On the classe of n-power normal,n-power quasi-normal operators on Hilbert space

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Abstract

in this paper we introduce the classe of n-power normal , n-power quasi norml operaors and prove some properties of n-power normal operators also we put some conditions which are necessitate the n-quasi normalite .

1. Introduction

A bounded linear operator on complex Hilbert space is n-power normal if $T^nT^* = T^*T^n$, we characterize some resultats about this classe of operators , also we will present another classe of operators which are n-power quasi normal operators satisfing $T^nT^*T = T^*TT^n$ for all positive integer n.

Definition 1 *$T \in B(H)$ is called an n-normal operator if $T^nT^* = T^*T^n$.*

Prosition 1 *Let$T \in B(H)$. Then T is n-normal if and only if T^n is normal where $n \in \mathbb{N}$.*

Proof: by appling Fugled-Putnam theorem.

Definition 2 *$T \in B(H)$ is called an n- quasi normal operator if $T^nT^*T = T^*T^{n+1}$.*

Lemme 1 *If $T \in B(H)$ is an n-normal operator,then T has finie acsent.*

Proof. Since T is n-normal, T^n is normal. Hence $N(T^n) = N(T^{2n})$.

$$\begin{aligned} Let\ x \in N(T^{n+1}) &\Longrightarrow T^{n+1}(x) = 0 \\ &\Longrightarrow T^{n-1+n+1}(x) = 0 \\ &\Longrightarrow T^{2n}(x) = 0 \\ &\Longrightarrow x \in N(T^{2n}) \\ &\Longrightarrow x \in N(T^n) \end{aligned}$$

Thus $N(T^n) = N(T^{n+1})$. Hence T has finite acsent .

2. The main resultas

Now, we can state the main result of this work:
Let$T \in B(H)$ be n-normal operator with closed range then:

1. $T^nTT^* = TT^*T^n$
2. $T^n(TT^*)^+ = (TT^*)^+T^n$
3. $T^nT^+ = T^+T^n$
4. $(T^n)^+T = T(T^n)^+$

Theorem 1 *Let$T \in B(H)$.$T = U + iV$ where $U = \frac{T+T^*}{2}$ and $V = \frac{T-T^*}{2}$*
let $B^2 = T^nT^*$ and $C^2 = T^*T^n$, if T satisfy:

1. B commutes with $U$and$V$.

2. $TB^2 = C^2T$

Then T is n-quasi normal operator .

Proof:
Since $BU = BU$ and $BV = VB$ Then: $B^2U = UB^2$ and $B^2V = VB^2$

$$\begin{aligned} B^2U = UB^2 &\Longrightarrow B^2\left(\frac{T+T^*}{2}\right) = \left(\frac{T+T^*}{2}\right)B^2 \\ &\Longrightarrow \frac{B^2T+B^2T^*}{2} = \frac{TB^2+T^*B^2}{2} \\ &\Longrightarrow B^2T + B^2T^* = TB^2 + T^*B^2 \\ B^2V = VB^2 &\Longrightarrow B^2\left(\frac{T-T^*}{2i}\right) = \left(\frac{T-T^*}{2i}\right)B^2 \\ &\Longrightarrow \frac{B^2T-B^2T^*}{2i} = \frac{TB^2-T^*B^2}{2i} \\ &\Longrightarrow B^2T - B^2T^* = TB^2 - T^*B^2 \\ &\Longrightarrow B^2T = TB^2 = C^2T \\ &\Longrightarrow T^nT^*T = T^*T^{n+1} \end{aligned}$$

So T is n-quasi nomal.

References

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