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EXTENDED SPECTRUM, NUMERICAL RANGE OF OPERATORS AND ALUTHGE TRANSFORMATION AND INTERACTIONS

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1-Summary

This thesis revolves around two themes : a transformation of $B(H)$ introduced by Aluthge, the first part is the subject of the study of the transformation of Aluthge which has had a significant impact in recent years in operator theory . Optimal results on the stability of a certain number of operator classes, such as the class of partial isometries and the classes associated with the asymptotic behavior of an operator, are provided. We also study the evolution of operator invariants, such as the minimum polynomial, the minimum function, the ascent and the descent, under the action of the transformation ; we compare more precisely the sequences of nuclei and images relating to the iterates of an operator and its Aluthge transform, and the second part will be devoted to the study of numerical range and a fairly recent spectral concept, called the extended spectrum operators. Let $T \in B(H)$, we say that a complex number λ is an extended eigenvalue of the operator T , if there exists an operator $X \in B(H) \setminus \{0\}$ such that $TX = \lambda XT$. In this case, the operator X is said to be an extended eigenvector associated with λ . We call extended spectrum of T the set of all extended eigenvalues of T , and we will denote it by $\sigma_{ext}(T)$. Then we will study the interactions between these notions. The behavior of Aluthge's transformation vis-à-vis the numerical range has been much studied. The objective of this thesis is the study of extended spectra, numerical range and their interactions more particularly with the Aluthge transformation. We develop more precisely the founding notion of Aluthge's transformation method. It has remarkable properties and, in fact, it is studied and used in many articles. The objective of this work is to study this object from various angles of observation. We define the Aluthge transformation of an operator T from its polar decomposition : if $T = U|T|$ is the polar decomposition of an operator $T \in B(H)$, we call Aluthge transformation of T , the operator of $B(H)$, noted $\Delta(T)$, defined by $\Delta(T) = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$. The transformation Δ thus defined is therefore an application of $B(H)$ in $B(H)$. More generally, we can define analogously the generalized Aluthge transformation of an operator T , with polar decomposition $T = U|T|$. For any real number $\lambda \in [0, 1]$, we call λ - Aluthge transformation of T the operator $\Delta_\lambda(T) = |T|^\lambda U|T|^{1-\lambda}$. This generalization covers, $\lambda = \frac{1}{2}$, the definition of the Aluthge transformation and, for $\lambda = 1$, the

definition of the Duggal transformation. The spectral properties of the Aluthge transformation show that it is a relevant tool in the study of $B(H)$.

2-Keywords

Aluthge transformation, Spectrum of an operator, Extended spectrum of an operator, Numerical range of an operator, Numerical radius.

3-Thesis plan

- Bibliographic research and training.
- Training + discretization of an article in the field.
- Preparation of an article.
- Submission and Preparation of another article.
- Submission and Preparation of the thesis.
- Writing of the thesis.

4-Bibliographic references

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