be adjusted to keep the terminal voltage VT constant.

$$\text{Voltage regulation} = \frac{\text{Terminal voltage under no load} - \text{Terminal voltage under full load}}{\text{Terminal voltage under full load}}$$

$$= \frac{E_f - V_t}{V_t} \times 100$$

Where,
E_f is the no-load voltage,
V_t is the full-load voltage.
At lagging condition, power factor $E_f>V_t$ and so the voltage regulation is positive.
At leading condition, power factor $E_f<V_t$ and so the voltage regulation is negative.

5.11.1. Methods to Measure Voltage Regulation of an Alternator

Various methods of determination of voltage regulation are,

- EMF or synchronous impedance method
- MMF or ampere turn method
- Zero Power Factor Method (ZPF) or Potier Method

5.12. Parallel operation of alternators

There are many advantages of operating the alternators in parallel:

- It helps in dealing with bigger load demands.
- Presence of many generators in the system increases the reliability of the system. The failure of one generator will not cause total black out.
- With more than one generator in the system, it becomes easy to remove the faulty generator from the system for the purpose of maintenance.
- With many generators operating in parallel, depending on the load condition, it is possible to run only a fraction of them, so that it is operated at near to full load, thus increasing the efficiency of the system.

5.12.1. Condition for Parallel Operation of Alternators

1. The rms line voltage of the two generators must be equal.
2. The two generators must have the same phase sequence.
3. The frequency of the incoming alternator must be slightly higher than the frequency of the running system.

5.13. Blondel's two-reaction theory

The theory which gives the method of analysis of the distributing effects caused by salient-pole construction is called two-reaction theory. Professor Andre Blondel has put forward the two-reaction theory. According to this theory, the armature m.m.f. can be divided into two components

- Components acting along the pole axis called **direct axis**
- Component acting at right angles to the pole axis called **quadrature axis.**

The component acting along direct axis can be magnetising or demagnetising. The component acting along quadrature axis is cross-magnetising. According to this theory, the disturbing effects are caused these components of armature m.m.f.

5.14. Harmonics

When the alternator is loaded, the waveform will not be sinusoidal. Such non-sinusoidal wave form is called a complex wave form. By using Fourier series representation, it is possible to represent complex non-sinusoidal waveform in terms of a series of sinusoidal components called harmonics, whose frequencies are integral multiples of the fundamental wave.

5.14.1. Minimisation of harmonics

- Distribution of stator winding.
- Short chording
- Fractional slot winding
- Skewing
- Larger air gap length.

References

[1]: Stephen J. Chapman, “Electric Machinery Fundamentals”

[2]: Ned Mohan, “Electric Machines and Drives: A First Course"

[3]: Theodore Wildi, “Electric Machines, Drives, and Power Systems"

[4]: Ion Boldea and Syed A. Nasar, “Introduction to Electric Machines and Drives"

[5]: S. K. Sahdev,Book, ‘’Electrical Machines ‘’; Cambridge University Press 2018

[6]: J.Fandino ; « Entraînement électrique », Volume 1, ISBN: 2-7462-1305-2, 2006

[7]: Francis Milsant , « Machines électriques » ; Ellipses, 1992

[8]: M.Kostenko et L.Piotrovski., « Machine électrique: machine à courant alternatif », Tome II, édition Mir 1979.

[9]: M.Kostenko et L.Piotrovski, « Machine électrique: machine à courant continu », Tome I, édition Mir 1979.

[10]: Francis Milsant, « Cours d’électrotechnique: Machine à courant continu » , Tome II, , Eyrolles, Paris 1981.

Abstract:

Electrical machines are a cornerstone of modern electrical engineering, enabling energy conversion and powering countless applications. This course provides a comprehensive understanding of electrical machines, organized into five core chapters. The first chapter introduces **magnetic circuits**, detailing the fundamental principles of magnetism, magnetic materials, and their applications in electrical machines. The second chapter focuses on **transformers**, exploring their design, operation, and critical role in energy transmission. The third chapter examines **DC machines**, covering their construction, working principles, and control methods. The fourth chapter delves into **induction motors**, highlighting their widespread industrial use, operational principles, and performance characteristics. Finally, the fifth chapter investigates **synchronous machines**, discussing their role in power generation and grid stability. By covering these topics, the course equips students with essential theoretical knowledge and practical insights into the operation and application of electrical machines.

Keywords: Magnetic circuits, Transformers, DC machines, Induction motors, Synchronous machines.

الملخص

تُعدّ الآلات الكهربائية حجر الزاوية في هندسة الكهرباء الحديثة، حيث تُمكّن تحويل الطاقة وتشغيل العديد من التطبيقات. يهدف هذا المقرر إلى تقديم فهم شامل للآلات الكهربائية، منظمًا في خمسة فصول أساسية. يتناول الفصل الأول الدوائر المغناطيسية، موضحًا المبادئ الأساسية للمغناطيسية، المواد المغناطيسية، وتطبيقاتها في الآلات الكهربائية. يركز الفصل الثاني على المحولات، مستعرضًا تصميمها، تشغيلها، ودورها الحاسم في نقل الطاقة. يستعرض الفصل الثالث آلات التيار المستمر، متناولًا بنيتها، مبادئ عملها، وطرق التحكم فيها. يغوص الفصل الرابع في المحركات الحثية، مبرزًا استخدامها الصناعي الواسع، مبادئ عملها، وخصائص أدائها. أخيرًا، يبحث الفصل الخامس في الآلات المتزامنة، متناولًا دورها في توليد الطاقة واستقرار الشبكة الكهربائية. من خلال تغطية هذه المواضيع، يُكسب المقرر الطلاب المعرفة النظرية الأساسية والرؤى العملية حول تشغيل الآلات الكهربائية وتطبيقاتها.

الكلمات المفتاحية: الدوائر المغناطيسية، المحولات، آلات التيار المستمر، المحركات الحثية، الآلات المتزامنة