ECHAHID HAMMA LAKHDAR UNIVERSITY - EL-OUED
Under the Supervision of the DGRSDT and in collaboration with the CRTI
International PluridisciplinaryPhD Meeting (IPPM'20)
1st Edition, February23-26, 2020
Theme: Modern Technology and Fineness Life
IPPM templat (En/Fr) 02 pages Max

Decay of solution in nonsimple for a porous thermoelastic

Afaf Ahmima, Abdelfeteh Fareh,
Faculté de sciences exactes, Université de El-oued, Laboratoire de théorie des opérateurs et EDP : fondement et Applications

January 18, 2020

Abstract: In this paper we consider a one-dimensional non simple system with porous thermoelasticity. We use the semigroup approach and prove a well-posedness result. An exponential decay result was obtained by the use of Gearhart theorem and the method developed by Zhung and Liu.

Keywords: nonsimple material, thermoelasticity, well-posedness, exponential decay, porous.

1 Introduction

In this work we consider the following problem

$$\begin{cases} \rho u_{tt} = \mu u_{xx} + b\varphi_x - k u_{xxxx} - d\varphi_{xxx} - \beta\theta_x & \text{in } (0,\infty)\times(0,L), \\ J\varphi_{tt} = d u_{xxx} + \alpha\varphi_{xx} - \xi\varphi - b u_x + m\theta - \tau\varphi_t & \text{in } (0,\infty)\times(0,L), \\ c\ \theta_t = \kappa\theta_{xx} - \beta T_0 u_{xt} - m T_0 \varphi_t & \text{in } (0,\infty)\times(0,L), \end{cases} \tag{1}$$

where u, φ are the displacement and volume fraction, θ is the temperature difference respectively and $\rho, \tau, J, c, \kappa, T_0$ are constitutive positive constants with $b, \mu, \xi, k, \alpha, d$ satisfying $\mu\xi \geq b^2$ and $k\alpha \geq d^2$.
We supplement system (1) by the following initial and boundary conditions

$$\begin{cases} u(0,x) = u_0(x), u_t(0,x) = u_1(x), \varphi(0,x) = \varphi_0(x), \varphi_t(0,x) = \varphi_1(x), \theta(0,x) = \theta_0(x) & \text{in } x \in (0,L), \\ u(t,0) = u(t,L) = u_{xx}(t,0) = u_{xx}(t,L) = \varphi_x(t,0) = \varphi_x(t,L) = \theta_x(t,0) = \theta_x(t,L) = 0 & \text{in } (0,\infty), \end{cases} \tag{2}$$

2 Well-posedness

Introducing the new variables $v = u_t$ and $\psi = \varphi_t$ the problem can be written

$$(u_t, v_t, \varphi_t, \psi_t, \theta_t)^T = A(u, v, \varphi, \psi, \theta)^T$$

where A is the operator defined on

$$\mathcal{H} := (H^2(0,L) \cap H_0^1(0,L)) \times L^2(0,L) \times H_*^1(0,L) \times L^2(0,L) \times L_*^2(0,L),$$

such that

$$H_*^m(0,L) := \left\{ \varphi \in H^m(0,L); \int_0^L \varphi dx = 0 \right\}, m = 0,1 \ \text{ et } \ H^0 = L^2$$

with domain

$$D(\mathcal{A}) = H^3 \cap H_0^1 \times H^2 \cap H_0^1 \times H^2 \times H_*^1 \times H^2,$$

The energy of system (1) is defined by

$$E(t) := \frac{1}{2} \int_0^L [\rho |u_t|^2 + \mu |u_x|^2 + k|u_{xx}|^2 + J\,|\varphi_t|^2 + \xi\,|\varphi|^2 + 2bu_x\varphi + 2du_{xx}\varphi_x + \alpha\,|\varphi_x|^2 + \frac{c}{T_0}|\theta|^2]dx,$$

We assume that the matrix

$$A = \begin{pmatrix} \mu & b \\ b & \zeta \end{pmatrix}, \quad B = \begin{pmatrix} \kappa & d \\ d & \alpha \end{pmatrix}$$

are positive difinites. Then, the energy $E(t)$ satisfies

$$\frac{dE}{dt}(t) = -\frac{\kappa}{T_0} \int_0^L |\theta_x|^2 dx - \tau \int |\varphi_t|^2 \leq 0.$$

Which shows the dissipativity of system (1).

Theorem 2.1. *For any $(u_0, u_1, \varphi_0, \varphi_1, \theta_0) \in \mathcal{H}$, the problem (1)-(2) has a unique mild solution $(u, v, \varphi, \psi, \theta)$ such that*

$$(u, v, \varphi, \psi, \theta) \in C\left([0, +\infty[\,; \mathcal{H}\right).$$

Moreover, if $(u_0, u_1, \varphi_0, \varphi_1, \theta_0) \in D(A)$, then the solution $(u, v, \varphi, \psi, \theta)$ satisfies

$$(u, v, \varphi, \psi, \theta) \in C^1\left([0, +\infty[\,; D(A)\right) \cap C\left([0, +\infty[\,; \mathcal{H}\right).$$

Proof. The proof of this theorem will be given by the use of the semigroup approach and the Lumer-Phillips theorem. □

3 Exponential Decay

Theorem 3.1. *For any $(u_0, u_1, \varphi_0, \varphi_1, \theta_0) \in \mathcal{H}$ the solution $(u, v, \varphi, \psi, \theta)$ of problem (1)-(2) decays exponentially,*

$$\|S(t)U_0\| \leq M \|U_0\| e^{-\eta t}, \ \forall t > 0$$

where $S(\cdot)$ is the semigroup generated by A.

The proof of this theorem is based on Gearhart's Theorem

Theorem 3.2 (Gearhart)**.** *A C_0-semigroup of contractions $S(t) = e^{At}$ on a Hilbert space is exponentially stable if and only if*

- $i\mathbb{R} \subset \rho(A)$,
- $\overline{\lim\limits_{|\lambda| \to +\infty}} \|(i\lambda I - A)^{-1}\| < \infty$.

The proof of Theorem (3.1) will be established through two lemmas

Lemme 3.1. *Let A be the operator associated with the system* (1)*, then*

$$\{i\lambda, \lambda \in \mathbb{R}\} \subset \rho\,(A)\,. \tag{3}$$

Proof. The proof will be given in three steeps.

- Shows that $0 \in \rho(A)$ and prove that $\left(\|A^{-1}\|^{-1}, \|A^{-1}\|^{-1}\right) \subset \rho(A)$.
- Prove that if $\sup\left\{\|(i\lambda I - A)^{-1}\|, |\lambda| < \|A^{-1}\|^{-1}\right\} = M < \infty$, then, $\left\{\lambda; |\lambda| < \|A^{-1}\|^{-1} + M^{-1}\right\} \subset \rho(A)$.
- Suppose that $\{i\lambda, \lambda \in \mathbb{R}\}$ is not in $\subset \rho\,(\mathscr{A})$ and obtained a contradiction.

$\square$

Lemme 3.2. *Let $\mathscr{A}$ be the operator associated with the system* (1)*, then*

$$\overline{\lim_{|\lambda| \to +\infty}} \|(i\lambda I - \mathscr{A})^{-1}\| < \infty. \tag{4}$$

Proof. The proof will be given by contradiction argument. Suppose that (4) is not satisfied, then there exists a sequence $(\lambda_n) \subset \rho(A), |\lambda_n| \longrightarrow \infty$ and a sequence of unit vectors $(U_n) \subset D(A)$ such that

$$\|(i\lambda_n I - A)U_n\| \longrightarrow 0.$$

We use the continuous embedding of H^{-2} in L^2 and prove that one of the compounds of U_n has to two different limits, which contradicted the unicity of the limit. the proof of (4) is complete. $\square$

References

[1] Aouadi, M. Stability aspects in a nonsimple thermoelastic diffusion problem, Appl. Anal. vol. 92 (9), (2013) 1816-1828.

[2] Aouadi, M. On uniform decay of a nonsimple thermoelastic bar with memory, J. Math. Anal. App. 402 (2013), 745-757.

[3] Fernández Sare, H. D., Muñoz Rivera, J. E., Quintanilla, R. Decay of solutions in nonsimple thermoelastic bars, Int. J. Eng. Sci. 48 (2010), 1233-1241.

[4] L. Gearhart, Spectral theory for contraction semigroups on Hilbert spaces, Trans. Amer. Math. Soc. 236 (1978), 385-394.

[5] Liu, Z., Magaña, A., Quintanilla, R. On the time decay of solutions for non-simple elasticity with voids, Z. Angew. Math. Mech. (2015), 1-17.

[6] Magaña, A., Quintanilla, R. Exponential decay in nonsimple thermoelasticity of type III, Math. Meth. Appl. Sci. 39 (2014), 225-235.

[7] Pata, V., Quintanilla, R. On the decay of solutions in nonsimple elastic solids with memory, J. MATH. Anal. App. 363 (2010), 19-28.

[8] J. Prüss, On the spectrum of C0-semigroups, Trans. Amer. Math. Soc. 284 (1984), 847-857.

[9] Quintanilla, R. Thermoelasticity without energy dissipation of nonsimple materials, ZAMM Z. Angew. Math. Mech. 83 (2003) 172-180.

[10] Lui, Z., Zheng. S. Semigroups associated with dissipative systems, Chapman & Hall/LCD, USA, 4-5 (1999).