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Faculty of Technology
Mechanical Engineering Department

Heat Transfer Course

Intended for 3rd-Degree in Energy

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PREFACE

The heat transfer course aims to:

- *Introduce the concepts of energy.*
- *Study the different modes of heat transfer (conduction, convection, radiation).*

Mastering these concepts allows us to determine how to assess, improve, or limit heat transfer within various systems such as heat exchangers, thermal insulation, etc.

This course material consists of lessons from the LMD Mechanical Engineering program, taught from 2008 to 2024 to third-year undergraduate students in Energy and Electromechanical Engineering at the Department of Mechanical Engineering at the University of El-Oued. It is the result of fifteen years of teaching. This part of the program is dedicated to the three modes of heat transfer: conduction, convection, and thermal radiation. Currently, it is intended for LMD undergraduate students in Electromechanical and Energy Engineering. We hope it will be highly useful in helping students better understand the principles of the three heat transfer phenomena. The course material is enriched with numerous examples and solved exercises.

The course is divided into five chapters:

The first chapter presents a general overview of the three modes of heat transfer, providing examples for each, with the aim of introducing the concepts in a simple and accessible manner.

The second and third chapters focus on heat transfer by conduction in steady and transient states. The general differential equation of conduction is developed in detail for various practical configurations (prismatic, cylindrical, and spherical).

The fourth chapter is dedicated to heat transfer by forced and natural convection, detailing the conservation equations (mass, momentum, and energy) necessary for understanding applications in specific cases. Common formulas used in free and forced convection, as well as dimensionless numbers, are also presented.

The fifth and final chapter focuses on thermal radiation, beginning with preliminary definitions of the various parameters related to this mode, followed by the laws governing heat transfer by radiation.

Each chapter is supplemented with solved exercises designed to cover all the theoretical knowledge developed in the course.

Abstract

This pedagogical document presents a comprehensive course in heat transfer. It has been written for third-year undergraduate students in energy engineering, mechanical engineering, and electromechanical engineering. The document reviews the fundamental concepts of heat transfer and focuses on the three primary modes of heat transfer: conduction, convection, and thermal radiation. The course begins with a general overview that includes definitions, heat flux, and the formulation of energy balance equations, which form the basic concepts for analyzing thermal systems.

The document also includes a detailed study of heat transfer by conduction under both steady-state and transient conditions, where the governing differential equations are derived and applied to various geometries such as plane walls, cylindrical systems, and spherical bodies. It also addresses practical applications including multilayer walls, thermal fins, and composite materials. Heat transfer by convection is studied according to Newton's law of cooling, with emphasis on natural and forced convection, as well as important concepts such as the boundary layer and dimensionless numbers (Reynolds, Prandtl, Nusselt, and Grashof) that aid in analysis and engineering application.

In addition, the document covers concepts of thermal radiation by presenting the fundamental definitions and laws, including blackbody radiation and emissivity. This work combines theoretical explanation with applied examples and solved exercises, which helps students understand and apply the concepts effectively. Overall, this course provides essential analytical tools for evaluating, designing, and optimizing thermal systems such as heat exchangers and insulating materials in real engineering applications.

Keywords

Heat transfer, conduction, convection, radiation, energy balance, steady-state, transient analysis, thermal conductivity, heat flux, dimensionless numbers

ملخص

تقدم هذه المطبوعة البيداغوجية مساقًا شاملاً في التحول الحراري، لقد تمت كتابتها لطلاب السنة الثالثة (ليسانس) طاقوية في الهندسة الميكانيكية والالكتروميكانيك، تستعرض هذه المطبوعة المفاهيم الأساسية للتحول الحراري، ويركز على الأنماط الثلاثة الرئيسية لانتقال الحرارة: التوصيل، والحمل، والإشعاع الحراري. يبدأ المساق بنظرة عامة تشمل التعريفات، وتدفق الحرارة، وصياغة موازنة معادلات الطاقة، التي تشكل المفاهيم الأساسية لتحليل الأنظمة الحرارية.

كما تتضمن المطبوعة دراسة مفصلة لانتقال الحرارة بالتوصيل في الحالتين المستقرة وغير المستقرة، حيث يتم اشتقاق المعادلات التفاضلية الحاكمة وتطبيقها على أشكال هندسية مختلفة مثل الجدران المستوية، والأنظمة الأسطوانية، والكروية. ويتناول أيضًا تطبيقات عملية تشمل الجدران متعددة الطبقات، والزعانف الحرارية، والمواد المركبة. أما انتقال الحرارة بالحمل فيتم دراسته وفق قانون نيوتن، مع التركيز على الحمل الطبيعي والقسري، بالإضافة إلى مفاهيم مهمة مثل الطبقة الحدية والأعداد اللابعدية (رينولدز، برانتل، نوسلت، وغراشوف) التي تساعد في التحليل والتطبيق الهندسي.

وبالإضافة الى ذلك تشمل المطبوعة مفاهيم على الإشعاع الحراري من خلال عرض التعاريف والقوانين الأساسية، بما في ذلك إشعاع الجسم الأسود ومعامل الانبعاث. ويجمع هذا العمل بين الشرح النظري والأمثلة التطبيقية والتمارين المحلولة، مما يساعد الطلبة على فهم وتطبيق المفاهيم بفعالية. وبشكل عام، يوفر هذا المقرر أدوات تحليلية أساسية لتقييم وتصميم وتحسين الأنظمة الحرارية مثل المبادلات الحرارية ومواد العزل في التطبيقات الهندسية الواقعية.

الكلمات المفتاحية:

انتقال الحرارة، التوصيل الحراري، الحمل الحراري، الإشعاع الحراري، موازنة الطاقة / اتزان الطاقة، الموصلية الحرارية، تدفق الحرارة.

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Nomenclature

D	Diameter (m).
d_i	Inner diameter (m).
d_e	Outer diameter (m).
$\dot{Q}$	Quantity of heat (J).
Q	Heat flux (W).
Q_e	Incoming heat flux (W).
Q_g	Generated heat flux (W).
Q_s	Outgoing heat flux (W).
Q_{st}	Stored heat flux (W).
$\dot{q}$	Volumetric energy density generated.
h	Convection coefficient (W/m²°C).
λ	Thermal conductivity of a material (W/m°C).
ρ	Density (Kg/m³).
μ	Dynamic viscosity (Kg/ms).
υ	Kinematic viscosity (m²/s).
ϕ	Heat flux density (W/m²).
h_a	Enthalpy of humid air (J/kg).
$h_{g,w}$	Enthalpy of water vapor (J/kg).
T_P	Wall temperature (°C).
T_∞	Fluid temperature (°C).
T_m	Average temperature (°C).
T_0	Initial temperature (°C).
U	Fluid flow velocity (m/s).
R_e	Reynolds number.
Pr	Prandtl number.
NUT	Transfer unit number.
Gr	Grashof number.
R_a	Rayleigh number.
Bio	Biot number.
F_0	Fourier number.
N_u	Nusselt number.
θ	Temperature (°C).
Ω	Solid angle.
C_{pa}	Specific heat of air at constant pressure (J/kg.°C).
C_{pv}	Specific heat of water vapor (J/kg.°C).
C_{pw}	Specific heat of water at constant pressure (J/kg.°C).
$M_{\lambda T}$	Monochromatic energy emittance (W/m²).
M_T	Total energy emittance (W/m²).
I_x	Energy intensity in a direction.
J_i	Radiosity.
L_x	Energy luminance in a direction.
ε	Fin efficiency.
$\varepsilon_{\lambda T}$	Emission factor or emissivity.

C_f	Local friction coefficient.
τ_s	Shear stress at the surface.
r	Cylinder radius (m).
H	Height of a solid object (m).
e	Thickness of a solid object (m).
η	Fin performance.
C_1	Planck constant.
σ	Stefan-Boltzmann constant.
S_c	Fin base section (m^2).
S_s	Fin lateral surface (m^2).
V	Volume of a solid body (m^3).
S	Surface area (m^2).

Chapter I:

Basic overview of heat transfer

The objective of this chapter is to:

- Give an introduction to heat transfer rate, heat flux,
- Elaborate on three modes of heat transfer- conduction, convection, and thermal radiation,
- Offer an introduction to physical laws of heat transfer,
- Enlighten thermal conductivity, R-value, thermal conductors, and insulators.

I.1 Introduction:

Thermodynamics studies heat transfer during a system's transition between equilibrium states, focusing on heat transfer rates and temperature distribution. It is critical for designing equipment like boilers, condensers, and heat exchangers and controlling cooling rates in turbine blades and combustion chambers. Heat transfer analysis also plays a key role in preventing overheating in electronic components, transformers, and bearings.

WHAT IS HEAT TRANSFER?

Heat is defined as energy in transit caused by temperature differences. Heat transfer is energy transmission from one region to another due to their temperature differences. Temperature disparities in mediums or within a medium necessitate heat transfer.

The amount of heat transferred per unit of time is called the heat transfer rate and is denoted by Q. The heat transfer rate has unit J/s, which is equivalent to Watt.

When the rate of heat transfer Q is available, then total amount of heat energy transferred ΔU during a time interval Δt can be obtained from:

$$\Delta u = \int_0^{\Delta t} Q dt = Q\Delta t \quad (Joule) \tag{I.1}$$

I.2 Definitions

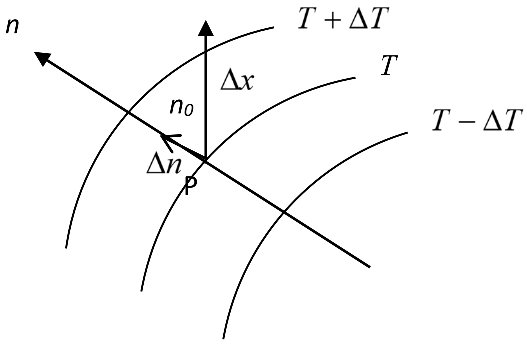
a)- **Temperature**: Temperature is an intensive thermodynamic state parameter that reflects, on a macroscopic level, the energetic state of matter at the microscopic scale. For fluids, it is determined by the level of molecular agitation. At the same time, solids arise from the vibration of atoms within a crystal lattice or the motion of electrons in specific materials (such as metals).

b)- Temperature Fields: Energy transfers are governed by the spatial and temporal changes in temperature. The instantaneous temperature value at any specific spatial point is called the temperature field. We can distinguish between two cases:

- Time-Independent Temperature Field: In this scenario, the temperature field remains constant with time, indicating a permanent or stationary regime.
- Time-Evolving Temperature Field: The temperature field changes over time, signifying a variable or transient regime.

c) - Isothermal Surface: An isothermal surface represents the geometric locus of material points that share the same temperature. It can be categorized into two types:

- <u>Stationary Isothermal Surface:</u> This type remains fixed in its spatial coordinates over time.
- <u>Unsteady Isothermal Surface:</u> This type continuously changes its position over time.



$$\overrightarrow{gradT} = \overrightarrow{\nabla T}$$
$$= \frac{\partial T}{\partial n}\overrightarrow{n_0}, [K/m]$$

Fig. I.1: Visualization of the Temperature Gradient.

d) - Temperature Gradient: The temperature gradient is a vector with a magnitude that determines the temperature variation in the direction perpendicular to the normal vector between two adjacent isothermal curves inside anybody.

Or:

n_0 : The unit vector of the common normal to isothermal surfaces.

$\frac{\partial T}{\partial n}$: The derivative of the temperature along the normal.

n : It is the scalar quantity or magnitude of the temperature gradient. It is negative in the direction of the temperature drop. If we consider a tri-orthogonal system O_x, O_y, O_z, we can write:

$$\overrightarrow{gradT} = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$$

Where, the projections of the vector on the coordinate axes O_x, O_y, O_z are:

$$(gradT)_x = \frac{\partial T}{\partial x}, \ (grad)_y = \frac{\partial T}{\partial y}, \ (grad)_z = \frac{\partial T}{\partial z}$$

e) - Energy-Heat: Energy-heat represents a form of energetic interaction between a thermodynamic system and its external environment, which is manifested by a change in the system's temperature.

$$Q_{12} = \int_1^2 \delta\varphi = \int_1^2 mcdT$$

$$Q_{12} = m\bar{c}(T_2 - T_1)\ [J] \quad (I.2)$$

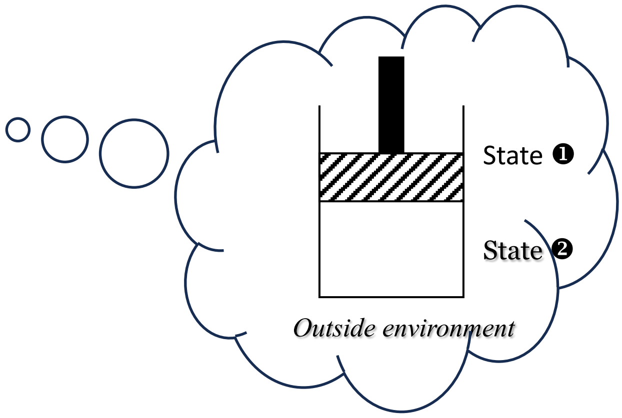


Fig.I.2: Thermodynamic system with two equilibrium states.

f) - Total Heat Flow: The total heat flux Q represents the amount of heat transferred through a given surface per unit of time.

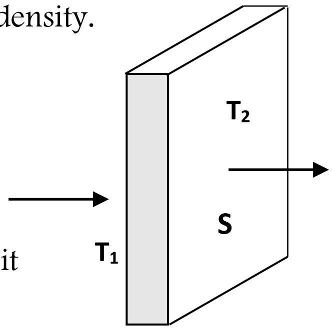
$$\varphi = \frac{d\phi}{dS};\left[W/m^2\right] \quad (I.3)$$

g) - Surface Heat Flux (Heat Flux Density): When the flux density is uniform on the heat transfer surface, we can express it as follows:

$$\varphi = \frac{\phi}{S};\left[W/m^2\right] \quad (I.4)$$

h)- Heat flux: The presence of a temperature gradient leads to heat transfer. The quantity of heat transferred per unit of time per unit area is referred to as heat flux density.

$$q = \frac{1}{S}\frac{dQ}{dt};\left[W/m^2\right]$$



The quantity of heat transferred through the entire surface S per unit of time is referred to as heat flux:

$$\dot{Q} = \frac{dQ}{dt};[W] \quad (I.5)$$

I.3 Formulation of a Heat Transfer Problem

I.3.1 Energy Balance

We must first define a system (S) by specifying its spatial boundaries and then establish the balance of the various heat flows that affect the state of the system and can be:

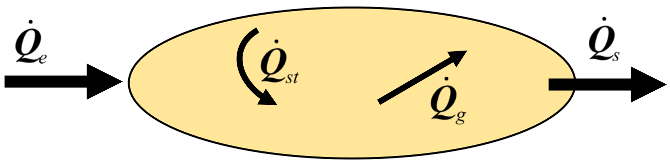


Fig. I.3: Different Heat Flows in a Thermal System.

There are four main heat flows

$\dot{Q}_e$: Incoming heat flow.

$\dot{Q}_s$: Outgoing heat flow.

$\dot{Q}_{st}$: Stored heat flow.

$\dot{Q}_g$: Heat flow generation.

We apply the first law of thermodynamics, which describes the conservation of energy, and we obtain:

$$\dot{Q}_e + \dot{Q}_g = \dot{Q}_{st} + \dot{Q}_s \qquad \text{(I.6)}$$

I.3.2 Expression of Energy Flows

We must now establish the expressions for the various energy flows. By incorporating these expressions into the energy balance, we will derive the differential equation whose solution will enable us to understand the temperature evolution at each point within the system. Given that there are three modes of heat transfer, we will have three expressions for heat flow.

a)- Conduction:

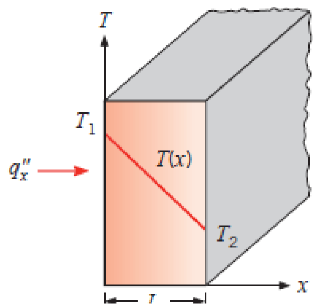
It is transferring heat to an opaque medium without moving matter under the influence of a temperature difference. The propagation of heat by conduction inside a body occurs through two mechanisms:

- Transmission by the vibrations of atoms or molecules.
- Transmission by free electrons.

The French mathematician and physicist Jean Baptiste Joseph Fourier proposed the fundamental law of heat transmission by conduction in 1822. The heat flux density is proportional to the temperature gradient and is given by the following expression:

$$\vec{\phi} = -\lambda \overrightarrow{gradT} \Rightarrow \vec{Q} = -\lambda S \overrightarrow{gradT} \qquad \text{(I.7)}$$

λ: Is the thermal conductivity. It is a material property dependent on its internal structure, describing the degree of heat flow transmission. In the case of one-dimensional conduction, we can express it as follows:



$$\dot{Q} = -\lambda S \frac{dT}{dx} \qquad \text{(I.8)}$$

$$\dot{Q} = qS = -\lambda S \frac{\Delta T}{\Delta x} = -\lambda S \frac{T_2 - T_1}{L}$$

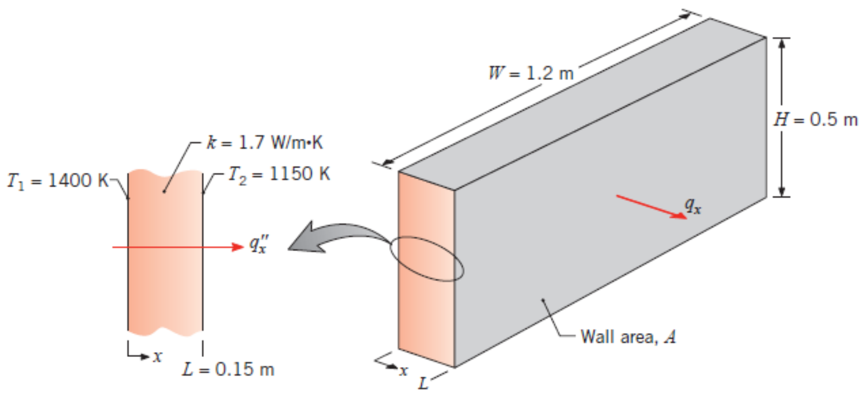
Table I.1 below provides thermal conductivity values for various materials at room temperature (20°C).

Table I.1: Thermal Conductivity Values of Metals at Room Temperature (20°C).

Matter	Thermal conductivity in (W/mK)
Air	0,0251
Aluminum	225,94
Aluminum (liquid)	92,048
Argon (gas)	0,0179
Bronze	117,15
Carbon (diamond)	543,92
Carbon (graphite)	167,36
Copper	397,48
Glass	1,0460
Gold	317,98
Gold (liquid)	167,36
Ice (H2O) solid	2,0920
Iron	71,965
Lead	34,309
magnesium	150,62
Mercury (liquid)	8,3680
Nickel	87,864
Silicon	125,52
Zinc	111,71

Example

The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m × 1.2 m on a side?



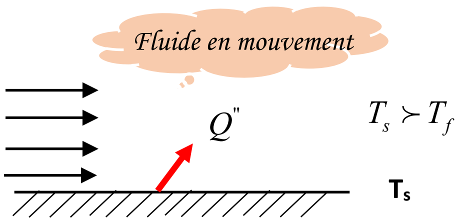
Since heat transfer through the wall is by conduction, the heat flux may be determined from Fourier's law:

$$\phi = \lambda \frac{\Delta T}{L} = 1,7.\frac{250}{0,15} = 2833\left(W/m^2\right)$$

$$Q = S.\phi = (1,2x0,5).2833 = 1700\ W$$

b)- Convection:

Convection is the transfer of heat within a fluid or between a solid surface and a fluid at different temperatures, with energy transmitted through the movement of the fluid. The convection mode of heat transfer consists of two mechanisms: in addition to energy transfer resulting from molecular motion through diffusion (conduction), energy is also transferred via the bulk movement of the fluid (advection).
Regardless of the specific nature of the heat transfer process via convection, the fundamental law of convection is Isaac Newton's law (1643–1727), which is expressed through the empirical relationship of heat flow exchanged by convection between a fluid and a solid wall. The formula for heat flow is as follows:



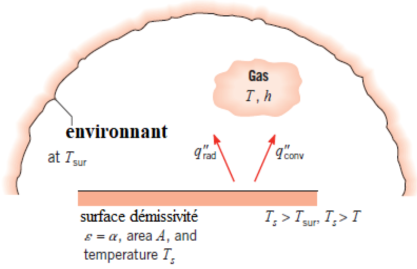
$$\dot{Q} = hS\left(T_s - T_f\right) \quad \text{(I.9)}$$

The value of 'h,' which represents the heat transfer coefficient through convection, depends on the nature of the fluid, its temperature, its velocity, and the geometric characteristics of the solid/fluid contact surface.

c) - Radiation: The third mode of heat transfer is thermal radiation. All surfaces at finite temperatures emit energy in the form of electromagnetic waves. It is the heating transfer by electromagnetic waves between two surfaces at different temperatures, even in a vacuum. The expression for the heat flow is as follows:

$$\dot{Q} = \sigma S\left(T_1^4 - T_2^4\right) \quad \text{(I.10)}$$

Or: $\sigma = 5,67.10^{-8}$ the constant of Stephan Boltzmann.



I.3.3 Energy Storage: The storage of energy in a body corresponds to an increase in its internal energy over time, thus (at constant pressure and in the absence of a change of state):

$$\dot{Q} = \rho C_P V \frac{\partial T}{\partial t} \quad \text{(I.11)}$$

Where: ρ Density, V Volume, C_P Specific heat.

I.3.4 Energy Generation: Energy generation occurs when another form of energy (chemical, electrical, mechanical, nuclear) is converted into thermal energy. It can be expressed in the following form:

$$\dot{Q}_g = \dot{q} V \quad \text{(I.12)}$$

$\dot{q}$: Volume density of generated energy. (The amount or concentration of a quantity per unit volume).

Chapter II

Steady-State Heat Conduction

II.1 Introduction

Conduction transfers heat from hot to cold regions within solids, liquids, or gases in contact. It involves the transmission of kinetic energy from high-temperature particles to low-temperature ones. In heat transfer problems within solid bodies, understanding the temperature field and heat flux is crucial for optimizing insulation thickness. This is facilitated by the energy equation consolidating key parameters of heat conduction in solid materials.

I.2 The General Energy Equation

The energy equation, also known as the heat diffusion equation or the differential heat equation, aims to facilitate the determination of the temperature field in the space and time of the studied body. Once the temperature distribution is known, we can easily calculate the heat flow transmitted by conduction.

In a Cartesian coordinate system, (O,x,y,z) as shown in Fig. II.1, the heat conduction equation can be derived by considering an infinitesimal volume V and conducting a heat balance over this volume over a period of time, dt. We assume the presence of a heat source within this volume, which generates volumetric thermal energy $\dot{Q}_g$.

Consideration of the Following Hypotheses:

- The studied environment is homogeneous. (λ is only a function of temperature (T).) and Isotropic Medium $\left(\lambda = \lambda_x = \lambda_y = \lambda_z\right)$.
- Physical parameters $\left(\rho, C_P, \lambda\right)$.
- The deformation of the elementary volume due to the temperature variation is negligible.

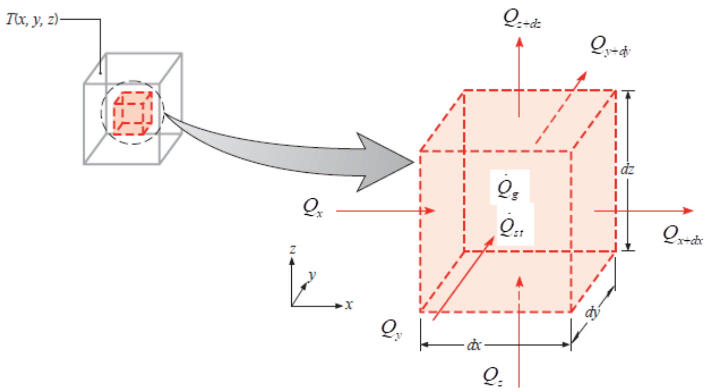


Fig. 11.1: Control Volume in Cartesian Coordinates.

The control volume is subjected to a set of energy quantities at its bordering surfaces (incoming and outgoing energies) as well as inside (generated and stored energy). These are summarized as follows:

- The quantities of energy entering the control volume are designated as the heat flow. Q_x, Q_y, Q_z .
- The quantities of energy leaving through the opposite facets $(x + dx, y + dy, z + dz)$ represent heat flow: $Q_{x+dx}, Q_{y+dy}, Q_{z+dz}$.
- By performing a Taylor series expansion of these quantities and neglecting the high-order terms, we obtain:

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x} dx$$

$$Q_{y+dy} = Q_y + \frac{\partial Q_y}{\partial y} dy \qquad \text{(II.1)}$$

$$Q_{z+dz} = Q_z + \frac{\partial Q_z}{\partial z} dz$$

Inside the control volume, a certain amount of energy is generated due to the presence of a heat source. This generated energy is expressed as:

$$\dot{Q}_g = qdV = q.dxdydz \qquad \text{(II.2)}$$

where: q is a flow generated per unit of volume in [W/m^3].

Inside the control volume, stored energy is represented in the following form:

$$\dot{Q}_{st} = \rho \frac{dU}{dt} dV \qquad \text{(II.3)}$$

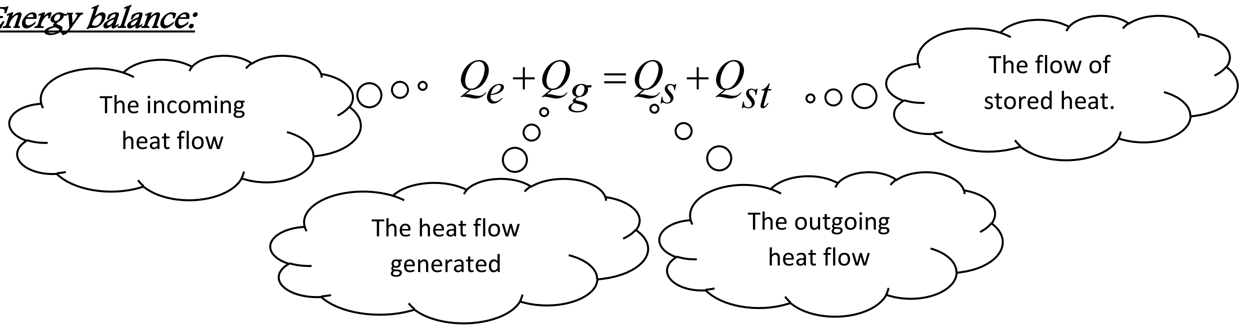
The change in internal energy of the control volume over time is equal to:

$$\frac{dU}{dt} = C \frac{dT}{dt} \qquad \text{(II.4)}$$

The energy stored in the control volume is thus equal to:

$$\dot{Q}_{st} = \rho C_P \frac{dT}{dt} dxdydz \qquad \text{(II.5)}$$

Energy balance:



$$Q_x + Q_y + Q_z - \left(Q_{x+dx} + Q_{y+dy} + Q_{z+dz}\right) + \dot{q}dV = \rho C_P \frac{dT}{dt} dV \qquad \text{(II.6)}$$

Relative to the section perpendicular to the x-axis, the quantities of heat are expressed using Fourier's law:

$$dQ_x = -\lambda\left(dydz\right)\frac{\partial T}{\partial x}$$

$$dQ_{x+dx} = dQ_x + \frac{\partial}{\partial x}\left(dQ_x\right)dx + ...\left(Taylor\ series\right)$$

$$dQ_{x+dx} = -\lambda\left(dydz\right)\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left[-\lambda\left(dydz\right)\frac{\partial T}{\partial x}\right]dx$$

$$dQ_{x+dx} = -\lambda\frac{\partial}{\partial x}\left[T + \frac{\partial T}{\partial x}dx\right]dydz$$

The heat balance relative to the x-axis is given by:

$$dQ_x - dQ_{x+dx} = -\lambda\frac{\partial T}{\partial x}dydz + \lambda\frac{\partial T}{\partial x}dydz + \lambda\frac{\partial^2 T}{\partial x^2}dxdydz = \frac{\partial}{\partial x}\left[\lambda\frac{\partial T}{\partial x}\right]dxdydz$$
$$\Rightarrow dQ_x - dQ_{x+dx} = \frac{\partial}{\partial x}\left[\lambda\frac{\partial T}{\partial x}\right]dxdydz \qquad \text{(II.7)}$$

The balance of the quantities of heat transmitted by conduction through the volume dV is expressed as follows:

$$dQ_x - dQ_{x+dx} + dQ_y - dQ_{y+dy} + dQ_z - dQ_{z+dz} =$$
$$\left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)\right]dxdydz \qquad \text{(II.8)}$$

The final energy balance is given by:

$$\rho C_P\frac{\partial T}{\partial t}dxdydz = \left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)\right] + dxdydz + \dot{q}dxdydz$$

$$\rho C_P\frac{\partial T}{\partial t} = \left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)\right] + \dot{q} \qquad \text{(II.9)}$$

By dividing by λ and introducing the heat diffusivity coefficient, $a = \dfrac{\lambda}{\rho C_P}$, we obtain the heat conduction equation in its general form:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{\lambda} = \frac{1}{a}\frac{\partial T}{\partial t} \qquad \text{(II.10)}$$

- If the conduction is stationary $\left(\dfrac{\partial T}{\partial t} = 0\right)$ we find the Poisson equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{\lambda} = 0 \qquad \text{(II.11)}$$

- If there is no energy generation within the system $\left(\dot{q} = 0\right)$, we can find the equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a}\frac{\partial T}{\partial t} \qquad \text{(II.12)}$$

- If conduction is stationary $\left(\frac{\partial T}{\partial t}=0\right)$ and there is no heat generation, the equation is referred to as Laplace:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0 \qquad \text{(II.13)}$$

For the one-dimensional conduction case, equation (II.13) becomes:

$$\frac{\partial^2 T}{\partial x^2}=0 \qquad \text{(II.14)}$$

II.2.1: The Energy Equation in Cylindrical Coordinates

The transition to cylindrical coordinates is accomplished using the following relationships: $\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$

Applying the energy balance to the control volume yields the heat conduction equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\lambda\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_P\frac{\partial T}{\partial t} \qquad \text{(II.15)}$$

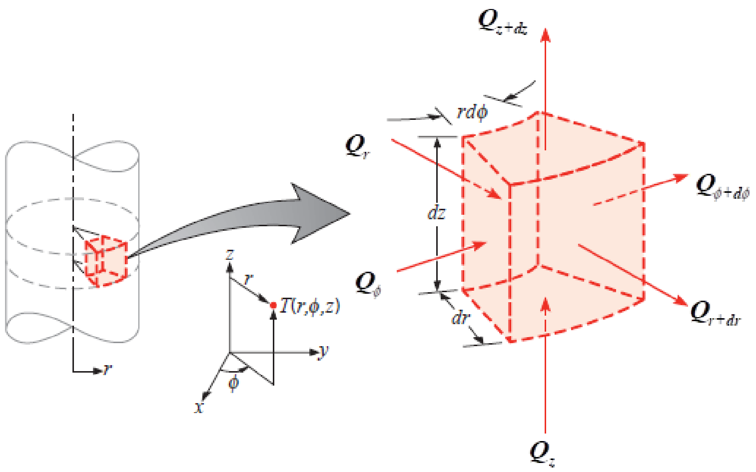


Fig.II.2: Cylindrical Coordinate System.

II.2.2: Energy Equation in Spherical Coordinates

The transition from Cartesian coordinates to spherical coordinates is accomplished using the following relationships:

$$\begin{cases} x = r\cos\theta\sin\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

The new parameters are: r, ϕ et θ

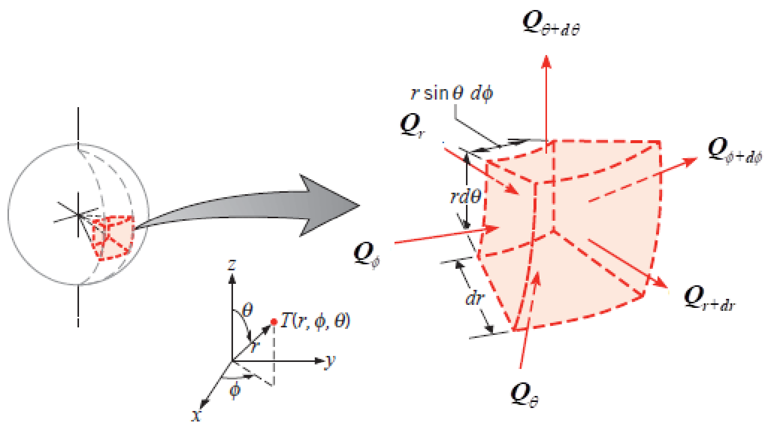


Fig.II.3: Spherical coordinate system.

Applying the energy balance to the control volume yields the equation for heat conduction in spherical coordinates in the following form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(\lambda\frac{\partial T}{\partial\phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\lambda\sin\theta\frac{\partial T}{\partial\theta}\right)+\dot{q}=\rho C_P\frac{\partial T}{\partial t} \qquad \text{(II.16)}$$

II.3 Unidirectional Heat Transfer

II.3.1 Flat Wall

We are considering a wall (see Fig. 11.4) with a thickness 'e' and thermal conductivity λ, featuring large transverse dimensions. The extreme faces of this wall are at temperatures 'T_1' and 'T_2.' In the absence of a heat source and under steady-state conditions, the heat conduction equation simplifies to:

$$\frac{d^2T}{dx^2}=0$$

$$\int\frac{d}{dx}\left(\frac{dT}{dx}\right)dx=\frac{dT}{dx}=C_1$$

$$\int\frac{d}{dx}(T)dx \quad\Rightarrow\quad T(x)=C_1x+C_2$$

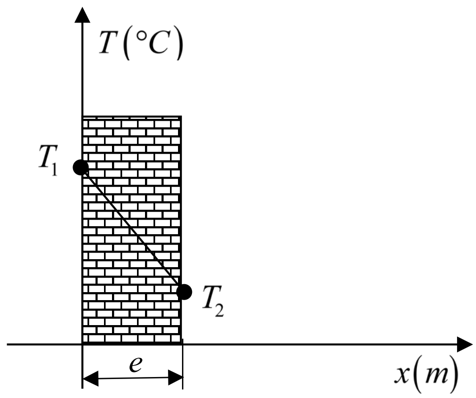


Fig. II.4: Flat Wall.

The integration constants C_1 and C_2 are determined from the following boundary conditions:

- at $x=0 \quad \Rightarrow T(0)=T_1=C_2$
- at $x=e \quad \Rightarrow T(e)=C_1(e)+C_2 \quad \Rightarrow T_2=C_1e+T_1 \quad \Rightarrow C_1=\frac{T_2-T_1}{e}$

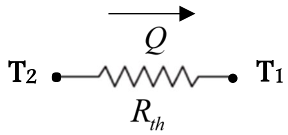
The solution is written: $$T(x)=\frac{T_2-T_1}{e}x+T_1 \qquad \text{(II.17)}$$

In steady-state, heat flow remains constant, and Fourier's law expresses:

$$Q = S.\phi = -\lambda S\frac{dT}{dx} = -\lambda S\frac{T_2 - T_1}{e} \qquad \text{(II.18)}$$

The slope of Figure II.4 is represented by: $\frac{dT}{dx} = \frac{T_2 - T_1}{e}$

The expression for thermal resistance is defined as: $R_{th} = \frac{e}{\lambda S}$, which can be represented by the following electrical analog diagram:

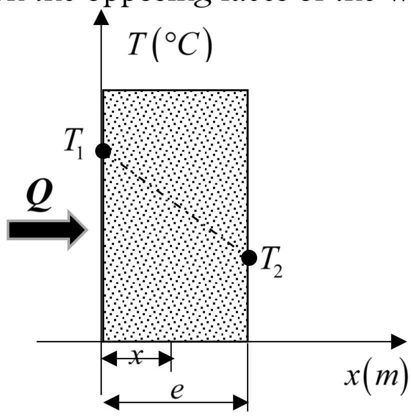


Example II.1

The wall of a building is constructed with bricks having a thickness of 38 cm and a thermal conductivity of 0.78 (W/m·K). The temperatures on the opposing faces of the wall are 18 °C and -15 °C, respectively.

Determine:

1- The heat flux across the wall's surface.
2- The thickness of the wall where the temperature is 0°C.



Solution:

1- The Heat Flow per Unit Area:

$$\phi = \lambda\frac{T_1 - T_2}{e} \Rightarrow \phi = 0,78\frac{18_1 - (-15)}{38.10^{-2}} = 67,73\left(W/m^2\right)$$

2- The variation of temperature as a function of x is given by the linear law:

$$T(x) = T_1 - \frac{x}{e}(T_1 - T_2)$$

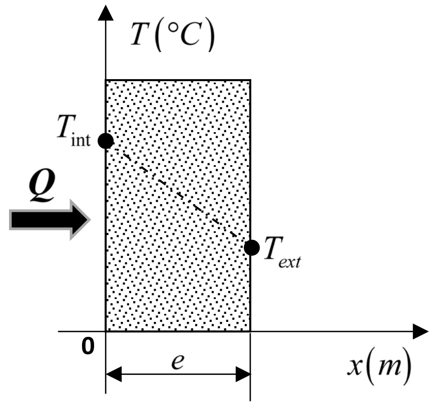
So: $T(x) = 0°C$

$$x = \frac{18-0}{18+15}.38.10^{-2} \Rightarrow x = 0,207m$$

Example II.2:

What is the steady-state temperature distribution within the walls of a house? Let's consider a scenario where the wall separates a room heated to 22°C from an interior space at 0°C. We will assume that there is negligible heat generation inside the wall (q = 0).

Solution:



$$\frac{d^2T}{dx^2}=0 \Rightarrow \frac{d\left(\frac{dT}{dx}\right)}{dx}=0 \Rightarrow \frac{dT}{dx}=A$$

$$dT = Adx \Rightarrow \int dT = \int Adx \Rightarrow \int dT = A\int dx$$

$$T(x)=Ax+B$$

The calculation of coefficients A and B involves the use of boundary conditions:

- For $x=0$:

$$T(x=0)=A(0)+B \Rightarrow B=T_{int}=22°C$$

- For $x=e$:

$$T(x=e)=A(e)+B \Rightarrow T_{ext}=A(20)+T_{int}=0=A20+22$$

$$\Rightarrow A=\frac{0-22}{20} \Rightarrow A=-\frac{22}{20}=-1{,}1(°C/cm)$$

Finally, the temperature gradient inside the wall is as follows: $\frac{dT}{dx}=-1{,}1(°C/cm)$

II.2.3 Flat Wall in Contact with Two Fluids:

We are considering a planar wall in contact with two fluids, and we know the temperatures of these fluids, T_{f1} and T_{f2}, in contact with the two wall surfaces.

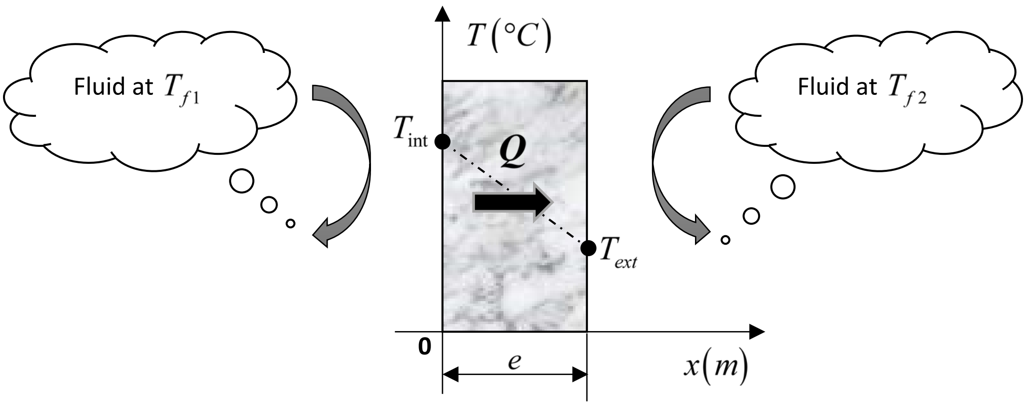


Fig.11.5: Flat Wall Adjacent to Two Fluids.

In a steady-state, heat flow is conserved when crossing the wall and can be expressed as follows:

$$Q=Q_{f1}=Q_{mur}=Q_{f2}$$

$$h_1S(T_{f1}-T_1)=\frac{(T_1-T_2)}{e/\lambda S}=h_2S(T_2-T_{f2})$$

$$Q=\frac{T_{f1}-T_{f2}}{\frac{1}{h_1 S}+\frac{e}{\lambda S}+\frac{1}{h_2 S}} \quad \text{(II.19)}$$

To determine the temperature at a distance x from the wall:

$$T_{f1}-T(x)=\left[R_{thf1}+R(x)\right]Q \qquad \text{Or} \quad R(x)=\frac{x}{\lambda S}$$

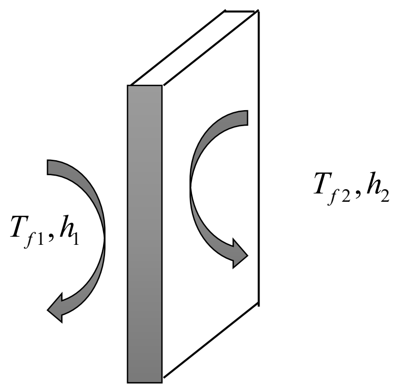
That's to say:

$$T(x)=T_{f1}-\left(R_{thf1}+\frac{x}{S}\right)\frac{T_{f1}-T_{f2}}{R_{thf1}+R_{th\_mur}+R_{thf2}} \quad \text{(II.20)}$$

Example II.3:

The wall of a heat exchanger is made of a 9.5mm thick copper plate. The heat exchange coefficients on the two sides of the plate are 2340 and 6100 $Kcal/hm^2°C$ corresponding, respectively, to the fluid temperatures of 82°C and 32°C. Assuming that the thermal conductivity of the wall is 344.5 $Kcal/hm^2°C$, evaluate the heat flux density and calculate the surface temperatures.

Solution :



$$Q=Q_{f1}=Q_{plaque}=Q_{f2}$$

- The heat flux density :

$$Q=\frac{T_{f1}-T_{f2}}{\frac{1}{h_1 S}+\frac{e}{\lambda S}+\frac{1}{h_2 S}}=80793Kcal/hm^2, \phi=93922,5W/m^2$$

- The temperatures T_1 et T_2 :

$$\phi=\frac{T_{f1}-T_1}{1/h_1}\Rightarrow T_1=47,5°C$$

$$\phi=\frac{T_2-T_{f2}}{1/h_2}\Rightarrow T_2=45,5°C$$

II.3.3 Multi-layer wall

We are considering a wall composed of several layers of different materials, and we only know the temperatures T_{f1} and T_{f2} of the fluids in contact with the two faces of the wall, with a lateral surface area S.

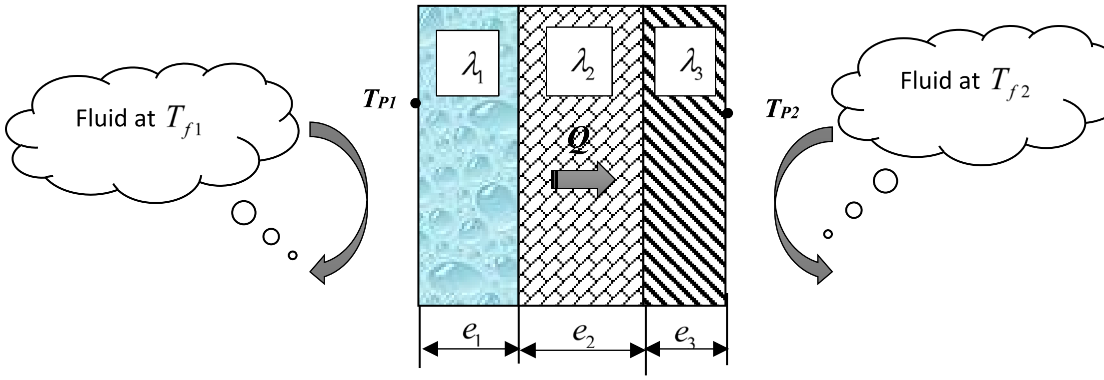


Fig.II.6: Multilayer wall adjacent to two fluids.

In a steady state, heat flow is conserved when crossing the wall and can be expressed as follows:

$$Q = Q_{f1} = Q_{layer1} = Q_{layer2} = Q_{layer3} = Q_{f2}$$

$$Q = h_1 S\left(T_{f1} - T_{P1}\right) = \frac{\left(T_{P1} - T_1\right)}{e_1 / \lambda_1 S} = \frac{\left(T_1 - T_2\right)}{e_2 / \lambda_2 S} = \frac{\left(T_2 - T_{P2}\right)}{e_3 / \lambda_3 S} = h_2 S\left(T_{P2} - T_{f2}\right)$$

$$Q = \frac{\left(T_{f1} - T_{f2}\right)}{\frac{1}{h_1 S} + \frac{e_1}{\lambda_1 S} + \frac{e_2}{\lambda_2 S} + \frac{e_3}{\lambda_3 S} + \frac{1}{h_2 S}} \quad \text{(II.21)}$$

In the case of an n-layer wall, we derive the expression for heat flow as follows:

$$Q = \frac{T_1 - T_{i+1}}{\sum_{i=1}^{n} \frac{e_i}{\lambda_i S}} \quad \text{(II.22)}$$

II.3.4 Composite wall

In practice, we often encounter walls composed of non-homogeneous layers that vary in height (see Figure II.7). Thus, each layer itself consists of different materials.

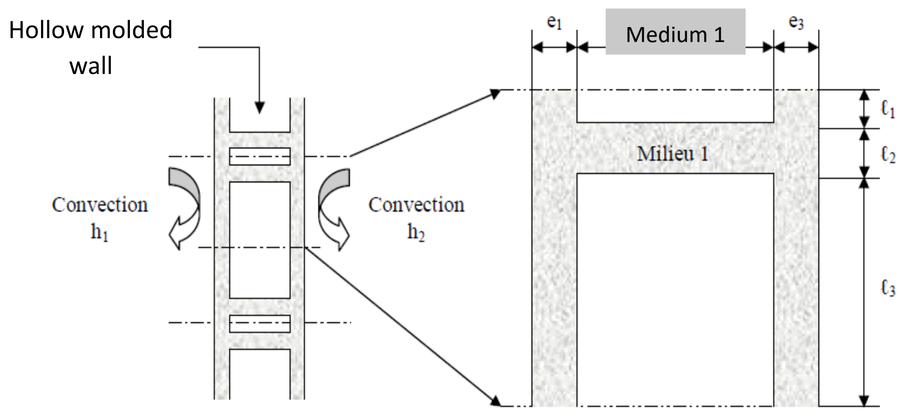


Fig. II 7: Composite wall diagram.

Using the laws of resistance association in series and in parallel to calculate the thermal resistance R_{th}, equivalent to a section of the wall with a width L and a height $l = l_1 + l_2 + l_3$, which can be expressed as follows:

$$R_{th} = R_1 + R_2 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} + R_6 + R_7 \quad \text{(II.23)}$$

with: $R_{th} = R_1 = \frac{1}{h_1 l L}, R_2 = \frac{e_1}{\lambda_1 l_1 L}, R_3 = \frac{e_2}{\lambda_2 l_1 L}, R_4 = \frac{e_2}{\lambda_1 l_2 L}, R_5 = \frac{e_2}{\lambda_2 l_3 L}, R_6 = \frac{e_3}{\lambda_1 l L}, R_7 = \frac{1}{h_2 l L}$

This can be represented by the equivalent electrical diagram shown in Figure II.8.

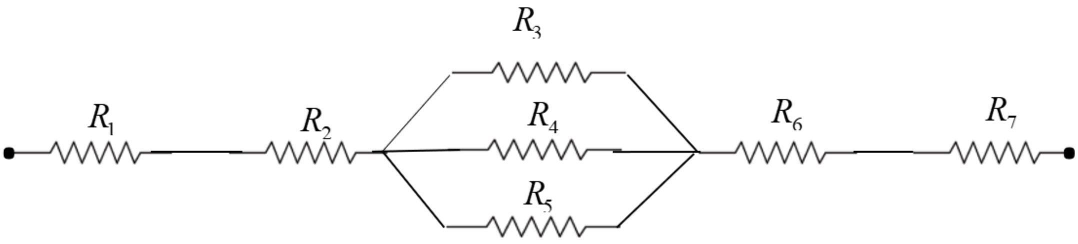


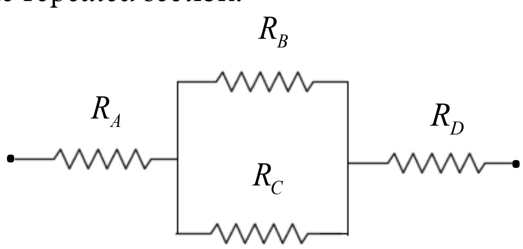
Fig. II.8: Equivalent electrical diagram of the composite wall.

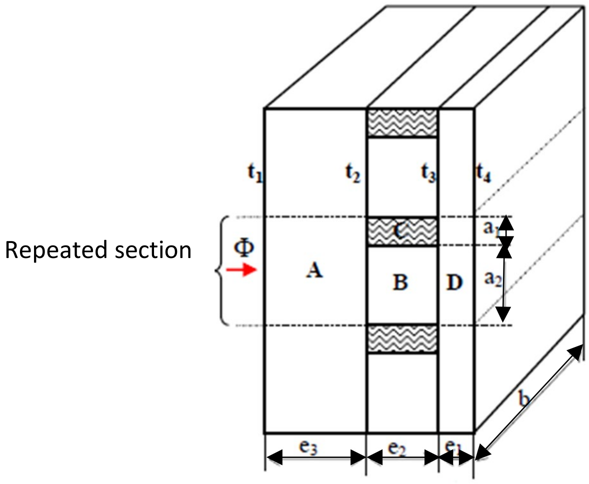
Example II.4:

A wall panel consists of three layers of different materials (see the figure below). The middle layer itself comprises repeating sections. Layer A has a thickness of 10 cm and $\lambda_1 = 0,45(W/m°C)$. Material B has a height of 37.2 cm, a thickness of 8.9 cm, and $\lambda_2 = 0,0251(W/m°C)$. Material C has a height of 3.8 cm, a thickness of 8.9 cm, and $\lambda_3 = 0,15(W/m°C)$. Layer D has a thickness of 1.3 cm, and $\lambda_4 = 0,814(W/m°C)$. The width of the wall is 3 m. Determine the heat flux transmitted through one of the repeated wall sections if $T_1 = 25°C$ and $T_4 = 0°C$.

<u>**Solution:**</u>

The equivalent electrical diagram from the repeated section:





Let's calculate the resistances of the different layers:

$$R_A = \frac{e_3}{\lambda_1 S_A} = \frac{0,1}{0,45(0,41.3)} = 0,18(°C/W)$$

$$R_B = \frac{e_2}{\lambda_2 S_B} = \frac{0,089}{0,251(0,373.3)} = 3,18(°C/W)$$

$$R_C = \frac{e_2}{\lambda_3 S_C} = \frac{0,089}{0,15(0,038.3)} = 5,2(°C/W)$$

$$R_D = \frac{e_3}{\lambda_4 S_D} = \frac{0,013}{0,814(0,41.3)} = 0,013(°C/W)$$

$$\Rightarrow R_{th\_éq} = R_A + \frac{1}{\frac{1}{R_B}+\frac{1}{R_C}} + R_D = 0{,}181 + \frac{1}{\frac{1}{3{,}18}+\frac{1}{5{,}2}} + 0{,}013 = 2{,}164\left(°C/W\right)$$

$$\Rightarrow Q = \frac{\left(T_1 - T_4\right)}{R_{th\_éq}} = \frac{25-0}{2{,}164} = 11{,}55\,W$$

II.3.5 Hollow Cylinder with Isothermal Side Surfaces

In the case where heat transfer occurs in a single direction, denoted as 'r,' and there are no heat sources, the heat conduction equation in cylindrical coordinates reduces to:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \qquad \text{(II.24)}$$

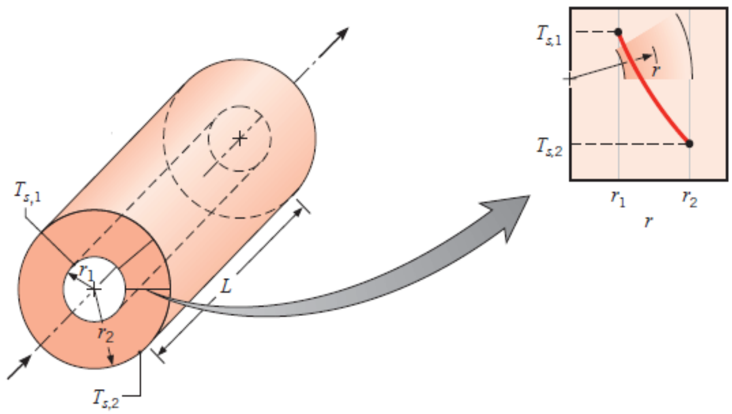


Fig.II.9: Conduction in a hollow cylinder with isothermal side surfaces.

The equation (II.24) becomes:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\int \frac{d}{dr}\left(r\frac{dT}{dr}\right)dr = r\frac{dT}{dr} = C_1$$

$$\int dT = C_1 \int \frac{dr}{r} \Rightarrow T\left(r\right) = C_1 \ln\left(r\right) + C_2$$

The integration constants C1 and C2 are determined from the following boundary conditions:

- For $r = r_1 : T\left(r_1\right) = T_1 = C_1 \ln\left(r_1\right) + C_2$ (a)
- For $r = r_2 : T\left(r_2\right) = T_2 = C_1 \ln\left(r_2\right) + C_2$ (b)

When : $\left(a\right)-\left(b\right):\ T_1 - T_2 = C_1\left(\ln\left(r_1\right) - \ln\left(r_2\right)\right) = C_1.\ln\left(\frac{r_1}{r_2}\right) \Rightarrow C_1 = \frac{\left(T_1 - T_2\right)}{\ln\left(\frac{r_1}{r_2}\right)}$

By substituting the expression of C_1 from (a), we obtain C_2 as follows:

$$C_2 = T_1 - \ln(r_1).\frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}$$

If we replace C_1 with its expression from (b), we can determine C_2 as follows:

$$C_2 = T_2 - \ln(r_2).\frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}$$

$$T(r) = \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}\ln(r) + T_1 - (T_1 - T_2).\frac{\ln(r_1)}{\ln\left(\frac{r_1}{r_2}\right)}$$

$$= \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}\ln\left(\frac{r}{r_1}\right) + T_1$$

$$\Rightarrow T(r) = T_1 + \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}.\ln\left(\frac{r}{r_1}\right) \qquad \text{(II.25)}$$

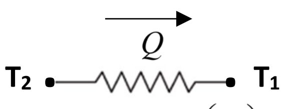
- The expression for the heat flux density is as follows:

$$\phi = -\lambda\frac{dT}{dr} = -\lambda\frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}.\frac{\frac{1}{r_1}}{\frac{r}{r_1}} = -\lambda\frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}.\frac{1}{r} \qquad \text{(II.26)}$$

- The expression for the heat flow is as follows:

$$Q = \phi.S = -\lambda\frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)}.\frac{1}{r}.2\pi rL = 2\pi\lambda L\frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \qquad \text{(II.27)}$$

- The thermal resistance for a hollow cylinder is given by $R_{th} = \frac{\ln\left(\frac{r_1}{r_2}\right)}{2\pi\lambda L}$, Which can be represented by the following electrical analog diagram:



$$R_{th} = \frac{\ln\left(\frac{r_1}{r_2}\right)}{2\pi\lambda L}$$

II.3.6 Multi-layer hollow cylinder:

Consider a tube covered with one or more layers of different materials, where we only know the temperatures T_{f1} and T_{f2} of the fluids in contact with the internal and external faces of the cylinder. h_1 and h_2 represent the heat transfer coefficients by convection between the fluids and the internal and external faces.

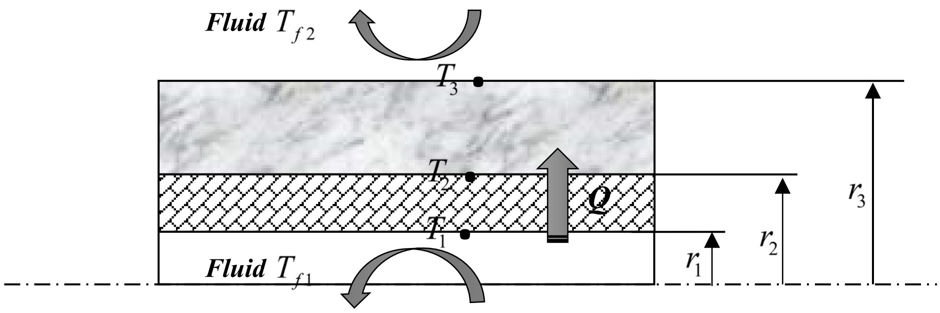


Fig.II.8: Conduction in a multilayer hollow cylinder.

In a steady state, heat flow (Q) is conserved as it passes through the various layers and is expressed as follows:

$$Q = Q_{f1} = Q_{couche1} = Q_{couche2} = Q_{f2}$$

$$Q = h_1 2\pi r_1 L\left(T_{f1} - T_1\right) = \frac{\left(T_1 - T_2\right)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1 L}} = \frac{\left(T_2 - T_3\right)}{\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2 L}} = h_2 2\pi r_3 L\left(T_3 - T_{f2}\right)$$

$$Q = \frac{\left(T_{f1} - T_{f2}\right)}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2 L} + \frac{1}{h_2 2\pi r_3 L}} \qquad \text{(II.28)}$$

The heat flow transmitted between fluid 1 and fluid 2 through the multilayer wall is expressed by the following relationship:

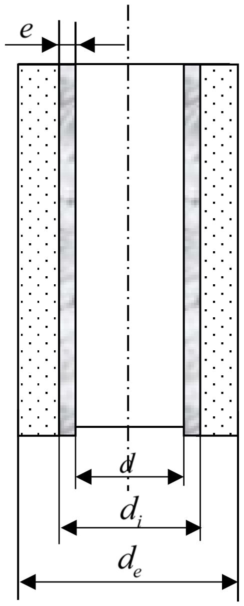
$$Q = \frac{\left(T_{f1} - T_{f2}\right)}{\frac{1}{h_1 2\pi r_1 L} + \sum_{i=1}^{n} \frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{2\pi\lambda_i L} + \frac{1}{h_2 2\pi r_{n+1} L}} \qquad \text{(II.29)}$$

Example II.4:

A reinforced concrete chimney with a thermal conductivity of 1.1 (W/m°C) has an internal diameter of 600 mm, and an external diameter of 1000 mm. It must be lined from the inside with a refractory material with a conductivity of 0.5 (W/m°C). Knowing that the losses do not exceed 2000 (W/m) and that the temperature of the interior surface of the reinforced concrete wall does not exceed 200°C, and the temperature of the interior surface of the refractory material is equal to 425°C, determine:

a) The thickness of the refractory material.

b) The temperature of the exterior surface of the chimney



Solution:

a) - The heat flow through the filling is expressed:

$$Q = \frac{T_p - T_{pi}}{R_{th}}$$

With: $R_{th} = \dfrac{\ln\left(\dfrac{d_i}{d}\right)}{2\pi\lambda_2 L}$

$$Q = \frac{T_P - T_{pi}}{\dfrac{\ln\left(\dfrac{d_i}{d}\right)}{2\pi\lambda_2 L}} \Rightarrow Q = \frac{2\pi\lambda_2\left(T_P - T_{pi}\right)}{\ln\left(\dfrac{d_i}{d}\right)};[W/m]$$

$$d = \frac{d_i}{\exp\left(\dfrac{\left(T_P - T_{pi}\right)2\pi\lambda_2}{Q}\right)} \Rightarrow d = \frac{600}{\exp\left(\dfrac{(425-200)2\pi.0,5}{2000}\right)} = 421,36(mm)$$

$$e = \frac{600 - 421,36}{2} = 89\,(mm)$$

b) - The temperature of the exterior surface of the chimney:

$$Q = \frac{2\pi\lambda_1}{\ln\left(\dfrac{d_e}{d_i}\right)}\left(T_{Pe} - T_{pi}\right) \Rightarrow T_{Pe} = T_{pi} - \frac{\ln\left(\dfrac{d_e}{d_i}\right)Q}{2\pi\lambda_1} = 200 - \frac{\ln\left(\dfrac{1000}{600}\right)2000}{2\pi 1,1} = 52°C$$

Example II.5:

A steel pipe with inner and outer radii of 10 cm and 11 cm, respectively, is used to transport superheated steam at 400 °C. To minimize heat loss, the pipe is insulated with two layers: a high-temperature-resistant insulator (expensive) is applied directly to the steel, followed by a second layer of plastic (more affordable but less resistant). The maximum allowable temperature for the plastic layer is 200°C, and its exterior surface reaches 50°C, with an external radius of 20 cm. The ambient air temperature is 20°C. Key parameters include convection coefficients 200 (W/m²K) for the Steel/Steam interface and 40 (W/m²K) for the plastic/Air interface. The steel pipe, plastic, and insulator conductivities are 60, 0.5, and 0.08 (W/mK), respectively.

Questions:

1) Why did we use two layers of insulation instead of just one? (For three reasons).
2) Calculate the heat flux lost per meter of length.
3) Calculate the thickness of each layer of insulation.
4) Calculate the temperature at the interior and exterior surfaces of the steel.

Solution:

1- We use two layers instead of one for the following reasons: good resistance to high temperatures, effective insulation, and reduced cost.

2- Calculation of heat flow per meter of length:

$\dot{Q} = h_e S(T_2 - T_\infty) = h_e(2\pi r_4 L)(T_4 - T_{air}) \quad AN: \dot{Q} = 1508W$.

3- Calculation of the thickness of the insulation layers:

- For plastic we have :

$$\dot{Q} = \frac{2\pi\lambda_P(T_3 - T_4)}{\ln\left(\frac{r_4}{r_3}\right)} \Rightarrow \ln\left(\frac{r_4}{r_3}\right) = \frac{2\pi\lambda_P(T_3 - T_4)}{\dot{Q}}$$

So : $r_3 = 14,64\ cm$, and the thickness of the plastic $e_p = 20 - 14,64 = 5,36\ cm$

- For the other insulator we have: $r_2 = 11cm,\ So: e_{is} = 14,64 - 11 = 3,64\ cm$

4- Calculation of temperature of the interior and exterior face of the steel:

$$\dot{Q} = h_i 2\pi r_1 L(T_{vap} - T_1)$$

We have:

$$\Rightarrow T_1 = T_{vap} - \frac{\dot{Q}}{h_i 2\pi r_1 L} \qquad AN: T_1 = 388°C$$

$$\Rightarrow \dot{Q} = \frac{2\pi\lambda_{ac}(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \Rightarrow T_2 = T_1 - \frac{\dot{Q}}{2\pi\lambda_{ac}}\ln\left(\frac{r_2}{r_1}\right) \qquad AN: T_2 = 387,6\ °C$$

II.3.7 Hollow sphere with isothermal surface:

With the previous assumptions and assuming that heat transfer occurs in a single direction (r), the equation in spherical coordinates (II-10) simplifies to:

$$\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2} = \frac{1}{r}\frac{d^2(rT)}{dr^2} = 0$$

$$\frac{1}{r}\frac{d^2(rT)}{dr^2} = \frac{1}{r}\frac{d}{dr}\left[\frac{d}{dr}(rT)\right] = \frac{1}{r}\frac{d}{dr}\left[T + r\frac{dT}{dr}\right]$$

$$\frac{1}{r}\frac{d}{dr}\left[T + r\frac{dT}{dr}\right] = \frac{1}{r}\left[\frac{dT}{dr} + \frac{d}{dr}\left(r\frac{dT}{dr}\right)\right]$$

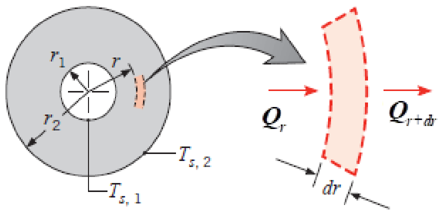


Fig.II.9: geometry of a hollow sphere.

$$\frac{1}{r}\left[\frac{dT}{dr} + \frac{dr}{dr}\cdot\frac{dr}{dr} + \frac{d^2T}{dr^2}\right] = 0$$

$$\frac{1}{r}\left[2\frac{dT}{dr} + \frac{d^2T}{dr^2}\right] = 0 \Rightarrow \frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

We can find the solution by changing variables.

Posing : $u = \frac{dT}{dr} \Rightarrow u' = \frac{d^2T}{dr^2}$

The equation becomes: $ru' + 2u = 0$

$$r\frac{du}{dr}+2u=0\Rightarrow\int\frac{du}{u}=-2\int\frac{dr}{r}\Rightarrow\ln(u)=-2\ln(r)+\ln(C_1)$$

$$\ln(u)=-\ln(r^2)+\ln(C_1)\Rightarrow\ln(u)=\ln\left(\frac{C_1}{r^2}\right)$$

$$\Rightarrow u=\frac{C_1}{r}=\frac{dT}{dr}\Rightarrow\int dT=C_1\int\frac{dr}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

To determine the constants C_1 and C_2, we must apply the following boundary conditions:

$$\text{à}\begin{cases}r=r_1\Rightarrow T(r_1)=T_1=-\dfrac{C_1}{r_1}+C_2\cdots\cdots(a)\\ r=r_2\Rightarrow T(r_2)=T_2=-\dfrac{C_1}{r_2}+C_2\cdots\cdots(b)\end{cases}$$

$(a)-(b)$ given:

$$(T_1-T_2)=-\frac{C_1}{r_1}+\frac{C_1}{r_2}=-C_1\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\Rightarrow C_1=-\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

Replacing C1 with its expression in (a), we find the expression for C2:

$$T_1=-\frac{C_1}{r_1}+C_2=\frac{\dfrac{T_1-T_2}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}}{r_1}+C_2\Rightarrow C_2=T_1-\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)r_1}$$

By introducing the two constants into the expression T(r) we obtain the final expression for the temperature:

$$T(r)=-\frac{C_1}{r_1}+C_2=\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}.\frac{1}{r}+T_1-\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)r_1}$$

$$T(r)=T_1+(T_1-T_2)\frac{\left(\frac{1}{r}-\frac{1}{r_1}\right)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

The expression for the heat flux density is given by:

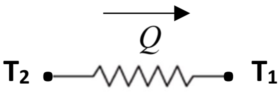
$$\frac{dT}{dr}=\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}.\left(-\frac{1}{r^2}\right)\Rightarrow\phi=-\lambda\frac{dT}{dr}\Rightarrow\phi=\lambda\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}.\frac{1}{r^2}\qquad\text{(II.30)}$$

The heat flux expression is given by:

$$Q=\phi S=-\lambda S\frac{dT}{dr}\Rightarrow\phi=4\pi r^2\lambda\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}.\frac{1}{r^2}\Rightarrow Q=4\pi\lambda\frac{(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}\qquad\text{(II.31)}$$

The expression for thermal resistance is given by: $R_{th}=\dfrac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi\lambda}$

Which can be represented by the following electrical analog diagram:



$$R_{th} = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi\lambda}$$

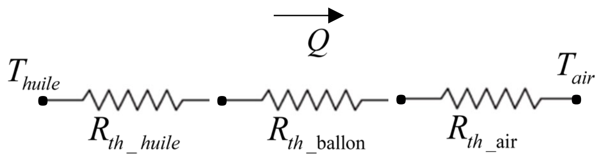
Example II.6:

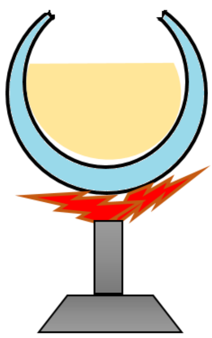
A flask used in the laboratory, containing engine oil, is subjected to a heat source (see figure below). The glass balloon, with a thermal conductivity of 0.80 W/m°C, is compared to a sphere whose interior and exterior diameters are respectively 20 cm and 21 cm. Knowing that the oil and the surrounding air have convection coefficients of 15 and 10 W/m²°C, respectively.

- Calculate the heat flux that must be supplied by the heat source to maintain a constant temperature of 80°C. We will assume the ambient temperature is 20°C, and we will neglect losses from the top of the tank.

Solution:

The heat input required to keep the oil temperature constant at 80°C must be at least equal to the losses experienced by the tank.





The heat flow is expressed by: $Q = \dfrac{T_{huile} - T_{air}}{R_{th\_huile} + R_{th\_ballon} + R_{th\_air}}$

$$Q = \frac{T_{huile} - T_{air}}{\frac{1}{h_{huile} S_{in}} + \frac{r_2^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi r_1^2} + \frac{1}{h_{air} S_{ex}}} = \frac{T_{huile} - T_{air}}{\frac{1}{h_{huile} 4\pi r_1^2} + \frac{r_2^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi r_1^2} + \frac{1}{h_{air} 4\pi r_2^2}}$$

$$Q = \frac{80 - 20}{\frac{1}{15.4\pi(0,1)^2} + \frac{1}{0,8.4\pi(0,1)^2} + \frac{1}{10.4\pi(0,105)^2}} = 46\,W$$

II.4 The fins:

Fins, effective heat conductors with one dimension larger than others, are employed in cooling electronic components and combustion engines. They aim to enhance heat exchange between a flat wall and an external fluid. Conduction facilitates transfer between the fin and the wall, while convection drives exchanges with the external fluid.

The fins are designed and manufactured in many shapes and forms, some of which are shown in Fig. II.12.

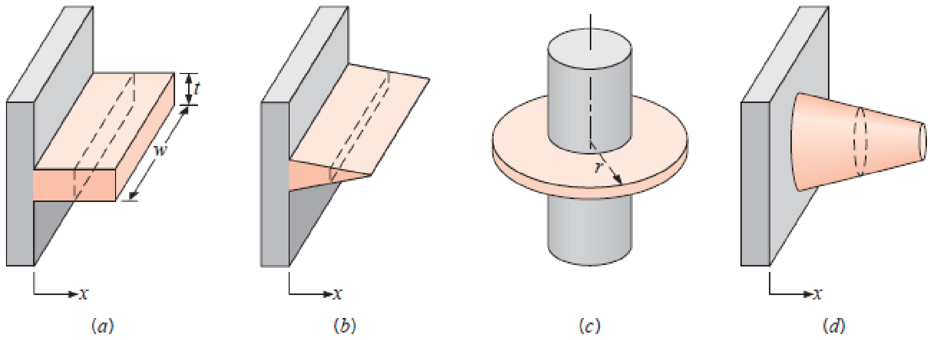
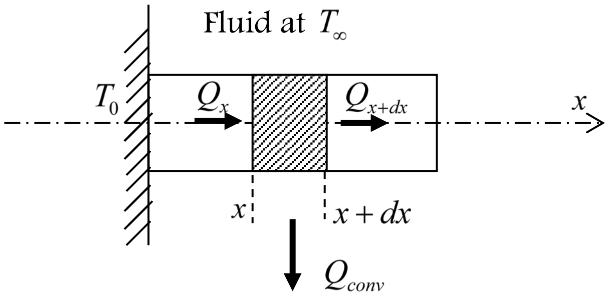


Fig.II.12: The different fin shapes.

The Hypotheses:

1) The Permanent Regime.
2) Neglecting Radiation.

The energy balance is conducted on the system constituted by the portion of the bar between the x and dx abscissae.



with:

Q_x : The heat flux transmitted by conduction to the abscissa x.

Q_{x+dx} : The heat flux transmitted by conduction at the abscissa x+dx.

Q_{conv} : The heat flux transmitted by convection to the periphery between x and x + dx.

Energy balance:

$$Q_x = Q_{x+dx} + Q_{conv} \tag{II.32}$$

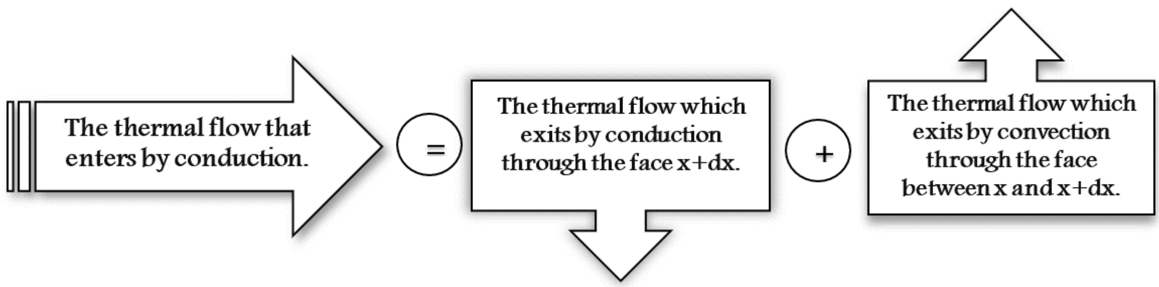
Such as:

$$Q_x = -\lambda S\left(\frac{dT}{dx}\right)_x \; ; \; Q_{x+dx} = -\lambda S\left(\frac{dT}{dx}\right)_{x+dx} \; ; \; Q_{conv} = hp.dx\left(T(x) - T_\infty\right) \; ; \; Q'_s = Q'_{\infty} S.dx$$

p: The perimeter of the straight section of the rod, $(p = \pi d)$

S: The surface of the cross-section of the rod, $(S = \pi d^2 / 4)$

That's to say:



$$-\lambda S\left(\frac{dT}{dx}\right)_x + \lambda S\left(\frac{dT}{dx}\right)_{x+dx} - hpdx\left[T(x)-T_\infty\right] + Q_s^{'}Sdx = 0$$

$$-\lambda S\frac{dT}{dx} + \lambda S\frac{dT}{dx} + \lambda S\frac{d}{dx}\left(\frac{dT}{dx}\right)dx - hpdx\left[T(x)-T_\infty\right] + Q_s^{'}Sdx = 0$$

After development, simplification and arrangement:

$$\frac{d^2T}{dx^2} - \frac{h.P}{\lambda S}\left[T(x)-T_\infty\right] + \frac{Q_s^{'}}{\lambda} = 0 \qquad (33)$$

Assuming $Q_s^{'} = 0$ and posing: $w^2 = \frac{hP}{\lambda S}, \theta = T - T_\infty$

We can write: $\frac{d^2\theta}{dx^2} - w^2\theta = 0$

If the section S is constant, it is a second-order equation with constant coefficients, and its general solution is of the form:

$$\theta(x) = A\exp(wx) + B\exp(-wx) \text{ Ou } \theta(x) = A_1 ch(wx) + B_1 sh(wx)$$

The quest for a fundamental solution to this heat exchange phenomenon relies on the boundary conditions linked to this element. Various scenarios may unfold.

The second boundary condition, specified at the fin tip, the free end of fin, that may correspond to any of four different physical situations given below:

- Case 1: The fin is very long, and the temperature at the fin tip approaches that of the surrounding fluid.
- Case 2: The finite long fin and with negligible heat loss from fin tip.
- Case 3: Finite long fin with convection heat loss from its fin tip.
- Case 4: The finite long fin with a specified temperature at its fin tip.

II.4.1 The fin is extremely long, and the temperature at the fin tip approaches that of the surrounding fluid:

In the scenario of an extended fin, our hypothesis is that:

$T(x = L) = T_\infty$ (*L*: fin length)

- Boundary conditions:

$$for\begin{cases} x = 0 \Rightarrow \theta(0) = T_0 - T_\infty \cdots\cdots(b) \\ x = L \Rightarrow \theta(L) = 0 \qquad \cdots\cdots(a) \end{cases}$$

Of (b) $\Rightarrow \quad A = 0$

and (a) $\Rightarrow \quad B = T_0 - T_\infty$

From where: $\quad \theta(x) = T_0 - T_\infty \exp(-wx)$

$T(x) - T_\infty = T_0 - T_\infty \exp(-wx)$, The dissipated flux over the entire surface of the fin can be calculated by integrating the local convection flux: $Q = \int_0^L hP\left[T(x) - (T_\infty)\right]dx$

In the case of steady-state conditions: $\quad Q = Q_{conv}(x = 0)$

$$\varphi_c = -\lambda S\left(\frac{dT}{dx}\right)_{x=L} = -\lambda S(T_0 - T_\infty)(-w)\exp(-wx)$$

With : $w = \sqrt{\dfrac{hP}{\lambda S}}$

$$Q = \sqrt{hP\lambda S}.(T - T_\infty) \qquad \text{(II.34)}$$

II.4.2 Constant Cross-Section Fin with Insulated Tip:

The general solution obtained is identical to the previous case.

With the boundary conditions:

$$\begin{cases} T(x=0) = T_0 \\ -\lambda S\left(\dfrac{dT}{dx}\right)_{x=L} = 0 \end{cases}$$

The solution is written as: $\dfrac{T(x) - T_\infty}{T_0 - T_\infty} = ch(wx) + th(wL)sh(wx)$

And the heat flux dissipated by the fin:

$$\varphi_P = w\lambda S.th(wL)(T_0 - T_\infty) \qquad \text{(II.35)}$$

II.4.3 Constant Cross-Section Fin with Heat Transfer at the Tip:

The boundary conditions:

$$\begin{cases} T(x=0) = T_0 \\ -\lambda S\left(\dfrac{dT}{dx}\right)_{x=L} = hS\left[T(x=L) - T_\infty\right] \end{cases}$$

The solution is written as: $\dfrac{T(x) - T_\infty}{T_0 - T_\infty} = \dfrac{ch[w(L-x)] + \left[\dfrac{h}{w\lambda} sh[w(L-x)]\right]}{ch(wL) + \dfrac{h}{w\lambda} sh(wL)}$

The heat flux dissipated by the fin:

$$Q = w\lambda S(T_0 - T_\infty)\frac{th(wL) + \dfrac{h}{w\lambda}}{1 + \dfrac{h}{w\lambda} th(wL)} \qquad \text{(II.36)}$$

II.4.4 Fin with Specific Temperature $T_{x=L}$:

The equation gives the temperature variation:

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = \frac{\left[\left(\dfrac{T_{x=L} - T_\infty}{T_0 - T_\infty}\right) sh(wx)\right] + shw(x-L)}{sh(wL)}$$

The heat flux dissipated by the fin:

$$Q=\sqrt{hp\lambda S}\left(T_0-T_\infty\right).\frac{chwL-\left(\frac{T_{x=L}-T_\infty}{T_0-T_\infty}\right)}{sh\left(wx\right)} \quad \text{(II.37)}$$

II.4.5 Fin with Infinite Length Tip $\left(T=T_\infty \ à \ x=\infty\right)$:

The temperature variation is expressed as:

$$\frac{T\left(x\right)-T_\infty}{T_0-T_\infty}=\exp\left(-wx\right)$$

The heat flux dissipated by the fin:

$$Q=\sqrt{hp\lambda S}\left(T_0-T_\infty\right) \quad \text{(II.38)}$$

II.4.6 Calculation of Fin Performance:

Fin performance can be evaluated using two main criteria: efficiency and effectiveness. ***a)-Efficiency*** is the ratio between the heat flux dissipated by the extended surface and the heat flux that would be dissipated without the fin. Denoted by the letter « ε », it is generally greater than or equal to 2 ($\varepsilon \geq 2$).

$$\varepsilon=\frac{Q_{ailette}}{Q_{c\_ailette}}=\frac{Q_{ailette}}{S_c h\left(T_{base}-T_\infty\right)} \quad \text{(II.39)}$$

Where: S_c is the cross-sectional area at the base of the fin.

For an infinitely long fin, the efficiency is given by the following equation: $\varepsilon=\sqrt{P\lambda / hS_c}$

b)- Fin effectiveness: It is the ratio of the heat flux dissipated by the fin to the maximum possible flux. The maximum dissipated flux is calculated by assuming that the temperature along the entire lateral surface is constant at:

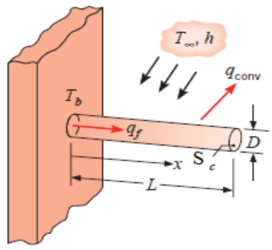
Where: S_s is the lateral surface area of the fin.

The following equation expresses the effectiveness: $\eta=\frac{Q_{ailette}}{Q_{max}}$

For a fin with an infinite-length tip: $\eta=\sqrt{\frac{P\lambda S_c}{hS_s}}$

c)- Fin Dimensions :

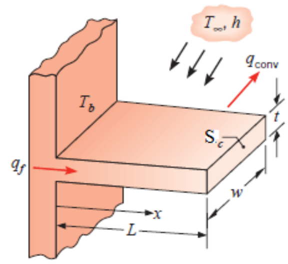
- *Cylindrical Fin* :



$P=\pi D$

$S_c=\frac{\pi D^2}{4}$

- *Rectangular Fin* :



$P=2w+2t$

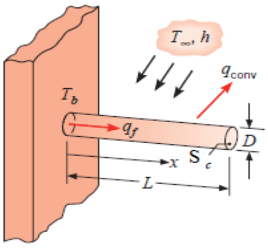
$S_c=wt$

Exercise 01:

An aluminum rod with a thermal conductivity of 200 W/mK, a constant cross-section of 4 cm in diameter, and a length of 13 cm is embedded in a wall maintained at a temperature of 238°C (see the adjacent figure). The rod is exposed to an environment at 21°C. The heat transfer coefficient by convection is 14 W/m²K.

- Calculate the heat flux lost by this rod.

Solution:



$$Q = \lambda w S.(T_0 - T_\infty).\frac{th(wL) + \dfrac{h}{\lambda w}}{1 + \dfrac{h}{\lambda w}.th(wL)}$$

$$S = \frac{\pi D^2}{4} = \frac{\pi (0,04)^2}{4} = 0,001256\ m^2$$

$$w = \sqrt{\frac{hP}{\lambda S}} = \sqrt{\frac{14.\pi.0,04^2}{200.0,001256}} = 2,645\ m^{-1}$$

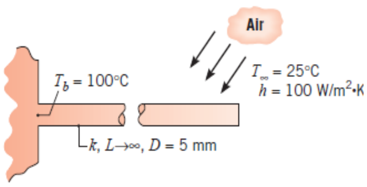
$$\frac{h}{\lambda w} = \frac{14}{200.2,645} = 0,02645$$

$$Q = 200.2,6464.0,001256(238-21).\frac{th(2,6464.0,13) + 0,02645}{1 + 0,02645 th(2,6464.0,13)} = 51,1243\ W$$

Exercise 02:

A very long rod with a diameter of 5 mm has an infinite tip maintained at a temperature of 100°C. The surface of the rod is exposed to ambient air at 25°C, with a heat transfer coefficient by convection of 100 W/m²K. Assuming that this rod is made of a copper alloy with a thermal conductivity of 398 W/mK:

- Determine the temperature distribution along the rod.
- Determine the heat flux lost by the rod.



Solution :

1- According to the assumption of an infinitely long fin, the temperature distribution is determined from the following expression:

$$T(x) = T_\infty + (T_b - T_\infty).\exp(-wx)$$

$$w = \sqrt{\frac{hP}{\lambda S_c}} = \sqrt{\frac{4h}{\lambda D}} = \sqrt{\frac{4.100}{398.5.10^{-3}}} \Rightarrow w = 14,177$$

$$T(x) = 25 + (100-25).\exp(-14,177x) \Rightarrow T(x) = 100.\exp(-14,177x).$$

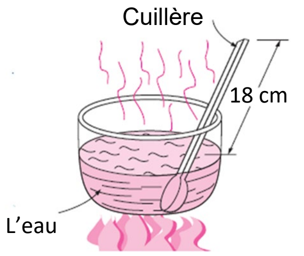
2- Determination of the heat flux through the rod:

$$Q = \sqrt{hP\lambda S}.(T_0 - T_\infty) \Rightarrow Q = \sqrt{100.\pi.D.\lambda(\pi D^2)/4}.(100-25)$$

$$Q = \sqrt{100.\pi^2.D^3.\lambda/4}.(100-25) \Rightarrow Q = \sqrt{100.\pi^2.(0,005)^3.398/4}.(100-25) \Rightarrow Q = 8,3W$$

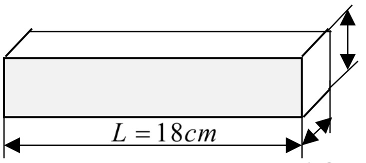
Exercise 03:

Let's take a stainless-steel spoon (λ = 15 W/m°C), partially immersed in boiling water at 93°C in a kitchen at 24°C. The spoon handle has a cross-sectional area of 0.2 x 1.3 cm and extends 18 cm into the air from the free surface of the water. If the heat transfer coefficient at the surface of the spoon exposed to the air is 17 W/m²°C:



☞ State your assumptions. Determine the temperature difference at the surface of the spoon's handle, knowing that the following relation expresses the temperature variation at the tip of the spoon:

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh w(L-x)}{\cosh(wL)}$$



Solution :

Assumptions :

1- The temperature of the immersed part of the spoon is equal to the temperature of the water.
2- The temperature varies along the spoon as T(x).
3- The heat transfer at the tip of the spoon is negligible.
4- The heat transfer coefficient is constant and uniform across the entire surface of the spoon.
5- The thermal properties of the spoon are constant.
6- Heat transfer by radiation is assumed to be negligible.
7- The cross-sectional area of the spoon is constant, and the temperature variation along the spoon can be expressed as: $\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh w(L-x)}{\cosh(wL)}$

Or :

$$P=2(l+t)=2\left(0,2.10^{-2}+1,3.10^{-2}\right)\Rightarrow P=0,030\,m$$

$$S_c=\left(0,2.10^{-2}\right).\left(1,3.10^{-2}\right)=0,000026\,m^2$$

$$w=\sqrt{\frac{hP}{\lambda S_c}}=\sqrt{\frac{17.0,030}{0,000026.15}}=36\,m^{-1}$$

The temperature at the tip of the spoon is determined by:

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh w(L-x)}{\cosh(wL)}$$

$$\frac{T(x=L)-T_\infty}{T_b-T_\infty}=\frac{\cosh w(L-L)}{\cosh(wL)}\Rightarrow T(L)=T_\infty+(T_b-T_\infty).\frac{1}{\cosh wL}$$

AN :

$$T(L)=24+(93-24).\frac{1}{\cosh(36.0,18)}=24,2°C$$

Therefore, the temperature difference across the entire exposed handle of the spoon is:

$$\Delta T=T_b-T(L)=93-24,2=68,8°C$$

Chapter III

TRANSIENT HEAT CONDUCTION

III.1 General presentation:

The general conduction equation, in the case where there is no internal heat generation, is given as:

$$\lambda \nabla^2 T = \rho C_P \frac{\partial T}{\partial t}, \quad \lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) = \rho C_P \frac{\partial T}{\partial t} \qquad \text{(III.1)}$$

If the heat flux is unidirectional, the partial differential equation is:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \qquad \text{Where} \qquad a = \frac{\lambda}{\rho C_P}$$

Practical problems most commonly encountered in transient conduction are divided into two main groups:

- Processes that tend to reach a thermal equilibrium state; for example: quenching of a mechanical part.
- Periodic thermal processes; Example: internal combustion engine components undergo periodic heating and cooling.

In this study, we will consider the first case, where the process tends to reach thermal equilibrium over time.

Sometimes, in less precise practical calculations, it is assumed that the internal resistance of the studied body is negligible, and the temperature is uniform throughout the body (applicable for thin bodies). However, in precise calculations, it is important to determine the temperature at different points in the body.

III.2 Dimensionless Numbers:

Two numbers are particularly important in transient conduction: the Biot number and the Fourier number.

III.2.1 Biot Number:

The Biot number expresses the ratio between the internal thermal resistance of the medium and the external thermal resistance.

$$Bio = \frac{Internal\ resistance}{External\ resistance}$$

III.2.2 Fourier Number:

The Fourier number expresses the ratio between the rate of heat transfer and the rate of heat storage.

$$F_0 = \frac{heat\ \ flow\ rate\ througth\ a\ surface\ A}{Velocity\ of\ \ storage\ in\ the\ volume\ \ V}$$

Thermal Balance:

$Q_{cond} = Q_{conv}$

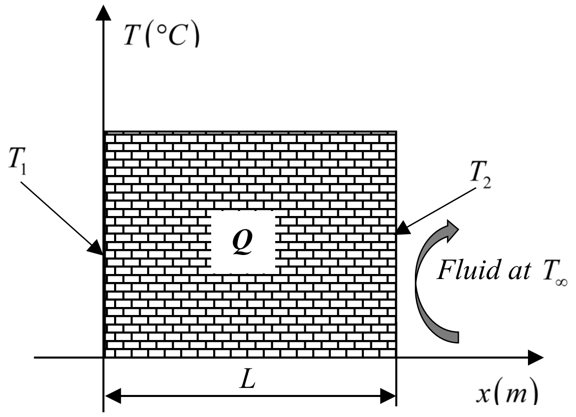
$-\lambda S \frac{dT}{dx} = hS(T_2 - T_\infty)$

$-\lambda S dT = LhS(T_2 - T_\infty)$

$-\lambda S(T_1 - T_2) = LhS(T_2 - T_\infty)$

$\frac{(T_2 - T_1)}{(T_2 - T_\infty)} = \frac{LhS}{\lambda S}$

$\frac{LhS}{\lambda S} = \frac{\frac{LhS}{L}}{\frac{\lambda S}{L}} = \frac{\frac{L}{\lambda S}}{\frac{1}{hS}} = \frac{R_{th\_cond}}{R_{th\_conv}} = \frac{Lh}{\lambda}$



The Biot number is defined as: $Bio = \frac{Lh}{\lambda}$

- If $Bio \to 0$ so $(Bio < 0,1)$ internal resistance is negligible, and the body cools or heats uniformly.
- If $Bio \to \infty$ so $(Bio > 100)$ convection resistance is negligible.

A characteristic length for the solid-fluid system is defined $L^* = \frac{V}{S}$.

For a plate of thickness e, the study of the variable regime can be divided into two groups:

- Those of thermally thin bodies, $(Bio < 0,1)$ $(h \ et \ L^* \ weak \ and \ \lambda \ is \ grand)$
- Those of thermally thick bodies, $(Bio > 100)$ $(h \ et \ L^* \ grand \ et \ \lambda \ it's \ weak)$

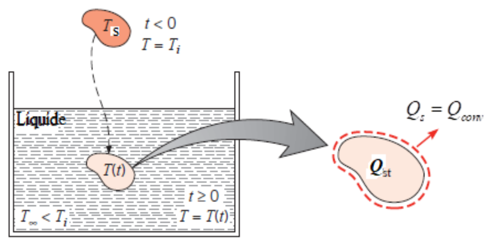
III.3 Thermally Thin Medium:

a) Quenching of a Body:

This problem assumes negligible internal thermal resistance. $(R_{th} = 0)$.

Assumptions :

- The temperature is the same throughout the solid object (heat conduction in the solid is very good, meaning $\lambda \to \infty$).
- $T = T(t)$ Control Volume: It is the entire object.



Thermal balance :

$$Q_e + Q_g = Q_s + Q_{st} \quad \text{(III.3)}$$

According to the assumptions $(Q_e = Q_g = 0)$, the balance equation reduces to:

$$-Q_s = Q_{st}$$

$$-hS(T_s - T_\infty) = \rho C_p \frac{dT}{dt} \quad \text{(III.4)}$$

where:

S: Exchange Surface or Separation Surface.

at $t=0$: $T_s = T_0$ Let us assume: $\begin{cases}\theta = T_S - T_\infty \\ \theta_0 = T_0 - T_\infty\end{cases}$

Equation (III.4) becomes:

$$-hS\theta = \rho C_P V \frac{d\theta}{dt} \qquad \text{This is a differential equation.}$$

$$\theta = \frac{\rho C_P V}{-hS}\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{\theta} = -\frac{hS}{\rho C_P V}dt$$

By integrating this equation, we obtain:

$$\int_{\theta_0 = T_i - T_\infty}^{\theta} \frac{d\theta}{\theta} = -\frac{hS}{\rho C_P V}\int_0^t dt \Rightarrow \ln\left(\frac{\theta}{\theta_0}\right) = -\frac{hS}{\rho C_P V}t$$

Therefore: $\frac{\theta}{\theta_0} = \exp\left(-\frac{hS}{\rho C_P V}t\right)$, Let us set $\tau = \frac{\rho C_P V}{hS}$ (thermal time constant)

Then :

$$\frac{\theta}{\theta_0} = \exp\left(-\frac{1}{\tau}t\right) \qquad \text{(III.5)}$$

We can express the Biot number in the exponent:

$$\frac{hS}{\rho C_P V}t = \frac{hS}{\rho C_P V}.\frac{\lambda.L^*}{\lambda.L^*} = \frac{hL^*}{\lambda}\frac{\lambda S}{\rho C_P V L^*}; a = \frac{\lambda}{\rho C_P}, et\ V = SL^*$$

$$\frac{hS}{\rho C_P V}t = Bio.\frac{a}{L^{*2}}t \Rightarrow \frac{hS}{\rho C_P V}t = Bio.F_0$$

Where: $F_0 = \frac{a}{L^{*2}}t$, This is the dimensionless Fourier number.

Thus, the general solution of the differential equation can also be written as:

$$\frac{\theta}{\theta_0} = \exp(-Bio.F_0), \text{ where } \frac{T - T_\infty}{T_0 - T_\infty} = \exp(-Bio.F_0) \qquad \text{(III.6)}$$

b) Quantity of heat exchanged with the medium:

The energy exchanged by the solid between the initial moment and a moment t can be written as:

$$Q = \int_0^t hS\theta dt = \theta_0 hS \int_0^t \exp\left(-\frac{hS}{\rho C_P V}t\right)dt$$

$$\frac{\theta}{\theta_0} = \exp\left(-\frac{hS}{\rho C_P V}t\right), \quad donc: \theta = \theta_0 \exp\left(-\frac{hS}{\rho C_P V}t\right)$$

$$Q = hS\theta_0\left(-\frac{hS}{\rho C_P V}\right).\left[\exp\left(-\frac{hS}{\rho C_P V}t\right)\right]_0^t$$

$$\Rightarrow Q(t) = \rho C_P V\left(T_i - T_\infty\right) = \left[1 - \exp\left(-\frac{1}{\tau}t\right)\right]$$

if $t \to 0$:

$$Q = \rho C_P V\left(T_0 - T_\infty\right) \quad \text{(III.7)}$$

Table III.1: Example of characteristic dimensions of some bodies.

Geometric shape of the body	Characteristic dimension L*
Long cylinder with radius « r»	L
Sphere with radius « r»	$r/2$
Cube with side length « a»	$r/3$
Long cylinder with radius « r»	$a/6$

Example III.1:

When taken out of the refrigerator, an apple was at a temperature of 4°C. For consumption, it is left in the open air at a temperature of 23°C. How long must one wait for its temperature to reach 20°C? The convection coefficient between the apple and the air is 6 W/m²°C. The apple is spherical with a diameter of 10.5 cm. Its thermal conductivity coefficient is 2.47 W/m°C. Its density is 998 kg/m³, and the specific heat is 2 kJ/kg°C.

For the sphere : $\begin{cases} V = \frac{4}{3}\pi r^3 \\ S = 4\pi r^2 \end{cases}$

Solution :

First, let's calculate the Biot number (Bi) to determine whether the internal resistance is negligible or not for solving this problem.

Since we are dealing with a sphere, the characteristic length is: $L^* = \frac{r}{3}$

$Bio = \frac{hL^*}{\lambda} = 0,0425 < 0,1 \Rightarrow$ The internal resistance is negligible.

Thus, we apply the following expression: $\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-Bio.F_0\right)$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-Bio.F_0\right) = \frac{20-23}{4-23} = 0,15789$$

$$\ln\left(0,15789\right) = -1,84585 = -Bio.F_0$$

$$F_0 = 43,4317 = \frac{a\tau}{L^{*2}},\ a = \frac{\lambda}{\rho C_P} = 1,237.10^{-6}\left(m^2/s\right)$$

$$\Rightarrow \tau = 10752,172s \simeq 179,2\min \simeq 2,9h$$

Example III.2:

Determine the time required for a small piece of aluminum, initially at 16 °C, to be heated to 510 °C by the gases of a blast furnace at 1204 °C. The characteristic dimension of the piece t is equal to 15 cm, and the convection coefficient between the piece and the gases is 85 W/m²K. The thermal conductivity of the aluminum alloy is taken as 210 W/mK, the density of aluminum is 2700 kg/m³, and the specific heat is 940 J/KgK.

Solution :

- Calculate the Biot number :

$$Bio = \frac{hL^*}{\lambda} = \frac{85.0,15}{210} = 0,0607 < 0,1 \Rightarrow$$ *The internal resistance is negligible.*

Let's take the formula for a solid subjected to an impulse response.

$$\frac{T(t)-T_\infty}{T_0-T_\infty} = \exp\left(-\frac{1}{\tau}t\right) \Rightarrow t = -\tau \ln\left(\frac{T(t)-T_\infty}{T_0-T_\infty}\right)$$

Calculate τ :

$$\tau = \frac{\rho C_P V}{hS} = \frac{\rho L^* C_P}{h} = \frac{2700.0,15.940}{85} = 4479s$$

Calculate t :

$$t = -\tau \ln\left(\frac{T(t)-T_\infty}{T_0-T_\infty}\right) = -(4479)\ln\left(\frac{510-1204}{16-1204}\right) = 2408s = 0,668h$$

Example III.3:

Determine the evolution of the temperature of a copper wire with a diameter of 0.8 mm, initially at a temperature of 149°C, when it is immersed in water with a convection coefficient equal to 73 Kcal/hm²°C.

- Plot the temperature evolution curve as a function of time. The properties of copper are given: $\lambda = 322(Kcal/hm°C)$,$C_P = 0,091(Kcal/Kg°C)$ $\rho = 8940Kg/m^3$

Solution :

- Calculate the Biot number:

$$L^* = \frac{V}{S} = \frac{\pi DL}{\pi\frac{D^2}{4}L} = \frac{D}{4} = \frac{0,8}{4} = 0,2\ mm$$

$$Bio = \frac{hL^*}{\lambda} = \frac{73,4.0,2.10^{-3}}{322} = 4,55.10^{-5} << 0,1 \Rightarrow$$ Is strictly less than, therefore the body is thermally thin, and the internal resistance is negligible.

$$\rho C_P V\frac{dT}{dt} = hS(T_\infty - T_r) \Rightarrow \frac{dT}{(T_\infty - T_r)} = -\frac{hS}{\rho C_P V}dt$$

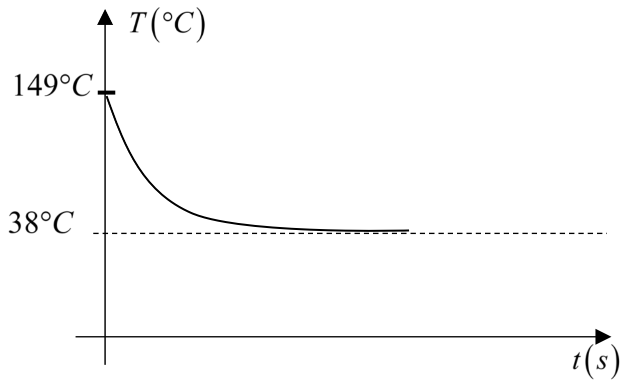
The solution of the differential equation: $\ln\left(T_r - T_\infty\right) = -\dfrac{hS}{\rho C_P V}t + C_1$

$$at\ t \to 0 :\Rightarrow T_r = T_0$$
$$\Rightarrow \ln\left(T_0 - T_\infty\right) = C_1$$
$$\tau = -\frac{hS}{\rho C_P V}$$
$$\ln\left(T_r - T_\infty\right) = -\tau t + \ln\left(T_0 - T_\infty\right)$$
$$\ln\left(\frac{T_r - T_\infty}{T_0 - T_\infty}\right) = -\tau t$$

$$\boldsymbol{AN:}\tau = \frac{hS}{\rho C_P V} = \frac{4.73,4}{0,8.10^{-3}.8940.0,021} = 4511,11$$

From the balance equation, we obtain:

$$T_r - T_\infty = \left(T_0 - T_\infty\right)\exp\left(-\tau t\right)$$
$$T\left(r\right) = T_\infty + \left(T_0 - T_\infty\right)\exp\left(-\tau t\right)$$
$$T_0 = 149°C\ et\ T_\infty = 38°C$$
$$T\left(r\right) = 38 + 111\exp\left(-4511,11t\right)$$



III.4 Thermally Thick Medium:

The study of heat transfer by conduction, in transient regime, for the case of non-negligible internal resistance, aims to determine the temperature distribution of a body as a function of time. The energy equation, in one dimension, takes the following form:

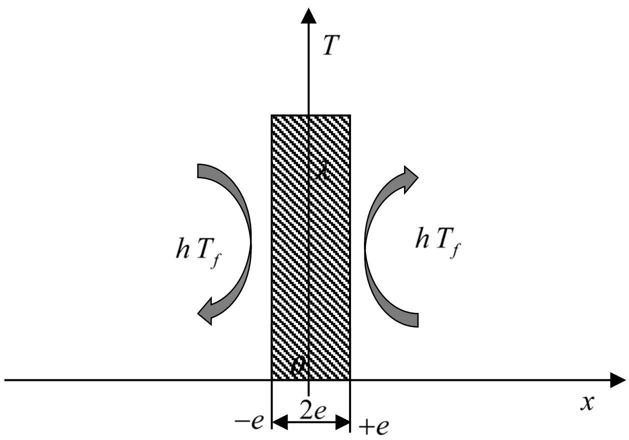
$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \qquad \text{(III.8)}$$

It is a second-order linear homogeneous differential equation with partial derivatives. This physical-mathematical equation can be solved using both classical analytical methods and approximate numerical methods. We will use the analytical method of separation of variables to solve a concrete problem.

III.4.1 Conduction dans une plaque indéfinie

Consider a plate of infinite dimensions along the y and z axes. The physical properties of the plate are constant: $\left(\rho, C_P, \lambda\right)$. At the initial time t_0, the plate, which is at temperature T_0, is immersed in a liquid at temperature: $\left(T_f > T_0\right)$.

Le coefficient de transfert de chaleur par convection h est constant. La distribution de température est unidimensionnelle car y et z sont très grand par rapport à x.



The symmetry of the boundary conditions with respect to the midplane ensures that at all times the thermal field is also symmetric about this plane. Thus, we set the origin of the coordinates at the center of the plate. For the convenience of calculation, the temperature is measured from the temperature of T_f. Hence, we can write : $\theta = T - T_f$

Given the mathematical formulation of this problem, the following expressions:

$$\frac{\partial T}{\partial t} = -\frac{\partial \theta}{\partial t} \text{ , et } \frac{\partial^2 T}{\partial x^2} = -\frac{\partial^2 \theta}{\partial t}$$

The heat equation is expressed in the following form:

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2}$$

- The initial condition : $\begin{cases} t = 0 \Rightarrow \theta = \theta_0 = T_f - T_0 \\ t = \infty \Rightarrow \theta \to 0, For : -e \le x \le +e \end{cases}$
- The boundary conditions :
  - For : $x = +e : \Rightarrow -\lambda\frac{\partial \theta}{\partial x} = h\theta \Rightarrow \left(\frac{\partial \theta}{\partial x}\right)_{x=+e} = -\frac{h\theta}{\lambda}$
  - For : $x = -e : \Rightarrow -\lambda\frac{\partial \theta}{\partial x} = h\theta \Rightarrow \left(\frac{\partial \theta}{\partial x}\right)_{x=-e} = \frac{h\theta}{\lambda}$
  - For : $x = 0 : \Rightarrow \frac{\partial \theta}{\partial x} = 0$

We apply the method of separation of variables to solve this equation: the sought function θ is in the form of a product of two functions:

$$\theta = T(t).\psi(x)$$

$$\psi(x)\frac{\partial T(t)}{\partial t} = a\frac{\partial^2 \psi}{\partial x^2}T(t) \tag{III.9}$$

By separating the variables, we obtain:

$$\frac{1}{a}\frac{T^{'}}{T} = \frac{\psi^{''}}{\psi}$$

If we fix the argument x and vary t, for every value of t, the left side will always be constant, and vice versa.

$$\frac{1}{a}\frac{T'}{T} = C_{const} \; et \; \frac{\psi''}{\psi} = C_{const}$$

By setting: $C_{const} = -w^2$ to ensure that $(\theta \to 0 \; et \; t \to \infty)$

$$\frac{1}{a}\frac{T'}{T} = -w^2 \; et \; \frac{\psi''}{\psi} = -w^2$$

The solution to these equations gives:

$$T(t) = C_1 \exp(-aw^2 t)$$
$$\psi(x) = C_2 \sin(wx) + C_3 \cos(wx) \quad \text{(III.10)}$$

After a lengthy demonstration, we finally obtain the expression that expresses the temperature as a function of time:

$$\theta^* = \frac{2\sin\gamma}{\gamma + \sin\gamma\cos\gamma}\cos(\gamma\overline{X}).\exp(-\gamma^2 F_0) \quad \text{(III.11)}$$

Where :

$\theta^* = \frac{\theta}{\theta_0} = \frac{T_f - T}{T_f - T_0}$: The temperature in a dimensionless form.

γ: Root of the equation.

$\tan\gamma = Bio$: Data from table III.2 as a function of Bio

$F_0 = \frac{at}{e^2}$: Fourier criterion. $a = \frac{\lambda}{\rho C_P}$: Thermal diffusivity

$\overline{X} = \frac{x}{e}$: Dimensionless spatial coordinate, it represents on the X-axis the position of the point for which we are calculating the temperature.

We set: $Y = \frac{2\sin\gamma}{\gamma + \sin\gamma\cos\gamma}$, Y is given in Table III.2 as a function of Bio.

$$\Rightarrow \theta^* = \frac{\theta}{\theta_0} = \frac{T_f - T}{T_f - T_0} = Y\cos(\gamma\overline{X})\exp(-\gamma^2 F_0) \quad \text{(III.12)}$$

This solution is valid for: $(F_0 \geq 0,3)$

- *Case of the temperature at the center of the plate*: $\left(\overline{X} = \frac{x}{e} = \frac{0}{e} = 0\right)$

$$\theta^* = \frac{\theta}{\theta_0} = \frac{T_f - T}{T_f - T_0} = Y\cos(\gamma 0)\exp(-\gamma^2 F_0) = Y\exp(-\gamma^2 F_0) \quad \text{(III.13)}$$

- *Case of the temperature at the surface of the plate:* $\left(\overline{X} = \frac{x}{e} = \frac{e}{e} = 1\right)$

$$\theta^* = \frac{\theta}{\theta_0} = \frac{T_f - T}{T_f - T_0} = Y\cos(\gamma)\exp(-\gamma^2 F_0) \quad \text{(III.14)}$$

Example III.4:

To cook the potato, we use the oven of the stove. We turn on the oven, and when its temperature reaches 176.6°C, we introduce the potato, which is at a temperature of 18.4°C. The convection coefficient between the surface of the potato and the hot air in the oven is equal to 13.135 W/m²K. We assume that the potato is spherical with a diameter of 7.62 cm. The thermal conductivity coefficient of the potato is 0.5 W/mK, and its thermal diffusivity coefficient is equal to 1.33 x 10^{-7} m²/s. Calculate the cooking time of the potato if we consider it cooked when its temperature reaches 112.7°C.

Solution :

☞ First, we calculate the Biot number to determine in which case (negligible internal resistance or not) this problem can be solved.

Given that the potato has a spherical shape: $L^* = \dfrac{r}{3}$

$Bio = \dfrac{hL^*}{\lambda} = \dfrac{h(r/3)}{\lambda} = \dfrac{13{,}135.1{,}27.10^{-2}}{0{,}5} = 0{,}3 > 0{,}1 \Rightarrow$ Non-negligible internal resistance; the potato is cooked when the temperature at the center is equal to $112{,}6°C \Rightarrow X^* = 0$

$$\theta^* = Y\exp\left(-\gamma^2 F_0\right), et\ \theta^* = \frac{T - T_f}{T_0 - T_f} = 4{,}404$$

According to the table, for the case of the sphere: $\left(L = r_0\right)$

$$Bio = \frac{hL^*}{\lambda} = \frac{hr_0}{\lambda} = \frac{13{,}135.3{,}81.10^{-2}}{0{,}5} = 1{,}00 \Rightarrow \gamma = 1{,}5708\ \ et\ Y = 1{,}2732$$

$$\Rightarrow F_0 = a\frac{t}{L^{*2}} = a\frac{t}{r_0^2} = 0{,}4653 \Rightarrow t = 5076\ s = 1{,}41\ heurs$$

Table III.2: Value of the coefficients as a function of Bio.

	Infinite flat wall $L=e$		**Infinite cylindrical wall** $L=r_0$		**Sphere** $L=r_0$	
Bio	$\gamma(rad)$	Y	$\gamma(rad)$	Y	$\gamma(rad)$	Y
0,01	0,0998	1,0017	0,1412	1,0025	0,1730	1,003 0
0,02	0,1410	1,0033	0, 1995	1,0050	0,2445	1,0060
0,03	0,1732	1,0049	0,2439	1,0075	0,2998	1,0090
0,04	0,1987	1,0066	0,2814	1,0099	0,3450	1,0120
0,05	0,2217	1,0082	0,3142	1,0124	0,3852	1,0149
0,06	0,2425	1,0098	0,3438	1,0148	0,4217	1,0179
0,0 7	0,2615	1, 0114	0,3 708	1,0173	0,4550	1,02 09
0,08	0,2791	1, 013 0	0,3960	1,0197	0,4860	1,0239
0,09	0,2956	1, 0145	0,4195	1,0222	0,5150	1,0268
0,10	0,3111	1, 0 160	0,4417	1,0246	0, 5423	1,0298
0,15	0,3779	1,0237	0,5376	1,0365	0,6608	1,0445
0,20	0,4328	1,0311	0,6170	1,0483	0,7593	1,0592
0,25	0,4801	1,0382	0,6856	1,0598	0,8448	1,0 737
0,30	0,5218	1 ,045 0	0,7465	1,071 2	0,9208	1,0880
0,40	0,5932	1,0580	0,8516	1,0932	1, 0528	1, 1164
0,50	0,6533	1,070 1	0,9408	1, 1143	1, 1656	1,14 41
0,60	0,7051	1,0814	1,0185	1, 1346	1,2644	1,1713
0,70	0,7506	1,0919	1,0873	1,1539	1,35 25	1,1978
0,80	0,791 0	1,1016	1,1490	1,1725	1,4320	1,2236
0,90	0,8274	1,1107	1,2048	1,1902	1,5044	1,2488
1,0	0,8603	1,1191	1,2558	1,2071	1,5708	1,2732
2,0	1,0 769	1, 1795	1, 5995	1,3384	2,0288	1,4793
3,0	1, 192 5	1,2102	1,788 7	1, 419 1	2 ,2889	1,6227
4,0	1,2646	1, 2287	1,9 081	1, 4698	2,4556	1, 7201
5,0	1,3138	1,2 402	1,9898	1, 5029	2,5704	1,7870
6,0	1,3496	1,2479	2,0490	1,5253	2,6537	1,8338
7,0	1,3766	1,2532	2,0937	1,5411	2,7165	1,8674
8,0	1,3978	1,2570	2,1286	1,5526	2,7654	1,8921
9,0	1,4149	1,2598	2,1566	1,5611	2,8044	1,9106
10,0	1, 4 289	1,2620	2,1795	1,5677	2,8363	1,9249
20,0	1,4961	1,2699	2,2881	1,5919	2,985 7	1,9781
30,0	1,5202	1,2717	2,3261	1,5973	3,0372	1,9898
40,0	1, 53 25	1,2723	2,3455	1,5993	3,0632	1,994 2
50,0	1,5400	1,2727	2,3572	1,60 02	3,0788	1,9962
100,0	1,5552	1,2733	2,3809	1,6015	3,1102	1,9990
∞	1,5707	1,2731	2,4050	1,6018	3,1415	2,0000

The linear interpolation formula : $f(c)=f(a)+(c-a)\dfrac{f(b)-f(a)}{b-a}$

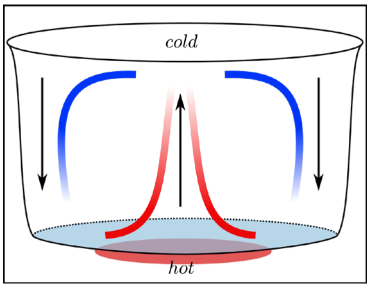
Chapter IV

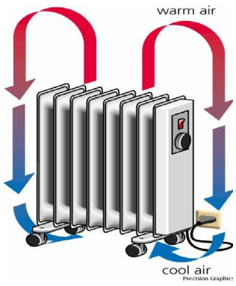
HEAT TRANSFER BY CONVECTION

IV.1 Generalities and definitions:

Convective heat transfer refers to the transfer of thermal energy between a solid surface and a fluid (liquid or gas) in motion. This process occurs through the bulk movement of the fluid, which can be driven by natural temperature differences (natural convection) or externally induced forces, such as fans or pumps (forced convection).

Example:





The quality of heat exchanged per unit of time depends on several parameters:

- The temperature difference between the wall and the fluid.
- The speed of the fluid.
- The specific heat capacity of the fluid.
- The state of the surface of the solid, as well as its size.

Depending on the mechanism that generates the movement of the fluid, distinctions can be made:

- *Natural (free) convection.*
- *Forced convection.*

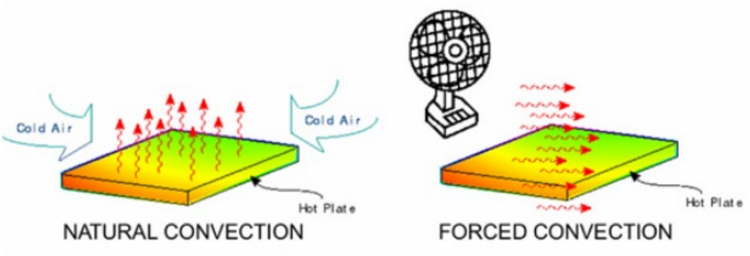
a) - *Natural (free) convection :*

In natural convection, fluid motion arises from buoyancy forces caused by density variations due to temperature differences between the fluid and adjacent surface. A greater temperature disparity leads to increased buoyancy forces, intensifying convection currents and elevating the heat transfer rate.

b) - *Forced convection:*

Forced convection is a heat transfer process in which the motion of a fluid (liquid or gas) is induced or compelled by external means, such as pumps, fans, or blowers. Unlike natural convection, which relies on buoyancy forces generated by temperature differences, forced convection involves a deliberate and controlled effort to move the fluid, enhancing heat transfer between the solid surface and the fluid. This method is commonly employed in various engineering applications to optimize cooling and heating processes.

Example:



Some practice examples of heat convection.

IV.2 Newton Law (1643-1727):

The expression for the quantity of dQ exchanged between the surface of a solid at (T_s) and the fluid at (T_f).

a) – Heat convection coefficient:

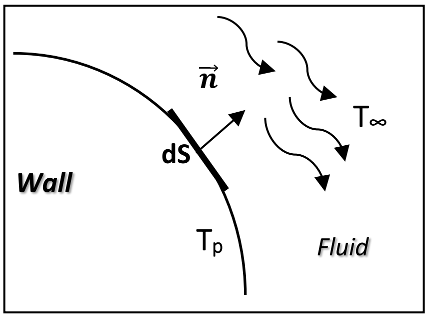
The study of convective heat transfer enables the determination of heat exchanges between a fluid and a wall.

The amount of heat dQ that passes through dS during the time interval dt can be expressed as:

$$dQ = h\left(T_P - T_\infty\right) dSdt$$

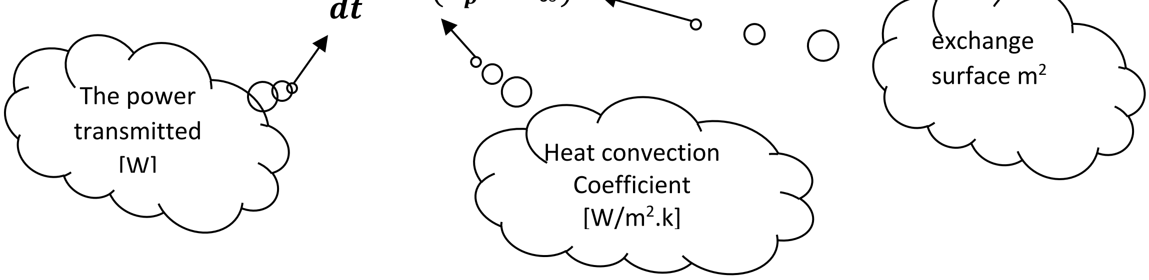
where:

h: Heat convection coefficient on [W/m^2.k].



Observed:
Irrespective of the convection type (free or forced) and the fluid's flow regime (laminar or turbulent), the heat flow transmitted is governed by the relationship known as Newton's law.

$$\frac{dQ}{dt} = h(T_p - T_\infty)ds$$



In convection mode, the central challenge in calculating heat flow involves determining the coefficient 'h,' which is contingent upon several parameters:

- Fluid characteristics.
- Nature of the flow (laminar or turbulent).
- Temperature.
- Geometry of the exchange surface.

Estimating the order of magnitude of the coefficient 'h' is crucial for diverse configurations.

Configurations :	*h on [w/m².k]*
Naturel Convection :	
- Vertical plate of height 0.3 m in the air.	4,5
- Horizontal cylinder with a diameter of 5 cm in the air.	6,5
- Horizontal cylinder with a diameter of 5 cm in water.	890
Forced Convection:	
- Air flow at 2 m/s over a square plate with sides of 2 m.	12
- Air flow at 35 m/s over a square plate with sides of 0.75 m.	75
- Water at 0.5 kg/s in a tube with a diameter of 2.5 cm.	3500
- Air flow at 50 m/s perpendicular to a tube with a diameter of 5 cm.	180

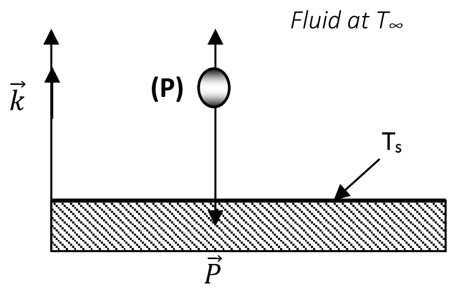
Order of magnitude of the coefficient 'h' ($W/m^2 \cdot K$):

- Free convection (air) from 5 to 25.
- Free convection (water) from 100 to 900.
- Forced convection (air) from 10 to 500.
- Forced convection (water) from 100 to 1500.
- Forced convection (oil) from 50 to 2000.

IV.3 Naturel Convection:

Definition: When the hot particle begins to move, it directly facilitates the transfer of heat towards the colder medium, leading to a convective regime.

Let's consider a horizontal surface (s) at temperature Ts in contact with a stationary fluid at temperature T_∞. A particle (P) of the fluid with volume (V) in contact with the surface (s) has a temperature close to T_s.



Force balance acting on particle (P):

- Archimedes' buoyant force : $\vec{A} = \rho(T_f).V.g.\vec{k}$

The buoyant force exerted on an object is equal to the weight of the displaced fluid:

The weight: $$\vec{P} = -m\vec{g} = -\rho(T_s).V.g.\vec{k} \quad \text{(III.2)}$$

as : $T_s > T_f$

So : $\rho(T_f) > \rho(T_s) \Rightarrow |\vec{A}| > |\vec{P}|$

The equation of particle motion in the immediate vicinity of S is written according to the fundamental principles of dynamics.

$$\sum \vec{F_{ex}} = m.\vec{a}$$

$$\left[\rho(T_f) - \rho(T_s)\right].g.V = \rho(T_s).V.\frac{dz^2}{dt^2} \quad \text{(III.3)}$$

where: $\frac{dz^2}{dt^2} = \frac{\rho(T_f) - \rho(T_s)}{\rho(T_s)}.g$

The mechanical equilibrium requires that denser particles be located below less dense ones. This promotes fluid movement: it is the phenomenon of natural convection.

IV.4 Study of convection phenomenon:

To investigate convection, we will address the following points:

1. Boundary layers.
2. Determination of the convection coefficient (h) through dimensional analysis.
3. Practical method for calculating h.
4. Experimental determination of h.
5. Nature of the convection coefficient.
6. Surface thermal resistances.

IV.5 Reminders on dimensional analysis

IV.5.1. Fundamental quantities

For the sake of convenience, we are led to arbitrarily choose a certain number of independent quantities as fundamental quantities; all other quantities will be expressed as functions of these and referred to as derived quantities. The fundamental quantities of the international system are:

- The mass M
- Length L
- Time T
- Temperature θ

For heat transfer problems, we add the heat quantity Q, which is expressed in terms of the fundamental dimensions M, L, and T per: $Q = M.LT^{-2}$

The dimensional analysis method is based on the principle of dimensional homogeneity of the terms in an equation. In a thermal convection problem, the physical quantities involved are grouped in the table below.

Physical Quantity	Symbol	SI Unit	Dimensional Quantity
Temperature	T	K	θ
Diameter	D	m	L
Flow velocity	U	m/s	LT^{-1}
Density	ρ	Kg/m^3	$M.L^{-3}$
Thermal conductivity	λ	W/mK	$M.L.T^{-3}.\theta^{-1}$
Specific heat	C_P	$J/Kg.K$	$L^2.T^{-2}.\theta^{-1}$
Convection coefficient	h	W/m^2K	$M.T^{-3}.\theta^{-1}$
Dynamic viscosity	μ	Kg/ms	$M.L^{-1}.T^{-1}$

IV.4.2 The method of dimensional analysis:

Let's suppose that h is a function of the variables:

- C_P : Specific heat in $[J/KgK]$
- λ : Thermal conductivity in $[W/mK]$
- μ : Dynamic viscosity in $[Kg/ms]$
- V : Velocity in $[m/s]$
- d : Characteristic dimension in $[m]$
- v : Kinematic viscosity in $[m^2/s]$

$$h = f\left(C_P, \lambda, \mu, d, v\right); in\left[W/m^2K\right] \quad \text{(III.4)}$$

h It is also an implicit function of the temperature T since λ, C_P and λ depend on it. These variables do not all come into play simultaneously.

Let's use the dimensional equations for each term represented in the table above. Starting from Newton's law, the dimensional equation for h obtained by:

$$h = \frac{[\delta Q]}{\left[\left(T_P - T_f\right)\right][dS][dt]} = \frac{M.L^2.T^{-2}}{\theta.L^2.T} = M.T^{-3}.\theta^{-1} \qquad \text{(III.5)}$$

By writing h in the form:

$$[h] = [C_P]^a.[\lambda]^b.[\mu]^c.[d]^d.[v]^e$$

$$[h] = \left(L^2.T^{-2}.\theta^{-1}\right)^a.\left(M.L.T^{-3}.\theta^{-1}\right)^b.\left(M.L^{-1}.T^{-1}\right)^c.(L)^d.\left(L.T^{-1}\right)^e$$

The fundamental quantities involved in the calculation of h are: the mass M, the time T, the length L, the temperature θ. Identifying the exponents in the equation with dimensions of h provides a linear system of equations for calculating a, b, c, d and e.

$$\left(L^2.T^{-2}.\theta^{-1}\right)^a.\left(M.L.T^{-3}.\theta^{-1}\right)^b.\left(M.L^{-1}.T^{-1}\right)^c.(L)^d.\left(L.T^{-1}\right)^e = M.T^{-3}.\theta^{-1}$$

Thus:

- The exponent of $M: b + c = 1$
- The exponent of $\theta: a + b = 1$
- The exponent of $L: 2\text{a} + b - c + d + e = 0$
- The exponent of $T: 2\text{a} + 3b + c + e = 3$

Solving the dimensional equations reveals dimensionless numbers that are very useful in the study of fluid mechanics, especially in convective phenomena. These numbers are in particular :

- **Reynolds Numbers :**

Characterizes the flow regime in the pipeline, can be laminar or turbulent.

$$R_e = \frac{\rho.U.D}{\mu} = \frac{U.D}{\upsilon} \qquad \text{(III.6)}$$

Therefore:

υ: is the kinematic viscosity of the fluid, equal to $\upsilon = \frac{\mu}{\rho}$

The Reynolds number is verified by a critical value. If the flow in:

- A pipe: $R_{ec} = 2200$
- A flat plate: $R_{ec} = 3.10^5$

- **Nusselt Numbers :**

It characterizes the importance of convection relative to conduction, it is the ratio of the heat exchanged by convection to the heat exchanged by conduction:

$$Nu = \frac{hS\Delta T}{\lambda S \frac{\Delta T}{d}} = \frac{hD}{\lambda} \qquad \text{(III.7)}$$

- **Prandtl Numbers :**

It represents the ratio of momentum diffusivity to thermal diffusivity, and is given by:

$$P_r = \frac{\mu C_P}{\lambda} \qquad \text{(III.8)}$$

- **Grashof Numbers :**

It represents the ratio of buoyancy forces to viscous forces, and is given by:

$$Gr = \frac{g.\beta.(T_s - T_0).L^3}{\upsilon^2} \qquad \text{(III.9)}$$

IV.4.3 Empirical correlations

A large number of empirical formulas are available to determine the heat transfer coefficient h through the expression of the Nusselt number. These formulas depend on the type of convection (natural or forced) as well as the flow regime (laminar or turbulent).

☞ ***Forced convection :***

1- <u>Flow over a flat plate</u>

- *When the flow regime is turbulent:*

$$Nu = 0,0035.\mathrm{R}_e^{0,8}.\mathrm{Pr}^{1/3} \quad \text{for } (R_e > 3.10^5, et\ \mathrm{Pr} \geq 0,5) \qquad \text{(IV.10)}$$

- *When the flow regime is laminar:*

$$Nu = 0,628.\mathrm{R}_e^{0,5}.\mathrm{Pr}^{1/3} \quad \text{for } (R_e > 3.10^5, et\ 0,5 \leq \mathrm{Pr} \geq 10) \qquad \text{(IV.11)}$$

2- <u>Flow inside cylindrical tubes</u> :

- *When the flow regime is laminar:* $\mathrm{R_e} \leq 2200$
  - *<u>Hausse formula</u> :*

$$Nu = 3,66 + \frac{0,0668.R_e.\mathrm{P}_r(D/L)}{1+0,04\left[R_e.\mathrm{P}_r(D/L)\right]^{2/3}}\left(\frac{\mu_m}{\mu_p}\right)^{0,14} \qquad \text{(IV.12)}$$

  - *<u>Sieder et Tate formula</u> :*

$$Nu = 1,86(R_e.\mathrm{P}_r)^{1/3}(D/L)^{1/3}\left(\frac{\mu_m}{\mu_p}\right)^{0,14}, \text{ for :}\left(R_e P_r (D/L)\right) > 10 \qquad \text{(IV.13)}$$

  - *<u>Kays formula</u> :*

$$Nu = 3,66 + \frac{1,104.R_e.\mathrm{P}_r(D/L)}{1+0,016\left[R_e.\mathrm{P}_r(D/L)\right]^{0,8}}, \text{ for :}\left(R_e P_r (D/L)\right) > 100 \qquad \text{(IV.14)}$$

- *When the flow regime is turbulent:* $\mathrm{R_e} > 2200$
  - *<u>Colbum formula</u> :*

$$Nu = 0,023.R_e^{0,8}\,\mathrm{P}_r^{1/3} \qquad \text{(IV.15)}$$

Pour : $(L/D) > 60$ *and* $(0,7 \leq P_r \leq 100)$ *and* $(10^4 > R_e > 1,2.10^3)$

  - *<u>Sieder et Tate formula</u> :*

$$Nu = 0,023R_e^{0,8}\,\mathrm{P}_r^{1,2}\left(\frac{\mu_m}{\mu_p}\right)^{0,14} \qquad \text{(IV.16)}$$

  - *<u>Mc-Adams formula</u> :*

$$Nu = 0,023 R_e^{1/5} P_r^{1/3} \left(\frac{\mu_m}{\mu_p}\right)^{0,14} \left[1+\left(D/L\right)^{0,7}\right] \qquad \text{(IV.17)}$$

Where:

D: It is the diameter of the tube in [m].

μ_m et μ_p: Dynamic viscosity is defined at T_m et T_P

T_m: Average temperature: $T_m = \dfrac{T_P + T_f}{2}$

T_p: Internal wall temperature of the tube.

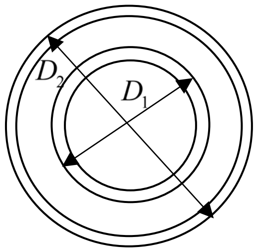
T_f: Fluid temperature.

3- *Flow in an annular space:*

$$Nu = 0,023.R_e^{0,8} P_r^{n} \qquad \text{(IV.18)}$$

with :

$$R_e = \frac{U_m H}{\upsilon} \text{ et } Nu = \frac{hH}{\lambda}; \left(H = D_2 - D_1\right)$$



- In the case of heating $n = 0,4$
- In the case of cooling $n = 0,3$

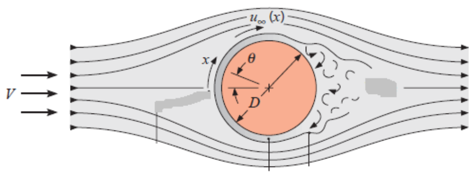
4- *Perpendiculaire flow to a tube :*

$$Nu = C.\left(R_e\right)^m \qquad \text{(IV.19)}$$

where:

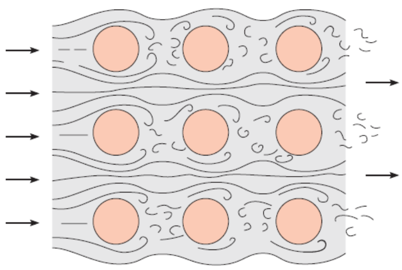
C et m: They are constants chosen based on the Reynolds number, which characterizes the flow regime. These values are provided in the table below :

Reynolds Numbers	C	m
1-4	0,981	0,330
4-40	0,821	0,385
40-4000	0,615	0,466
4000-40000	0,174	0,618
40000-250000	0,0239	0,805



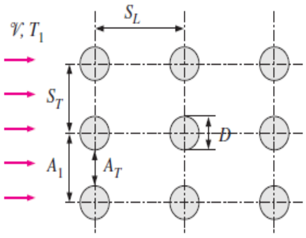
5- *Perpendicular flow to a row of tubes:*

$$Nu = 0,33.R_e.P_r^{1/3} \qquad \text{(IV.20)}$$



a) *To a row of regular tubes:*

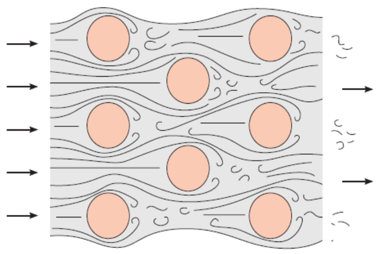
The maximum velocity is determined based on the mass conservation required for a steady, incompressible flow. For an inline arrangement, the maximum velocity occurs in the region of minimum flow between the tubes, and mass conservation can be expressed as:



$$\rho V A_1 = \rho V_{max} A_T \ ou\ V S_T = V_{max}(S_T - D)$$

$$V_{max} = \frac{S_T}{(S_T - D)} V \qquad \text{(IV.21)}$$

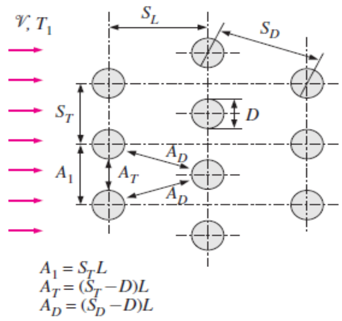
b) To a staggered row of tubes:



In the staggered arrangement, the fluid approaching through zone A1 passes through zone A_T, then through zone 2AD, wrapping around the pipe of the next row. if$(2A_D > A_T)$, The maximum velocity will always occur at A_T between the tubes, so the Vmax relationship can also be used for staggered tubes. but if $(2A_D < A_T)$ *or if* $2(S_D - D) < (S_T - D)$

where: S_D : It is the diagonal pitch.

$S_D < (S_T + D)/2$



The maximum velocity will occur in the diagonal cross-sectional areas, and the maximum velocity in this case becomes:

$$V_{max} = \frac{S_T}{2(S_D - D)} V \qquad \text{(IV.22)}$$

since $\rho V A_1 = \rho V_{max}(2A_D)$, *ou* $V S_T = 2V_{max}(S_D - D)$

Arrangement	R_{e_D}	Correlation
Regular rows	0-100	$Nu_D = 0,9 R_{e_D}^{0,4} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$
	100-1000	$Nu_D = 0,52 R_{e_D}^{0,5} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$
	1000-2.10⁵	$Nu_D = 0,27 R_{e_D}^{0,63} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$
	2.10⁵-2.10⁶	$Nu_D = 0,033 R_{e_D}^{0,8} \Pr^{0,4} (\Pr/\Pr_s)^{0,25}$
Staggered rows	0-500	$Nu_D = 1,04 R_{e_D}^{0,4} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$
	500-1000	$Nu_D = 0,71 R_{e_D}^{0,5} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$
	1000-2.10⁵	$Nu_D = 0,35 (S_T/S_L)^{0,2} R_{e_D}^{0,6} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$
	2.10⁵-2.10⁶	$Nu_D = 0,031 (S_T/S_L)^{0,2} R_{e_D}^{0,8} \Pr^{0,36} (\Pr/\Pr_s)^{0,25}$

In both representations, after the first rows, the heat transfer coefficient remains constant. Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross-flow over tube arrays. More recently, ZUKAUSKAS proposed correlations, the general form of which is given by the following relation:

$$N_{u_D}=\frac{hD}{\lambda}=CR_{e\_D}^{m}P_r^{n}\left(P_r/P_{r_s}\right)^{0,25} \qquad \text{(IV.23)}$$

where: *C, m* and *n* the values of the constants depend on the Reynolds number.

These correlations are detailed in the table above for: $0,7<P_r<500$ *and* $0<R_{e_D}<2.10^6$

The uncertainty in the values of the Nusselt number obtained from these correlations is 15%. Note that all properties, except for s, Pr should be evaluated at the average temperature of the fluid, which is determined from:

$$T_m=\frac{T_i-T_e}{2} \qquad \text{(IV.24)}$$

The outer diameter of the tube D is taken as the characteristic length. The arrangement of the tubes in the row is characterized by the transverse pitch S_T, the longitudinal pitch S_L, and the diagonal pitch S_D between the centers of the tubes. The diagonal pitch is determined from the following expression:

$$S_D=\sqrt{S_L^2+\left(S_T/2\right)^2} \qquad \text{(IV.25)}$$

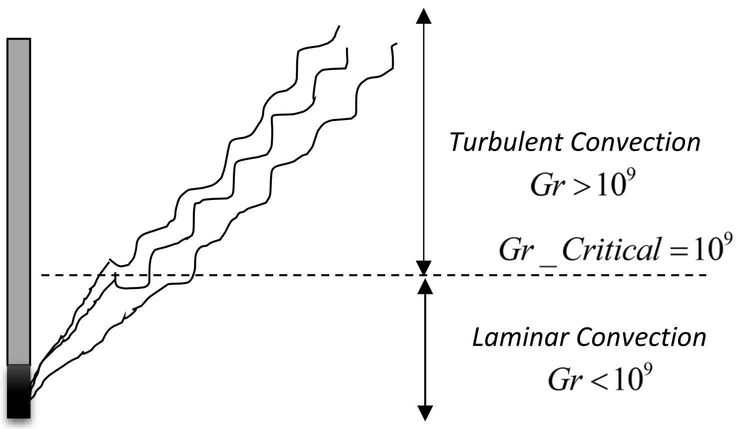
When the fluid enters the tube array, the flow area decreases by:

$$\left(A_1=S_T L\right)\ \grave{a}\ \left(A_T=\left(S_T-D\right)/L\right)$$

☞ ***In natural (free) convection:***

The Grashof number is therefore the ratio of the gravitational forces that act to set the fluid in motion to the viscous forces that tend to dampen this motion. The larger the Grashof number (Gr), the more intense the natural convection will be.

Example:



Experiments show that the flow in natural convection is initially laminar and then becomes turbulent in the upper section, with the transition occurring at a height corresponding to a Grashof number of the order of 10^9.

Experimental results related to heat transfer by natural convection can be correlated by expressions of the form:

$$Nu=\phi\left(Gr\right).\psi\left(\Pr\right)=C\left(Gr.\Pr\right)^m \qquad \text{(IV.26)}$$

1- Vertical plates and cylinders :

A vertical cylinder can be treated as a vertical plate when: $D\geq\frac{35L}{Gr_L^{1/4}}$ the characteristic length is L.

- *When the flow regime is laminar:* $R_a = 10^4 - 10^9$

$$Nu = 0,59 R_{a_L}^{1/4} \quad \text{(IV.27)}$$

- *When the flow regime is turbulent:* $R_a = 10^9 - 10^{13}$

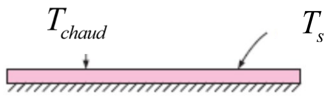
$$Nu = 0,10 R_{a_L}^{1/3} \quad \text{(IV.28)}$$

2- ***Horizontal plates :***

- If the upper wall surface is heated, the characteristic length is given by:

  $L = A_s / \text{p}$

  Where : P : *perimetre* , A_s : *surface*



- *When the flow regime is laminar:* $R_a = 10^4 - 10^7$

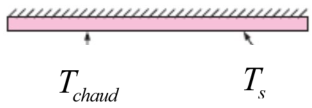
$$Nu = 0,54 R_{a_L}^{1/4} \quad \text{(IV.29)}$$

- *When the flow regime is turbulent:* $R_a = 10^7 - 10^{11}$

$$Nu = 0,15 R_{a_L}^{1/3} \quad \text{(IV.30)}$$

- If the lower wall surface is heated, the characteristic length is given by:

  $L = A_s / \text{p}$



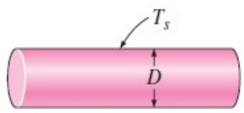
- *When the flow regime is laminar:* $R_a = 10^5 - 10^{11}$

$$Nu = 0,27 R_{a_L}^{1/4} \quad \text{(IV.31)}$$

3- ***Horizontal cylinders :***

The characteristic length is the diameter D.

The Rayleigh number is: $R_{a_D} \leq 10^{12}$

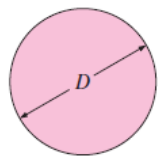


$$Nu = \left[0,6 + \frac{0,387 R_{a_L}^{1/6}}{\left[1 + \left(0,559 / \text{Pr}\right)^{9/16}\right]^{8/27}} \right]^2 \quad \text{(IV.32)}$$

4- Spheres :

La longueur caractéristique est le diamètre D.

The Rayleigh number is : $R_{a_D} \leq 10^{11}$



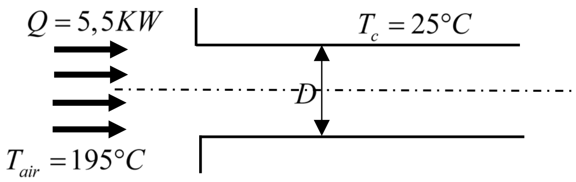
$$Nu = 2 + \frac{0,589 R_{a_L}^{1/4}}{\left[1 + \left(0,469 / \text{Pr}\right)^{9/16}\right]^{4/9}} \quad \text{(IV.33)}$$

Exercise N° 01:

In a cylinder with a diameter of 2.8 cm and a length of 3 m, air flows at a temperature of 195°C. The cylinder, maintained at a temperature of 25°C, receives a heat flux of 5500W.

1. Determine the convective heat transfer coefficient *h*.
2. Deduce the Nusselt number of the flow, knowing that: $\lambda = 0,026 \left(W / m°C\right)$.
3. Calculate the Reynolds number of the flow. Assuming that: $Nu = 0,023 R_e^{0,8} \,\text{Pr}^{0,4}$

Solution :



1. *The heat transfer coefficient :*

$$Q = hS(T_{air} - T_c) \Rightarrow h = \frac{Q}{\pi DL(T_{air} - T_c)} = \frac{5500}{\pi.2,8.10^{-2}.3.(195-25)} = 122,6\left(W/m^2°C\right)$$

1- Nusselt number (N_u) :

$$Nu = \frac{hD}{\lambda} = \frac{122,6.2,8.10^{-2}}{0,026} = 132,03$$

2- Reynolds number (R_e) :

$$Nu = 0,023R_e^{0,8}\Pr^{0,4} \Rightarrow R_e = \sqrt[0,8]{\frac{132,3}{0,023(0,73)^{0,4}}} = 58481,64$$

Exercise N° 02:

A thin plate with a length of 3 m and a width of 1.5 m is subjected to an airflow with a velocity of 2.0 m/s and a temperature of 20°C, in the longitudinal direction. The temperature of the plate's surfaces is 84°C. The following calculations are requested:

1. The convective heat transfer coefficient along the length of the plate;
2. The heat flux transferred from the plate to the air. The air properties at 20°C are:

$$\rho = 1,175\left(Kg/m^3\right)\ ,\ \mu = 1,8.10^{-5}\left(Kg/ms\right)\ ,\ \lambda = 0,026\left(W/mK\right)\ ,\ C_P = 1006\left(J/KgK\right)$$

Solution :

1- The convective heat transfer coefficient (h):

$R_e = \frac{\rho UL}{\mu} = \frac{1,175.3.2}{1,80.10^{-5}} = 391666,67 < 3.10^5$, Therefore, the flow regime is laminar, in this case, the following Nusselt correlation is applied: $Nu = 0,66R_e^{0,5}\Pr^{0,33}$

$$Nu = \frac{hL}{\lambda} = 0,66R_e^{0,5}\Pr^{0,33} \Rightarrow h = \frac{0,66R_e^{0,5}\Pr^{0,33}.\lambda}{L}$$

$$h = \frac{0,66R_e^{0,5}\Pr^{0,33}.\lambda}{L} = \frac{0,66(391666,67)^{0,5}(0,71)^{0,33}.0,026}{3} = 3,1972\left(W/m^2K\right)$$

2- The heat flux transferred from the plate (which has two surfaces, the upper and lower surfaces) to the air is given by:

$$Q = 2hS(T_P - T_a) = 2.(3,1972).(3.1,5)(357-293) = 1841,587W$$

Exercise N° 03:

Water flows under forced conditions through a coil made of a tube with a diameter of 18 mm. The water flow rate is 0.24 kg/s, and its temperature is 120°C. The temperature of the inner wall of the pipe, which is 3 m long, is assumed to be constant and equal to 110°C.

1- Calculate the flow velocity of the water inside the coil.
2- Specify the nature of the water flow inside the coil.
3- Calculate the amount of heat transferred by the water, knowing that the Nusselt number is correlated by the following relations:

$$-\ for\ a\ \text{laminar}\ flow\ Nu = 3,66 + \frac{0,065(D/L)R_e \Pr}{1+0,04\left[(D/L)R_e \Pr\right]^{2/3}}$$

$$-\ for\ a\ \ turbulent\ \ flow\ Nu = 0,023R_e^{0,8}\Pr^{1/3}$$

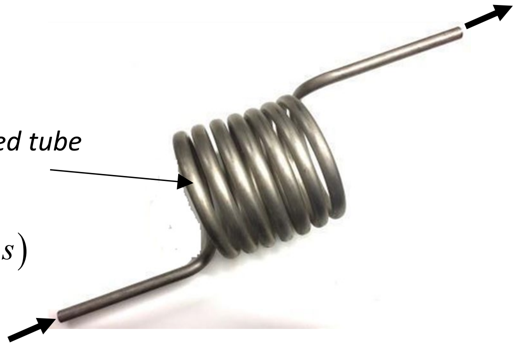
The properties of water at 120°C are given as:

$$\rho_{water} = 945,3\left(Kg/m^3\right); \mu_{water} = 2,34.10^{-4}\left(Kg/m^2\right); \lambda_{water} = 0,685\left(W/mK\right)$$

$$C_p = 4250\left(J/Kg.K\right), \mu_p = 2,57.10^{-4}\left(Kg/m^2\right)$$

Solution :

1- Flow Velocity :



$$V = \frac{m_{\_water}}{\rho\frac{\pi D^2}{4}} = \frac{\frac{14,5}{60}}{945,3.\frac{\pi\left(18.10^{-3}\right)^2}{4}} = 1,0046\left(m/s\right)$$

2- Specify the nature of the water flow inside the coil:

a) -Reynolds number :

$$R_e = \frac{\rho.U.D}{\mu} = \frac{945,3.1.18.10^{-3}}{2,34.10^{-4}} \Rightarrow R_e = 7,3053.10^4$$

$R_e > 2200$, Therefore, the flow regime of water inside the coil is turbulent.

b)- Prandtl number:

$$\Pr = \frac{\mu C_P}{\lambda} = \frac{2,34.10^{-4}.4250}{0,685} \Rightarrow \Pr = 1,4518$$

c)- Nusselt number:

$$Nu = 0,023R_e^{0,8}\Pr^{1/3} = 0,023\left(7,3053.10^4\right)^{0,8}\left(1,45\right)^{1/3} \Rightarrow Nu = 202,5863$$

d)- The convection coefficient equals:

$$h = \frac{Nu.\lambda}{D} = \frac{202,5863.0,685}{18.10^{-3}} \Rightarrow h = 7,7095.10^3\left(W/m^2K\right)$$

3- Calculate the heat flux lost inside the coil (Helix-shaped tube):

$$Q = hS\left(T_{\_water} - T_P\right) = 7,7095.10^3\left(\pi.18.10^{-3}.3\right)\left(120-110\right) \Rightarrow Q = 1,3079.10^4\left(W\right)$$

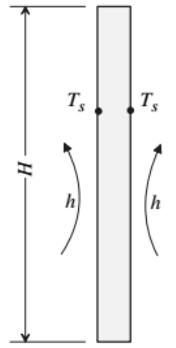
Exercise N° 04:

An electric convector with a power of 1500W has the shape of a vertical flat plate with a width of 1.5m. The ambient air surrounding the convector is at a temperature of 20°C.

The thermophysical properties of air at 50°C are:

$\upsilon = 17,95.10^{-6}\ m^2/s$; $\lambda =\ 0,0283\ W/mK$; P_r=0,698 ; $\beta = 0,0031\ K^{-1}$.

What should be the height H of the convector so that its surface temperature T_s does not exceed a certain value? Verify the height H for both convection regimes. Given that the Nusselt number is correlated by the following relationships:



- For a laminar regime : $N_u = 0,59 R_a^{\ 0,250}$
- For a turbulent regime : $N_u = 0,10 R_a^{\ 0,333}$

Solution :

- *The data :*

$$Q = 1500\,W; l = 1,5\,m; T_{air} = 20°C.$$

The thermophysical properties of air at 50°C are:

$$\upsilon = 17,95.10^{-6}\left(m^2/s\right); \lambda = 0,0283\left(W/mK\right); \Pr = 0,698; \beta = 0,0031K^{-1}$$

- *Calculate the height H of the convector so that its surface temperature Ts does not exceed a certain value.*

$$T_m = \frac{\left(T_{air} - T_s\right)}{2} = \frac{20+80}{2} = 50°C.$$

- *Calcule the Grashof number :*

$$Gr_{air} = \frac{g\beta_{air}\left(T_s - T_{air}\right)H^3}{\upsilon_{air}^2} = \frac{9,81.0,0031\left(80-20\right)H^3}{\left(17,95.10^{-6}\right)^2} = 5,656.10^9 H^3$$

We verify the height H for both convection regimes:

- *Assuming the convection is turbulent, we apply the correlation:*

  $$Nu_{\_air} = 0,10\left(G_r P_r\right)_{air}^{1/3} = 0,10\left(5,656.10^9 H^3.0,698\right)_{air}^{1/3} = 158H$$

  We deduce the heat transfer coefficient:

  $$h = Nu_{air}\frac{\lambda_{air}}{H} = 158.0,0283 = 4,47\left(W/m^2K\right)$$

In a steady-state regime, the power supplied by the convector is dissipated by convection into the surrounding ambient air:

$$Q = hS\left(T_s - T_{air}\right)$$

We calcule H :

$$Q = hlH\left(T_s - T_{air}\right) \Rightarrow H = \frac{\left(T_s - T_{air}\right)}{hl} = \frac{1500}{2.1,5.4,47.\left(80-20\right)} = 1,86\,m$$

- *Assuming the convection is laminar, we apply the correlation:*

$$Nu_{\_air} = 0,59\left(G_r P_r\right)_{air}^{1/4} = 0,59\left(5,656.10^9 H^3.0,698\right)_{air}^{1/4} = 147,89H^{3/4}$$

We deduce the heat transfer coefficient:

$$h = Nu_{air}\frac{\lambda_{air}}{H} = 147,89H^{3/4}.0,0283 = 4,18H^{-1/4}\left(W/m^2K\right)$$

We calcule H :

$$Q = hlH\left(T_s - T_{air}\right) \Rightarrow H = \frac{\left(T_s - T_{air}\right)}{hl} = \frac{1500}{2.1,5.4,18.\left(80-20\right)} = 2,51\,m$$

Exercise N° 05:

Calculate the heat transfer coefficient by convection as well as the heat flux generated during the forced flow of oil at a speed of 0.5 m/s in a tube with a diameter of 10 mm and a length of 1 m, if the average temperatures of the oil and the wall are 80°C and 20°C, respectively.

The characteristics of the oil used at the temperature at which it flows, i.e., 80°C, are: $\rho = 844\left(Kg/m^3\right);\mu_m = 30,8.10^{-4}\left(Kg/ms\right);\lambda = 0,10\left(W/m°C\right);C_P = 1,846\left(KJ/Kg°C\right)$.

At the wall temperature of 20°C, the viscosity of the oil is: $\mu_p = 198,2.10^{-4}\,Kg/ms$

Solution :

- *The Reynolds number:* $R_e = \frac{\rho VD}{\mu} = \frac{844.0,5.0,01}{30,8.10^{-4}} = 1370 < 2300$, Therefore, the flow regime is laminar, and the Haussner relation can be applied.
  Let's begin by calculating the following relation: $R_e P_r\left(D/L\right)$

$$R_e P_r\left(D/L\right) = R_e.\left(\frac{\mu C_P}{\lambda}\right).\left(D/L\right) = \frac{1370.30,8.10^{-4}.1846.0,01}{0,108.1} = 721$$

- *Calculate the convection coefficient :*

$$N_u = \frac{hD}{\lambda} = Nu = 3,66 + \frac{0,0668.R_e.P_r\left(D/L\right)}{1+0,04\left[R_e.P_r\left(D/L\right)\right]^{2/3}}\left(\frac{\mu_m}{\mu_p}\right)^{0,14}$$

$$\Rightarrow h = \frac{\lambda}{D}\left[3,66 + \frac{0,0668.R_e.P_r\left(D/L\right)}{1+0,04\left[R_e.P_r\left(D/L\right)\right]^{2/3}}\left(\frac{\mu_m}{\mu_p}\right)^{0,14}\right]$$

$$\Rightarrow h = \frac{\lambda}{D}\left[3,66 + \frac{0,0668.721}{1+0,04\left[721\right]^{2/3}}\left(\frac{30,8}{198,2}\right)^{0,14}\right] = 134,6\left(W/m^2°C\right)$$

- *Calculate the heat flux exchanged:*

$$Q = hS\left(T_p - T_f\right) = h\left(\pi DL\right)\left(T_p - T_f\right) = 134,6.\left(\pi.0,01.1\right)\left(80-20\right) = 253,7\,W$$

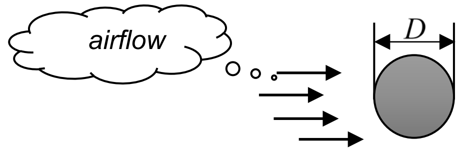
Exercise N° 06:

Consider a bar with a circular cross-section and a diameter of 15 mm, cooled by an air flow with a velocity of 1 m/s and a temperature of 20°C. Calculate the amount of heat transferred by the air per unit length of the bar if the surface temperature of the bar is 80°C.

Solution:

The flow is perpendicular to the bar, and the Hilpert Correlation can be applied: $R_e = \frac{\rho VD}{\mu} = \frac{1,20.1.0,015}{1,8.10^{-5}} = 1000$;

According to the table, we select the constants: $\begin{cases} C = 0,615 \\ m = 0,466 \end{cases}$



$$N_u = \frac{hD}{\lambda} = C.\left(R_e\right)^m \text{Þ}h = \frac{\lambda}{D}C.\left(R_e\right)^m = \frac{0,615\left(1000\right)^{0,466}}{0,015} = 26,55\left(W / m^2K\right)$$

$$Q = hS\left(T_P - T_f\right) = h\pi DL\left(T_P - T_f\right) = 26,55.\pi.0,015.\left(80-20\right) = 75\,W$$

Exercise N° 07:

Hot water flows through a cylindrical tube with a length of 6 meters and a diameter of 8 cm, as shown in Figure I. This tube is oriented horizontally and passes through a large room with an ambient temperature of 20°C. Assuming that the surface temperature of the tube's exterior is 70°C and the film temperature (wall-air) is 45°C, the following properties are given: $\lambda = 0,0269W / m°C$; $\upsilon = 1,947.10^{-5}m^2 / \text{s}$; $P_r = 0,724$.

- Determine the heat flux lost by the tube due to natural convection.

The Nusselt number is correlated by the following expressions:

- For a laminar regime : $Nu = \left\{0,6 + \frac{0,387R_a^{1/6}}{\left[1+\left(0,559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2$
- For a turbulent regime : $Nu = 0,10\left(Gr\Pr\right)^{0,333}$

Solution :

Given Data :

$$L = 6\ m; D = 8m; T_s = 20°C; T_p = 70°C$$

The properties of air at the film temperature (wall-air) of 45°C are given as:

$$\lambda = 0,0269\ W / m°C\ ; \upsilon = 1,947.10^{-5}m^2 / s\ ; \Pr = 0,724$$

Calculate the Grashof number:

$$T_f = \frac{T_s + T_{air}}{2} = \frac{70+20}{2} = 45°C = 318K$$

$$\beta = \frac{1}{T_f\left(K\right)} = \frac{1}{318} \Rightarrow \beta = 3,144.10^{-3}$$

$$Gr = \frac{g\beta\left(T_s - T_f\right)D^3}{\upsilon^2} = \frac{9,81.3,144.10^{-3}\left(70-20\right).\left(0,08\right)^3}{1,947.10^{-5}} \Rightarrow Gr = 2,5787.10^6$$

Calculate the Rayleigh number:

$$R_a = Gr.\Pr = 2,5787.10^6.0,7241 \Rightarrow\ R_a = 1,867.10^6$$

From the value of the Rayleigh number, it is found that the flow regime is laminar$\left(R_a < 10^9\right)$,The Nusselt number is calculated using the following correlation:

$$Nu = \left\{0,6 + \frac{0,387R_a^{1/6}}{\left[1+\left(0,559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Calculate the Nusselt number :

$$Nu = \left\{ 0,6 + \frac{0,387\left(1,867.10^{6}\right)^{1/6}}{\left[1+\left(0,559/0,724\right)^{9/16}\right]^{8/27}} \right\}^{2} \Rightarrow Nu = 17,39$$

Calculate the heat transfer coefficient by convection:

$$Nu = \frac{hD}{\lambda} \Rightarrow h = \frac{Nu.\lambda}{D} = \frac{17,39.0,0269}{0,08} \Rightarrow h = 5,867\left(W/m^{2}K\right)$$

Calculate the heat flux rate:

$$Q = hS\left(T_s - T_{air}\right) = h\pi DL\left(T_s - T_{air}\right) \Rightarrow Q = 5,867.(1,508)(70-20) \Rightarrow Q = 442\,W$$

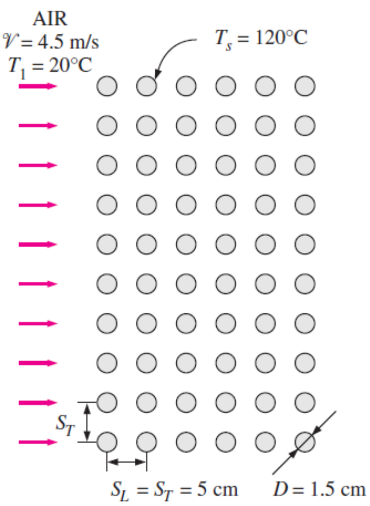
Exercise N° 08:

In an industrial setup, the air must be preheated before entering a furnace using geothermal water at 120°C, which flows through the tubes of a tube bank located in a duct. The air enters the duct at 20°C and 1 atm, with an average velocity of 4.5 m/s, and flows over the tubes in the normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in a series, both longitudinally and transversely. The pitch between the tubes is $S_L=S_T=5$ cm. There are 6 rows in the flow direction, with 10 tubes in each row, as shown in the figure below. Determine the thermal power per unit length of the tubes. The properties of the air at the assumed average temperature of 60°C are given by:

$$\lambda = 0,02808\left(W/mK\right);\ \rho = 1,06\left(Kg/m^{3}\right);\ P_r = 0,7202\ ,\ P_{r_s} = 0,7073$$

$$C_P = 1,007\left(KJ/KgK\right);\ \ \mu = 2,008.10^{5}\ \ (\text{Kg/m.s})$$

The density of air at 20°C is equal to: $\rho_a = 1,204\left(Kg/m^{3}\right)$



Solution:

$D = 0,015\,m\,;S_L = S_T = 0,05\,m$

The maximum velocity and the Reynolds number based on V_{max}

$$V_{max} = \frac{S_T}{S_T - D}V = \frac{0,05}{0,05-0,015}.(4,5) = 6,43\left(m/s\right)$$

$$R_{e_D} = \frac{\rho V_{max} D}{\mu} = \frac{(1,06).(6,43).(0,015)}{2,008.10^{-5}} = 5091$$

The average Nusselt number is determined using the following correlation: $Nu_D = 0,27R_{e_D}^{0,63}\,\text{Pr}^{0,36}\left(\text{Pr}/\text{Pr}_s\right)^{0,25}$

$$Nu_D = 0,27\left(5091\right)^{0,63}\left(0,7202\right)^{0,36}\left(0,7202/0,7073\right)^{0,25} = 52,2$$

This Nusselt number applies to tube banks with: 6 rows (N_L = 6), and the correction factor is F = 0.945. Therefore, the average Nusselt number and the heat transfer coefficient for all the tubes in the tube bank are given by:

$$Nu_{D,N_L} = FNu_D = \left(0,945\right)\left(52,2\right) = 49,3$$

$$h = \frac{Nu_{D,N_L}\lambda}{D} = \frac{49,3\left(0,02808\right)}{0,015} = 92,2\left(W/m^{2}°C\right)$$

The total number of tubes is: $N = N_L X\ N_T = 6X10 = 60$. For a unit tube length: $(L = 1\,m)$ The heat transfer surface and the mass flow rate of air (evaluated at the inlet) are:

$$A_s = N\pi DL = 60\pi(0,015).1 = 2,827 m^2$$

$$\dot{m} = \dot{m}_1 = \rho_1 V(N_T S_T L) = 1,024.4,5.10.0,05.1 = 2,709(Kg/s)$$

Next, the outlet temperature of the fluid, the logarithmic mean temperature difference, and the thermal power become:

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_P}\right)$$

$$= 120 - (120 - 20)\exp\left(-\frac{2,827.92,2}{2,709.1007}\right) = 29,11°C$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln((T_s - T_e)/(T_s - T_i))} = \frac{(120 - 29,11) - (120 - 20)}{\ln((120 - 29,11)/(120 - 20))} = 95,4°C$$

$$\dot{Q} = hA_s\Delta T_{\ln} = 92,2.2,827.95,4 = 2,49.10^4 W$$

The thermal power can also be determined more simply from:

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_P(T_e - T_i) = 2,709.1007.(29,11 - 20) = 2,49.10^4 W$$

Chapter V

Heat Transfer by Radiation

V.1 Introduction:

Most solid, liquid, or gaseous bodies at a temperature above 0 K emit electromagnetic radiation. When this radiation is absorbed, it is transformed into thermal energy. Anybody that emits this type of radiation is capable of absorbing radiation of the same nature. Thus, there will be a heat exchange by radiation between two bodies capable of emitting this type of radiation. This type of exchange exists even when the two bodies are at the same temperature; in this case, the net heat flow exchanged is zero (the two bodies are said to be in thermal equilibrium). The heat flow increases as the temperature difference between the two environments increases, but it also depends on the temperature levels. It can be stated that radiation exchanges increase and become predominant at high temperatures.

Radiation propagates in a straight line at the speed of light and consists of radiations of different wavelengths, as demonstrated by W. Herschell's experiment.

V.2 Definitions :

V.2.1 Classification

Physical quantities will be distinguished according to:

- The spectral composition of the radiation
  - If the quantity relates to the entire spectrum, it is called total.
  - If it concerns a narrow spectral interval $d\lambda$ around a specific wavelength λ, it is called monochromatic G_λ.
- The spatial distribution of the radiation

If the quantity relates to all directions in space, it is called hemispherical. If it characterizes a specific direction of propagation, it is called directional.

V.1.2 Definitions Related to Sources

V.2.2.1 Flux

The flux of a source S is defined as the power radiated, denoted φ, by S into all the surrounding space, across all wavelengths. The flux is expressed in Watt (W).

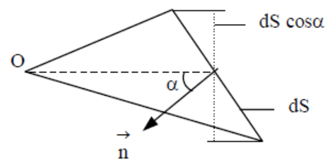
- The flux emitted by a surface element dS into an elemental solid angle $d\Omega$ is denoted $d^2\varphi$.
- The flux emitted into all space by a surface element dS is denoted $d\varphi$.
- The flux emitted by a surface S into the solid angle $d\Omega$ surrounding the direction O_x is denoted $d\varphi_x$.

We therefore have the following relationships:

$$d\varphi = \int_\Omega d^2\varphi \text{ and } \varphi = \int_s d\varphi = \int_\Omega d\varphi_x \quad (V.1)$$

Elementary solid angles:

☞ The solid angle under which a surface S is viewed from a point O is, by definition, the area of the intersection surface of the unit sphere and the cone with vertex O that rests on the boundary of surface S. The solid angle under which the boundary of a small surface dS (treated as a flat surface) is viewed from a point O can be calculated by:



$$d\Omega = \frac{dS\cos\alpha}{r^2} \qquad (V.2)$$

V.2.2.1 Energy Emittance

- *Monochromatic :*

A surface element dS emits a certain flux of energy by radiation in all directions of the half-space. This flux is distributed over a range of wavelengths. If we consider the energy flux emitted $d\varphi_{\lambda}^{\lambda+d\lambda}$ between the two wavelengths λ1 and $\lambda + d\lambda$, we define the monochromatic emittance of a source at temperature T as:

$$M_{\lambda T} = \frac{d\varphi_{\lambda}^{\lambda+d\lambda}}{dSd\lambda} \quad \left[W/m^2\right] \qquad (V.3)$$

- *Total :*

It is the density of heat flux emitted by radiation from dS over the entire spectrum of wavelengths. It depends only on the temperature T and the nature of the source:

$$M_T = \int_{\lambda=0}^{\lambda=\infty} M_{\lambda T} d\lambda = \frac{d\varphi}{dS} \quad \left[W/m^2\right] \qquad (V.4)$$

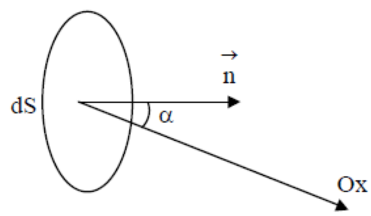
V.2.2.2 Energetic intensity in a direction

We call the energetic intensity I_x the flux per unit solid angle. It is the flux emitted by a surface element dS over a solid angle $d\Omega$.

$$I_x = \frac{d^2\varphi_x}{d\Omega} \qquad (V.5)$$

V.2.2.3 Energetic luminance in a direction

Let it be the angle formed by the normal $\vec{n}$ to the emitting surface S, in the direction O_x, along which the energetic intensity I_x is defined. The projection of S onto the plane perpendicular to O_x is called the apparent surface Σ.



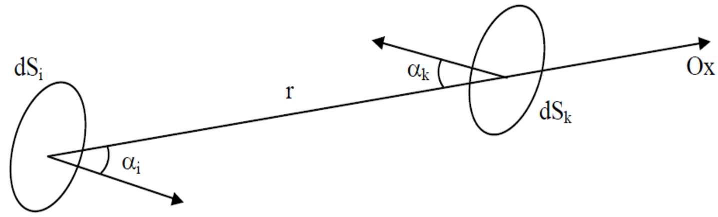
And the energetic intensity in the direction O_x, it is the flux per unit surface area emitted by the apparent surface. This is called the energetic luminance L:

$$L_x = \frac{I_x}{dS_x} = \frac{I_x}{dS\cos\alpha} = \frac{d^2\varphi_x}{d\Omega dS\cos\alpha} \qquad (V.6)$$

Application: Bouguer's Formula

We deduce from the previous definitions the expression $d^2\varphi_x$

- Sent by an element dS_k
- Of luminance L_x



$$d^2\varphi_x = I_x d\Omega = L_x dS_i \cos\alpha_i d\Omega$$

where: $d\Omega$ is the solid angle from which, since the surface dS_i we see the surface dS_k , therefore:

$$d\Omega = \frac{dS_k \cos\alpha}{r^2}$$

Hence the Bouguer formula :

$$d^2\varphi_x = L_{ix}\frac{dS_i \cos\alpha_i dS_k \cos\alpha_k}{r^2} \qquad \text{(V.7)}$$

V.2.3 Definitions relative to a receiver:

V.2.3.1 Eclairement :

It is the analogue of emittance for a source. Illumination is the flux received per unit surface receiver, coming from all directions.

V.2.3.2 Reception of radiation by a solid:

When incident radiation strikes a body at temperature *T*, part of the energy $\varphi_\lambda\rho_{\lambda T}$ is incident, $\varphi_\lambda\alpha_{\lambda T}$ absorbed, and the rest $\varphi_\lambda\tau_{\lambda T}$ is transmitted and continues its path.
Obviously :

$$\varphi_\lambda = \varphi_\lambda\rho_{\lambda T} + \varphi_\lambda\alpha_{\lambda T} + \varphi_\lambda\tau_{\lambda T} \qquad \text{(V.8)}$$

Where : $\rho_{\lambda T} + \alpha_{\lambda T} + \tau_{\lambda T} = 1$

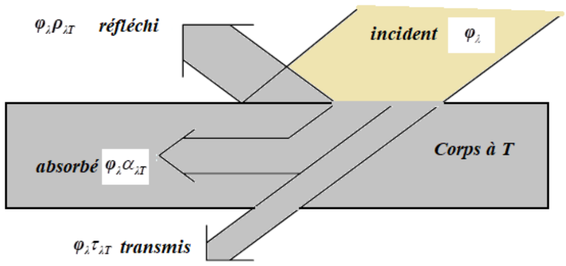


Fig. V.1: Reflection, transmission, and absorption of radiation.

We define the monochromatic reflective $\rho_{\lambda T}$, absorptive $\alpha_{\lambda T}$, and filtering powers $\tau_{\lambda T}$, which depend on the nature of the body, its thickness, its temperature T, the wavelength of the incident radiation, and the angle of incidence λ.

When considering the incident energy across the entire spectrum of wavelengths, we obtain the total reflective $\rho_{\lambda T}$, absorptive $\alpha_{\lambda T}$, and filtering powers $\tau_{\lambda T}$.

V.2.3 Definitions of Black and Gray Bodies

V.2.4.1 Black Body

A black body is a body that absorbs all the radiation it receives, regardless of its thickness, temperature, angle of incidence, and the wavelength of the incident radiation. It is defined as $\alpha_{\lambda T} = 1$ an ideal absorber with an absorptivity of 1 across all wavelengths. A surface coated with lamp black (soot) approximates a black body.

Properties of a Black Body:

- All black bodies radiate in the same manner.
- A black body emits more radiation than a non-black body at the same temperature.

V.2.4.2 Gray Bodies

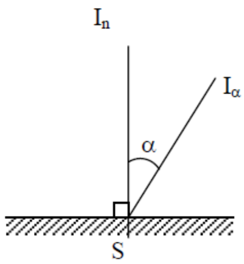
A **gray body** is a body whose absorptive power $\alpha_{\lambda T}$ is independent of the wavelength λ of the radiation it receives. It is defined as $\alpha_{\lambda T} = \alpha_T$.

V.3 Radiation Laws

V.3.1 Lambert's Law

In the case where the source is isotropic, the luminance is independent of direction: $L = L_x$

$$L_n = \frac{I_n}{S} \; ; \; L_\alpha = \frac{I_\alpha}{S\cos\alpha}$$



From this equality $L_n = L_\alpha$, we derive Lambert's law for an isotropic source.

$$I_\alpha = I_n \cos\alpha \qquad \text{(V.9)}$$

Thus, the indicatrix of intensity is a sphere tangent at point O to the emitting surface when it follows Lambert's law.

Fig. V.2: Luminance and energetic intensity of an isotropic source.

When a body follows Lambert's law, it is shown that emittance and luminance are proportional:

$$M = \pi L \qquad \left[W/m^2\right] \qquad \text{(V.10)}$$

V.3.2 Kirchhoff's law

At a given temperature T and for a given wavelength λ, the ratio $\frac{M_{\lambda T}}{\alpha_{\lambda T}}$ is the same for all bodies.

For a black body: $\alpha_{\lambda T} = 1$, the ratio $\frac{M_{\lambda T}}{\alpha_{\lambda T}}$ is the same $M_{O\lambda T}$ calling it the monochromatic emittance of the black body, thus:

$$M_{\lambda T} = M_{O\lambda T}\alpha_{\lambda T} \quad \left[W/m^3\right] \tag{V.11}$$

The monochromatic emittance of any body is equal to the product of its absorbing power. Hence the monochromatic emittance of the black body, which is why it is important to know the radiation emitted by the black body.

Case of Gray Bodies: Generalized Kirchhoff's Law

In the case of a gray body, we can generalize this law, which facilitates applications. Indeed, for a gray body, we have :

$$M_T = \int_{\lambda=0}^{\lambda=\infty} M_{\lambda T} d\lambda = \int_{\lambda=0}^{\lambda=\infty} \alpha_{\lambda T} M_{O\lambda T} d\lambda = \alpha_{\lambda T} \int_{\lambda=0}^{\lambda=\infty} M_{O\lambda T} d\lambda \tag{V.12}$$

By calling $M_{O\lambda T}$ the total emittance of a black body at temperature T, we obtain for a gray body:

$$M_T = M_{O\lambda T} \qquad \left[W/m^2\right] \tag{V.13}$$

The total emittance $M_{O\lambda T}$ of a gray body at temperature T is equal to the product of its absorptivity α_T and the total emittance M_{OT} of a black body at the same temperature.

V.3.1 Wien's Laws

The wavelength λ_m at which emission is maximal varies with the temperature of the source; it is expressed by Wien's displacement law:

$$\lambda_m = \frac{2898.10^{-6}}{T} \quad \left[\mu m\right] \tag{V.14}$$

The second law of Wien expresses the maximum value of the emittance of a black body as a function of its temperature:

$$Mo_{\lambda MT} = 1287.10^{-8}.T^5 \tag{V.15}$$

V.4 Black Body Radiation

V.4.1 Monochromatic Emittance :

It is given by Planck's law:

$$M_{O\lambda T} = \frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1} \qquad \left[W/m^{-3}\right] \tag{V.16}$$

With : $\begin{cases} C_1 = 3,742.10^{-16} \ \left[W.m^{-2}\right] \\ C_2 = 1,4385.10^{-2} \ \left[m.K\right] \end{cases}$

Planck's law allows us to plot isothermal curves representing the variations of emittance as a function of wavelength λ_m for various temperatures:

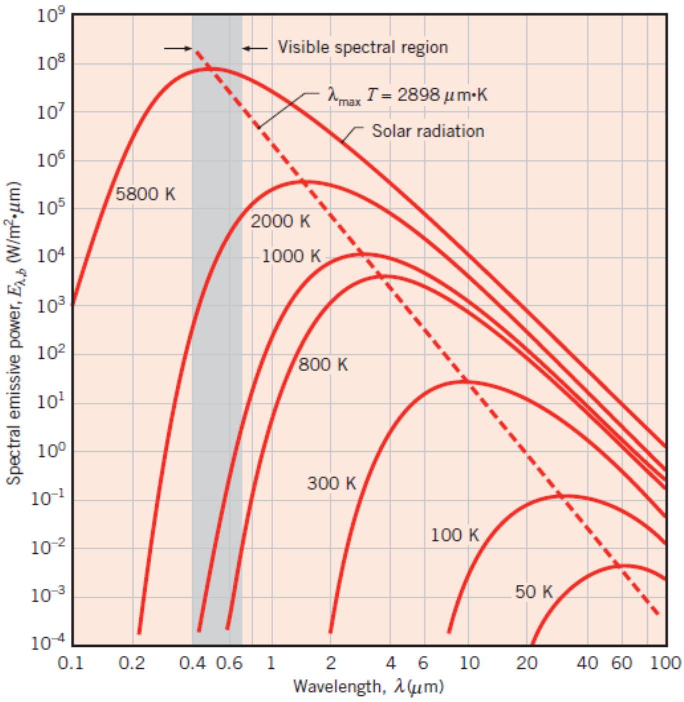


Fig. V.3: Variations of $M_{O\lambda T}$ as a function of wavelength for various temperatures.

The wavelength M_{λ} at which emission is maximal varies with the temperature of the source:

$$\lambda_M = \frac{2,897.10^{-3}}{T} \quad [\mu m] \tag{V.17}$$

$$M_{O\lambda_M T} = 0,410\left(\frac{T}{10}\right)^5 \quad [W/m^3] \tag{V.18}$$

☞ For the sun $(T \simeq 5777K)$, 90% of the energy is emitted between 0.31 and 2.5 µm, with the maximum located in the visible spectrum. In contrast, a black body at 373 K (100°C) has its maximum emission around $\lambda = 8\mu m$ in the infrared.

V.4.2 Total Emittance :

The integration of Planck's formula over all wavelengths gives the total emittance M_{O_T} of the black body, which depends only on the temperature T. This results in the Stefan-Boltzmann law:

$$M_{O_T} = \sigma T^4 \quad [W/m^2] \tag{V.19}$$

with: $\sigma = 5,675.10^{-8}\left[W.m^{-2}.K^{-4}\right]$

In calculations, we will often write: $\sigma = 5,675\left(\frac{T}{100}\right)^4$

V.4.3 Fraction of emittance:

It is the fraction of flux emitted per unit surface of the black body at temperature T between the wavelengths λ_1 *and* λ_2 :

$$F_{\lambda_1 T-\lambda_2 T}=\frac{\int_{\lambda_1}^{\lambda_2} Mo_{\lambda T}d\lambda}{\int_0^{\infty} Mo_{\lambda T}d\lambda}=\frac{\int_{\lambda_1}^{\lambda_2} Mo_{\lambda T}d\lambda}{\sigma T^4}=\frac{\int_0^{\lambda_2} Mo_{\lambda T}d\lambda-\int_0^{\lambda_1} Mo_{\lambda T}d\lambda}{\sigma T^4}$$

$$\Rightarrow\quad F_{\lambda_1 T-\lambda_2 T}=\frac{\int_0^{\lambda_2} Mo_{\lambda T}d\lambda}{\sigma T^4}-\frac{\int_0^{\lambda_1} Mo_{\lambda T}d\lambda}{\sigma T^4} \qquad (V.20)$$

Which can also be written as: $F_{\lambda_1 T-\lambda_2 T}=F_{0-\lambda_2 T}-F_{0-\lambda_1 T}$

Let us calculate $F_{0-\lambda T}$ at T constant:

$$F_{0-\lambda T}=\frac{1}{\sigma T^4}\int_0^{\lambda}\frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1}d\lambda=\frac{1}{\sigma}\int_0^{\lambda}\frac{C_1(\lambda T)^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1}Td\lambda=\frac{1}{\sigma}\int_0^{\lambda}\frac{C_1(\lambda T)^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1}d(\lambda T) \qquad (V.21)$$

☞ We observe that $F_{0-\lambda T}$ depends only on the product λT . Therefore, it is sufficient to create a table once and for all λT , giving $F_{0-\lambda T}$ and use it for the calculation of $F_{\lambda_1 T-\lambda_2 T}=F_{0-\lambda_2 T}-F_{0-\lambda_1 T}$

V.5 Radiation from Non-Black Bodies

V.5.1 Emission Factor or Emissivity:

We define the emissive properties of real bodies in relation to the emissive properties of black bodies under the same temperature and wavelength conditions. These properties are characterized using coefficients called emission factors or emissivities. These monochromatic or total coefficients are defined by :

$$\varepsilon_{\lambda T}=\frac{M_{\lambda T}}{Mo_{\lambda T}}\quad et\quad \varepsilon_T=\frac{M_T}{Mo_T} \qquad (V.22)$$

According to Kirchhoff's law, it can be shown that:

$$\alpha_{\lambda T}=\varepsilon_{\lambda T} \qquad (V.23)$$

V.5.1 Case of gray bodies :

They are characterized by $\alpha_{\lambda T}=\alpha_T$ according to what has been said above: $\varepsilon_{\lambda T}=\varepsilon_T$

Or: $M_T=\varepsilon_T Mo_T$, we deduce the emittance of the gray body at temperature *T*:

$$M_T=\varepsilon_T\sigma T^4 \qquad (V.24)$$

V.6 Reciprocal radiation of multiple surfaces:

Hypothesis :

The surfaces considered will be assumed to be homogeneous, opaque, isothermal, and gray.

Radiosity and Net Flux Lost

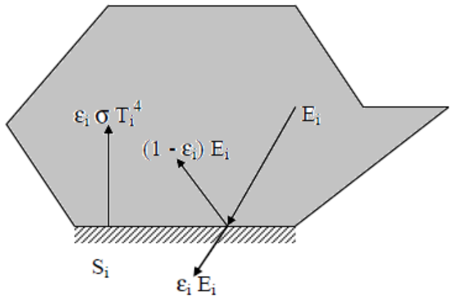
The radiation leaving a surface is the sum of its own emission and the reflection of a portion of the incident radiation on that surface S_i. The apparent emittance of the surface is called radiosity, denoted as J_i, thus:

$$J_i = \varepsilon_i \sigma T_i^4 + (1-\varepsilon_i) E_i \quad \left[W/m^2\right] \tag{V.25}$$

With :

E_i : Illumination of the surface S_i in [W/m²].

Let us now consider the surface S_i chosen among n isothermal and homogeneous surfaces that delimit a volume:



The net energy density lost by radiation S_i is written as:

$$\phi_{i_{net}} = \varepsilon_i \sigma T_i^4 - \varepsilon_i E_i \tag{V.26}$$

Introducing, the radiosity J_i par: $E_i = \dfrac{1}{(1-\varepsilon_i)}\left(J_i - \varepsilon_i \sigma T_i^4\right)$

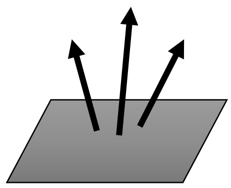
We obtain:

$$\phi_{i_{net}} = \frac{1}{(1-\varepsilon_i)}\left(\sigma T_i^4 - J_i\right) = \varepsilon_i\left(\sigma T_i^4 - E_i\right) = J_i - E_i \quad \left[W/m^2\right] \tag{V.27}$$

Exercice N° 01:

A surface of 1.5 cm² radiates like a black body at a temperature of 1600 °C. Calculate:

1) Total power in space.
2) Its energetic luminance.
3) The wavelength for which the radiation is maximum.



Black Bodies (S)

Solution :

1- Total power radiated in space:

$\varphi = M_{\lambda T} S = \sigma T^4 S$

<u>*AN*</u>*:*

$\varphi = M_{\lambda T} S = \sigma T^4 S = 5,67.10^{-8}(1873)^4 .1,5.10^{-4} = 104,67 W$

2- Energetic luminance :

$$L_T = \frac{\sigma}{\pi} T^4 = \frac{5,67.10^{-8}(1873)^3}{\pi} = 2,25.10^5 W/m^2 sr$$

3- The wavelength for which the radiation is maximal: According to Wien's law:

$$\lambda_{\max} = \frac{2900}{T} = \frac{2900}{1873} = 1,55 \mu m$$

For $2,3\mu m$ We have $x = \dfrac{\lambda}{\lambda_m} = \dfrac{2,3}{1,55} = 1,48$, On the reduced Planck layer, we read that for this value of x, y is approximately $y = 0,73$.

The spectral luminance then has the value:

$L_{\lambda T}^0 = bT^5 y = 4,1.10^{-8}(1873)^5 .0,73 = 6,9.10^8 W/m^2 sr \mu m$

Exercise N° 02:

A point radiation source emits a power of 200 W.
Calculate its energetic intensity.
It illuminates, at an incidence of 30°, a surface of 0.25 m² placed at 3 m.
Calculate the illumination of this surface, as well as the energetic flux it receives.

Solution:

- The source is uniformly distributed around it; the energetic intensity is:

$$I_e = \frac{\varphi}{4\pi} = \frac{200}{4\pi} = 15,9\, W/sr$$

- The energetic illumination of a surface element is given by:

$$E = \frac{I_e \cos\alpha}{L^2} = \frac{15,9.\sqrt{30}}{2.3^2} = 1,53\left(W/m^2\right)$$

Thus, the flux received by the surface is: $\varphi = E.S = 1,53.0,25 = 0,38\, W$

Exercise N° 3:

To heat a room in an apartment, a cylindrical radiator with a diameter of 2 cm and a length of 50 cm is used. This radiator radiates like a black body and emits a power of 1 kW:

1. Calculate its temperature.
2. Calculate the wavelength for which its luminance is maximal.
3. What should its temperature be so that this wavelength is 2 μm?
4. Calculate the power released.

Solution:

1- Calcul de sa température :

Le flux total rayonné est donné par la relation : $\varphi = Mo_T S = \sigma T^4 S$

$$S = \pi DL = 2\pi.10^{-2}.0,5 = 3,14.10^{-2}\, m^2$$

$$T = \sqrt[4]{\frac{\varphi}{\sigma S}} = \sqrt[4]{\frac{10^3}{5,67.10^{-8}.3,14.10^{-2}}} = 866K$$

2- La longueur d'onde λ_m pour laquelle sa luminance est maximale est :

$$\lambda_m = \frac{2900}{866} = 3,35\mu m\,.$$

3- Pour cette longueur d'onde soit de 2μm il faut une température :

$$T' = \frac{2900}{2} = 1450K$$

4- La puissance dégagée est proportionnelle à ma température à la quatrième puissance de la température, cette puissance serait :

$$\phi = \varphi\left(\frac{T'}{T}\right)^4 = 10^3\left(\frac{1450}{866}\right)^4 = 7,86.10^3 W$$

Exercise N° 04:

We consider a spherical object with a diameter of 23 cm at a temperature of 1500 K, suspended in the air as shown in the adjacent figure. Assuming that the sphere closely approximates a black body, determine:

a) The spectral emissive power of the black body at a given wavelength.

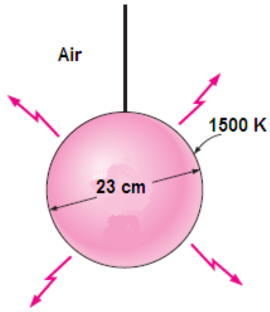
b) The total emissive power of the black body.

c) The wavelength at which the object's radiation emission reaches its maximum level, and deduce the maximum spectral emissive power.

d) The total amount of radiation emitted by this object in 5 minutes.

<u>Given</u>:

- Stefan-Boltzmann constant: $\sigma = 5,67.10^{-8} W/m^2K^4$
- Planck's law constants: $C_1 = 2\pi hc_0^2 = 3,742.10^8 W\,\mu m^4/m^2$



Solution :

An isothermal spherical object (∅=23 cm) at 1500 K is suspended in the air, assuming it behaves as a black body.

Its spectral emissive power at a wavelength of 1.93 μm will be determined using Planck's distribution law to be:

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.742\times10^8}{(1.93)^5\left[\exp\left(\frac{1.439\times10^4}{(1.93)(1500)}\right) - 1\right]}$$

$$= \mathbf{97638.27}\,\frac{\boldsymbol{W}}{\boldsymbol{m^2.\mu m}} \quad \text{Soit } \mathbf{97.64}\,\frac{\boldsymbol{kW}}{\boldsymbol{m^2.\mu m}}$$

(a) Its total emissive power will be determined using Stefan-Boltzmann's law to be:

$$E_b = \sigma T^4 = (5.67\times10^{-8})(1500)^4 = 287043.75\ W/m^2 \quad \text{Soit } 287\ kW/m^2$$

(b) The wavelength at which the radiation emission of this object reaches its maximum level will be determined using Wien's displacement law:

$$(\lambda T)_{puiss\,max} = 2897.8\ \mu m.K \longrightarrow \lambda_{puiss\,max} = \frac{2897.8}{1500} = 1.93\ \mu m$$

Thus, we can deduce that the maximum spectral emissive power at this wavelength is: $E_{b\lambda_{max}} = 97638.27\frac{W}{m^2.\mu m} = 97.64\frac{kW}{m^2.\mu m}$

(d) The total amount of radiation emitted by this black body in 5 minutes will be:

$$A_s = \pi\emptyset^2 = 3.141(0.23)^2 = 0.72243\ \text{m}^2$$

$$\Delta t = 5\times60 = 300\ \text{s}$$

$$Q_{ray} = E_b A_s \Delta t = (287\ \times10^3)(0.72243)(300) = 62201.22\ kJ$$

Exercise N° 05 :

A small surface of area $A = 10^{-3} m^2$ emits isotopically (see the figure below). The total luminance is measured in the direction normal to the surface as $L_n = 4500\ Wm^{-2}sr^{-1}$. The radiated flux is intercepted by four other surfaces of the same area A. These surfaces are oriented as shown in the figure below $(r_0 = 0,7m)$.

a) What is the total luminance in the four emission directions associated with the four receivers?

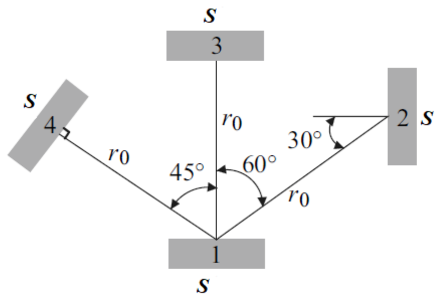
b) What are the values of the solid angles under which the four receiving surfaces are seen from the emitter?

c) What are the values of the flux intercepted by the four surfaces?

Assumptions:

(H1) The surfaces emit uniformly and isotopically.

(H2) The surfaces are small enough that we can consider: $A_i / r_i^2 \ll 1$.



Solution :

a) - If the radiation is isotropic, then its intensity does not depend on the emission direction. Thus, the luminance is:

$$L = L_n = 4500 \quad Wm^{-2}sr^{-1}$$

b) - The solid angle between surface i (2, 3, and 4) and surface 1 is given by the following relation: $d\Omega_{i\to 1} = dS\cos\alpha_i / r^2$. In this relation, θ_i represents the angle between surface i and the direction of the incident radiation coming from surface 1. However, based on hypothesis (H2), we can assume that r does not vary significantly compared to r0 when scanning across surface i. Therefore, an approximate value of the solid angles can be given by:

$$\Omega_{2\to 1} = \frac{S\cos\alpha_2}{r_0^2} = \frac{10^{-3}.\cos 30°}{(0,7)^2} = 1,767.10^{-3}\, sr$$

$$\Omega_{3\to 1} = \frac{S\cos\alpha_3}{r_0^2} = \frac{10^{-3}.\cos 0°}{(0,7)^2} = 2,041.10^{-3}\, sr$$

$$\Omega_{4\to 1} = \frac{S\cos\alpha_2}{r_0^2} = \frac{10^{-3}.\cos 45°}{(0,7)^2} = 1,443.10^{-3}\, sr$$

c)- To calculate the flux intercepted by surface i (2, 3, and 4) coming from surface 1, we use the following relation: $\phi_{1\to j} = LS\cos\alpha_1\Omega_{j\to 1}$. In this relation, α_1 represents the angle between the direction of the radiation emitted from surface 1 and incident on surfaces i with respect to surface 1. Thus, we obtain:

$$\phi_{1\to 2} = LS\cos\alpha_1\Omega_{1\to 2} = 4500.10^{-3}.\cos 60°.1,767.10^{-3} = 3,98.10^{-3}W$$

$$\phi_{1\to 3} = LS\cos\alpha_1\Omega_{3\to 1} = 4500.10^{-3}.\cos 0°.2,041.10^{-3} = 9,19.10^{-3}W$$

$$\phi_{1\to 4} = LS\cos\alpha_1\Omega_{4\to 1} = 4500.10^{-3}.\cos 45°.1,443.10^{-3} = 4,59.10^{-3}W$$

Chapitre VI

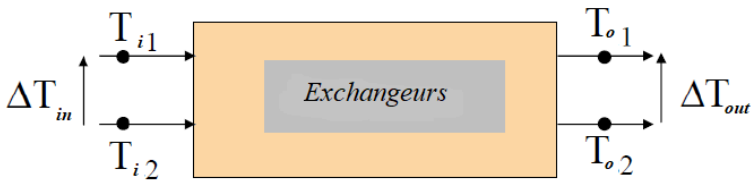
HEAT EXCHANGERS

VI.1 Introduction :

A heat exchanger is a device that transfers heat between two fluids at different temperatures, used in various applications such as car radiators and refrigerators. These devices facilitate the cooling of hot engine water or refrigerant by exposing them to cooler atmospheric air. Heat exchangers are also essential in applications like steam condensation, space heating, air-conditioning, and chemical processing. Two important aspects to consider in heat exchangers are :

- Thermal Aspect (T_{i1}, T_{e2}, T_{o1}, T_{o2}).
- Mechanical aspects (ρ_{i1}, ρ_{i2}, ... Q_{m1}, Q_{m2}).

  A heat exchanger = thermal quadripolar ⇔ 2 inputs and two outputs.



II.2 External Aspect:

Measure the essential operating characteristics of the heat exchanger at one of its four terminals, rather than undertaking more sensitive internal measurements. Parameters that are measurable and actually measured at the entry and exit of each are:

- State: liquid or gas;
- Mass flow rate, constant from entry to exit;
- Temperature, which varies within the exchanger;
- Pressure, which varies slightly.

Note: The thermo-physical characteristics of each of the two fluids are also known, specifically:

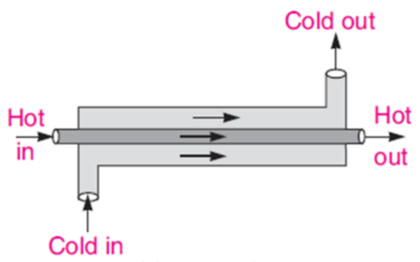
- The specific heat capacity (mass heat capacity) C_p;
- The density ρ;
- The thermal conductivity λ;
- The viscosity μ.

II.3 Classification:

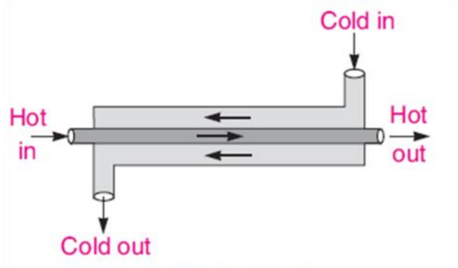
A classification can be established based on the relative direction of flow of the two fluids. We thus distinguish:

- Exchangers with parallel flow or anti-methodical exchanger ⇒ flow of the two fluids parallel and in the same direction.
- Counter-current exchangers or methodical exchanger ⇒ flow of the two fluids parallel and in opposite directions.

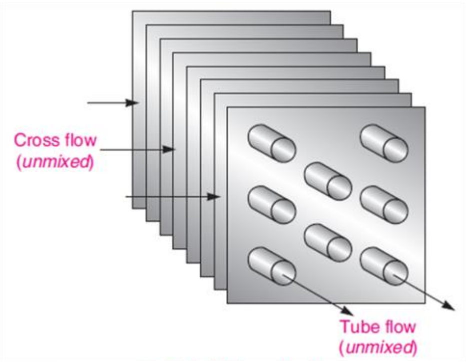
- <u>Crossflow exchangers</u>, with or without mixing ⇒ flow of the two fluids perpendicular to each other.
- <u>Coaxial exchanger</u> or double-tube exchanger: The flow of the fluids can occur in the same direction or in opposite directions.



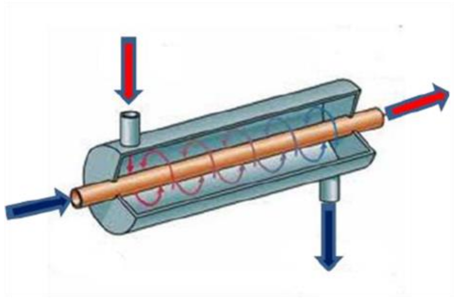
(a) Parallel flow



(b) Counter flow heat exchanger



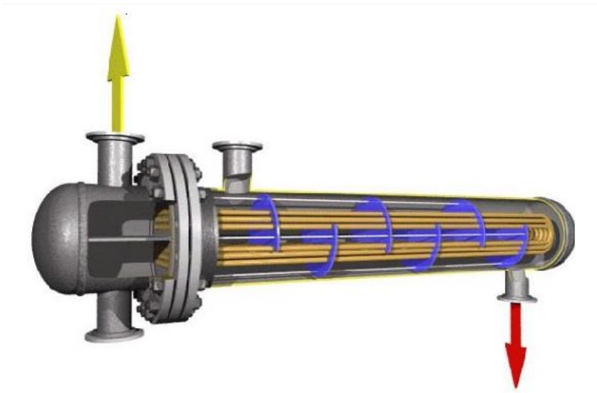
C) Crossflow exchangers



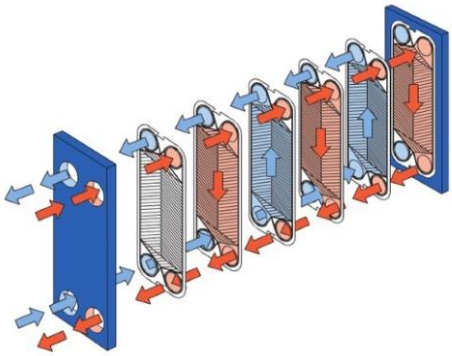
d) Coaxial exchanger

Another classification based on technological construction:

a) Tubular heat exchanger:



b) Plate heat exchanger:



c) Compact finned heat exchangers:

II.4 Overall Heat Transfer Coefficient (U):

A tubular heat exchanger typically consists of two fluids flowing separately, divided by a solid wall. Heat transfer occurs in three steps: initially from the hot fluid to the wall by convection, then through the wall by conduction, and finally from the wall to the cold fluid by convection again.

The total thermal resistance associated with this heat transfer process includes two convective and one conductive resistance, as illustrated in the figure below.

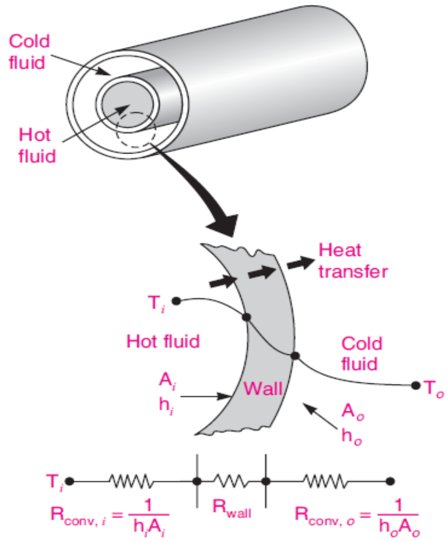
In this context, the subscripts 'i' and 'o' refer to the inside and outside surfaces of the inner tube of the heat exchanger, respectively.

For double pipe heat exchanger, $A_i = \pi d_i L$, and $A_o = \pi d_o L$, and the thermal resistance of the tube wall is given by: $R_{th\_Wall} = \dfrac{In\left(\dfrac{d_o}{d_i}\right)}{2\pi L\lambda}$

The total resistance for a double pipe heat exchanger with clean surfaces can be expressed as:

$$R_{th\_\text{total}} = R_{th\_conv,i} + R_{th\_wall} + R_{th\_conv,\text{o}}$$

$$R_{th\_\text{total}} = \frac{1}{h_i A_i} + \frac{In\left(\dfrac{d_o}{d_i}\right)}{2\pi L\lambda} + \frac{1}{h_o A_o}$$



Thermal resistance network for heat transfer in a double-pipe heat exchanger

In a well-insulated heat exchanger, the rate of heat transfer from the hot fluid is equal to the rate of heat transfer to the cold fluid. This means that :

$$Q = \dot{m}_c C_{Pc}\left(T_{ce} - T_{ci}\right) = C_c\left(T_{ce} - T_{ci}\right)$$
$$Q = \dot{m}_h C_{Ph}\left(T_{hi} - T_{he}\right) = C_h\left(T_{hi} - T_{he}\right)$$

where the subscripts c and h stand for the cold and hot fluids, respectively, and the product of the mass flow rate and the specific heat of a fluid $\dot{m}C_p$ is called the heat capacity rate.

the heat transfer rate is expressed as: $Q = \frac{\Delta T}{R_{th}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$

where U is **overall heat transfer coefficient.** It is measured in $\left[W/m^2K\right]$.

The overall heat transfer coefficient can be expressed as:

$$\sum R_{th} = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

The overall heat transfer coefficient based on outside tube surface can be expressed as:

$$U_o = \frac{1}{\sum R_{th} A_o}$$

$$= \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o}{2\pi L\lambda} In\left(\frac{d_o}{d_i}\right) + \frac{1}{h_o}}$$

$$= \frac{1}{\frac{d_o}{d_i h_i} + \frac{d_o}{2\lambda} In\left(\frac{d_o}{d_i}\right) + \frac{1}{h_o}}$$

Similarly, overall heat transfer coefficient based on inside tube surface can be expressed as:

$$U_i = \frac{1}{\sum R_{th} A_i}$$

$$= \frac{1}{\frac{A_i}{A_o h_o} + \frac{A_i}{2\pi L\lambda} In\left(\frac{d_o}{d_i}\right) + \frac{1}{h_i}}$$

$$= \frac{1}{\frac{d_i}{d_o h_o} + \frac{d_i}{2\lambda} In\left(\frac{d_o}{d_i}\right) + \frac{1}{h_i}}$$

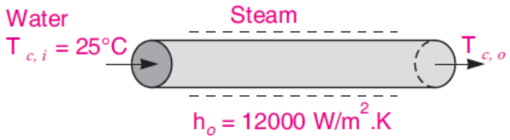
When the tube in a heat exchanger has a small wall thickness and high thermal conductivity, the thermal resistance of the tube material is negligible $\left(R_{thwall} \approx 0\right)$. This results in the inner and outer surfaces of the tube being essentially identical $\left(A_i \approx A_o\right)$. Consequently, the equation for the overall heat transfer coefficient simplifies.

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

Example VI-1:

Water at 25°C and 1.5 m/s enters a long brass ($\lambda = 110W/mK$) condenser tube with inner diameter of 1.34 cm and outer diameter of 1.58 cm. The heat transfer coefficient for condensation at outer surface of the tube is 12000 W/m².K.

- Calculate overall heat transfer coefficient based on outer surface of the tube.



VI.5 Heat Exchanger Analysis:

Heat exchangers often operate under consistent conditions for extended periods, allowing them to be modeled as steady-flow devices. The following assumptions are made in this model:

- The mass flow rate for each fluid remains constant.
- The properties of the fluids, such as temperature and velocity at the inlet and outlet, do not change.
- Variations in kinetic energy, potential energy, specific heat, and conduction along the length are considered negligible.
- The outer surface of the outer tube is well-insulated, preventing any heat transfer to the surroundings; thus, heat transfer occurs exclusively between the hot and cold fluids.
- There is no phase change occurring within the fluids in the heat exchanger.

According to the first law of thermodynamics, the rate of heat transfer from the hot fluid equals the rate of heat transfer to the cold fluid.

- The rate of heat transfer by hot fluid: $Q = \dot{m}_h C_{Ph} \left(T_{hi} - T_{ho}\right)$
- The rate of heat transfer to cold fluid: $Q = \dot{m}_c C_{Pc} \left(T_{ce} - T_{ci}\right)$

Usually, heat exchangers are analyzed either for their size using log mean temperature difference method or for their rating using effectiveness-NTU method.

VI.6 Log Mean Temperature Difference Method:

In a tubular heat exchanger, the temperature difference between the hot and cold fluids changes as they flow through it. To accurately assess this variability, the logarithmic mean temperature difference (ΔT_{LM}) is calculated. Additionally, the total heat transfer rate can be determined using the overall heat transfer coefficient and the available surface area of the exchanger.

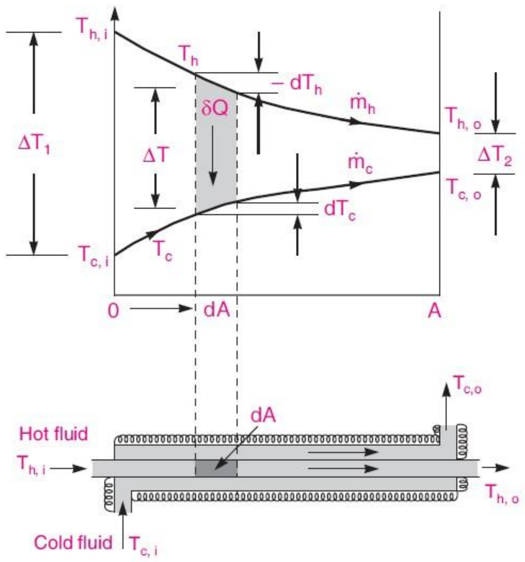
$$Q = UA\Delta T_{LM}$$

Where: ΔT_{LM} is the log mean temperature difference.

VI.6.1. Parallel Flow Heat Exchanger

In a parallel flow double-pipe heat exchanger, the temperature difference between the hot and cold fluids occurs at the inlet and diminishes exponentially toward the outlet.

The temperature of the hot fluid decreases and that of the cold fluid increases along the length of the exchanger, ensuring the cold fluid's temperature never surpasses that of the hot fluid.



An energy balance is applied to this system to analyze the thermal interactions between the fluids.

The rate of heat transfer δQ from hot fluid:

$$\delta Q = -\dot{m}_h C_{p,h} dT_h \tag{VI.1}$$

and for cold fluid:

$$\delta Q = \dot{m}_c C_{p,c} dT_c \tag{VI.2}$$

Let for differential surface area *d*A, the temperature difference ΔT between hot and cold fluid is expressed as: $\Delta T = T_h - T_c$

In differential form: $$d(\Delta T) = dT_h - dT_c \tag{VI.3}$$

Solving the eqn (1) and (2) for dT_h and dT_c as:

$$dT_h = -\frac{\delta Q}{\dot{m}_h C_{P,h}}$$

$$dT_c = -\frac{\delta Q}{\dot{m}_c C_{P,c}}$$

Substituting the values of dT_h and dT_c in differential form of eqn. (3), we get:

$$d(\Delta T) = -\delta Q\left[\frac{1}{\dot{m}_h C_{P,h}} + \frac{1}{\dot{m}_c C_{P,c}}\right] \tag{VI.4}$$

The heat transfer rate across the differential surface area dA of heat exchanger can also be expressed as:

$$\delta Q = U \Delta T dT$$

using δQ in eqn. (4) we get:

$$d(\Delta T) = -U\Delta T dT\left[\frac{1}{\dot{m}_h C_{P,h}} + \frac{1}{\dot{m}_c C_{P,c}}\right]$$

Rearranging,

$$\frac{d(\Delta T)}{\Delta T} = -UdAB \tag{VI.5}$$

Where: $B = \left[\frac{1}{\dot{m}_h C_{P,h}} + \frac{1}{\dot{m}_c C_{P,c}}\right]$

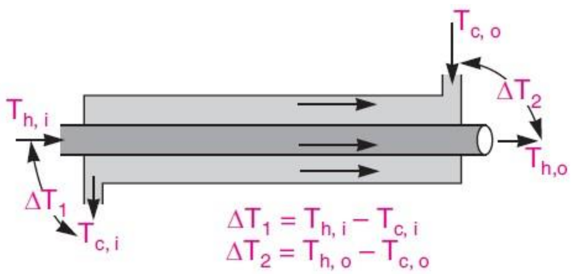
Integrating eqn. (5) from inlet to outlet conditions of the heat exchanger.

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -UB\int_0^A dA \tag{VI.6}$$

Or $$In\left[\frac{\Delta T_2}{\Delta T_1}\right] = -UBA$$

Substituting B, we get:

$$In\left[\frac{\Delta T_2}{\Delta T_1}\right] = -UA\left[\frac{1}{\dot{m}_h C_{P,h}} + \frac{1}{\dot{m}_c C_{P,c}}\right]$$



(a) Parallel flow heat exchanger

Substituting $\dot{m}_h C_{P,h}$ and $\dot{m}_c C_{P,c}$ for the cold and hot fluids, respectively, into the equations for the heat transfer rates, we obtain the following:

$$In\left[\frac{\Delta T_2}{\Delta T_1}\right] = -UA\left[\frac{T_{h,i}-T_{h,o}}{Q}+\frac{T_{c,o}-T_{c,i}}{Q}\right]$$

Or
$$Q=\frac{UA}{In\left[\frac{\Delta T_2}{\Delta T_1}\right]}\left[\left(T_{h,i}-T_{c,i}\right)-\left(T_{h,o}-T_{c,o}\right)\right]$$

Or
$$Q = UA\frac{\Delta T_1-\Delta T_2}{In\left[\frac{\Delta T_2}{\Delta T_1}\right]}$$

Comparing above expression with $Q = UA\Delta T_{LM}$, we get:

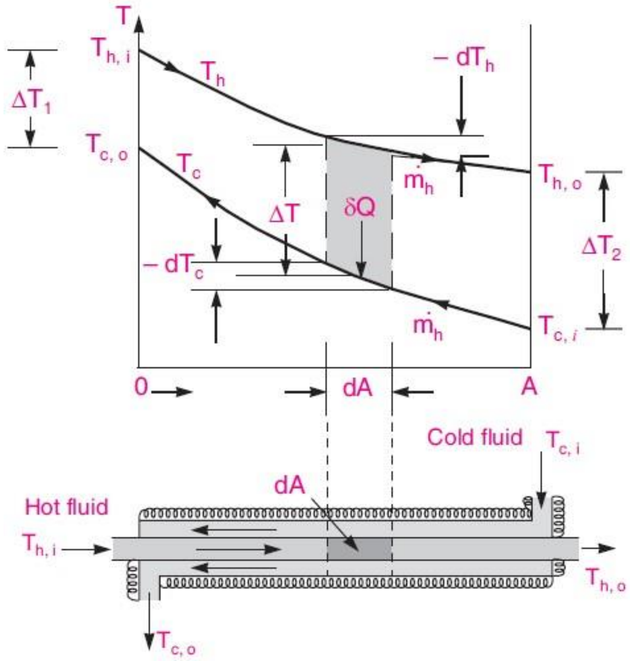
$$\Delta T_{LM} = \frac{\Delta T_1-\Delta T_2}{In\left[\frac{\Delta T_2}{\Delta T_1}\right]} = \frac{\Delta T_2-\Delta T_1}{In\left[\frac{\Delta T_1}{\Delta T_2}\right]} \quad \text{(VI.7)}$$

The ΔT_{LM} is called the log mean temperature difference, which is suitable form of temperature difference for tubular heat exchanger. the heat transfer rate in double pipe heat exchanger can be expressed as:

$$Q = UA\Delta T_{LM} = U_o A_o \Delta T_{LM} = U_i A_i \Delta T_{LM} \quad \text{(VI.8)}$$

VI.6.2. Counter Flow Heat Exchangers

In a counterflow arrangement, the hot and cold fluids enter the heat exchanger from opposite ends, flowing in opposite directions. As a result, the outlet temperature of the cold fluid can surpass that of the hot fluid. The temperature distribution for a counterflow heat exchanger is illustrated in the figure opposite.



Here,

$$\Delta T_1 = T_{h,i} - T_{c,o}$$
$$\Delta T_2 = T_{h,o} - T_{c,i}$$

Applying the energy balance to differential elements in hot and cold fluids.

The rate of heat transfer δQ from hot fluid:

$$\delta Q = -\dot{m}_h C_{p,h} dT_h$$

and for cold fluid:

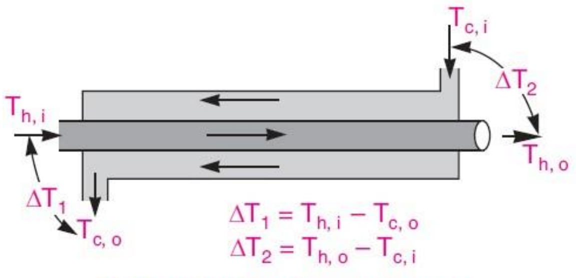
$$\delta Q = -\dot{m}_c C_{p,c} dT_c$$

The temperature difference ΔT between hot and cold fluids within the differential area *d*A can be expressed as:

$$\Delta T = T_h - T_c$$

In differential form:

$$d(\Delta T) = dT_h - dT_c$$



(b) Counter flow heat exchanger

By substituting the values of dT_c and dT_h into the differential form of the heat transfer rate equation for the hot and cold fluids, we obtain:

$$d(\Delta T) = -\delta Q\left[\frac{1}{\dot{m}_h C_{P,h}} - \frac{1}{\dot{m}_c C_{P,c}}\right]$$

Rearranging and integrating, we get:

$$In\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left[\frac{1}{\dot{m}_h C_{P,h}} - \frac{1}{\dot{m}_c C_{P,c}}\right]$$

Substituting $\dot{m}_h C_{P,h}$ and $\dot{m}_c C_{P,c}$ for the cold and hot fluids, respectively, into the equations for the heat transfer rates, we obtain the following:

$$In\left[\frac{\Delta T_2}{\Delta T_1}\right] = -UA\left[\frac{T_{h,i} - T_{h,o}}{Q} + \frac{T_{c,o} - T_{c,i}}{Q}\right]$$

Or

$$Q = \frac{UA}{In\left[\frac{\Delta T_2}{\Delta T_1}\right]}\left[\left(T_{h,i} - T_{c,i}\right) - \left(T_{h,o} - T_{c,o}\right)\right]$$

Or $\quad Q = U_o A_o \Delta T_{LM} = U_i A_i \Delta T_{LM}$

The term ΔT_{LM} refers to the logarithmic mean temperature difference in a counterflow heat exchanger and is defined as follows:

$$\Delta T_{LM,countre} = \frac{\left[\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)\right]}{In\left(\frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)} \quad \text{(VI.9)}$$

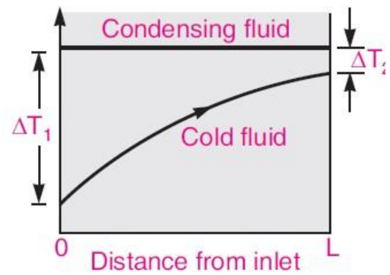
VI.6.3. Condenser

Temperature distribution for a condenser is shown in Figure below. The temperature of the condensing fluid (hot fluid) remains constant. Hence the temperature differences.

$$\Delta T_1 = T_h - T_{c,i}$$

$$\Delta T_2 = T_h - T_{c,o}$$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{In\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{\left[\left(T_h - T_{c,i}\right) - \left(T_h - T_{c,o}\right)\right]}{In\left(\frac{T_h - T_{c,i}}{T_h - T_{c,o}}\right)}$$



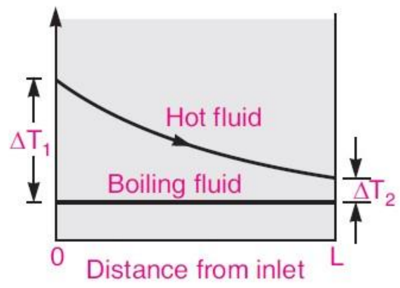
VI.6.4. Evaporator

Temperature distribution for an evaporator is shown in Fig. 14.8(c), the temperature of cold fluid (evaporating fluid) remains constant, hence, the log means temperature difference given by:

$\Delta T_1 = T_{h,i} - T_c$

$\Delta T_2 = T_{h,o} - T_c$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{In\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{\left[\left(T_{h,i} - T_c\right) - \left(T_{h,o} - T_c\right)\right]}{In\left(\frac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} \qquad \text{(VI.10)}$$



Example VI-2:

In a counter-flow double pipe heat exchanger, water is heated from 25°C to 65°C by an oil with a specific heat of 1.45 kJ/kg K and mass flow rate of 0.9 kg/s. The oil is cooled from 230°C to 160°C. If the overall heat transfer coefficient is 420 W/m2°C, calculate the following:

1) The rate of heat transfer,
2) The mass flow rate of water, and
3) The surface area of the heat exchanger.

VI.7. The Effectiveness-NTU Method

It is very simple to use LMTD method of heat exchanger analysis when inlet and outlet temperatures, mass flow rates for hot and cold fluids and overall heat transfer coefficient are available or easily be determined from specified relations. The heat transfer surface area and thus size of heat exchanger can easily be determined from:

$$Q = UA\Delta T_{LM} \qquad \text{(VI.11)}$$

When the heat exchanger type and size are fixed, and only inlet temperatures are known while outlet temperatures need determination, the LMTD method is ineffective, leading to impractical and tedious iterations. To simplify this scenario, Kays and London introduced the effectiveness-NTU method, eliminating the need for complex iterations.

VI.7.1 Heat Exchanger Effectiveness

It is a dimensionless parameter and defined as the ratio of actual heat transfer rate Q_{act} by heat exchanger to maximum possible heat transfer rate Q_{max}. It is denoted by ε and expressed as:

$$\varepsilon = \frac{Q_{act}}{Q_{\max}} = \frac{\dot{m}_c C_{P_c}\left(T_{ce} - T_{cs}\right)}{Q_{\max}} = \frac{\dot{m}_f C_{P_f}\left(T_{fs} - T_{fe}\right)}{Q_{\max}} \qquad \text{(VI.12)}$$

The maximum transferable heat: it depends on the difference between the inlet temperatures of the hot and cold fluids. This temperature difference is associated with the fluid that has the minimum heat capacity rate ($\dot{m}C_p$). Therefore:

$$Q_{\max} = \left(\dot{m}C_P\right)_{\min}\left(T_{ce} - T_{fe}\right) \qquad \text{(VI.13)}$$

The fluid with the minimum heat capacity rate ($\dot{m}Cp_{min}$) can be either the hot fluid or the cold fluid.

In both cases, we will have respectively:

$$\varepsilon_c = \frac{\left(\dot{m}_c C_{P_c}(T_{ce} - T_{cs})\right)}{\left(\dot{m}_c C_{P_c}(T_{ce} - T_{fe})\right)} = \frac{T_{ce} - T_{cs}}{T_{ce} - T_{fe}} \qquad (VI.14)$$

$$\varepsilon_c = \frac{\left(\dot{m}_f C_{P_f}(T_{fs} - T_{fe})\right)}{\left(\dot{m}_f C_{P_f}(T_{ce} - T_{fe})\right)} = \frac{T_{fs} - T_{fe}}{T_{ce} - T_{fe}} \qquad (VI.15)$$

Thus, we can generally write:

$$\varepsilon = \frac{\Delta T \text{ (minimum fluid)}}{\text{maximum temperature difference in the heat exchanger}} \qquad (VI.16)$$

Using equation (22), which was previously demonstrated:

$$\ln\left(\frac{T_{cs} - T_{fs}}{T_{ce} - T_{fe}}\right) = -K_g S\left(\frac{1}{\dot{m}_c C_{P_c}} + \frac{1}{\dot{m}_f C_{P_f}}\right)$$

This can be written as:

$$\ln\left(\frac{T_{cs} - T_{fs}}{T_{ce} - T_{fe}}\right) = -\frac{K_g S}{\dot{m}_f C_{P_f}}\left(1 + \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\right) \qquad (VI.17)$$

Or:

$$\frac{T_{cs} - T_{fs}}{T_{ce} - T_{fe}} = \exp\left[\ -\frac{K_g S}{\dot{m}_f C_{P_f}}\left(1 + \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\right)\right] \qquad (VI.18)$$

Equation (22) allows expressing T_{cs} as:

$$Q = -\dot{m}_c C_{P_c}(T_{ce} - T_{cs}) = \dot{m}_f C_{P_f}(T_{fe} - T_{fs})$$

$$T_{cs} = T_{ce} + \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\left(T_{fe} - T_{fs}\right) \qquad (VI.19)$$

Substituting equation (34) into the left-hand side of equation (33):

$$\frac{T_{cs} - T_{fs}}{T_{ce} - T_{fe}} = \frac{T_{ce} + \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\left(T_{fe} - T_{fs}\right) - T_{fs}}{T_{ce} - T_{fe}} \qquad (VI.20)$$

By adding and subtracting T_{fe} in the numerator of equation (35), simplifying and rearranging, we obtain:

$$\frac{T_{cs} - T_{fs}}{T_{ce} - T_{fe}} = 1 - \varepsilon_f - \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\varepsilon_f = 1 - \left(1 + \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\right)\varepsilon_f \qquad (VI.21)$$

Hence:

$$\varepsilon_f = \frac{1 - \left(\frac{T_{cs} - T_{fs}}{T_{ce} - T_{fe}}\right)}{1 + \left(\frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\right)} \qquad (VI.22)$$

Substituting the temperature ratio in equation (37) by its expression from equation (33), the effectiveness of a parallel-flow heat exchanger where the cold fluid is the one with the minimum heat capacity rate is:

$$\varepsilon_f = \frac{1 - \exp\left[NUT - \left(\frac{K_g S}{\dot{m}_f C_{P_f}}\right)\left(1 + \frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\right)\right]}{1 + \left(\frac{\dot{m}_f C_{P_f}}{\dot{m}_c C_{P_c}}\right)} \qquad (VI.23)$$

Note:

If the hot fluid has the minimum heat capacity rate ($\dot{m}_c C_{P_c}$), it can be shown that the same expression for effectiveness remains valid except that the subscript for cold fluid 'f' must be replaced by the subscript for hot fluid 'c'.

In general, we can write:

$$\varepsilon = \frac{1 - \exp\left[\ -\left(\frac{K_g S}{C_{\min}}\right)\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{1 + \left(\frac{C_{\min}}{C_{\max}}\right)} \qquad (VI.24)$$

Where:

$C = \dot{m} C_P$: is the heat capacity rate.

$NTU = \frac{K_g S}{C_{\min}}$: is the number of transfer units, representing the size of the heat exchanger.

b) Case of the counter-flow heat exchanger:

A demonstration similar to that carried out in the previous case (parallel flow) will lead to the expression of effectiveness for a counter-flow heat exchanger:

$$\varepsilon = \frac{1 - \exp\left[\ -\left(\frac{K_g S}{C_{\min}}\right)\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{1 - \left(\frac{C_{\min}}{C_{\max}}\right)\exp\left[\ -\left(\frac{K_g S}{C_{\min}}\right)\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]} \qquad (VI.25)$$

In both cases, we have:

$$\varepsilon_c = \frac{\dot{m}_c C_{P_c}(T_{ce} - T_{cs})}{\dot{m}_c C_{P_c}(T_{ce} - T_{fe})} = \frac{T_{ce} - T_{cs}}{T_{ce} - T_{fe}} \qquad (VI.26)$$

$$\varepsilon_f = \frac{\dot{m}_f C_{P_f}(T_{fs} - T_{fe})}{\dot{m}_f C_{P_f}(T_{ce} - T_{fe})} = \frac{T_{fs} - T_{fe}}{T_{ce} - T_{fe}} \qquad (VI.27)$$

Example:

Consider a heat exchanger with a heat transfer surface area of 11 m². The exchanger is used to heat oil flowing at a rate of $\dot{m}_f = 0.725$ kg/s, with a specific heat capacity $C_{P_f} =$ 1900 J/(kg·K) and inlet temperature $T_{fe} = 15$ °C. The hot fluid is water vapor flowing at $\dot{m}_c = 5.2$ kg/s, with a specific heat capacity $C_{P_c} = 1860$ J/(kg·K) and inlet temperature $T_{ce} = 130$ °C. The overall heat transfer coefficient is $K_g = 275$ W/(m²·°C).

- Calculate the heat transfer rates for both configurations (parallel flow and counter-flow).

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