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A dynamic contact problem between elasto-viscoplastic piezoelectric bodies with normal compliance adhesion and damage

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Abstract

We consider a dynamic frictionless contact problem between two elasto-viscoplastic piezoelectric bodies with damage. The evolution of the damage is described by an inclusion of parabolic type. The contact is modelled with normal compliance condition. The adhesion of the contact surfaces is considered and is modelled with a surface variable, the bonding field whose evolution is described by a first order differential equation:
$\dot{\beta} = H_{ad}(\beta, \xi_\beta, R_\nu([u_\nu]), \boldsymbol{R}_\tau([\boldsymbol{u}_\tau]))$. We derive variational formulation for the model and prove an existence and uniqueness result of the weak solution. The proof is based on arguments of nonlinear evolution equations with monotone operators, a classical existence and uniqueness result on parabolic inequalities, differential equations and fixed-point arguments.

Keywords : Electro-elasto-viscoplastic materials, normal compliance, adhesion; damage, differential equations, fixed point.

1 Introduction

The piezoelectric materials have been used in numerous fields taking advantage of the flexible characteristics of these polymers. Some of the applications of these polymers include Audio device-microphones, high frequency speakers, tone generators and acoustic modems; Pressure switches-position switches, accelerometers, impact detectors, flow meters and load cells; Actuators-electronic fans and high shutters. Since piezoelectric polymers allow their use in a multitude of compositions and geometrical shapes for a large variety of applications from transducers in acoustics, ultrasonic's and hydrophone applications to resonators in band pass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments.

The piezoelectricity was discovered by the brothers Curie in 1880 (Curie and Curie, 1880); it consists on the apparition of electric charges on the surfaces of some crystals after their deformation. The reverse effect was outlined in 1881; it consists on the generation of stress and strain in crystals under the action of electric field on the boundary. A deformable material which undergoes piezoelectric effects is called a piezoelectric material. Different models have been developed to describe the interaction between the electric and mechanical fields (see, e.g.[2]). Therefore there is a need to extend the results on models for contact with deformable bodies which include coupling between mechanical and electrical properties. General models for elastic materials with piezoelectric effects can be found in [9] and, more recently, for viscoelastic piezoelectric materials in [3]. Elasto-viscoplastic piezoelectric materials have been also studied [1].

The importance of this paper is to make the coupling of the piezoelectric problem and a frictionless contact problem with adhesion. The adhesive contact between deformable bodies, when a glue is added to prevent relative motion of the surfaces, has received recently increased attention in the mathematical literature. Analysis of models for adhesive contact can be found in [1, 2] and recently in the monographs [10]. In all these papers is the introduction of a surface internal variable, the bonding field, denoted in this paper by β, it describes the point wise fractional density of adhesion of active bonds on the contact surface, and some times referred to as the intensity of adhesion. Following [7], the bonding field satisfies the restriction $0 \leq \beta \leq 1$, when $\beta = 1$ at a point of the contact surface, the adhesion is complete and all the bonds are active, when $\beta = 0$ all the bonds are inactive, severed, and there is no adhesion, when $0 < \beta < 1$ the adhesion is partial and only a fraction β of the bonds is active. The novelty of this work lies in the analysis of a system that contains strong couplings in the multivalued boundary conditions: both the normal compliance contact condition and tangential contact condition depend on the adhesion, and the adhesion be written by the differential equation of the general form

$$\dot{\beta} = H_{ad}\big(\beta, \xi_\beta, R_\nu([u_\nu]), \boldsymbol{R}_\tau([\boldsymbol{u}_\tau])\big).$$

The aim of this paper is to study the coupling of a elasto-viscoplastic piezoelectric problem with normal compliance, adhesion and damage. Then the constitutive laws considered here are of

$$\begin{cases} \boldsymbol{\sigma}^\ell = \mathcal{A}^\ell \boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}^\ell) + \mathcal{G}^\ell \boldsymbol{\varepsilon}(\boldsymbol{u}^\ell) + (\mathcal{E}^\ell)^* \nabla \varphi^\ell + \\ \displaystyle\int_0^t \mathcal{F}^\ell \Big(\boldsymbol{\sigma}^\ell(s) - \mathcal{A}^\ell \boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}^\ell(s)) - (\mathcal{E}^\ell)^* \nabla \varphi^\ell(s), \boldsymbol{\varepsilon}(\boldsymbol{u}^\ell(s)), \zeta^\ell(s) \Big)\, ds \\ \boldsymbol{D}^\ell = \mathcal{E}^\ell \varepsilon(\boldsymbol{u}^\ell) - \mathcal{B}^\ell \nabla \varphi^\ell \end{cases} \tag{1}$$

where $\boldsymbol{u}^\ell$ the displacement field, $\boldsymbol{\sigma}^\ell$ and $\boldsymbol{\varepsilon}(\boldsymbol{u}^\ell)$ represent the stress and the linearized strain tensor, respectively, $\boldsymbol{D}^\ell$ is the electric displacement vector. Here $\mathcal{A}^\ell$ and $\mathcal{G}^\ell$ are nonlinear operators describing the purely viscous and the elastic properties of the material, respectively, $\mathcal{F}^\ell$ is a nonlinear constitutive function describing the viscoplastic behaviour of the material. We also consider that the viscoplastic function $\mathcal{F}^\ell$ depends on the internal state variable ζ^ℓ describing the damage of the material caused by plastic deformations. $\mathcal{B}^\ell$ denotes the electric permittivity tensor, $E(\varphi^\ell) = -\nabla\varphi^\ell$ is the electric field, $\mathcal{E}^\ell = (e_{ijk})$ represents the third order piezoelectric tensor, $(\mathcal{E}^\ell)^*$ is its transposition.
Dynamic contact problems with Kelvin-Voigt materials of the form (**??**) can be found in [4]. The differential inclusion used for the evolution of the damage field is

$$\dot{\zeta}^\ell - \kappa^\ell \Delta \zeta^\ell + \partial \psi_{K^\ell}(\zeta^\ell) \ni \phi^\ell\big(\boldsymbol{\sigma}^\ell - \mathcal{A}^\ell \boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}^\ell) - (\mathcal{E}^\ell)^* \nabla \varphi^\ell, \boldsymbol{\varepsilon}(\boldsymbol{u}^\ell), \zeta^\ell \big),$$

where K^ℓ denotes the set of admissible damage functions defined by

$$K^\ell = \{\xi \in H^1(\Omega^\ell);\ 0 \le \xi \le 1,\ \text{a.e. in } \Omega^\ell\}, \tag{2}$$

κ^ℓ is a positive coefficient, $\partial\psi_{K^\ell}$ represents the subdifferential of the indicator function of the set K^ℓ and ϕ^ℓ is a given constitutive function which describes the sources of the damage in the system. General models for damage were derived in [**?**, 6] from the virtual power principle. Mathematical analysis of one-dimensional problems can be found in [5]. The three-dimensional case has been investigated in [8]. In all these papers the damage of the material is described with a damage function ζ^ℓ, restricted to have values between zero and one. When $\zeta^\ell = 1$, there is no damage in the material, when $\zeta^\ell = 0$, the material is completely damaged, when $0 < \zeta^\ell < 1$ there is partial damage and the system has a reduced load carrying capacity. Contact problems with damage have been investigated in [5].

With these assumptions, the classical formulation of the dynamic problem for frictionless contact problem with normal compliance and adhesion between two electro-elastic-viscoplastics bodies with damage is the following.

2 Our approach

The paper is organized as follows. In section 2 we present notation and some preliminaries. The model is described in section 3 where the variational formulation is given. In section 4, we present our main result stated in Theorem 4.1 and its proof which is based on arguments of time-dependent variational inequalities, parabolic inequalities, differential equations and fixed point.

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