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Theme:

**Study the mechanical behavior of materials under the effect of the
number of notches**

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THANKING AND APPRECIATION

WE ARE GREATFULL TO "ALLAH" THE ALMIGHTY
FIRST AND FOR
MOST WHOME PLESSED US BY COMPLETING THIS
HUMBLE WORK

THEN OUR FRIENDS AND FAMILLY WHO SUPPORT US IN
OUR JOURNEY



*Thank you for everyone who helped us
physically, mentally, even spiritly. we
most appreciate you and hope for this
work to make you proud in us*

Abstract

It is necessary to study the effect of notch quantity on material behavior for the sake of preserving the structural integrity and safety. Notches are stress concentrators and have a great impact on properties like strength, toughness and fatigue life. Detailed knowledge of these influences allows for accurate failure prediction, design improvement by eliminating stress concentrations (for instance by using fillets or radii) and material selection (for example, utilization of alloys with high notch toughness or composite materials). Some of the key aspects that have been examined are stress concentration factors, sites of fracture initiation, and fatigue crack propagation. These insights are important in many areas such as aerospace (fuselage connections, wing assemblies), automotive engineering (chassis, engine components), civil construction (steel connections, reinforced concrete) and manufacturing (machined and welded structures). Eventually, such information helps to create safer, lighter, and more durable products in a wide range of engineering domains.

ملخص

من الضروري دراسة تأثير عدد الثقوب على سلوك المواد بهدف الحفاظ على السلامة الهيكلية والأمان. تعتبر الثقوب مُركّزات للإجهاد ولها تأثير كبير على خصائص مثل القوة والمتانة وعمر الإجهاد. تسمح المعرفة التفصيلية لهذه التأثيرات بالتنبؤ الدقيق بالفشل، وتحسين التصميم عن طريق إزالة مُركّزات الإجهاد (على سبيل المثال باستخدام التقوسات أو الأنصاف) واختيار المواد (على سبيل المثال، استخدام سبائك ذات مقاومة عالية للكسر في وجود الشقوق أو المواد المركبة). بعض الجوانب الرئيسية التي تم فحصها هي معاملات تركيز الإجهاد، ومواقع بدء الكسر، وانتشار شقوق الإجهاد. هذه الرؤى مهمة في العديد من المجالات مثل الفضاء الجوي (وصلات جسم الطائرة، وتجميعات الأجنحة)، وهندسة السيارات (الشاسيه، ومكونات المحرك)، والإنشاءات المدنية (وصلات الصلب، والخرسانة المسلحة) والتصنيع (الهيكل المشغولة والملحمة). في النهاية، تساعد هذه المعلومات في إنشاء منتجات أكثر أماناً وأخف وزناً وأكثر متانة في مجموعة واسعة من المجالات الهندس

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List of Symbols and Notations

U	Potential energy of the cracked body.
U_0	Potential energy of uncracked body.
U	Elastic energy due to the presence of the crack
U	Elastic-surface energy due to the formation of crack surfaces. a = One-half crack length
$2(2aB)$	Total surface crack area
γ_s	Specific surface energy
E	Modulus of elasticity.
σ	applied stress and ν = Poisson's ratio
a	Crack size or half crack length
A	Area
A	Crack size or half crack length.
A/w	Ratio of crack length to specimen's width.
A_0	Original area.
AL	Instantaneous area and length.
B	Specimen thickness
d	A dimension used in the calculation of expansion E for cold-expanded holes.
D_0	Initial diameter.
E	Modulus of elasticity or Young's modulus.
e	The nominal stress or strain
ϵ (epsilon)	Strain. Defined as deformation of the material
ϵ_c	Compressive strain.
ϵ_0	Initial rapid elongation in a typical creep curve.
ϵ_t	Tensile strain. The elongation per unit length.
ϵ_T	True strain.
ϵ_v	Volumetric strain.
$\epsilon_x, \epsilon_y, \epsilon_z$	Strain components in x, y, z directions.
f	Fringe constant.
$f(\theta)$	A function of theta (cylindrical polar co-ordinates).
F	Applied force.
F_I, F_{II}	Functions of the ratio of crack length to specimen's width (A/w) and the type of load applied.
γ (gamma)	Shear strain.
γ	Surface energy per unit area.
G	Strain energy release rate.
G_c	Critical strain energy release rate.
h	Model thickness.

K	Bulk modulus.
K	Strength coefficient.
K	Stress intensity factor.
K _c	Critical value of K (fracture toughness).
K _I	Stress intensity factor for Mode I fractures.
K _{II}	Stress intensity factor for Mode II fracture.
K _{IC}	Plane-strain fracture toughness.
K _t	Elastic stress concentration factor.
L ₀	Original length.
n	Strain hardening exponent.
p	Pressure.
P	Applied force.
r, θ	Cylindrical polar
R	Crack resistance.
S	Nominal stress or strain.
σ (sigma)	Stress
σ*	Characteristic stress.
σ ₀	Yield point.
σ _c	Compressive stress.
σ _f	Fracture stress.
σ _i	Stress component.
σ ₀	Yield point stress (same as σ ₀).
σ _t	Tensile stress.

GENERAL INTRODUCTION

Geometric discontinuities, such as holes or grooves collectively referred to as notches are unavoidable in the design and manufacture of many engineering components and structures. These notches fundamentally disrupt the uniform flow of stress in a loaded material, acting as stress concentrator, notch effects are crucially important for several reasons, they are vital for engineering stress analysis to predict actual stresses in structural components, enable accurate failure prediction under various loading conditions, guide material selection when components feature geometric discontinuities, and are essential for structural integrity assessment and maintenance protocols. Our work will be structured to build an understanding of these phenomena.

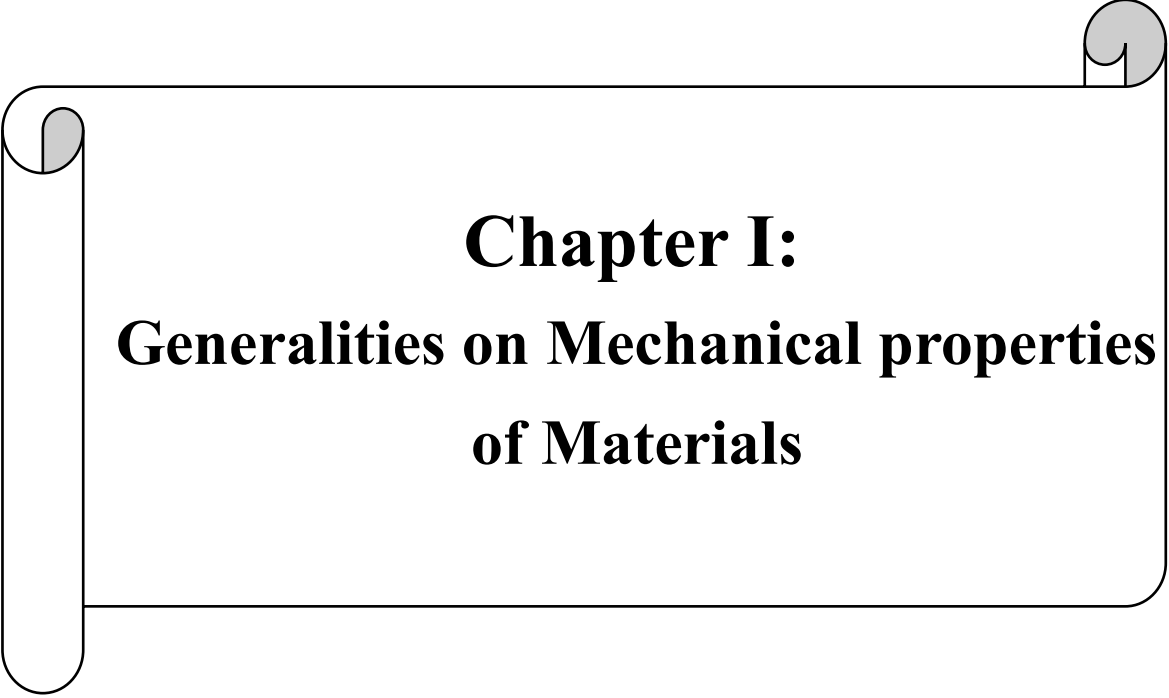
In working on this project, we will be focusing on providing a comprehensive study, moving from fundamental material properties and theoretical concepts to a numerical demonstration, to highlight the critical impact that the presence and increasing number of notches have on the mechanical integrity and performance of materials and components.

We did organize this memoire In III chapters in which we will discuss:

Chapter I: provides foundational knowledge on the mechanical properties of materials, defining concepts like stress and strain, elasticity and plasticity, creep, and fracture toughness. This chapter underscores the importance of studying notch effects and introduces the various approaches used for this purpose, including experimental, analytical, and numerical methods.

Chapter II: we focused specifically on how notches influence mechanical behavior. It elaborates on the causes of stress concentration around holes. and details the consequences for component performance. And conducting research on the most used theoretical approaches such as Griffith and Irwin's theories, and the stress intensity factor

Chapter III: presents a practical investigation using Numerical study and simulation in ABAQUS. It introduces the Finite Element Analysis (FEA) software ABAQUS. and the specific method used, XFEM (extended Finite Element Method), which is well-suited for modeling discontinuities like cracks and holes. this chapter involves a simulation and comparison of the tensile behavior of a 3D ASTM D638 specimen with **one**, two, and four holes.



**Chapter I:
Generalities on Mechanical properties
of Materials**

I.1 Introduction:

Material mechanical properties determine how a material will respond to applied forces, which then define its suitability for various engineering applications.

These properties are critical to structural integrity and functional performance; from the robust steel beams that support skyscrapers to the flexible polymers in everyday gadgets.

This field examines a material's response to external loads, and in terms of the strength, stiffness, ductility, hardness, and toughness. Engineers can make informed decisions about material selection, design components effectively, and predict service life of components under both normal and abnormal operating conditions by examining how materials deform, fracture, or resist permanent changes.

These generalities are explored in an attempt to gain a deeper understanding of material behavior, and how it plays such a critical role in the world around us.

I.1.1 Importance of Studying Notch Effects

For a given load, notches are stress concentrators and therefore cause a localized increase in stress. This stress concentration can have a number of critical consequences such as:

- **Mechanical Engineering:**

Reduced Strength: The stress at the notch tip may be greatly in excess of the nominal stress on the component. Premature yielding or fracture at loads lower than the material would normally absorb without a notch can result from this.

Fatigue Failure Initiation: Notches are preferentially sites for fatigue crack initiation in cyclic loading. Failure over time occurs due to the elevated stress levels and strain gradients at the notch root that accelerate damage accumulation.

Ductility Altered: In ductile materials, the triaxial stress state due to a notch can constrain plastic deformation at the notch tip, reducing the local specimen ductility, and potentially rendering the material brittle like. On the other hand, some ductile materials can show "notch strengthening" under tensile loading, as the constraint of plastic deformation by the surrounding elastic material.

Fracture Toughness Reduction: Fracture toughness of a material can be significantly reduced by a notch leading to the increased chances of crack propagation under a given applied stress intensity.

Notches cause a significant decrease in a material's resistance to impact loads. With its stress concentrator, rapid crack initiation and propagation occurs, with lower energy absorption, and potentially catastrophic failure.

Thus, it is important to understand how notches affect mechanical behavior. [6]

Engineering Stress Analysis: Due to the stress concentration factors related to notches, engineers must take into account these factors to predict actual stresses experienced by structural components.

Accurate Failure Prediction: Failure prediction that is reliable depends on taking into account notch effects for static, fatigue or impact loading.

Critical to safety is ensuring that the structures are designed with appropriate notch geometries, low stress concentration or that the materials used are low in notch sensitivity.

Material Selection: For materials subjected to such geometric discontinuities, it is an important consideration when choosing a material.

Structural Integrity Assessment: It is critical to the maintenance and safety protocols to assess the influence on remaining life and load carrying capacity of the structures due to existing notches or defects.

- **Aerospace Engineering:**

Aircraft Structures: Designing airframes, wing components, engine parts, and landing gear where weight reduction often leads to thin sections and complex geometries with stress concentrations. Fatigue and fracture of notched components are paramount concerns for aircraft safety and longevity.

Propulsion Systems: Turbine blades, shafts, and other engine components operate under extreme conditions and cyclic loading, making notch analysis vital for preventing fatigue failure.

Spacecraft Design: Similar to aircraft, spacecraft components must be lightweight and reliable, necessitating careful consideration of notch effects.

Drones: Engineers use Finite Element Analysis (FEA) and other simulation tools to predict stress distribution around notches in drone frames and arms. They apply principles like using **fillets (rounded corners)** instead of sharp angles, optimizing hole shapes (e.g., circular instead of sharp-angled), and reinforcing high-stress areas to mitigate stress concentration and prevent premature failure.

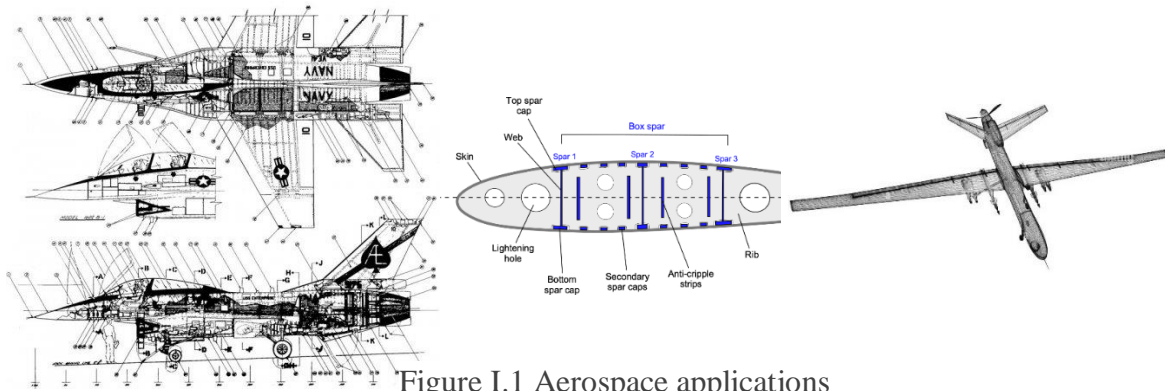


Figure I.1 Aerospace applications

- **Automotive Engineering:**

Chassis and Body Design: Designing vehicle frames, suspension components, and body panels that can withstand static and dynamic loads, including crash impacts, where notches or design discontinuities are common.

Engine and Drivetrain Components: Crankshafts, connecting rods, and gear teeth are subject to significant cyclic stresses, making notch fatigue a critical design consideration.

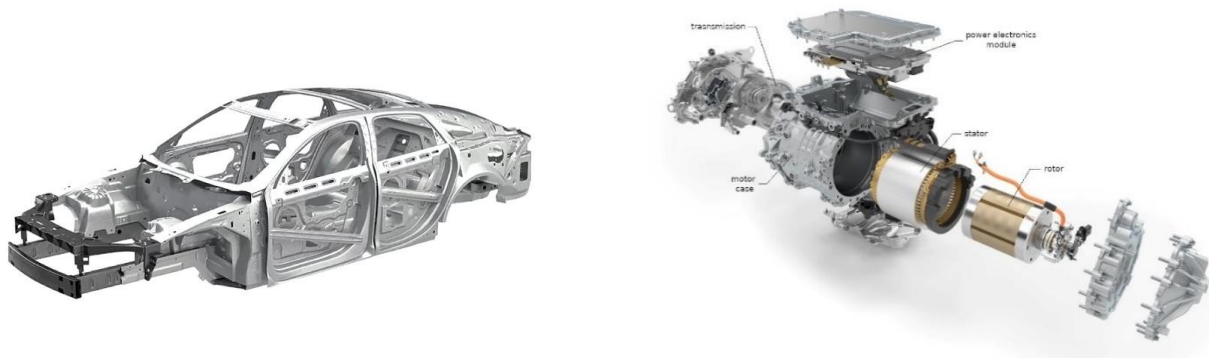


Figure I.2 Automotive applications

- **Biomedical Engineering:**

Implants and Prosthetics: Designing medical devices that are implanted in the body (e.g., hip implants, dental implants). These devices must withstand cyclic loading and biological environments, and any stress concentrators from design or manufacturing flaws must be minimized to prevent fatigue or fracture.

- **Nuclear Engineering:**

Reactor Components: Designing and assessing the integrity of pressure vessels, piping, and other critical components in nuclear power plants, which operate under high temperatures, pressures, and irradiation, making fracture mechanics and notch analysis essential for safety.

- **Materials Science and Engineering:**

Material Development: Understanding how alloying, heat treatment, and microstructure affect a material's notch sensitivity and fracture toughness to develop new, more resilient materials.

Failure Analysis: Investigating why components failed, often tracing the origin of failure to a notch or a defect that caused stress concentration.

Characterization: Developing and refining test methods (like notched tensile tests, Charpy, Izod) to accurately measure material properties related to notch behavior.

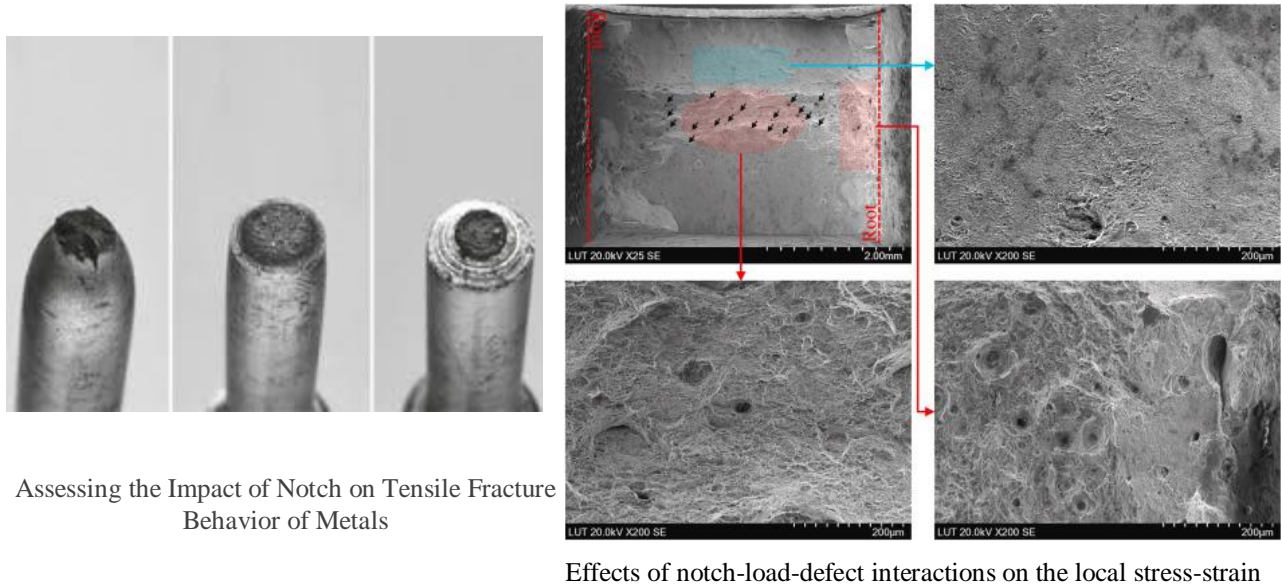


Figure I.3 Applications on Materials Science and Engineering

I.1.2 Approaches to Studying Notch Effects.

Various experimental, analytical and numerical approaches for investigating the effect of notches on the mechanical behavior of materials are developed.

Experimental Testing:

Comparing tensile strength and ductility of notched and un notched specimens with varying notch geometry (shape, size, root radius) aids in quantifying the notch sensitivity of the material.

Fatigue Testing of Notched Specimens: Fatigue tests (axial, bending, torsional) on notched samples are performed to determine fatigue strength reduction factors and to evaluate the effects of notch on fatigue life.

Standardized tests such as Charpy and Izod impact tests, as well as static fracture toughness tests on specimens with machined notches or fatigue pre-cracks are used to evaluate a material's resistance to crack initiation and propagation in the presence of sharp discontinuities.

Digital Image Correlation (DIC): This optical method enables measurement of the full field surface displacements and strains around the notch under load, and gives the details of the local deformation behavior as well as stress concentrations.

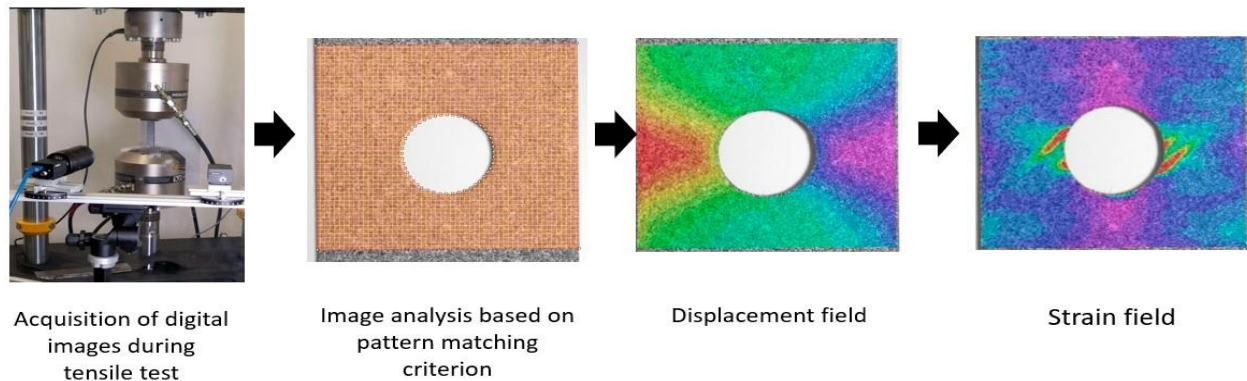


Figure I.4 Scheme of digital image correlation (DIC) procedure.

I.1.3 Analytical Methods.

Stress Concentration Factors (Kt):

Elastic stress analysis provides theoretical stress concentration factors for different notch geometries under different loading conditions. The peak stress at the notch root is estimated using these factors. These semi empirical approaches are.

Neuber's Rule and other Notch Correction Methods: These attempt to relate the elastic stress concentration factor to the fatigue notch factor (K_f) or fatigue strength reduction factor ($K_{f,r}$), The effect of notch geometry and material properties is considered

Linear Elastic Fracture Mechanics (LEFM): This theory employs the stress intensity factor (K) as a characteristic of the stress field in the vicinity of the tip of a sharp crack emanating from a notch and predicts fracture in terms of a critical value of K (fracture toughness, K_{Ic}).

I.1.4 Numerical Methods.

Finite element analysis (FEA): FEA is a very useful tool for stress and strain distribution in complicated notch geometries under different loadings, elastic and plastic. Detailed analysis of stress concentrations, triaxiality and crack initiation sites are permitted.

Boundary Element Method (BEM): Another such numerical technique that nicely fits problems with complex geometries and stress concentration is Boundary Element Method (BEM), commonly used in fracture mechanics studies.

Computational Fracture Mechanics (CFM): The use of numerical techniques such as FEA for crack initiation and propagation from notches and prediction of fracture behaviors and fatigue life.

Local Approach Methods: are those that involve considering the material behavior at a critical distance or volume ahead of the notch tip to predict failure. Usually, materials specific parameters and failure criteria based on local stresses or strains are involved.

I.1.5 Brief History of Composite Materials.

➤ Stat of art: older studies on materials with numbers of notches

Composite materials, i.e. materials comprising two or more distinct constituents with different properties, combine to create a material with superior properties, have been around for millennia:

Early Examples (7000 BCE - Classical Era): Ancient Mesopotamia and Egypt are among the first examples of using mud mixed with straw for stronger bricks. Wood strips were glued at different angles, around 3400 BCE, when Mesopotamians developed early forms of plywood. Egyptians also utilize composites because weight and papyrus with plate (cartonnage). Composite bows made of wood, horn, and sinew were developed in the Medieval Period and provided superior energy storage over traditional wooden bows.

Portland cement: The development of Portland cement in the 19th Century led to the development of reinforced concrete (a composite of concrete and steel reinforcing bars), which revolutionized the construction industry because of its high strength and fire resistance.

Modern era of composites started with the development of synthetic polymers and fiber reinforcement in the 20th Century. Key milestones include:

Significant step: Invention of Bakelite, one of the first synthetic thermosetting resins, early 1900s.

Owens Corning introduced the glass fibers in 1930s and unsaturated polyester resins were developed for the fiber reinforced polymer (FRP) industry in 1930s.

In the 1960s, high strength and high stiffness carbon fibers enabled new lightweight and high-performance composite applications in aerospace and other harsh environments.

Continuous advancement in the resin systems (epoxies, vinyl esters, etc.), fiber types (aramid, boron, etc.), and manufacturing processes (layup, molding, pultrusion, etc.) has resulted in a wide array of composite materials with designed properties for a wide range of applications including aerospace and automotive, sports equipment, and construction. The more recent and rapidly evolving area within the field is nanocomposites with nanoscale reinforcements.

Professor L. Susmel Primarily 2000s: research on fatigue assessment of notched metallic and composite components. Develops new fatigue criteria. and increasingly uses machine learning for fatigue limit prediction.

Professor Filippo Berto: study on fracture mechanics and fatigue, especially for additively manufactured (3D-printed) metals. Investigates how manufacturing defects (porosity, roughness) interact with design notches to control fatigue and failure.

Professor Pedro Lazzarin: whom work is leading authority on the Theory of Critical Distances (TCD). Develops and applies TCD for predicting static strength and fatigue limits of both sharp and blunt notched components, bridging classical stress concentration and fracture mechanics.

A. Khechai & P.M. Mohite cited in papers in 2018: Research on stress concentration and strength degradation in **composite laminates** with various notch shapes. Utilizes techniques like Digital Image Correlation (DIC) and develops predictive models for notched composite tensile strength.

Jifa Chen, Hao Ni, Li Huang, Yu Yang, Zhuoyi Chen in recent publications in the 2020s: study to applying and refining the Critical Distance Theory to predict the fatigue life of notched components, often in steel structures, combining experimental data with Finite Element (FE) simulations.

Professor Michael R. Lovell (of University of Wisconsin-Milwaukee and Marquette University, USA - 2020): research involved the mechanical behavior of materials and components, which would inherently include considerations of stress concentrations in designs.

He published extensively in engineering journals. His work is involved in understanding and manipulating localized stress concentrations arising from surface features,

I.2 Properties of Mechanical Materials:

I.2.1 Stress and Strain:

Is a Fundamental concept which are important to understand how mechanical materials behave under load are stress and strain. Internal resistance of a material to an applied force is called stress ($\sigma=F/A$).

However, strain is deformation of the material, that is change in length compared to the original length ($\epsilon = \Delta L / L_0$). It is known that a material's stiffness and strength can be related to the relationship between stress and strain.

Also, the degree of stress and strain concentration is a factor in the fatigue strength of the notch parts

It is measured by the **elastic stress concentration factor**, K_t :

$$K_t = \frac{\sigma}{s} = \frac{\epsilon}{e} \quad (I. 1)$$

As long as $\sigma = \epsilon \text{ constant} = E$ Where: σ = the maximum stress or strain at the notch
 s or e = the nominal stress or strain.

I.2.1.1 Stress (σ)

When a material is subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point. [4]

In other words, A body under the action of external forces, undergoes distortion and the effect due to this system of forces is transmitted throughout the body developing internal forces in it. To examine these internal forces at a point O, inside the body, consider a plane MN passing through the point O. If the plane is divided into a number of small areas, and the forces acting on each of these are measured, it will be observed that these forces vary from one small area to the next. On the small area DA at point O, a force DF will be acting as shown in the from this the concept of stress as the internal force per unit area can be understood. Assuming that the material is continuous, the term "stress" at any point across a small area ΔA can be defined by the limiting equation as below. [1]

- It uses original cross section area of the specimen and also known as engineering stress or conventional stress.

Therefore, $\sigma = \frac{P}{A}$

- P is expressed in *Newton* (N) and A , original area, in square meters (m^2), the stress σ will be expressed in N/m^2 . This unit is called *Pascal* (Pa).

- As *Pascal* is a small quantity, in practice, multiples of this unit are used.
 $1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$ (kPa = Kilo Pascal)
 $1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$ (MPa = Mega Pascal)
 $1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$ (GPa = Giga Pascal)



Figure I.5 Stress

• Tensile stress (σ_t)

If $\sigma > 0$ the stress is tensile. i.e. The fibers of the component tend to elongate due to the external force. A member subjected to an external force tensile P and tensile stress distribution due to the force is shown in the given figure.I.6 [4]

Figure 1

• Compressive stress (σ_c)

If $\sigma < 0$ the stress is compressive. i.e. The fibers of the component tend to shorten due to the external force. A member subjected to an external compressive force P and compressive stress distribution due to the force is shown in the given figure I.7 [4]

• Shear stress (τ)

When forces are transmitted from one part of a body to other, the stresses developed in a plane parallel to the applied force are the shear stress. Shear stress acts parallel to plane of interest. Forces P is applied transversely to the member AB as shown. The corresponding internal forces act in the plane of section C and are called shearing [4]

forces. The corresponding average shear stress (τ) = $\frac{P}{A}$

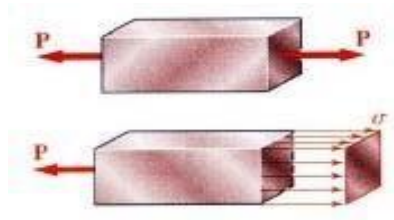


Figure I.6 Tensile stress

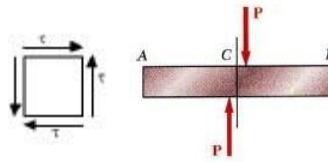


Figure I.8 Shear stress

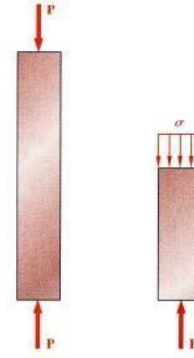


Figure I.7 Compressive stress

I.2.1.2 Strain (ϵ)

The displacement per unit length (*dimensionless*) is known as strain.

- **Tensile strain (ϵt)**

The elongation per unit length as shown in the figure is known as tensile strain.

$$\epsilon t = \Delta L / L_0$$

It is engineering strain or conventional strain. Here we divide the elongation to original length not actual length ($L_0 + \Delta L$)

- **Compressive strain (ϵc)**

If the applied force is compressive then the reduction of length per unit length is known as compressive strain. It is negative. Then $\epsilon c = (-\Delta L) / L$

I.2.1.3 True stress and True Strain:

- **True stress:**

The true stress is defined as the ratio of the load to the cross-section area at any instant.

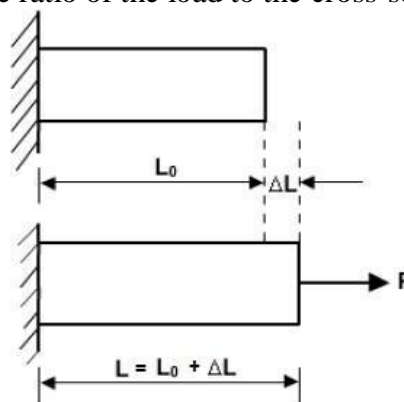


Figure I.9 Tensile strain

$$\sigma_t = \frac{\text{load}}{\text{instantaneous area}} = \sigma_{1+\varepsilon} \tag{I.2}$$

• **True strain:**

$$\varepsilon = \int_{L_0}^L \frac{dl}{l} = \ln\left(\frac{L}{L_0}\right) = \ln(1 + \varepsilon) = \ln\left(\frac{A_0}{A}\right) - 2\ln\left(\frac{d_0}{d}\right)$$

(I.3) or engineering strain (ε) = $e^{\varepsilon T} - 1$

The volume of the specimen is assumed to be constant during plastic deformation.

[$A_0 L_0 = AL$] It is valid till the neck formation

• **Comparison of engineering and the true stress-strain curves shown below:**

- The true stress-strain curve is also known as the **flow curve**.
- True stress-strain curve gives a true indication of deformation characteristics because it is based on the instantaneous dimension of the specimen.
- In engineering stress-strain curve, stress drops down after necking since it is based on the original area.
- In true stress-strain curve, the stress however increases after necking since the cross-sectional area of the specimen decreases rapidly after necking.
- The flow curve of many metals in the region of uniform plastic deformation can be expressed by the simple power law. $\sigma T = K(\varepsilon T)^n$ Where K is the strength coefficient n is

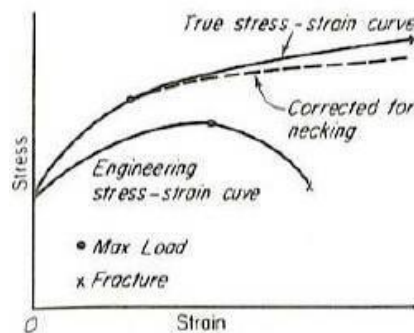


Figure I.10 true stress strain and engineering stress and strain curve with fracture point

the strain hardening exponent $n = 0$ perfectly plastic solid $n = 1$ elastic solid For most metals, $0.1 < n < 0$

- True stress-strain curve for typical ductile materials, i.e., aluminum, show that the stress strain relationship follows up the Hooke’s law up to the yield point, σ_0 . [2]

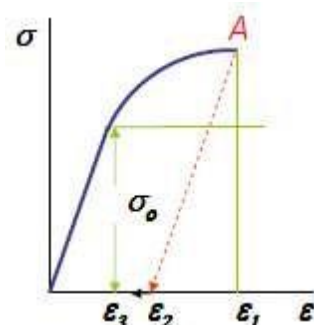


Figure I.11 true strain curve

- Beyond σ_0 , the metal deforms plastically with strain-hardening. This cannot be related by any simple constant of proportionality. [2]
- If the load is released from straining up to point A, the total strain will immediately decrease from ϵ_1 to ϵ_2 by an amount of σ/E . The strain $\epsilon_1 - \epsilon_2$ is the recoverable elastic strain. Also, there will be a small amount of the plastic strain $\epsilon_2 - \epsilon_3$ known as anelastic behavior which will disappear by
- The engineering stress – strain curve is based entirely on the original dimensions of the specimen (this cannot represent true deformation characteristic of the material). [2]
- The true stress – strain curve is based on the instantaneous specimen dimensions

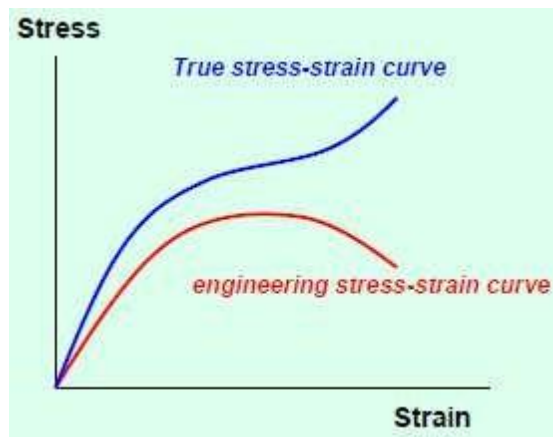


Figure I.12 true stress strain and engineering stress and strain

I.2.1.4 Volumetric strain (ϵ_v):

A relationship similar to that for length changes holds for three-dimensional (volume) change.

For volumetric strain (ϵ), the relationship is $(\epsilon) = (V - V_0)/V_0$ or $(\epsilon_v) = \Delta V/V_0$

- Where V is the final volume, V_0 is the original volume, and ΔV is the volume change.
- Volumetric strain is a ratio of values with the same units, so it also is a dimensionless quantity.
- $\Delta V/V_0 = \text{volumetric strain} = \epsilon_x + \epsilon_y + \epsilon_z = \epsilon_1 + \epsilon_2 + \epsilon_3$
- Dilation: The hydrostatic component of the total stress contributes to deformation by changing the area (or volume, in three dimensions) of an object. Area or volume change is called dilation and is positive or negative, as the volume increases or decreases, p respectively. $e = \text{Where } p \text{ is pressure.}$

I.2.1.5 Poisson’s Ratio (μ):

$$\frac{\text{transverse strain or lateral strain}}{\text{longitudinal strain}} = \frac{-\epsilon}{\epsilon_x} \tag{I.4}$$

(Under unidirectional stress in x-direction)

- The theory of isotropic elasticity allows Poisson's ratios in the range from -1 to 1/2.
- Poisson's ratio in various materials

Table I.1 Materials and its poisson ratio

Material	Poisson's ratio	Material	Poisson's ratio
Steel	0.25 – 0.33	Rubber	0.48 – 0.5
C.I	0.23 – 0.27	Cork	Nearly zero
Concrete	0.2	Novel foam	negative

- We use cork in a bottle as the cork easily inserted and removed, yet it also withstands the pressure from within the bottle. Cork with a Poisson's ratio of nearly zero, is ideal in this application.

I.2.1.6 Thermal or Temperature stress and strain:

- When a material undergoes a change in temperature, it either elongates or contracts depending upon whether temperature is increased or decreased of the material.
- If the elongation or contraction is not restricted, i. e. free then the material does not experience any stress despite the fact that it undergoes a strain.
- The strain due to temperature change is called *thermal strain* and is expressed as, ΔT
- Where α is co-efficient of thermal expansion, a material property, and ΔT is the change in temperature.
- Thermal stress produces the same effect in the material similar to that of mechanical stress. A compressive stress will produce in the material with increase in temperature and the stress developed is tensile stress with decrease in temperature.

I.2.1.7 Thermal stress on Mild steel coated with Brass:

A brass rod placed within a steel tube of exactly same length. The assembly is making in such a way that elongation of the combination will be same. To calculate the stress induced in the brass rod, steel tube when the combination is raised by $t^{\circ}\text{C}$ then the following analogy has to do

- (a) Original bar before heating.
- (b) Expanded position if the members are allowed to expand freely and independently after heating.

(c) Expanded position of the compound bar i.e. final position after heating.

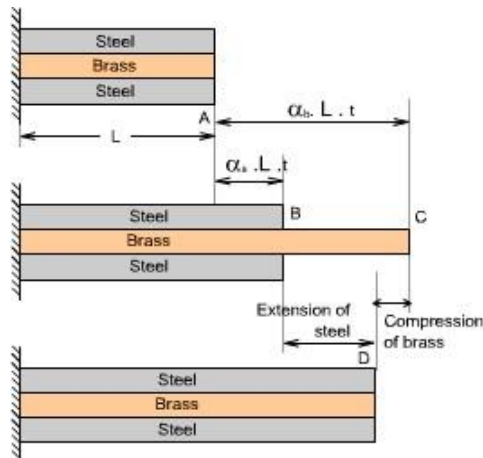


Figure I.13 stress on Mild steel coated with Brass

I.2.1.8 Creep:

When a member is subjected to a constant load over a long period of time it undergoes a slow permanent deformation and this is termed as “creep”. This is dependent on temperature. Usually at elevated temperatures creep is high. [2]

- **The materials have its own different melting point; each will creep when the homologous**

Testing temperature > 0.5 . Homologous temp $= > 0.5$ Melting temperature .A typical creep curve shows three distinct stages with different creep rates. After an initial rapid elongation ϵ_0 , the creep rate decreases with time until reaching the steady state.

a Primary creep is a period of transient creep. The creep resistance of the material increases due to material deformation.

b Secondary creep provides a nearly constant creep rate. The average value of the creep rate during this period is called the minimum creep rate. A stage of balance between competing strain hardening and recovery (softening) of the material.

C Tertiary creep shows a rapid increase in the creep rate due to effectively reduced cross-sectional area of the specimen leading to creep rupture or failure. In this stage intergranular cracking and/or formation of voids and cavities occur.

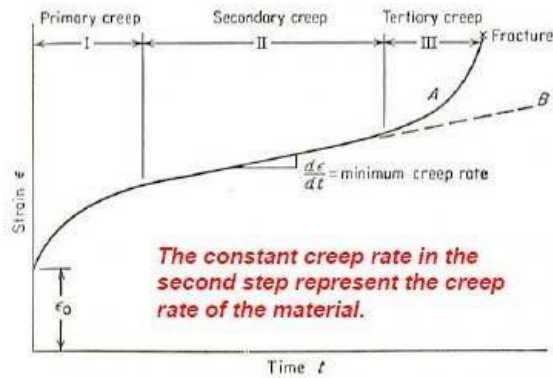


Figure I.14 creep rate

I.2.1.9 Hooke’s Law:

Linear elastic behavior in the tension test is well described by Hooke's law, namely

$$\sigma = E\varepsilon \tag{I.5}$$

where E is the modulus of elasticity or Young's modulus. For most materials, this is a large number of the order of 10^{11} Pa.

The statement that the component of stress at a given point inside a linear elastic medium are linear homogeneous functions of the strain components at the point is known as the generalized Hooke's law [4] Mathematically, this implies that.

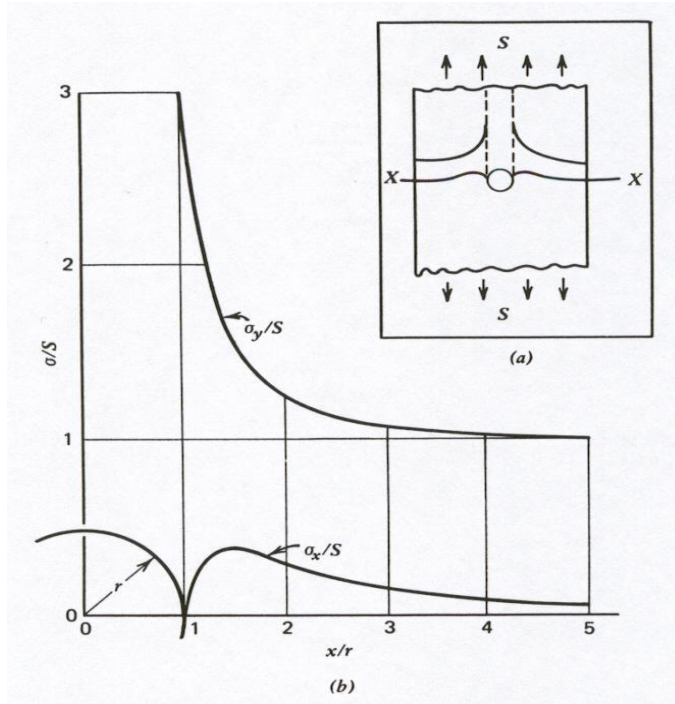
$$\sigma^{ij} = D^{ijkl} \epsilon_{kl} \tag{I.6}$$

Where σ^{ij} and ϵ_{ij} are, respectively the stress and strain tensor components. The quantity D^{ijkl} is the tensor of elastic constants and it characterizes the elastic properties of the medium.

The elastic strain energy W is defined as the form:

$$W = \frac{1}{2} \sigma^{ij} \epsilon_{kl} = \frac{1}{2} D^{ijkl} \epsilon_{ij} \epsilon_{kl} \tag{I.7}$$

And has the property that $\sigma^{ij} = \partial W / \partial \epsilon_{ij}$. Because of the symmetry of W , the actual number of elastics in most general cases is 21. This number is further more reduced in special cases that are of much interest in applications.



stress concentration factors. However, in calculating the stress intensity factor from the nominal stress we use the gross area as if the crack did not exist

• **CONCENTRATIONS AND GRADIENTS**

K_t plotted vs the ratio of hole diameter to sheet width. In the upper curve the nominal stress is defined as load divided by total or **gross area** (wxt).

In the lower curve the nominal stress:

• **net area.**

In this book we use the **net area** to define the nominal stress when using

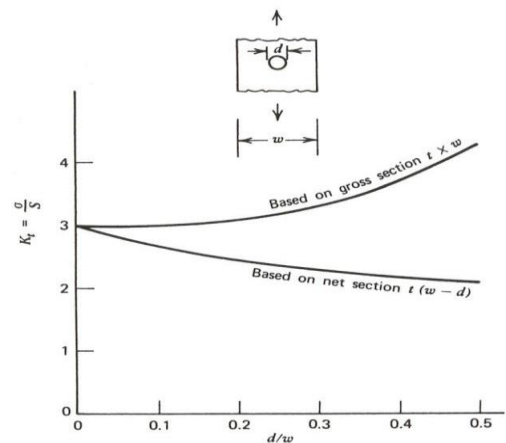


Figure I.16 stresses near a circular hole in the center of a wide sheet in tension

Figure I.15 K_t plotted vs the ratio of hole diameter to sheet width

I.3 Elasticity & Plasticity:

I.3.1 Elasticity:

All structural materials possess to a certain extent the property of elasticity. If external forces, producing deformation of a structure, don't exceed a certain limit; the deformation disappears with the removal of the forces. In this course it will be assumed that the bodies undergoing the action of external forces are perfectly elastic, that they resume their initial form completely after removal of forces. [1]

Also, can be described as the property of a material to regain its original shape after deformation when the external forces are removed. When the material is in elastic region the strain disappears completely after removal of the load, The stress-strain relationship in elastic region need not be linear and can be non-linear (example rubber). The maximum stress value below which the strain is fully recoverable is called the *elastic limit*. It is represented by point A in figure. All materials are elastic to some extent but the degree varies, for example, both mild steel and rubber are elastic materials but steel is more elastic than rubber. [2]

Real Definition:

Is the ability of a material to return to its original shape and size on the removal of external forces. [2]

I.3.2 Plasticity:

Is the property of a material of being permanently deformed by a force without breaking. Thus, if a material does not return to the original shape, it is said to be plastic. Within certain load limits, mild steel, copper, polythene and rubber are examples of elastic materials; lead and plasticine are examples of plastic materials. If a tensile force applied to a uniform bar of mild steel is gradually increased and the corresponding extension of the bar is measured, then provided the applied force is not too large, a graph depicting these results is likely to be as shown in below Figure I.14. Since the graph is a straight line, extension is directly proportional to the applied force. [2]

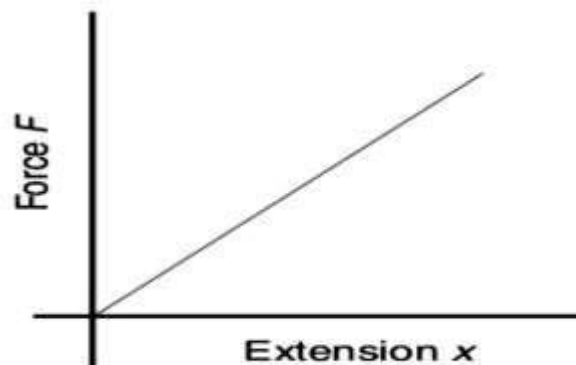


Figure I.17 tensile force applied to a uniform bar of mild steel

- When the stress in the material exceeds the elastic limit, the material enters into plastic phase where the strain can no longer be completely removed. Under plastic conditions materials ideally deform without any increase in stress. A typical stress strain diagram for an elastic-perfectly plastic material is shown in the figure. Mises-Henky criterion gives a good starting point for plasticity analysis. [1]

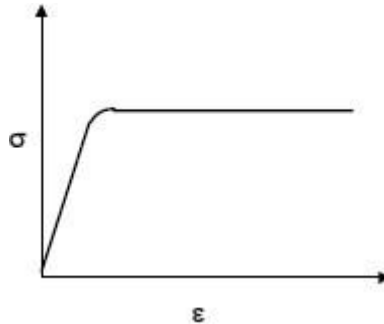


Figure I.18 Mises-Henky curve

I.3.3 Theoretical concepts of plasticity:

The classical theory of plasticity grew out of the study of metals in the late nineteenth century. It is concerned with materials which initially deform elastically, but which deform plastically upon reaching a yield stress. In metals and other crystalline materials, the occurrence of plastic deformations at the micro-scale level is due to the motion of dislocations and the migration of grain boundaries on the micro-level. In sands and other granular materials plastic flow is due both to the irreversible rearrangement of individual particles and to the irreversible crushing of individual particles. Similarly, compression of bone to high stress levels will lead to particle crushing. The deformation of micro voids and the development of micro-cracks is also an important cause of plastic deformations in materials such as rocks. [19]

A good part of the discussion in what follows is concerned with the plasticity of metals; this is the ‘simplest’ type of plasticity and it serves as a good background and introduction to the modelling of plasticity in other material-types. There are two broad groups of metal plasticity problem which are of interest to the engineer and analyst. The first involves relatively small plastic strains, often of the same order as the elastic strains which occur. Analysis of problems involving small plastic strains allows one to design structures optimally, so that they will not fail when in service, but at the same time are not stronger than they really need to be. In this sense, plasticity is seen as a material failure¹. [19]

The second type of problem involves very large strains and deformations, so large that the elastic strains can be disregarded. These problems occur in the analysis of metals manufacturing and forming processes, which can involve extrusion, drawing, forging, rolling and so on. In these latter-type problems, a simplified model known as perfect plasticity is

usually employed (see below), and use is made of special limit theorems which hold for such models.

Plastic deformations are normally rate independent, that is, the stresses induced are independent of the rate of deformation (or rate of loading). This is in marked contrast to classical Newtonian fluids for example, where the stress levels are governed by the rate of deformation through the viscosity of the fluid. [17]

Materials commonly known as “plastics” are not plastic in the sense described here. They, like other polymeric materials, exhibit viscoelastic behavior where, as the name suggests, the material response has both elastic and viscous components. Due to their viscosity, their response is, unlike the plastic materials, rate-dependent. Further, although the viscoelastic materials can suffer irrecoverable deformation, they do not have any critical yield or threshold stress, which is the characteristic property of plastic behavior. When a material undergoes plastic deformations, i.e. irrecoverable and at a critical yield stress, and these effects are rate dependent, the material is referred to as being viscoelastic. [4]

Plasticity theory began with Tresca in 1864, when he undertook an experimental program into the extrusion of metals and published his famous yield criterion discussed later on. Further advances with yield criteria and plastic flow rules were made in the years which followed by Saint-Venant, Levy, Von Mises, Hencky and Prandtl. The 1940s saw the advent of the classical theory; Prager, Hill, Drucker and Koiter amongst others brought together many fundamental aspects of the theory into a single framework. The arrival of powerful computers in the 1980s and 1990s provided the impetus to develop the theory further, giving it a more rigorous foundation based on thermodynamics principles, and brought with it the need to consider many numerical and computational aspects to the plasticity problem. [1]

- Plastic deformation is a non-reversible process where Hooke’s law is no longer valid.
- One aspect of plasticity in the viewpoint of structural design is that it is concerned with predicting the maximum load, which can be applied to a body without causing excessive yielding. [20]
- Another aspect of plasticity is about the plastic forming of metals where large plastic required to

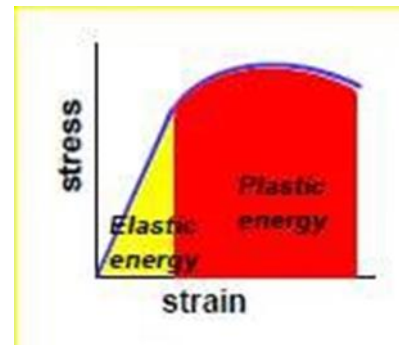


Figure I.19 stress and strain curve

I.4 Fracture Toughness:

I.4.1 Understanding fracture toughness:

Strength and toughness? Why bother? What's the difference? [20]

- Strength – resistance to plastic flow; onset at yield strength, max at tensile strength (work hardening); area under stress strain curve to fracture – work of fracture.
- Toughness – resistance to propagation of a crack. A tough material yields, work hardens, and absorbs energy – the crack is insignificant. A brittle (not tough) material
- Every material has a certain strength and a certain toughness. It is and/or (not either/or). Tests for toughness [20]
- Energy methods are good for acceptance and quality control – however they do not measure a material property we can use in design

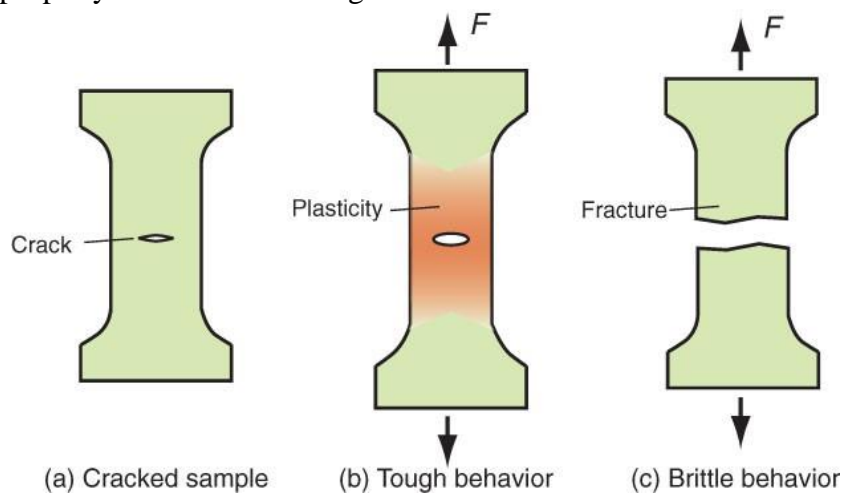


Figure I.20 Tough vs brittle behavior

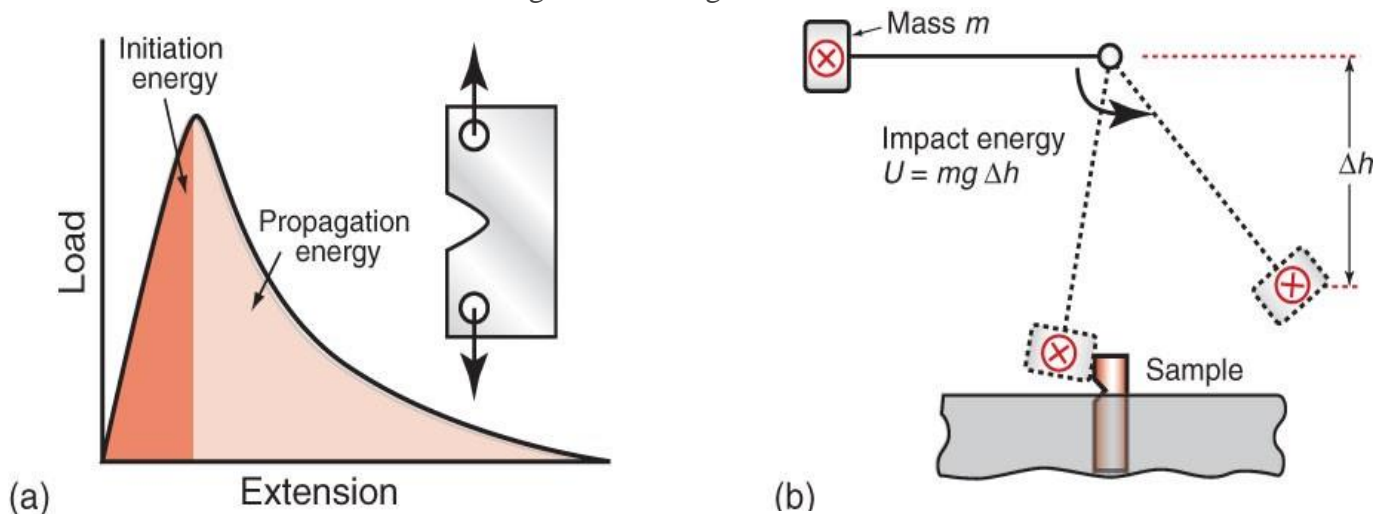


Figure I.21 (a) The tear test. (b) The impact test

- **A more precise definition for toughness would be:**

A resistance to impact and represent the ability of material to support load even after yielding or crack formation. Toughness enable material to endure shock or blows, or toughness is also considered to mean resistance to fracture when material is deformed above the elastic limit, an increment in toughness lead to increase in the amount of energy needed to produce damage. The area under stress – strain curve represents the **modulus of toughness**. [2]

Toughness is measured in units of joules per cubic meter (J/m³).

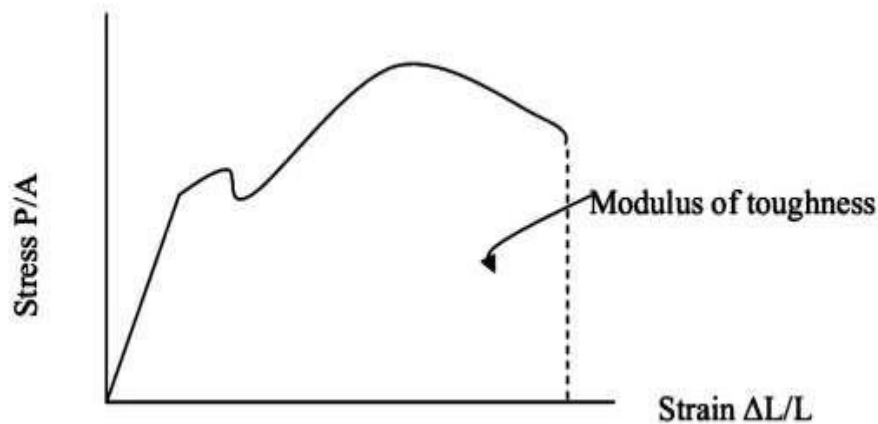


Figure I.22 stress – strain curve represents the modulus of toughness

- **fracture toughness true definition:**

Fracture toughness is an indication of the amount of stress required to propagate preexisting flaw. It is a very important material property since the occurrence of flaws is not completely avoidable in the processing, fabrication, or service of a material/component. Flaws may appear as cracks, voids, metallurgical inclusions, weld defects, design discontinuities, or some combination thereof. Since engineers can never be totally sure that a material is flaw free, it is common practice to assume that a flaw of some chosen size will be present in some number of components and use the linear elastic fracture mechanics (LEFM) approach to design critical components. This approach uses the flaw size and features, component geometry, loading conditions and the material property called fracture toughness to evaluate the ability of a component containing a flaw to resist catastrophic fracture. [6]

A parameter called the critical stress-intensity factor (K_C) is used to represent the fracture toughness of most materials. A Roman numeral subscript indicates the mode of fracture and the three modes of fracture are illustrated in the image to the right. Mode, I fracture is the conditioning which the crack plane is normal to the direction of largest tensile loading. This is the most commonly encountered mode and, therefore, for the remainder of the material we will consider K_I . The stress intensity factor is a function of loading, crack size, and structural geometry. The stress intensity factor may be calculated by the following equation:

$$K_I = \sigma \beta \sqrt{\pi a} \tag{I.8}$$

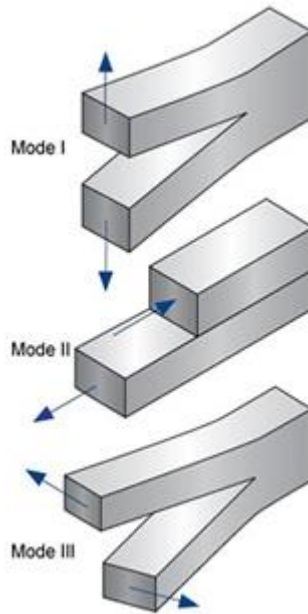


Figure I.23 Mode I opening crack, Mode II in plane shear, Mode III out of plane shear

- The fracture toughness K_C of a material is defined as the critical stress intensity factor for immediate crack growth under given conditions. Under prescribed-load boundary conditions, such crack growth will be immediately catastrophic. Any flaw large enough (length a) that its stress intensity K_I might reach K_C under the loading σ may cause immediate failure.
- In larger objects the applied load for catastrophic fracture may be significantly lower than expected from the the nominal ultimate strength of the material. As such, catastrophic fracture is often the limiting factor in engineering design. In addition, repeated stress intensities K_I which do not reach K_C can still cause slow crack growth due to fatigue. [14]

I.4.2 Role of Material Thickness:

Specimens having different thickness produce different values for K_C . Thin specimens cannot have stress components normal to their free surfaces, whereas the inside of thicker specimens may be constrained by surrounding material and develop stress components in the thickness dimension. Such thin specimens have a higher value of K_C because they have a much larger plastic zone area than thicker specimens, spreading the fracture energy over that larger area. The value of K_C decreases with thickness until the thickness exceeds some critical dimension. At this point the value of K_I becomes relatively constant and this value, K_{IC} , is a true material property which is called the plane-strain fracture toughness. [14]

K_{IC} is usually measured by the process specified in ASTM Standard E399. The relationship between stress intensity, K_I , and fracture toughness, K_{IC} , is similar to the relationship between stress and tensile stress

- . The stress intensity, K_I , represents the level of “stress” at the tip of the crack
- . The fracture toughness, K_{IC} , is the highest value of stress intensity that a material under very specific (plane-strain) conditions can withstand without fracture
- . As the stress intensity factor reaches the K_{IC} value, unstable fracture occurs. As with a material’s other mechanical properties, K_{IC} is commonly reported in reference books and other sources.

As the thickness of the specimen increases, fracture toughness decreases and the plain strain Strain - A measure of relative change in the size or shape of a body. “Linear strain” is change (increase or decrease) in a linear dimension. Usually expressed in inches per inch (in. /in.), or millimeters per millimeter (mm/mm). assumptions become more accurate. [14]

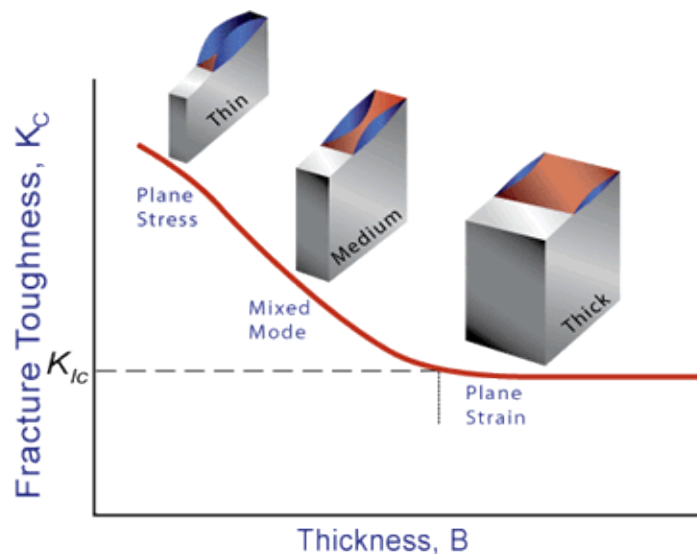


Figure I.24 Fracture toughness along specimen thickness

I.4.3 Plane-Strain Fracture Toughness Testing:

The problems associated with brittle fracture have received massive attention with the result that the mechanisms and controlling factors have been explicated satisfactorily in general terms. What is still lacking is the ability to translate test results quantitatively to the prediction of the service performance of a particular structure subjected to particular service conditions. This situation applies especially to materials that behave in an elastic-plastic mode at locations in the structure involving stress concentrations (flaws, fillets, connections, etc.). The nickel cryogenic steels fall into this category. Since the goal of test specimen design is to approximate the most severe stress pattern and loading conditions that are anticipated in the service structure, the trend has been toward full-section specimens containing flaws with fatigue-sharpened roots and dynamic loading. For the low-nickel steels, at least, the determination of quantitative fracture-toughness values is probably less significant than the temperature range in which transition from high to low values takes place because of the abruptness of the transition. [6]

a) The Charpy Test (ASTM E23 and A370):

Despite the objections that have been leveled at the Charpy test because of its small size and inadequate notch severity, it has persisted as the most commonly used test for notch toughness, mainly because it is economical, convenient, and familiar to engineers. The limitation of the standard V-notch Charpy test lies in the failure of the Charpy transition temperature range of a given steel to correlate exactly with the temperature below which a service structure of the same steel becomes unreliable in resisting brittle fracture. This shortcoming is shared more or less, however, by other notch-toughness tests. It can be used to estimate fracture toughness parameters of limited shear-energy absorption. Consequently, the Charpy test is often used as a check of material quality and uniformity after the toughness requirements of the application and the suitability of the type of steel have been established by other testing methods. [6]

When performing a fracture toughness test under ASTM E399,

the most common test specimen configurations are the single edge notch bend (SENB or three-point bend), and the compact tension (CT) specimens. From the above discussion, it is clear that an accurate determination of the plane-strain fracture toughness [14]

requires a specimen whose thickness exceeds some critical thickness (B). Testing has shown that plane-strain conditions generally prevail when:

$$B \geq 2.5 \left(\frac{K_{IC}}{\sigma_y} \right)^2 \quad (I.9)$$

When a material of unknown fracture toughness is tested, a specimen of full material section thickness is tested or the specimen is sized based on a prediction of the fracture toughness. If the fracture toughness value resulting from the test does not satisfy the requirement of the above equation, the test must be repeated using a thicker specimen. In addition to this thickness calculation, test specifications have several other requirements that must be met (such as the size of the shear lips) before a test can be said to have resulted in a K_{Ic} value.

When a test fails to meet the thickness and other test requirement that are in place to ensure plane-strain condition, the fracture toughness values produced is given the designation K_{Ic} . Sometimes it is not possible to produce a specimen that meets the thickness requirement. For example, when a relatively thin plate product with high toughness is being tested, it might not be possible to produce a thicker specimen with plain-strain conditions at the crack

tip.

NOTE: A thin object will experience a more ductile form fracture due to a large plastic zone with shear lips that occupy a large percentage of the thickness. For thick objects, the plastic zone in plane strain is much smaller and the shear lips occupy a much smaller portion of the thickness, giving more brittle fracture characteristics and lower K_{Ic} . [14]

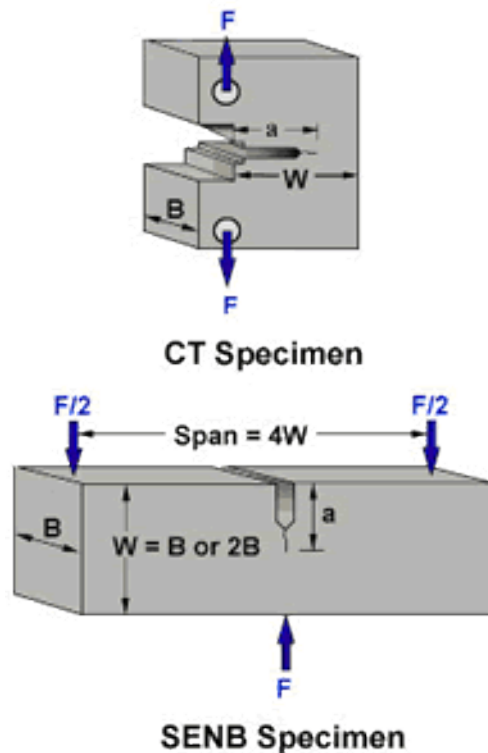


Figure I.25 fracture toughness test under ASTM E399

I.4.4

Notch Toughness:

Prior to 1955, the evaluation of the notch toughness of low-nickel steels for low-temperature service was conducted mostly with the keyhole-notch CharPY specimen. While the V-notch has been adopted, the Charpy test is still the principal means of measuring toughness and, of course, is specified in A593. While interest in various fracture- toughness tests has recently increased, their use is largely confined to laboratory studies and has not been extended to quality control or material specifications for the A203 steels. [6]

Notch toughness is the ability that a material possesses to absorb energy in the presence of a flaw. As mentioned previously, in the presence of a flaw, such as a notch or crack, a material will likely exhibit a lower level of toughness. When a flaw is present in a material, loading induces a triaxial tension stress state adjacent to the flaw. The material develops plastic strains as the yield stress is exceeded in the region near the crack tip. However, the amount of plastic deformation is restricted by the surrounding material, which remains elastic. [14]

When a material is prevented from deforming plastically, it fails in a brittle manner. Notch-toughness is measured with a variety of specimens such as the Charpy V-notch impact specimen or the dynamic tear test specimen. As with regular impact testing the tests are often repeated numerous times with specimens tested at a different temperature. With these specimens and by varying the loading speed and the temperature, it is possible to generate curves such as those shown in the graph. Typically, only static and impact testing is conducted but it should be recognized that many components in service see intermediate loading rates in the range of the dashed red line.

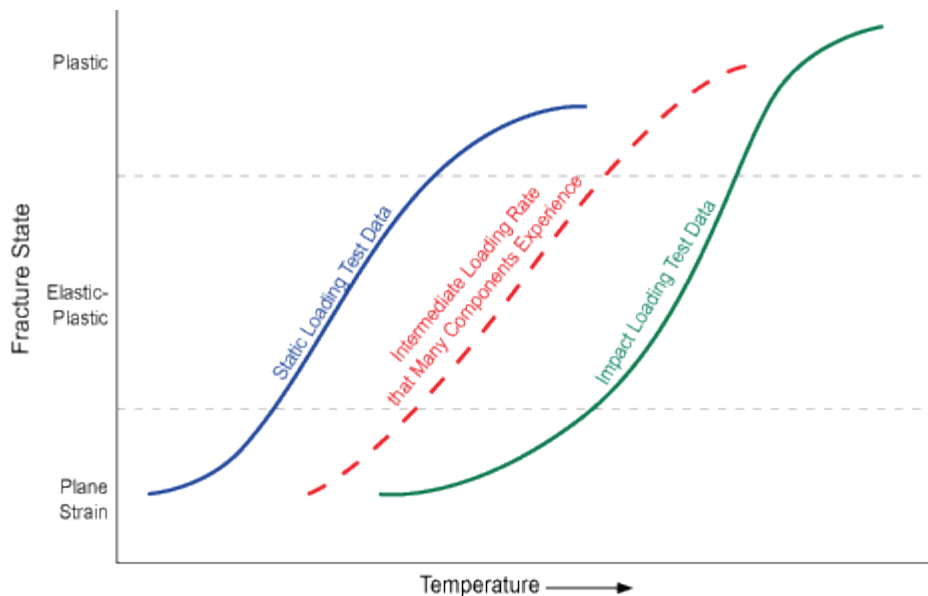
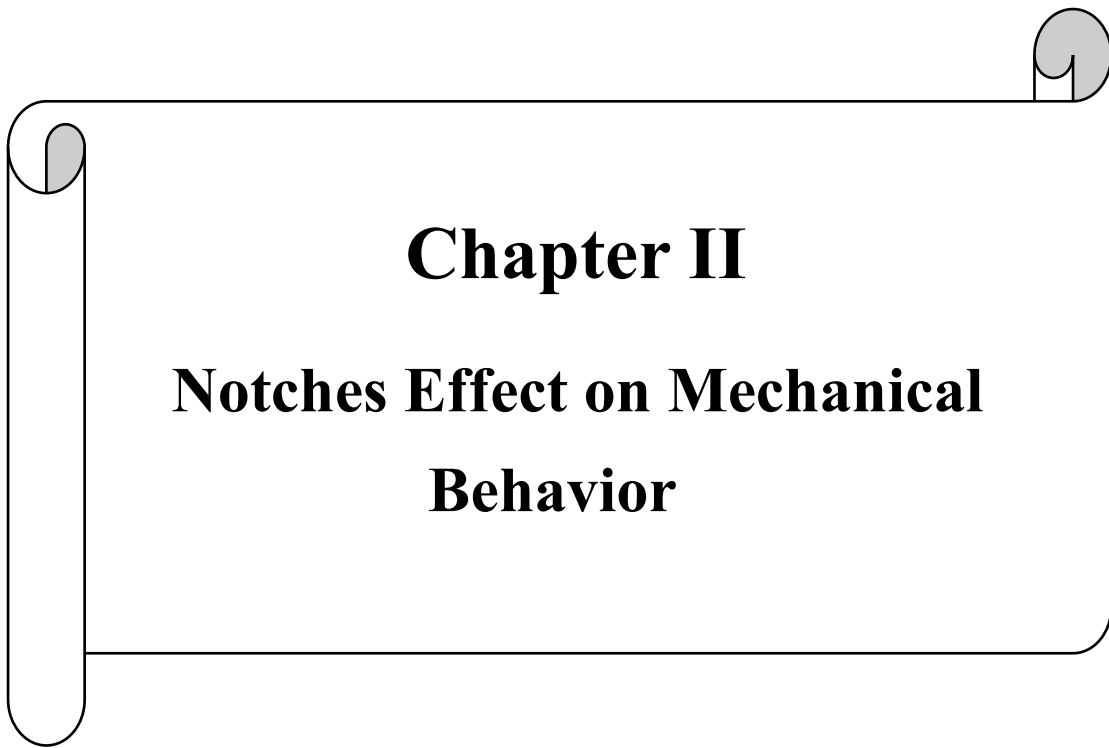


Figure I.26 the fracture state along the temperature curves

Conclusion

In Chapter I: Generalities on Mechanical properties of Materials:

provides foundational knowledge, defining key mechanical properties like stress and strain., elasticity and plasticity, creep, and fracture toughness, detailing their negative consequences such as reduced strength and fatigue life, and outlines various approaches to studying notch effects, including experimental, analytical (Stress Concentration Factors, Fracture Mechanics), and numerical methods. It also touches upon sources of notches.



Chapter II
Notches Effect on Mechanical Behavior

II.1 INTRODUCTION:

Intentional or unintentional geometric discontinuities, such as sharp corners, holes or grooves, are very important in the mechanical behavior of materials, and can be represented by notches. A cornerstone of the safety and reliability of engineered structures is studying their effects.

The study of the mechanical behavior of a material under the effect of a number of notches is of one of the most important assets in the field of engineering and materials science.

To study the mechanical behavior of the number of notched, different approaches can be used, from laboratory experiments to advanced numerical models and solutions.

Experimental tests, and numerical modeling that offers a more affordable and reproducible approach to predict the behavior of materials in different number of notches

Holes, are among the most common geometric features in the design and manufacturing of mechanical components and structures. They can exist in various shapes and sizes, serving multiple functional purposes, ranging from facilitating assembly with bolts and screws to providing channels for fluid and gas flow, and even reducing the weight of parts without significantly compromising their strength. [8]

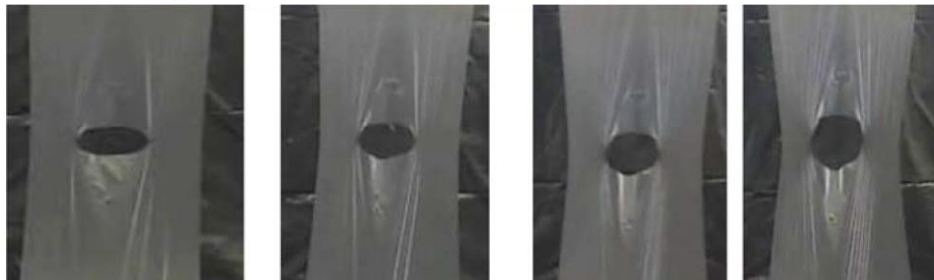


Figure II.1 different holes on elastic material

You will find twelve types of holes in engineering: blind hole, through hole, interrupted hole, simple hole, counterbored hole, spot face hole, countersink hole, counter drill hole, tapered hole, screw clearance hole, tapped hole, and threaded hole. Each type has unique characteristics and plays specific roles in engineering designs, from structural components in aerospace engineering to complex parts in electronic devices. [8]

Studying holes in mechanical analysis is important since their presence have a large influence on stress distribution in a material. A part with hole is a center of stress concentration, when a part is subjected to a load, the hole acts as a center of stress concentration. That is, the stresses are much higher around the hole edges than the nominal stresses in the part far from the hole. [9]

The fundamental principles of hole design in engineering revolve around precision, functionality and integration. A well-designed hole is not just about its size and shape; it is about the material it is made of and how it interacts with the purpose it is serving. This involves a thorough understanding of the material properties, the forces applied to the hole and how it fits the end-user requirements of the product. [9]

- A counterbored hole is an enlarged cavity in a material for recessing the head of a fastener.
- A spot face hole is similar to a counterbored hole, but is shallow. It allows a fastener to sit flat.
- As the name suggests, an interrupted hole is a hole where another feature intersects the hole. A new hole-making operation drills a sequence of
- A through hole is a hole made to go completely through the material. A through hole goes all the way through the workpiece. It is sometimes called a thru-hole.

These are some of the many holes shaped found in engineering

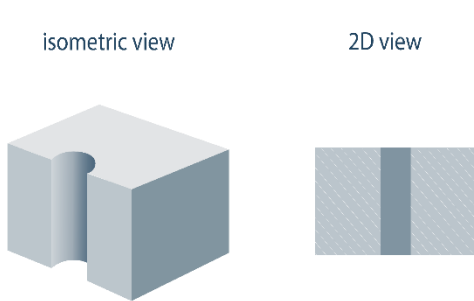


Figure II.2 counterbored hole

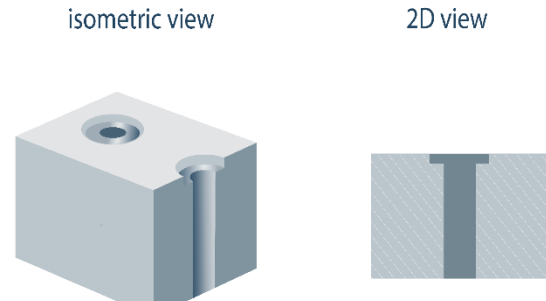


Figure II.5 through hole

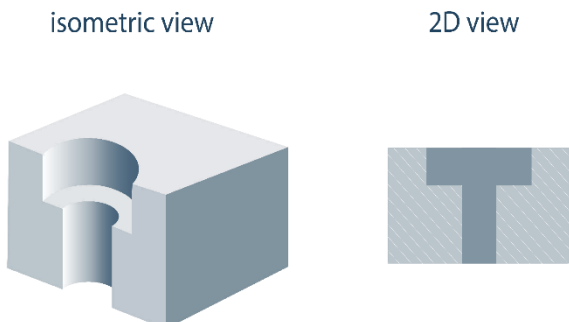


Figure II.3 spot face hole

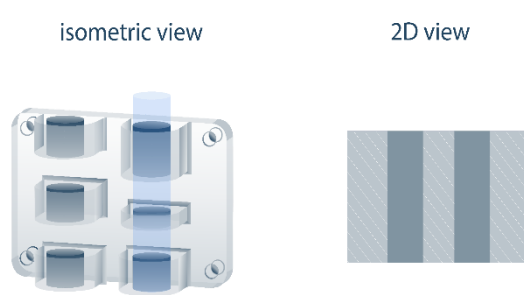


Figure II.4 interrupted hole

- **Factors that affect the magnitude of stress concentration include:**

Circular holes are usually less stressed and their stress concentration (stress concentration factor) is much lower than those with sharp corners or elliptical shape with a high aspect ratio. The relative size of the hole to the size of the part affects the extent of the stress concentration region.

Orientation of the load: The orientation of the applied load with respect to hole location determines the maximum stress fields. Several important consequences follow from the stress concentration around holes.

High local stresses: As the high local stresses can exceed the material's ultimate strength at the edge of the hole, cracks and fracture can initiate from the edge of the hole at loads which are lower than those expected from a solid part.

Fatigue failure: Fatigue causes stress concentration at the hole regions to become preferred sites for fatigue crack initiation and growth, greatly reducing service life of the component. [31]

Permanent deformation: The stress concentration can cause the material near the hole to exceed its yield strength, which means the part may be locally permanently deformed. For this reason, it is important in mechanical analysis to understand the stress behavior around holes. As a result, the attention of mechanical analysts to the stress behavior around holes is urgent for several reasons:

Construction Safety: Structural and mechanical components that include holes require special consideration to avoid failure due to excessive stress. Stress concentration factor (K_t) models are part of the calculations and estimates done for all holes in consideration. Its purpose is to estimate the highest possible stress circled a hole.

Failure Analysis: Most mechanical non-destructive failure analysis starts with a basic examination of holes and fractures in components, which is why it requires a different approach for broad range cracks. The ring like form of the fillet is an example of a rounded corner put on edges which makes it easier.

Design Optimization: Aside from complexity, the measures taken in trying to understand the intersection of stress distribution with the shape, size and the location of a hole can lead to elimination of stress concentration which increases material efficiency.

Inspection and Maintenance: Strategic knowledge of where stress concentration exists makes the prognosis of non-destructive testing accurate, which in turn makes identification of cracks that may lead to delicate structure failure possible. To conclude, even if from a functionality and assemble it becomes clear that a hole is imperative, it strongly suggests the need for a serious consideration mechanically.

II.2 Sources of notches and holes in materials:

Both notches and holes in materials can arise from a variety of causes, broadly categorized as:

- **Manufacturing Processes:**

Machining: Operations like drilling, milling, turning, and grinding can intentionally create holes and sometimes unintentionally introduce notches (e.g., tool marks, sharp corners).

Welding: Defects such as incomplete fusion, undercuts (grooves along the weld bead), and porosity can act as notches or effectively create internal "holes."

Casting: Shrinkage cavities, gas porosity, and incomplete filling can lead to internal or surface discontinuities resembling holes or notches.

Forming: Processes like stamping, punching, and deep drawing can create intended holes but also potential notches at sharp bends or sheared edges.

Additive Manufacturing (3D Printing): Imperfections like voids, incomplete fusion between layers, and surface roughness can act as stress concentrators similar to notches or small holes.

Surface Finishing: Grinding or polishing that is too aggressive or leaves sharp edges can create unintended notches.

- **Design Intent:**

Fastening: Holes are intentionally created for bolts, screws, rivets, and other fasteners.

Weight Reduction: Holes can be designed into components to reduce material usage and overall weight.

Access and Clearance: Holes may be necessary for access to internal parts or to provide clearance for other moving components.

Fluid Dynamics: Gaps and ducts are constructed specifically for fluid or gas flow. Stress Mitigation: Extreme overloads in some specific designs could forcefully attempt failure at 'notches' which if engineered carefully could create certain failure controls.

- **Damage During Operational Use:**

Fatigue: The passive notches created by low surface quality practically guarantee the evolution of cavities after a certain period in conjunction with cyclic loading when stress is concentrated in some places which originates from slight imperfections in design processes, surface taken or teeth, and preformed openings on the surface.

Impact Damage: Indentations or scratches can be punctured notches (holes) or even through-thickness cracks.

Corrosion: Stress raisers (notches) are developing on surface as a result of material degradation's electrochemical process. Also, some cavities (holes) which permit the notched material lose notched the surface will create.

Wear: Gradually the material may be removed by means of abrasive forces or erosion. As a result, it is likely to be formed also but more notched holes and gaining grooves.

Overloading: Stress concentration like detecting holes or developing notched almost all the time fracture will start from yielding of component after exceeding design load.

Environmental Factors: Notch cracks due to exposure to damage inducing bleeds of harsh chemicals and extremely low degrees raise to materials embrittlement.

Accidental scratches or nicks: Accidental mechanical damage during manufacturing, handling or use can create small notches or holes on the surface.

In short summary, notches and holes are signs of the manufacturing processes, whether deliberate or accidental, as well as from the myriads of stresses and environmental conditions a material or component undergoes during its service life. Knowing the origin of these features is important in anticipating the mechanical behavior and possible failure of the materials.

II.3 Fracture Mechanics:

Since World War II there has been great progress in understanding the ways in which materials fracture. Such knowledge has proved essential to better formulation of fracture mechanisms. Nevertheless, it is still not possible to use this knowledge, together with other material properties, for predicting fracture behavior in engineering terms with a high degree of confidence. and it is the intention of the present chapter to provide the necessary background information on fracture mechanisms. Metallic materials, especially structural engineering alloys, are highly complex. which shows various microstructural features (not all of which need be present in a particular material) and also the two main types of fracture path, trans granular and intergranular fracture. Of fundamental importance is the fact that almost all structural materials are polycrystalline, i.e. they consist of aggregates of grains, each of which has a particular crystal orientation. The only exceptions are single crystal turbine blades for high performance jet engines. [10]

II.3.1 stress intensity factor (K):

The stress intensity factor (K) is a crucial parameter in fracture mechanics used to characterize the stress state at the tip of a crack or notch in a material subjected to an applied load or residual stresses. It quantifies the magnitude of the stress field in the immediate vicinity of the crack tip and provides a criterion for predicting whether a crack will propagate and lead to fracture. [11]

For the determination of stress intensity factors the Boundary Collocation Method (BCM) was used and for the determination of the weight function the direct adjustment method, was applied. The stress intensity factor K is a measure for the singular stress term occurring near the tip of a crack and defined by:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f(\theta) + \dots, \quad (\text{II.1})$$

where r , T are the cylindrical polar co-ordinates of a point with respect to the crack tip, K is a quantity which gives the magnitude of the elastic stress field. It is called the stress intensity factor. Dimensional analysis shows that K must be linearly related to stress and directly related to the square root of a characteristic length [11]

for mode I:

$$f_{xx} = \cos\left(\frac{\varphi}{2}\right) \left[1 - \sin\left(\frac{\varphi}{2}\right) \sin\left(\frac{3\varphi}{2}\right)\right] \quad (\text{II.2})$$

$$f_{yy} = \cos\left(\frac{\varphi}{2}\right) \left[1 + \sin\left(\frac{\varphi}{2}\right) \sin\left(\frac{3\varphi}{2}\right)\right] \quad (\text{II.3})$$

$$f_{xy} = \cos\left(\frac{\varphi}{2}\right) \left[\cos\left(\frac{\varphi}{2}\right) \sin\left(\frac{\varphi}{2}\right) \cos\left(\frac{3\varphi}{2}\right)\right] \quad (\text{II.4})$$

and for mode II:

$$f_{xx} = \sin\left(\frac{\varphi}{2}\right) \left[2 + \sin\left(\frac{\varphi}{2}\right) \sin\left(\frac{3\varphi}{2}\right)\right] \quad (\text{II.5})$$

$$f_{yy} = \sin\left(\frac{\varphi}{2}\right) \cos\left(\frac{\varphi}{2}\right) \cos\left(\frac{3\varphi}{2}\right) \quad (\text{II.6})$$

$$f_{xy} = \cos\left(\frac{\varphi}{2}\right) \left[1 - \sin\left(\frac{\varphi}{2}\right) \cos\left(\frac{3\varphi}{2}\right)\right] \quad (\text{II.7})$$

K is the stress intensity factor. For the loading modes considered in this report the stress intensity factors KI and KII are expressed as. [11]

$$KI = \sigma * \sqrt{\pi a} FI(A/w) \quad (\text{II.8})$$

$$KII = \tau * \sqrt{\pi a} FII(A/w) \quad (\text{II.9})$$

where a is the crack length, W is the width of the component and σ^* , τ^* are characteristic stresses in the component, e.g. the outer fiber stress in a bending bar. FI and FII are functions of the ratio of the crack length to the specimen's width as well as of the type of load applied. [11]

II.4 Effects of the number of holes in material's properties and engineering:

II.4.1 Stress Concentration:

The number of holes in a material or engineering component can have several significant effects, primarily related to stress distribution, strength, and overall structural integrity. We are going to see the most common effects. [8]

Holes disrupt the uniform flow of stress in loaded material. The material surrounding the hole must now bear the load that would have been carried by the inside material, so that localized increase in stress is seen at the edges of the hole. This is stress concentration. It increases with sharp corners of holes or other geometric discontinuities. [8]

Grooves, fillets, holes, or other abrupt changes in cross section or any disruption of a smooth surface causes increased stresses around these areas. Notch is a general term meaning any or all of the above. [25]

Imagine that the flow of force is like the flow of water. An obstruction in a flow field would cause the water's velocity to increase around the obstruction. The same is true for force except that the stress increases around the "obstruction".

The stress is said to concentrate around these obstructions, or notches.

The amount of stress around a notch is: [25]

$$\sigma_{max} = Kt \sigma_{nom} \quad (\text{II.10})$$

- **stress concentration factor:**

K_t is called the theoretical stress concentration factor and σ_{nom} is called the nominal stress. In the case of shear stress:

$$\tau_{max} = K_t \tau_{nom} \quad (\text{II.11})$$

The nominal stress, whether shear or axial, is calculated using the net cross section.

the stress concentration factors (SCFs) are based on the nominal stress at the minimum diameter or width and defined: [32]

$$K_1 = \frac{\sigma_{max}}{\sigma_{nom}} \quad (\text{II.11.1})$$

For example, to calculate the nominal stress and the maximum stress of a plate. Maximum stress determination will require you to first calculate the stress concentration factor, K_t , that accounts for the reduction of cross - sectional area, and the notch sensitivity, q

As the holes increase, particularly when they are closely spaced, the stress concentration regions can tend to interact and overlap. Thereby producing even higher peak stresses in the material between the holes as compared to a single isolated hole. The maximum stress that will be experienced near multiple holes can be many times greater than the nominal stress that has been applied to the component. [32]

a. Here is an example on the effect of holes in stress concentration:

Effects of hole stress concentration and its mitigation on the tensile strength of sheet moulding compound (SMC-R50) composites:

the effects of hole stress concentration on the tensile strength of SMC-R50 composites, which are composed of 50% E-glass fibers in a vinyl ester resin. The study examines both centric and eccentric hole locations and explores a method to mitigate stress concentration by applying transverse normal pressure around the hole boundary. [13]

The SMC-R50 composite contains 50% by weight of 25 mm long chopped E-glass strands, randomly oriented in the plane of the SMC, with other ingredients like calcium carbonate, magnesium oxide, and a thermoplastic additive. [13]

Sheets of R50 material were compression molded into plaques of 2.7 mm thickness. Rectangular specimens (100 mm x 25 mm) were prepared with centered and off-centered holes of varying diameters (3.3 mm to 12.7 mm). [13]

- Static tension tests were conducted using a Universal testing machine.
- Transverse normal pressure was applied by clamping flat washers on both sides of the specimen and tightening the nuts to various torque levels.
- Both static and fatigue tests were performed on clamped and unclamped specimens.

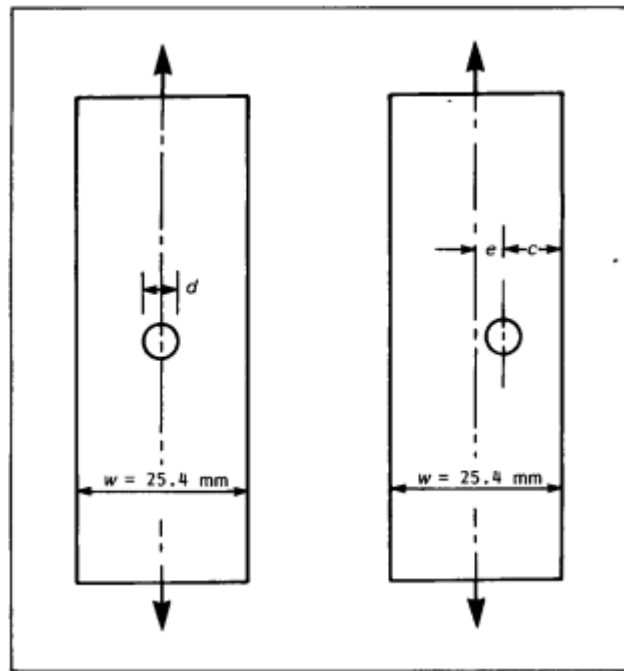


Figure II.6 Specimen configuration with (left) centered hole, and (right) off-centered hole

The results were:

- The notched tensile strength was calculated based on both gross and net cross-sectional areas.
- The R50 material showed low notch sensitivity, attributed to naturally occurring flaws like voids and fiber ends.
- The Whitney-Nuismer Point Stress model was used to predict the notch sensitivity.
- For off-centered holes, tensile strength varied with hole eccentricity and diameter.
- Applying transverse normal pressure increased the notched tensile strength, with clamped specimens showing higher endurance in fatigue tests compared to unclamped ones. [13]

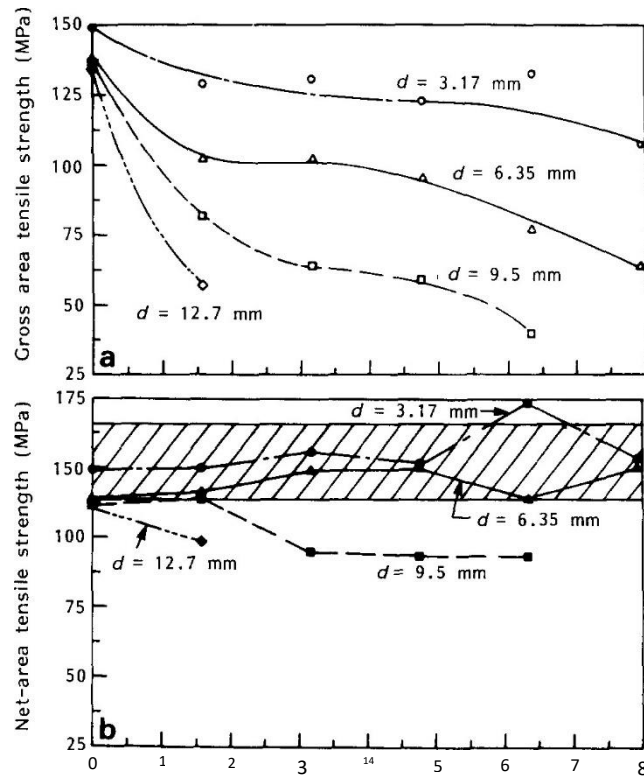


Figure II.7 Tensile strength variation in specimens with off-centered holes: (a) gross-area notched tensile strength; and (b) net-area notched tensile strength. (The cross-hatched area indicates the range of unnotched strengths observed for SMC-R50 laminates)

This experiment highlights the potential of using transverse normal pressure to enhance the mechanical properties of composites with stress concentrations due to holes.

II.4.2 Reduction in Strength (Load-Carrying Capacity):

Holes are material removed hence; the area available to carry the applied load is reduced. Therefore, for a given applied force smaller cross-sectional areas will experience higher average stresses. An additional hole further reduces the area available to bear the load. [13]

With an increase in number and size of holes, overall strength of the component decreases. The dimension of "net section" the remaining material after subtraction of holes load resisting thus, it is critical for component strength.

Engineers must calculate the net section area under hole design for component such that it suffices under applied load with adequate factor of safety. A higher number of holes necessitates a larger initial cross-section or a stronger material to maintain the required strength.

II.4.3 Impact on Fatigue Life:

Holes act as stress concentration, and under cyclic loading, they serve as fatigue crack initiation site. Elevated stresses at the edge of the hole facilitate initial microscopic crack growth progression toward macroscopic failure at stress levels well below the material static strength. [31]

Multiple holes increase count potential sites initiation fatigue cracks. In addition, closely spaced holes will lead to crack growth escalation due to stress field interaction. More holed components generally exhibit increased sensitivity to fatigue failure and will demonstrate reduced fatigue life under identical loading conditions when compared to less or not holed components. [31]

a. Here is an example on how holes can impact fatigue life:

- **An Experimental Study of Reaming Sizes on Fatigue Life of Cold-Expanded 7050-T7451 Aluminum Alloy:**

In this experiment the Objective of the study aims to determine the optimal reaming size to enhance the fatigue life of cold-expanded holes in 7050-T7451 aluminum alloy. This involves analyzing fatigue life, fracture morphology, microstructure, and residual stress under various reaming conditions. [15]

The experiment used 7050-T7451 aluminum alloy plates with initial hole diameters of 4.3 mm, 5.3 mm, 7.18 mm, and 11.1 mm. A split-sleeve cold expansion method was applied to introduce compressive residual stress around the holes. The cold-expanded holes were reamed to different final diameters to assess the impact on fatigue life. Fatigue tests were conducted using an INSTRON 8801 machine, with parameters such as stress amplitude and loading frequency controlled. [15]

To compare the effects of different reaming depths on the fatigue life of cold-expanded 7050-T7451 aluminum alloy plates, various specimens were cold-expanded with approximately 4% expansion. The expansion E was calculated as follows:

$$Er = \frac{(D0-2d)-D}{D} \quad (\text{II.12})$$

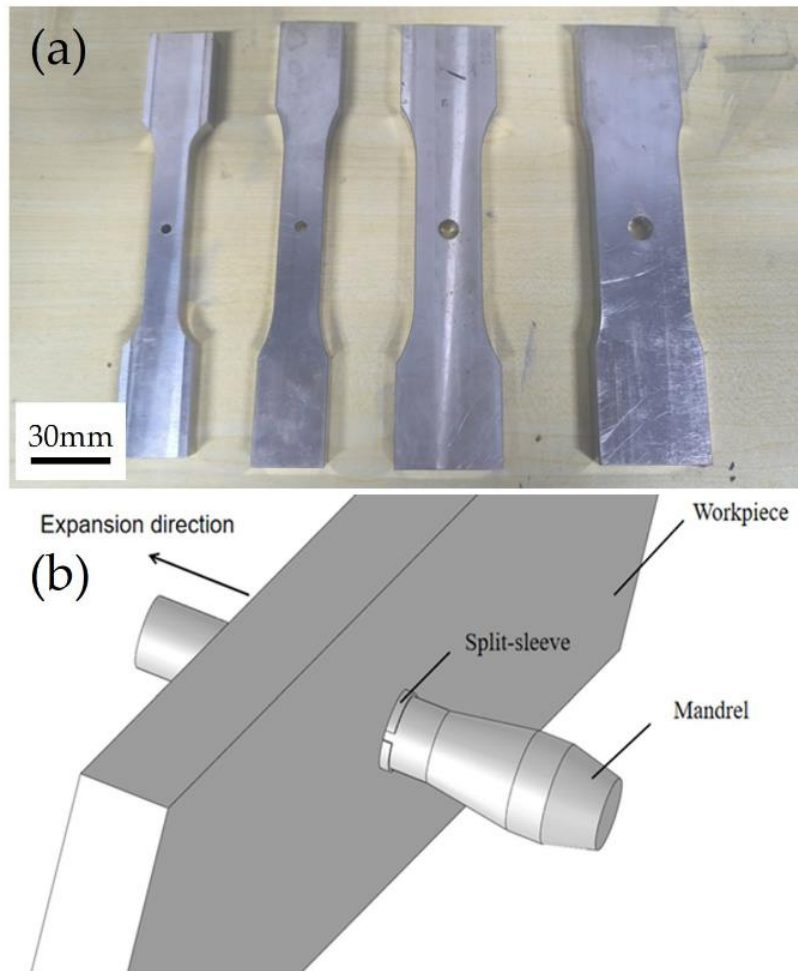


Figure II.8 (a,b) Cold-expanded specimens and the illustration of the split-sleeve cold expansion process

Initially, fatigue life increases with reaming size but decreases if the reaming is excessive. Optimal reaming sizes significantly enhance fatigue life compared to untreated specimens.

Cold expansion forms a dense grain structure around the hole, creating a crystal refinement layer. Excessive reaming removes this layer, reducing fatigue strength. Reaming reduces radial residual

stresses around the hole, which can decrease fatigue resistance. The study found that excessive reaming leads to a significant reduction in residual stress, weakening the material's ability to withstand fatigue [15]

- The study concludes that determining the optimal reaming size is crucial for enhancing the fatigue life of cold-expanded holes. Proper assessment of the plastic deformation layer's depth, formed by cold expansion, is essential in guiding reaming dimensions.
- Excessive reaming disrupts the beneficial residual stress field, reducing the fatigue life of the material. [15]



Figure II.9 Cold expansion specimens with different initial hole diameters after fatigue testing: (a) 4.3 mm; (b) 5.3 mm; (c) 7.18 mm; (d) 11.1 mm

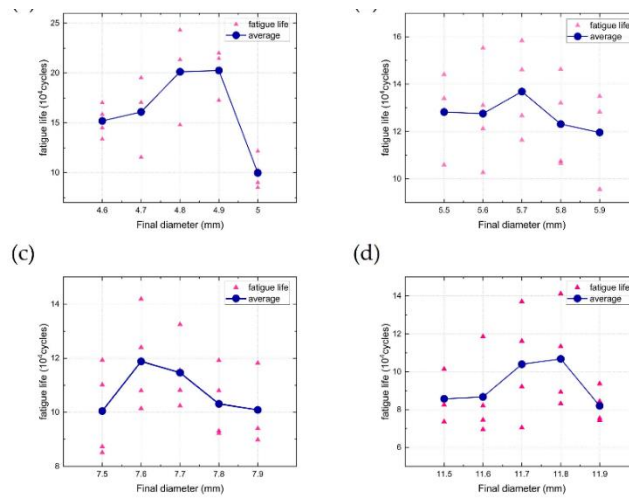


Figure II.10 Fatigue life of specimens with different initial hole diameters after being reamed to different final hole diameters

II.4.4 Influence on Fracture Toughness:

Fracture toughness refers to the property of material that resists the propagation of cracks. Holes can influence the stress state at the crack tip when a pre-existing crack is near a hole. The stresses due to the crack and hole may interact to either suppress or encourage crack extension depending on their geometry and loading conditions. [29]

Increasing number of holes in a part naturally increases probability of interaction between a crack and one or more holes, making prediction of such interaction in terms of fracture behavior rather difficult. Closely spaced holes can practically reduce material's resistance to fracture by providing multiple potential pathways for crack propagation or by increasing overall stress intensity in the region.

When designing critical structures where fracture is a concern, a multiple hole will necessitate a thorough fracture mechanics analysis to assess crack initiation and propagation potential from these stress concentrations. [10]

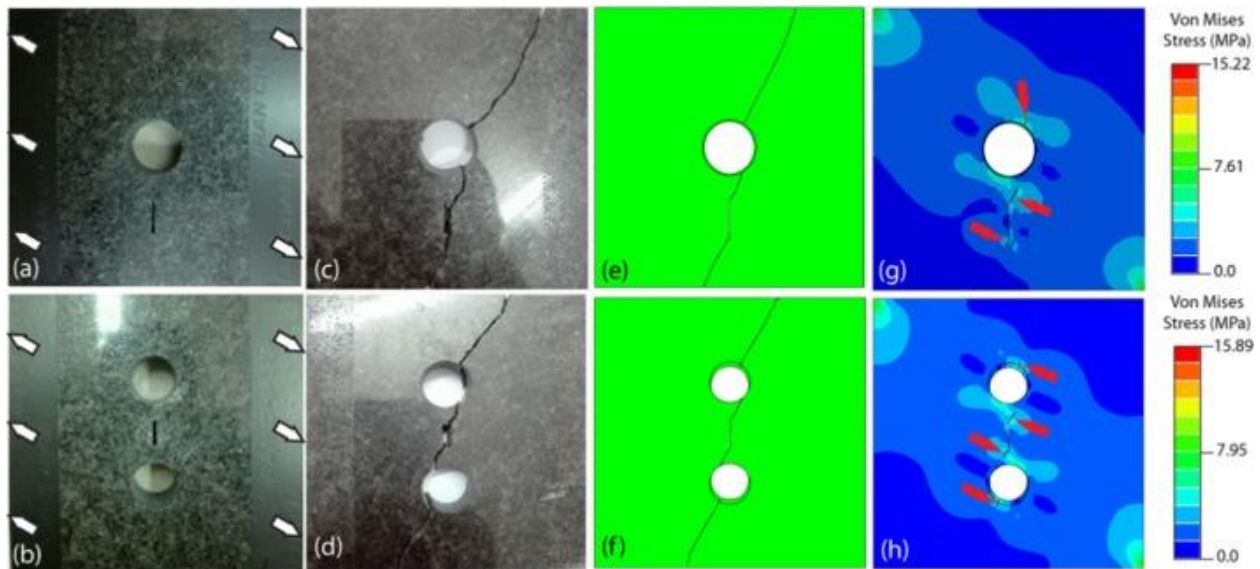


Figure II.11 influence of number of holes on crack growth

II.5 Theories for analyzing the effect of cracks:

II.5.1 Griffith's Theory of brittle fracture:

Griffith proposed that a brittle material contains a population of fine small cracks and flaws that have a variety of sizes, geometries and orientation which produces a stress concentration of sufficient magnitude so that the theoretical cohesive strength is reached in localized regions at a nominal stress which is well below the theoretical value. [19]

Griffith also noted in 1921 that when a stressed plate of an elastic material containing a crack, the potential energy decreased and the surface energy increased. Potential energy is related to the release of stored energy and the work done by the external loads. The “surface energy” results from the presence of a crack. [10]

- When one of the cracks spreads, it produces an increase in the surface area of the side of the crack.
- This requires energy to overcome the cohesive force of the atom
- It requires an increase in surface energy. Most read
- The source of increased in surface energy is the elastic strain energy which is released as the crack

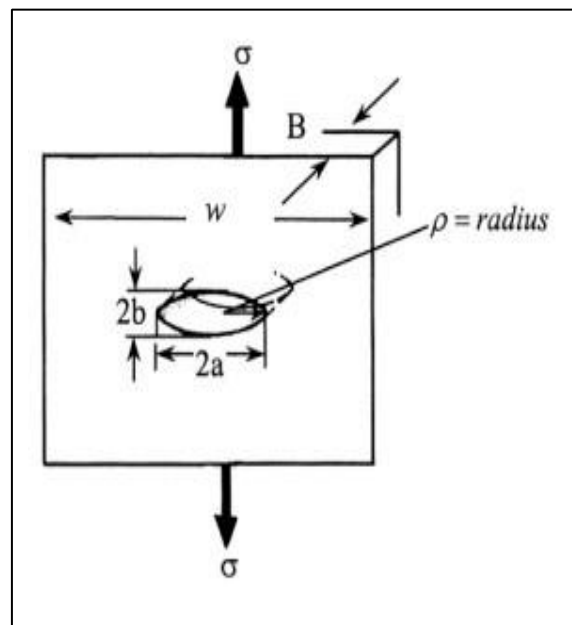


Figure II.12 Griffith's theory Crack parameters

The total potential energy of the system is given by [19]

$$U = U_o - U_a + U_\gamma \quad (\text{II.13})$$

$$U = U_o - \frac{\pi\beta a^2 \sigma^2 B}{E} + 2(2aB\gamma_s) \quad (\text{II.14})$$

The equilibrium condition of eq. (II.13) is defined by the first order partial derive. The equilibrium condition of eq. (II.14) is defined by the first order derivative with respect to crack length. This derivative is of significance because the critical crack size may be predicted very

easily. If $\frac{dU}{da} = 0$ the crack size and total surface energy are, respectively

$$a = \frac{(2\gamma_s)E}{\pi\beta\sigma^2} \quad (\text{II.14})$$

$$(\text{II.15}) \quad (2\gamma_s) = \frac{\pi\beta a \sigma^2}{E}$$

$$G = \frac{\pi a \sigma^2}{E}$$

$$= \frac{\pi a \sigma^2}{E} (1 - \nu^2)$$

where G is the strain energy release rate. While, Griffith criterion for fracture is:

$$G_c = \frac{\pi \sigma_f^2 a}{E} \quad (\text{II.16})$$

Where σ_f is the fracture stress, a is half crack length at the beginning of fracture. If $G \geq G_c$, this is the criterion for which the crack will begin to propagate.

Returning to equation (II.14) and rearranging it, we will get a significant expression in linear elastic fracture mechanics (LEFM)

$$\sigma\sqrt{\pi a} = \sqrt{\frac{(2\gamma_s)E}{\beta}} \quad (\text{II.17})$$

$$\boxed{K_I = \sigma\sqrt{\pi a}} \quad (\text{II.18})$$

The parameter K_I is called the stress intensity factor which is the crack driving force and its critical value is a material property known as fracture toughness, which in turn, is the resistance force to crack extension. K_{IC} is the fracture toughness, is important to recognize that fracture parameter K_{IC} has different values when measured under plane stress and plane strain.

Fracture occurs when $K_I \geq K_{IC}$.

II.5.2 Irwin's theory:

An alternative treatment of the elastic crack was developed by Irwin, who used a similar mathematical model to that employed by Griffith except in this case the remotely applied stress is biaxial. Irwin's theory obtained expressions for the stress components near the crack tip. The most elegant expression of the stress field is obtained by relating the Cartesian components of stress to polar coordinates based at the crack tip as shown below. [10]

Irwin also designated the left-hand side of the equation as the energy release rate, G , representing the energy per unit new crack area that is available for infinitesimal crack extension. The right-hand side of equation represents the surface energy increase per unit new crack area that would occur owing to infinitesimal crack extension and is designated the crack resistance, R . It follows that G must be larger than R before crack growth occurs. If R is a constant, this means that G must exceed a critical value $G_c = R = \text{constant}$. [10]

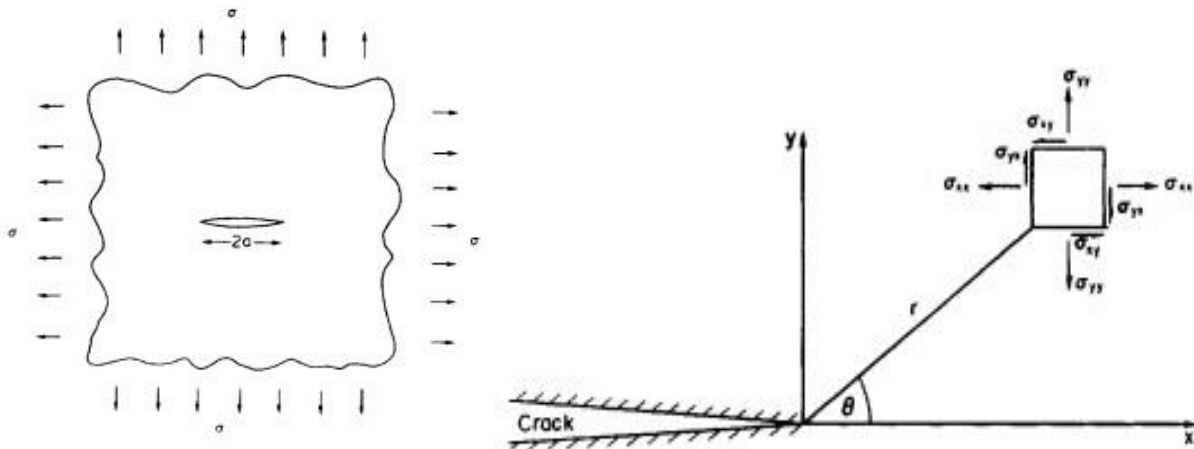


Figure II.13 A through-thickness crack in a loaded infinite plate

$$\begin{aligned}
 \sigma_{yy} &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \\
 \sigma_{xx} &= \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \\
 \tau_{xy} &= \frac{K}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}
 \end{aligned}
 \tag{II.19}$$

For plane stress, $\sigma_{zz} = 0$

For plane strain, $\epsilon_{zz} = 0$ $\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$

Example.1: -Steel tie in a girder bridge has a rectangular cross-section 200 mm wide and 20 mm deep. Inspection reveals that a fatigue crack has grown from the shorter edge and in a direction approximately normal to the edge. The crack has grown 23 mm across the width on one face and 25 mm across the width on the opposite face. If K_{Ic} for the material is 55 MPa. m^{1/2} estimate the greatest tension that the tie can withstand. Assume that the expression for K in a SEN specimen is applicable.

Solution

$a = (23+25)/2 = 24$ mm $a/W = 24/200 = 0.12$ from 9.5.b find Compliance function constants, then

$$Y = 1.99 - 0.41 \left(\frac{a}{W} \right) + 18.7 \left(\frac{a}{W} \right)^2 - 38.48 \left(\frac{a}{W} \right)^3 + 53.85 \left(\frac{a}{W} \right)^4$$

$$Y = 1.99 - 0.41(0.12) + 18.7(0.12)^2 - 38.48(0.12)^3 + 53.85(0.12)^4$$

$$Y = 2.154$$

At the onset of fracture $K = K_{Ic}$

$$K_I = Y\sigma\sqrt{a}$$

$$55 * 10^6 = 2.154 \frac{P}{0.02 * 0.2} \sqrt{0.024}$$

Hence failure load $P = 659.282$ kN

Example.2: - A thin cylinder has a diameter of 1.5 m and a wall thickness of 100 mm. The working internal pressure of the cylinder is 15 MN/m² and K_{IC} of the material is 38 MPa.m^{1/2}. Estimate the size of the largest flaw that the cylinder can contain. Assume that for this physical configuration ($K = \sigma \sqrt{\pi a}$) and that the flaw is sharp, of length 2a, and perpendicular to the hoop stress. [18]

Solution

$$\sigma = \frac{Pd}{2t} = \frac{15 \times 1.5}{2 \times 0.1} = 112.5 \text{ MN/m}^2$$

$$K = \sigma \sqrt{\pi a}$$

$$38 = 112.5 \sqrt{\pi a}$$

$$a = \mathbf{64.3 \text{ mm}}$$

H.W.

A photoblastic model shows that a fringe has a maximum distance of 2.2 mm from the crack tip. If the fringe constant is 11 N/mm²/fringe-mm and the model thickness is 5 mm, determine the value of the stress-intensity factor under this applied load.

II.6 Study on Stress concentration factor around different number of holes in the plate by Mr. CHORFI Sofiane [24]

II.6.1 Infinite plate having a central hole:

An infinite plate featuring a central circular hole with a diameter $d=15\text{mm}$ and a thickness $t=5\text{mm}$, subjected to normal stresses in a plane.

- **Traction effect:**

Two cases of tension can be encountered:

The first case consists of a uniaxial tensile stress along X, with:

$\sigma_1 \neq 0$ et $\sigma_2 = 0$, and we find the stresses at the edge of the holes at points A and B, as well as their respective stress concentration factors (K_t) as follows:

$\sigma_A = 3\sigma_1$, and a **stress** intensity factor of: $K_{tA} = 3$ to A.

$\sigma_B = -\sigma$, and a **stress** intensity factor of: $K_{tB} = -1$ to B.

The first case consists of a biaxial tensile stress along X, with:

$\sigma_1 \neq 0$ et $\sigma_2 \neq 0$ we find that:

for $\sigma_2 = \sigma_1$ (traction) $\Rightarrow \sigma_A = \sigma_B = 2\sigma_1$ et $K_{tA} = K_{tB} = 2$;

for $\sigma_2 = -\sigma_1$ (disablement) $\Rightarrow \sigma_A = -\sigma_B = 4\sigma_1$, et $K_{tA} = K_{tB} = 4$;

and for $-1 \leq \sigma_2/\sigma_1 \leq 1 \Rightarrow K_t = 3 - \sigma_2/\sigma_1$

The different values of stress concentration factors obtained can be compared with those obtained theoretically as a function of the tensile stress ratios (σ_1/σ_2) for the different cases.

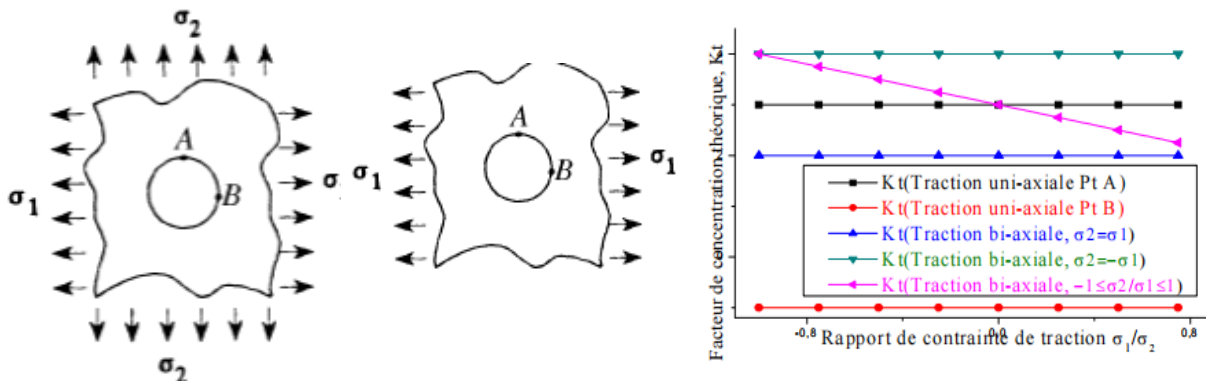


Figure II.14 Variation of K_t for a holed plate under the effect of normal stresses in the plane

II.6.2 Finite width plate having an eccentric circular hole:

The finite width plate having an eccentric circular hole with a diameter $d = 15\text{mm}$ and a thickness $t = 5\text{mm}$ is represented in Figure II.14. Similarly, the plate can be subjected to axial tensile forces or bending forces in the plane.

a) Effect of axial tension:

For an axial tensile stress, the maximum normal stress (σ_{\max}) in the AB section of the plate, as well as its relationship with the stress concentration factor (K_t) are given by:

$$\sigma_{\text{nom}} = \frac{\sigma \sqrt{1 - (d/2c)^2}}{1 - (d/2c)} \frac{1 - c/D}{1 - (c/d) \left[2 - \sqrt{1 - (d/2c)^2} \right]} \quad (\text{II.20})$$

where $\sigma_{\max} = \sigma_B = K_t \sigma_{\min}$

Consequently, the stress concentration factor K_t as a function of the ratio $(d/2c)$ is represented in Figure II.15

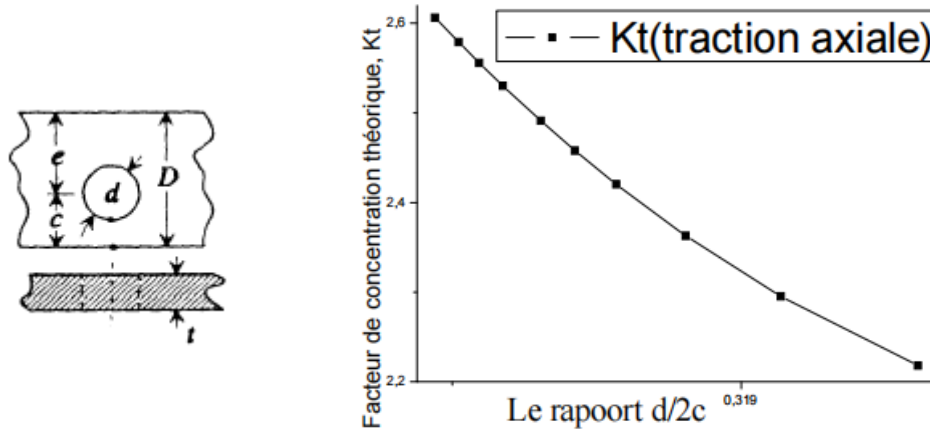


Figure II.15 Variation of K_t under axial tension as a function of the ratio $(d/2c)$

b) Effect of bending in a plane:

The bending of the plate in a plane (Fig. II.15) induces bending at the edge of the plate and at the edge of the hole, we have:

$$\sigma_B = K_{tB} \times \sigma_{min} \quad \sigma_{min} = 6M/(D^2 \cdot t) \quad \sigma_{max} = \max(\sigma_A, \sigma_B) \quad (\text{II.21})$$

The theoretical factor K_t exhibits two curve behaviors. For K_{tA} at the plate edge, it is nearly a straight line. However, the factor K_{tB} at the hole edge increases as a function of $d/2c$, where it takes the same value as K_{tA} at the point $d/2c=0.3$.

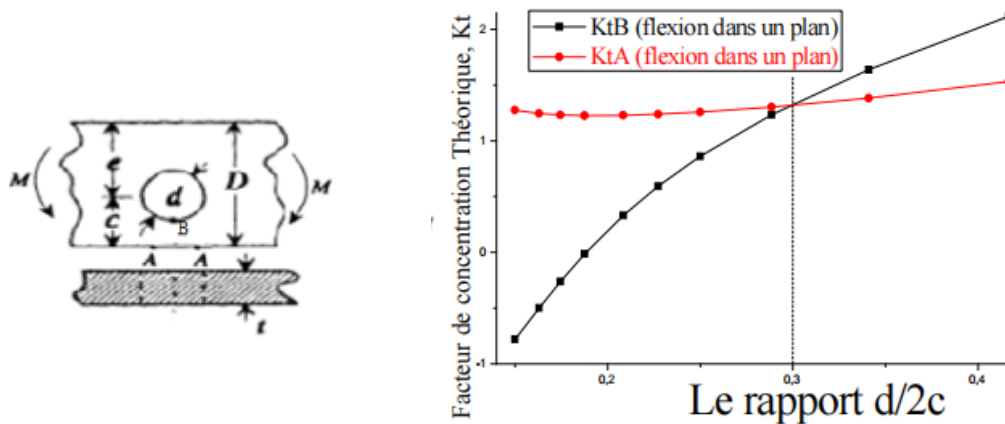


Figure II.16 Variation of K_t under in-plane bending as a function of $(d/2c)$

II.6.3 Infinite width plate having two equal circular holes:

The plate is subjected to a single normal stress in a plane. For this type of loading, we encounter two cases of tension:

a) Uniaxial tension parallel to the rows of holes That is:

$$\sigma_2 = \sigma \text{ and } \sigma_1 = 0 \quad (\text{II.23})$$

b) Biaxial tension (tension in both directions):

$$\sigma_1 = \sigma_2 = \sigma \quad (\text{II.24})$$

And in these cases, we have:

$$\sigma_{max} = \sigma_B = Kt \cdot \sigma_{nom} \quad \text{and} \quad \sigma_{nom} = \frac{\sigma \sqrt{1 - (\frac{d}{L})^2}}{1 - (\frac{d}{L})} \quad \text{for } 0 \leq d/L \leq 1. \quad (\text{II.25})$$

The theoretical stress concentration factor Kt is represented in Figure II.16. It decreases as a function of the ratio (d/L) for both tension cases; however, in the case of biaxial tension, it takes small values compared to the uniaxial Kt (Fig. II.17).

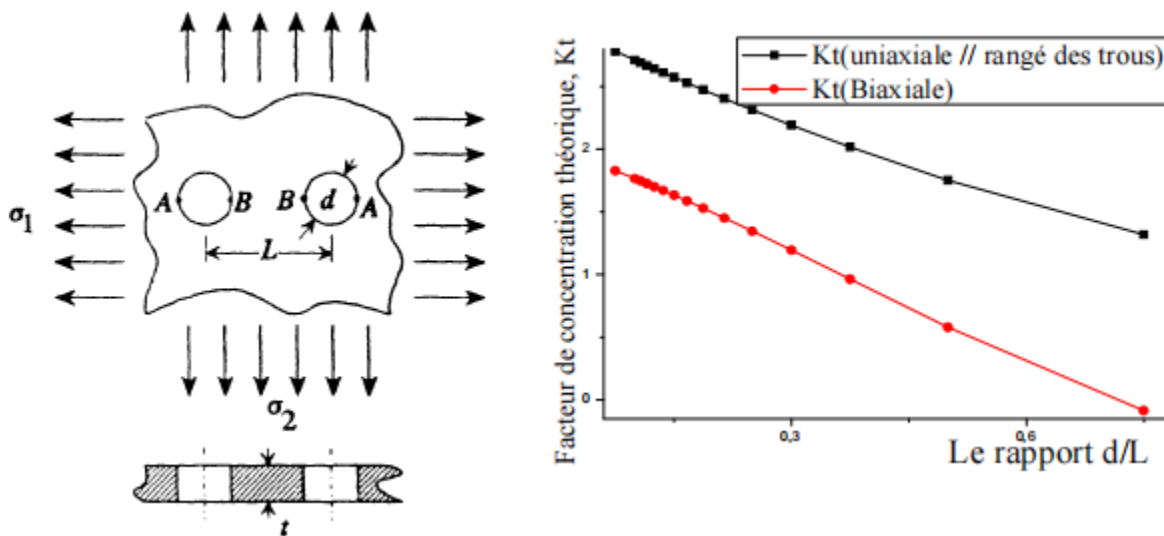


Figure II.17 Variation of Kt under normal stress as a function of d/L

II.6.4 Infinite width plate having a row of circular holes:

An infinite width plate comprising a row of several circular holes with a diameter $d=15\text{mm}$, a distance L between the centers of the holes, and a thickness $t = 5\text{mm}$.

a) Effect of normal stress in a plane

The in-plane loading shown in Fig. II.17 induces three cases of tension:

- Normal axial tension in the row of holes

$$\sigma_2 = \sigma ; \sigma_1 = 0 \quad \text{for } 0 \leq d/L \leq 1 \quad \text{and} \quad \sigma_{max} = \sigma_B = Kt \cdot \sigma$$

- Uniaxial tension parallel to the row of holes

$$\sigma_2 = 0; \sigma_1 = \sigma \text{ so, } \sigma_{min} = \frac{\sigma}{1 - (\frac{d}{L})} \text{ and } \sigma_{max} = \sigma_A = K_t \times \sigma_{min}$$

- Biaxial tension (II.26)

Where, $\sigma_1 = \sigma_2 = \sigma$, $\sigma_{max} = \sigma_A = K_t \times \sigma_{min}$ and for $0 \leq d/L \leq 1$; $\sigma_{min} = \frac{\sigma}{1 - (\frac{d}{L})}$

According to Figure II.17, we observe that the stress concentration factor K_t increases as a function of the ratio (d/L) for the case of normal axial tension in the row of holes, but it decreases for the case of tension parallel to the row of holes. Whereas for the effect of biaxial tension, it decreases slightly and remains almost stable as a function of d/L (Fig. II.18).

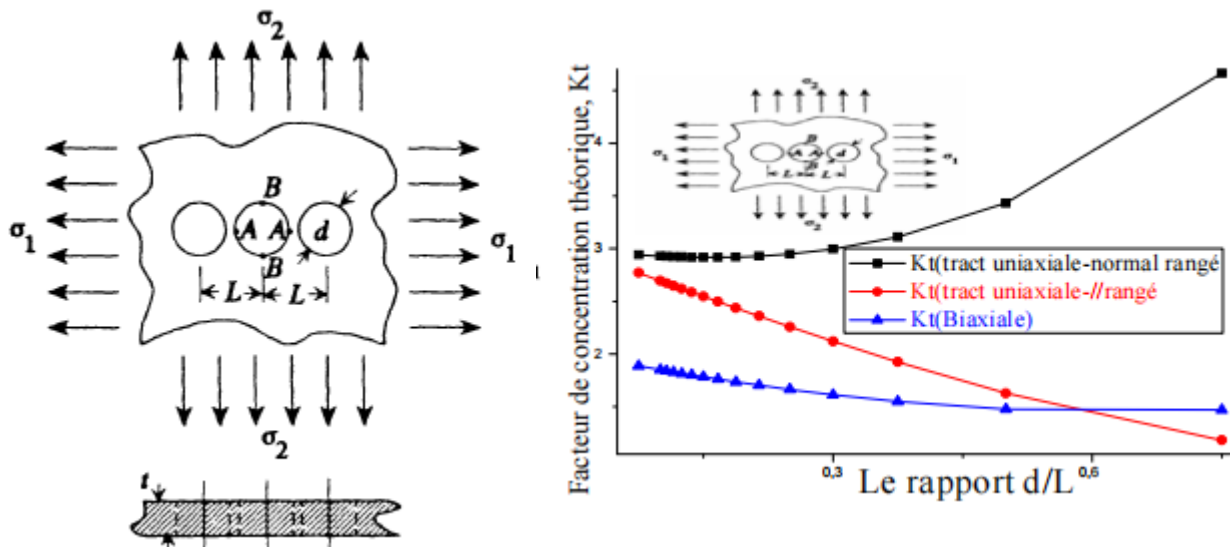
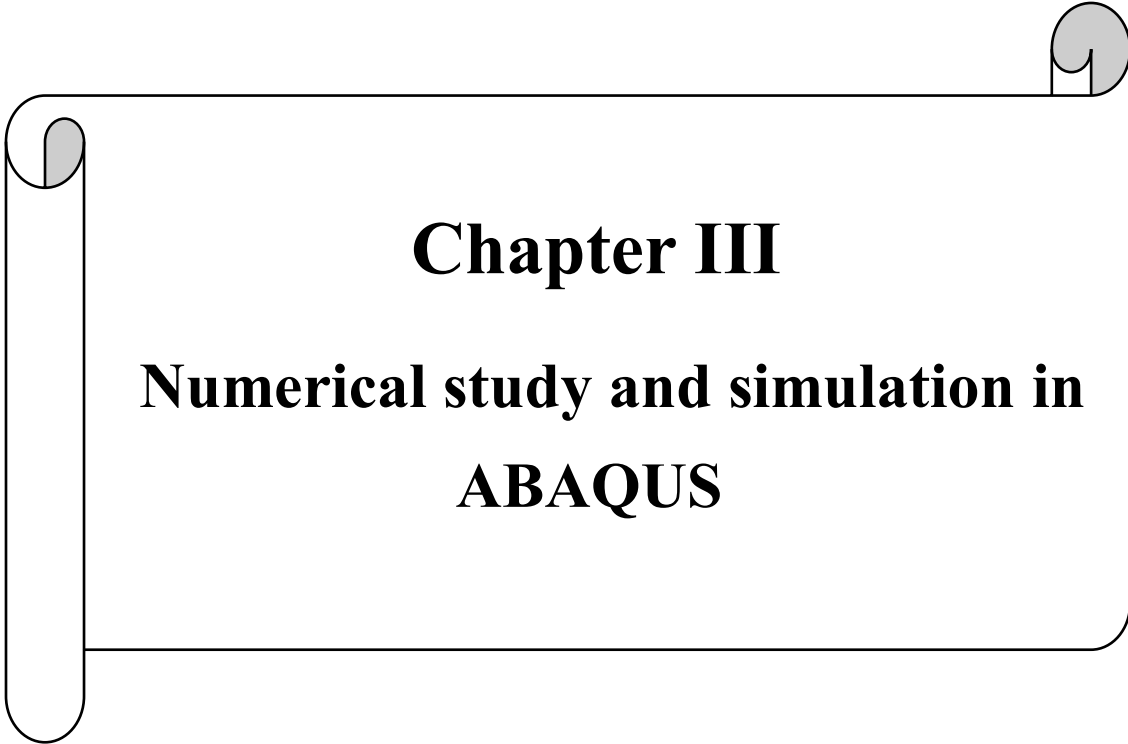


Figure II.18 Variation of K_t for a row of holes under normal stress as a function of (d/L)

Conclusion

In Chapter II: Notches Effect on Mechanical Behavior:

specifically focused on how notches, particularly holes, influence mechanical behavior. It reiterates that hole's cause stress concentration, discusses factors affecting its magnitude, and highlights important consequences and the need for mechanical analysis around holes for construction safety, failure analysis, design optimization, and inspection. This chapter also delves into Fracture Mechanics, defining the stress intensity factor (K) and presenting Griffith's and Irwin's theories for analyzing crack behavior. A key theoretical result presented here is the principle that closely spaced multiple holes lead to interaction and overlapping of stress concentration regions, causing higher peak stresses between holes.



Chapter III
Numerical study and simulation in
ABAQUS

III.1 Definition and a general introduction:

III.1.1 Definition of ABAQUS:

ABAQUS is a finite element analysis software developed by DASSAULT. It is composed of three products: ABAQUS/Standard, ABAQUS/Explicit, and ABAQUS/CAE. The ABAQUS software suite is particularly renowned for its technology, quality, and reliability. It has established itself as an integral part of the design processes of world-renowned companies in all industrial sectors. ABAQUS offers the best solutions for linear, non-linear, explicit, and dynamic problems. The software provides an unparalleled environment for finite element analysis, offering a large number of alternatives to operations involving multiple suppliers and products. In our study we are going to use the standard version for our simulation process. [23]

III.2 Description of ABAQUS Capabilities and Overview of ABAQUS Features:

ABAQUS is a solver (Standard, Explicit, Implicit, etc.) that performs the resolution of a problem described by an input file or data file with the extension. and that writes the solution into an output file or results file with the extension. [23]

Table III-3

The file of data	The file of results
<ul style="list-style-type: none">• Extension: inputs• Content: keywords that describe the geometries, the materials, the boundary conditions	<ul style="list-style-type: none">• Extension: outputs• Content: contours and curves of results

Performing a numerical simulation is done by creating its own file input thanks to a text editor by knowing the various keywords or well by using the graphical interface of ABAQUS CAE.ABAQUS covers almost all the domains of physics and all the types of problems. [23]

Physical domains:

- Mechanics
- Thermal
- Electrical (piezo and thermal)
- Coupled problems

Problems:

- Static and dynamic
- Linear and non-linear

ABAQUS offers a very wide choice of elements (more than 100) with the possibility of programming new elements (in FORTRAN, on ABAQUS/Standard).

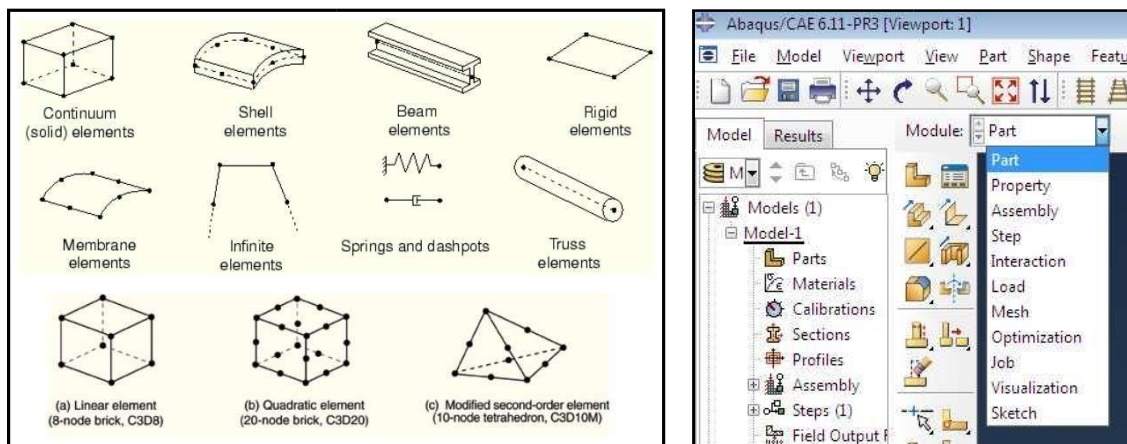


Figure III.1 ABAQUS shapes modeling and design

III.3 FEM the method used in ABAQUS:

Most modern software systems use the finite element method which is basically a numerical technique for solving complex engineering problems involving stress analysis.

There are two major FEM in the modern technology which are XFEM (extended Finite Element Method), which is an extension of the conventional Finite Element Method that allows for the modeling of discontinuities, such as cracks, voids, and interfaces, within elements *without* the need for the mesh to conform to these features. and UFEM (Unfitted Finite Element Method), and it is a broader class of finite element methods that, like XFEM, allows the computational mesh to be independent of the geometry of the domain or any internal interfaces and discontinuities. XFEM is often considered a subset or a specific type of UFEM.

ABAQUS uses the XFEM for the reason that the mesh must be carefully aligned with any discontinuities, which can be cumbersome and require remeshing as the discontinuity evolves (e.g. as a crack grows).

Here's a breakdown of key aspects of XFEM:

Unity partition: XFEM derives from the unity partition concept, permitting the standard finite element approximation to be enriched by adding specific enrichment functions that reveal discontinuous behavior.

Enrichment Functions: From standard FEM formulation, additional degrees of freedom and enrichment functions are added for elements intersected by the discontinuity.

These typically include:

- Heaviside function: To portray leap discontinuities at the crack faces.
- Asymptotic crack-tip functions: To seize the singular stress field close to the crack tip in issues related to fracture mechanics.
- Modeling Discontinuities: XFEM is capable of modeling: Crack initiation as well as propagation along arbitrary, solution-dependent paths without the need for remeshing. Multiple cracks and branching cracks along with cracks that are interacting with other features. Material interfaces between two different materials. Voids and inclusions.

Advantages:

- **No remeshing:** Simplifies the modeling of evolving discontinuities.
- **Complex geometries:** Can handle discontinuities in complex geometries more easily.
- **Crack initiation:** Can model crack initiation without a pre-defined crack location.
- **Implementation in ABAQUS:** XFEM capabilities for fracture analysis have been implemented in ABAQUS, through which the user can define crack domains and simulate crack growth based on different criteria. It allows static analysis and dynamic analysis but only in some limited cases with XFEM.

III.3.1 Core Principles of FEM in ABAQUS:

- **Discretization (Meshing):**

The first important step involves the process of dividing the actual geometry of the structure or domain into some finite elements, which are a collection of smaller and also simpler parts. Nodes exist as specific points at which these elements are linked. The finite element mesh is formed via the entire collection of elements, as well as nodes. The solution's degree of accuracy is influenced by the level of density of such mesh (number of elements for each unit area or volume). Several meshes (e.g., structured, unstructured) of differing element shapes (e.g., hexahedral, tetrahedral, triangular, quadrilateral) are generatable. Tools that are wide-ranging for this certain purpose are offered by ABAQUS now. [23]

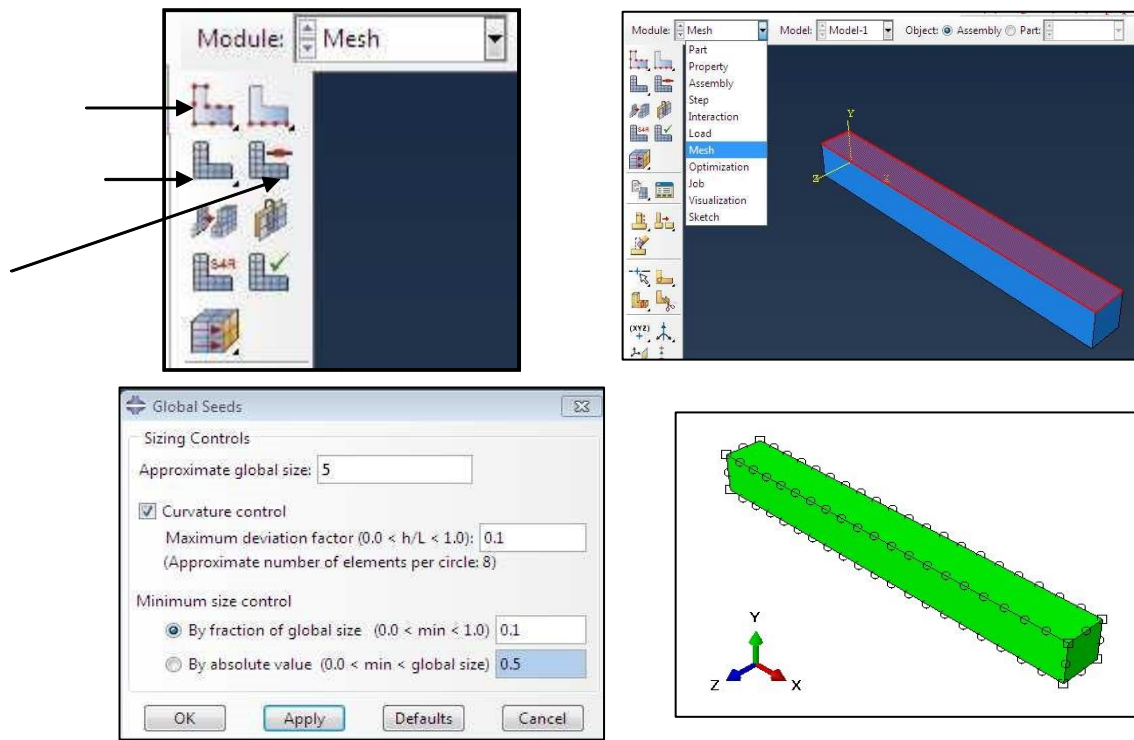


Figure III.2 Seeding edges process

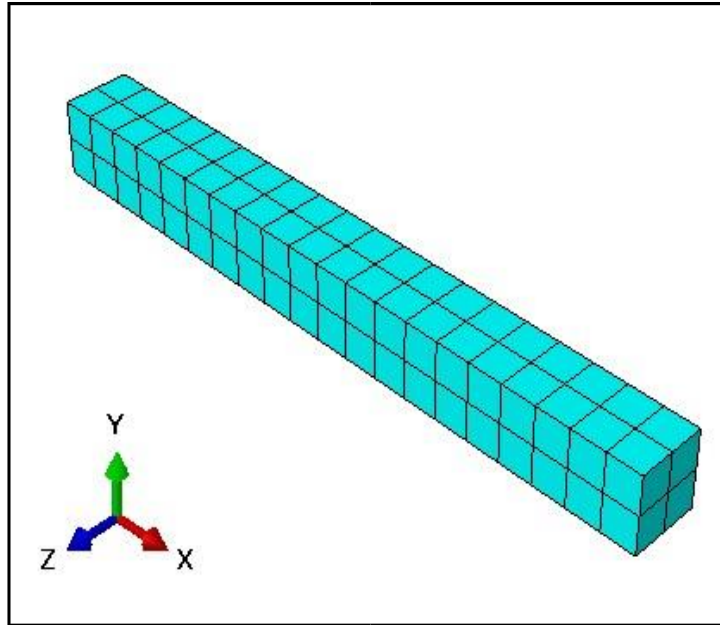


Figure III.3 MESH module

- **Applying Boundary Conditions and Loads:**

Users specify boundary conditions (restrictions regarding the nodal rotations and displacements) and loads (thermal loads, pressures, forces, etc.) on the finite element model to mimic actual situations. The nodes as well as elements belonging to the mesh have these specific conditions applied to them. These mesh elements also have all of these loads applied to them. [23]

- **Assembling the System of Equations:**

ABAQUS forms the System of Equations based on the element formulations, material properties, boundary conditions, and loads. These equations usually relate the nodal displacements to the applied forces for stress analysis

- **Solving the System of Equations:**

ABAQUS employs various **solvers** (Standard, Explicit, Implicit) to solve this system of equations

• **Analysis and visualization:**

ABAQUS/CAE (graphical user interface) offers very powerful analysis tools. It includes displaying stress and strain contours, displacement fields, animations of deformed shapes, and plotting results over time or along some specific paths. The output database file (.odb) is usually written as the solution.

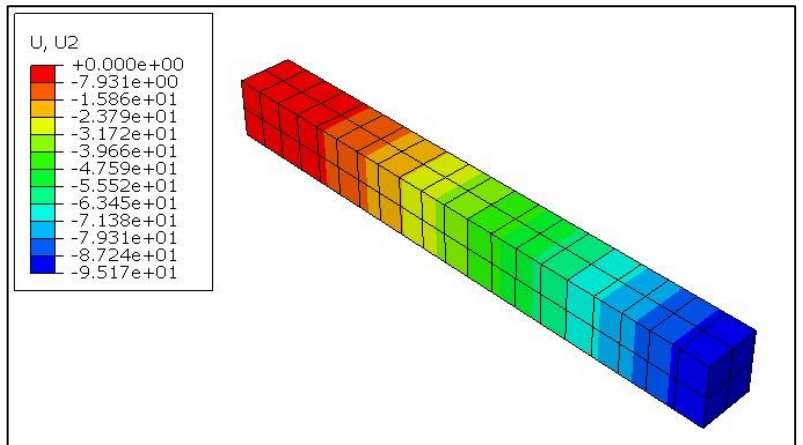
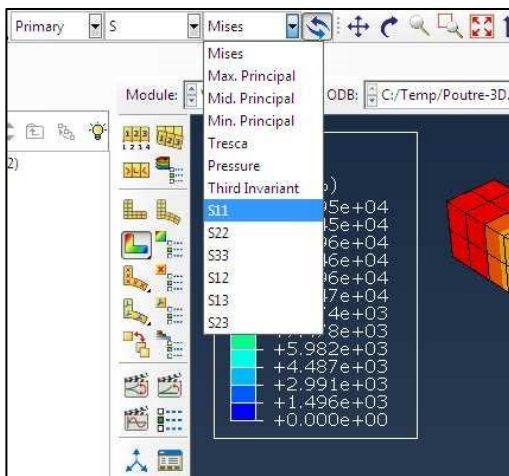
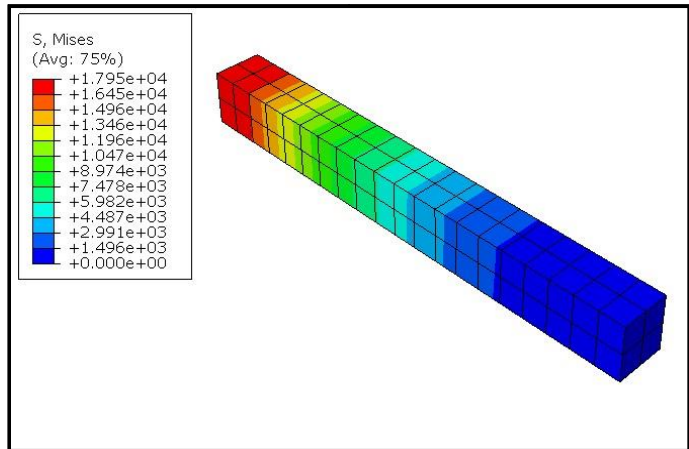
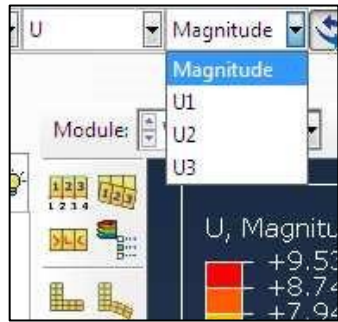
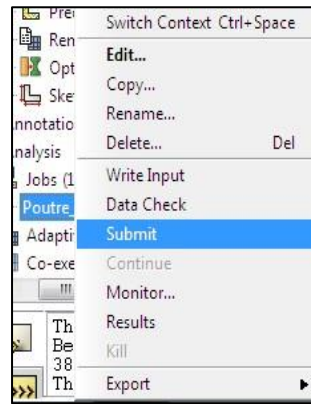
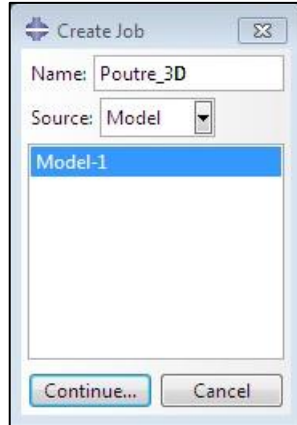
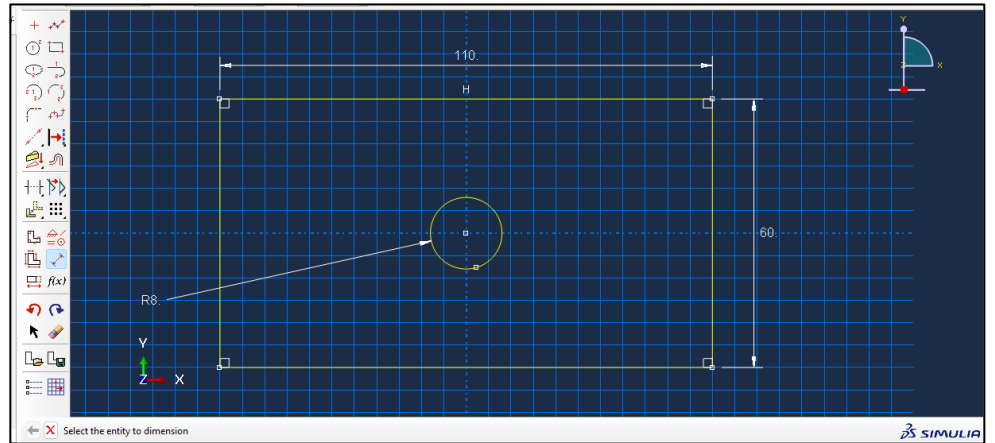


Figure III.4 visualizations and results

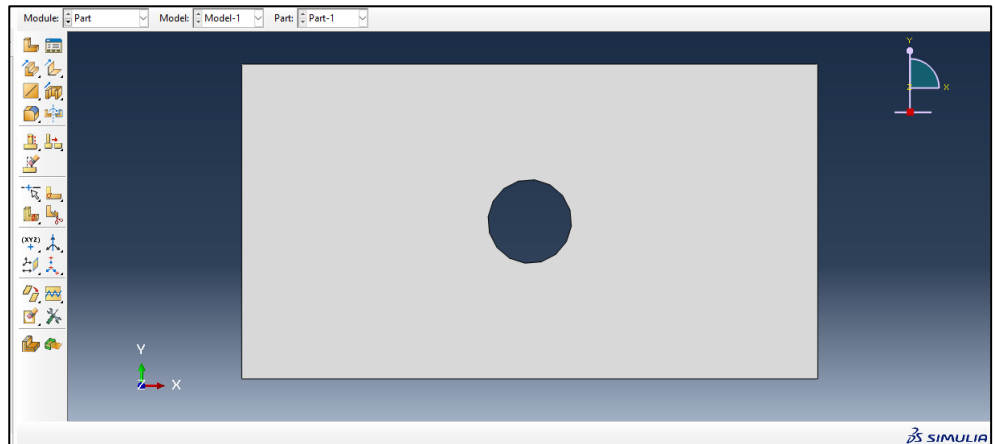
III.4 Simulation Using ABAQUS:

III.4.1 Analysis of a Simple 2D plate with A Hole:

- first thing we designed the plate and drew it in Abaqus



- the full plate part



- after splitting the part for better meshing

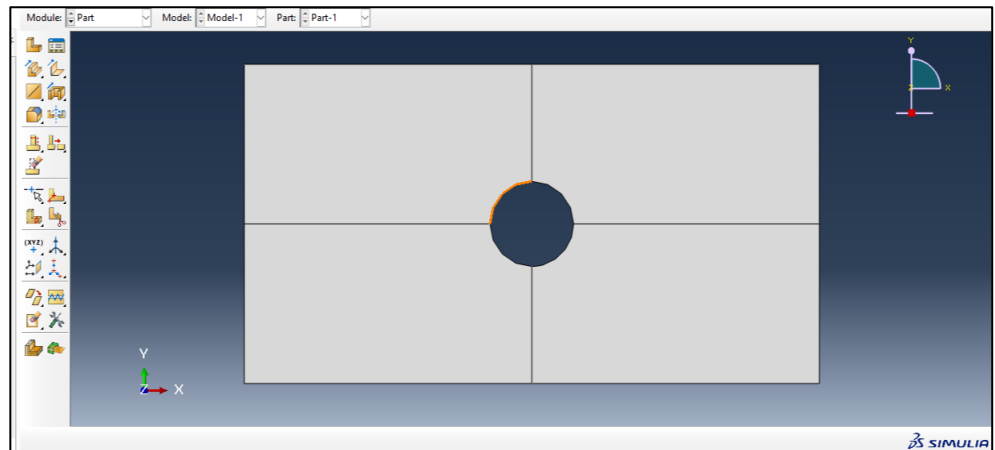


Figure III.5 Part section

➤ seeding the edges

using an element size of 1mm. Seeding edges in ABAQUS is a way for us to directly control the density of the mesh along specific edges of the geometry. By seeding, we specified the number of elements that will be placed along an edge or the approximate size of the elements along that edge.

Therefore, we seeded edges, particularly those associated with stress concentrators the holes, to increase the mesh density in these critical areas. A finer mesh allows the XFEM solver to more accurately calculate and represent the sharp stress and strain. This leads to more reliable results for peak stresses, stress intensity factors, and overall structural performance While ABAQUS uses XFEM to model discontinuities within elements, the initial meshing process (including seeding) is fundamental for setting up the analysis and accurately capturing the stress state, especially around designed features like holes and notches.

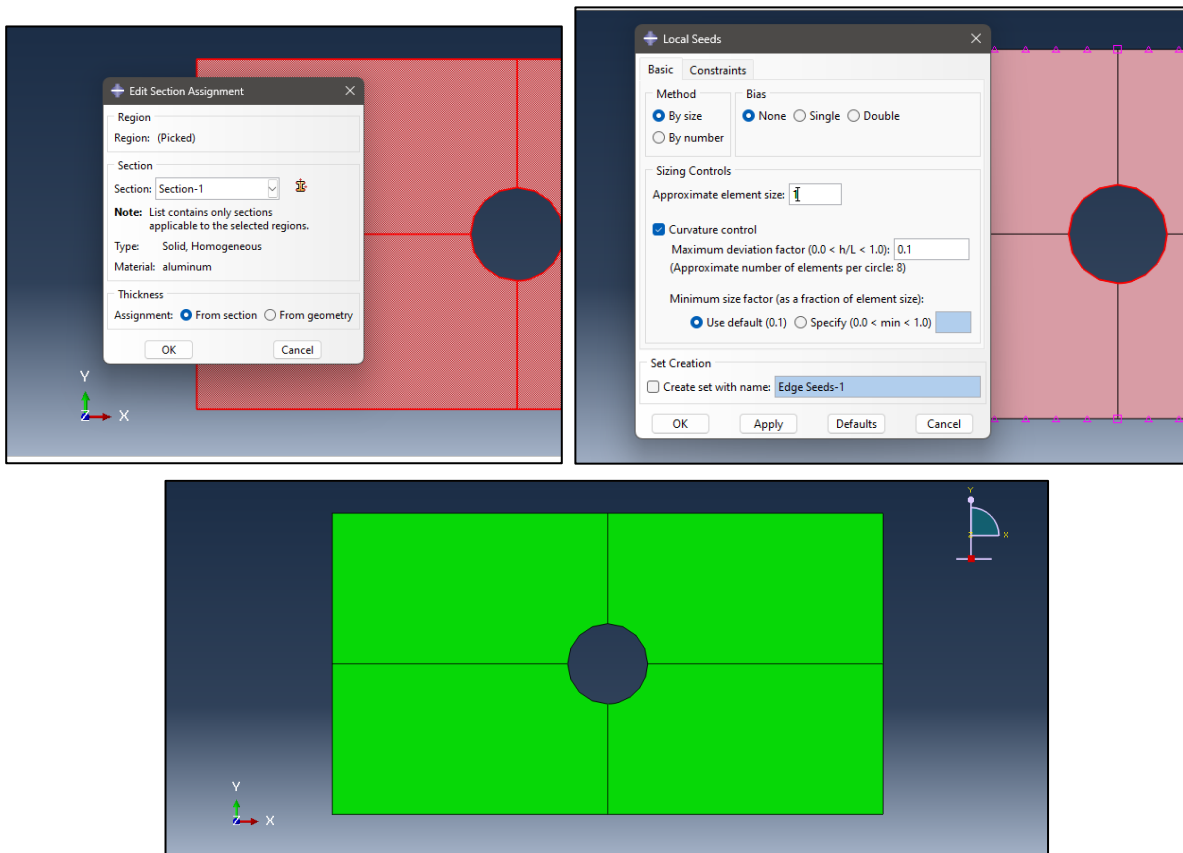


Figure III.6 edging the module

➤ Creating mesh

meshing is the process of breaking down the model's geometry into smaller pieces so that ABAQUS can apply the Finite Element Method to solve the problem, and the quality and density of this mesh directly impact the accuracy of the simulation results. While ABAQUS uses advanced techniques like XFEM to handle certain discontinuities within elements without needing the mesh to conform to the discontinuity's path, the initial meshing of the overall geometry remains a core step in defining the model for analysis.

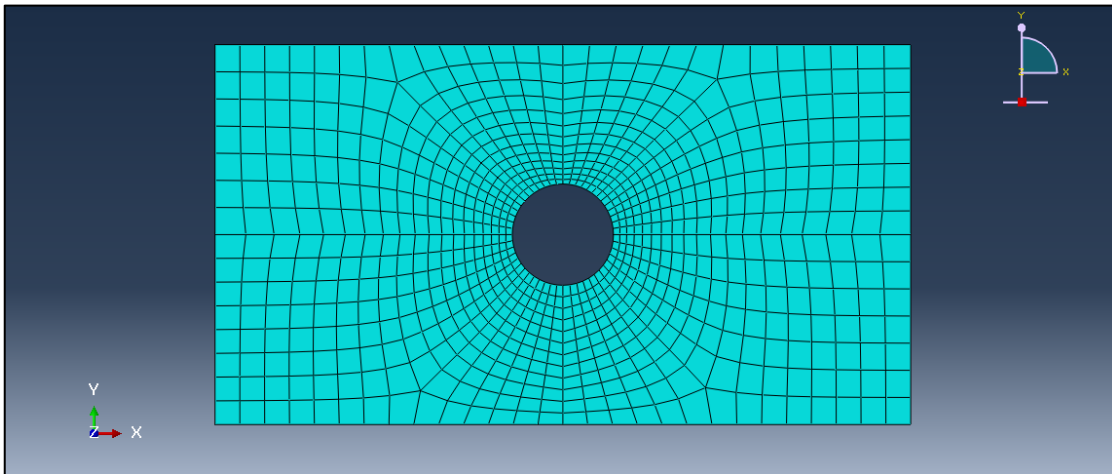
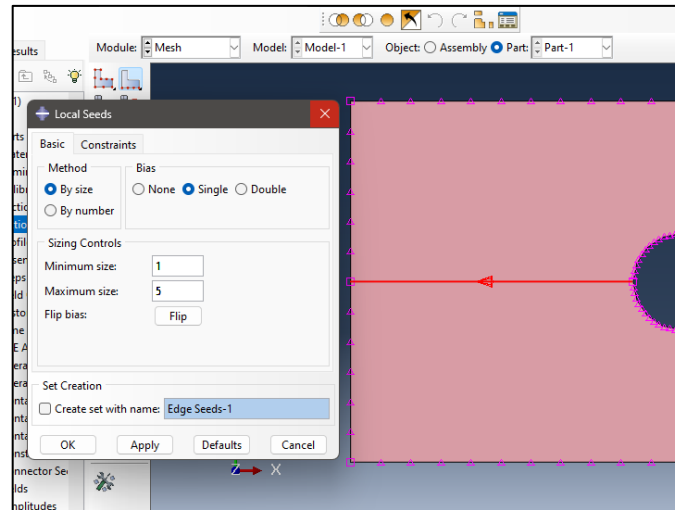


Figure III.7 Mesh part

➤ **applying load and boundary conditions:**

boundary conditions and loads are essential inputs that tell ABAQUS how the structure is supported and what forces it's subjected to. Without them, the simulation cannot represent the real-world scenario and therefore cannot provide meaningful results about the structure's integrity or performance under operating conditions.

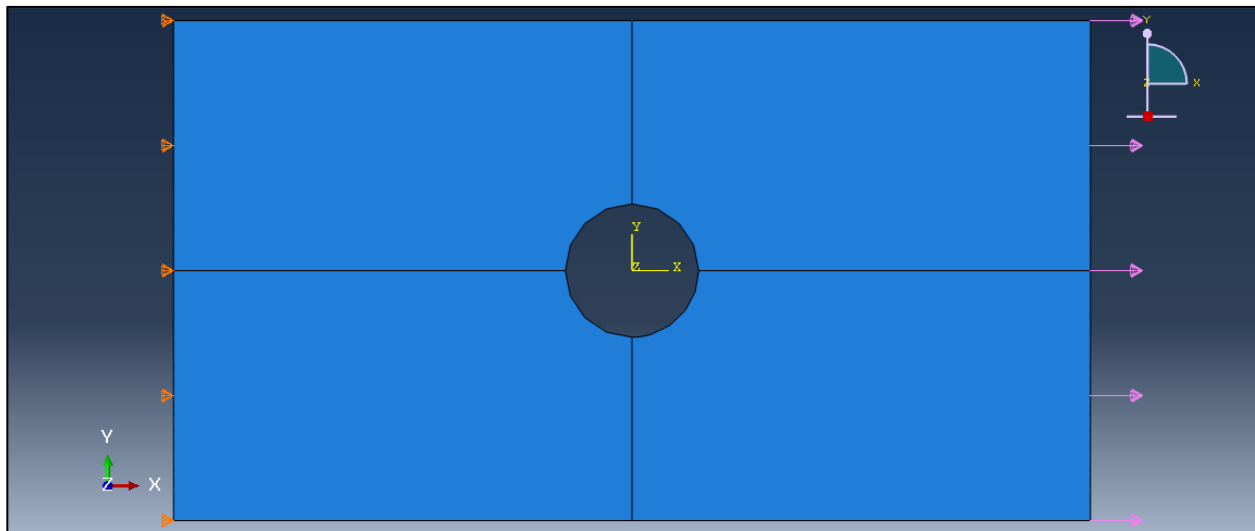
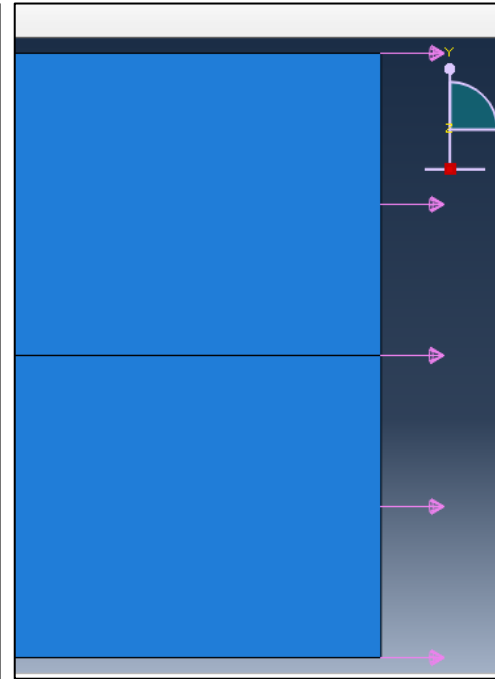
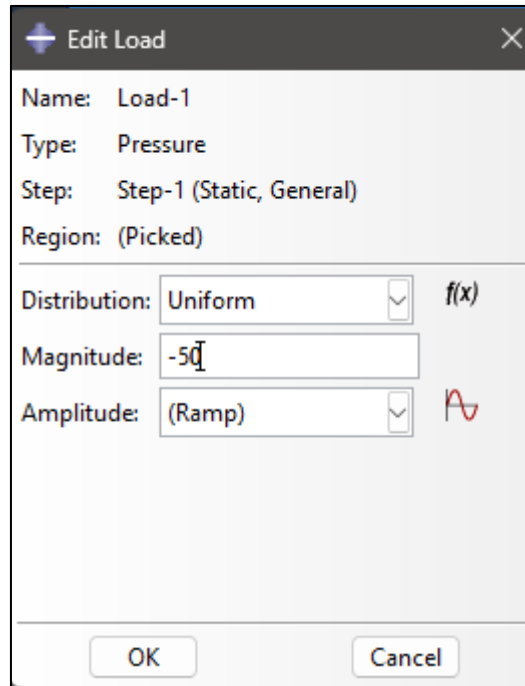


Figure III.8 boundary condition and loads application

➤ **editing step and element type:**

editing the step and element type is necessary to precisely define the problem for the ABAQUS FEM solver. The step tells the solver the nature of the analysis (static, dynamic, etc.) and how the external conditions (loads, BCs) are applied over time. The element type tells the solver the mathematical formulation to use for each small piece of the mesh, governing how it responds internally to these conditions based on material properties. Both are essential for an accurate and meaningful simulation that can mimic actual situations and predict material behavior.

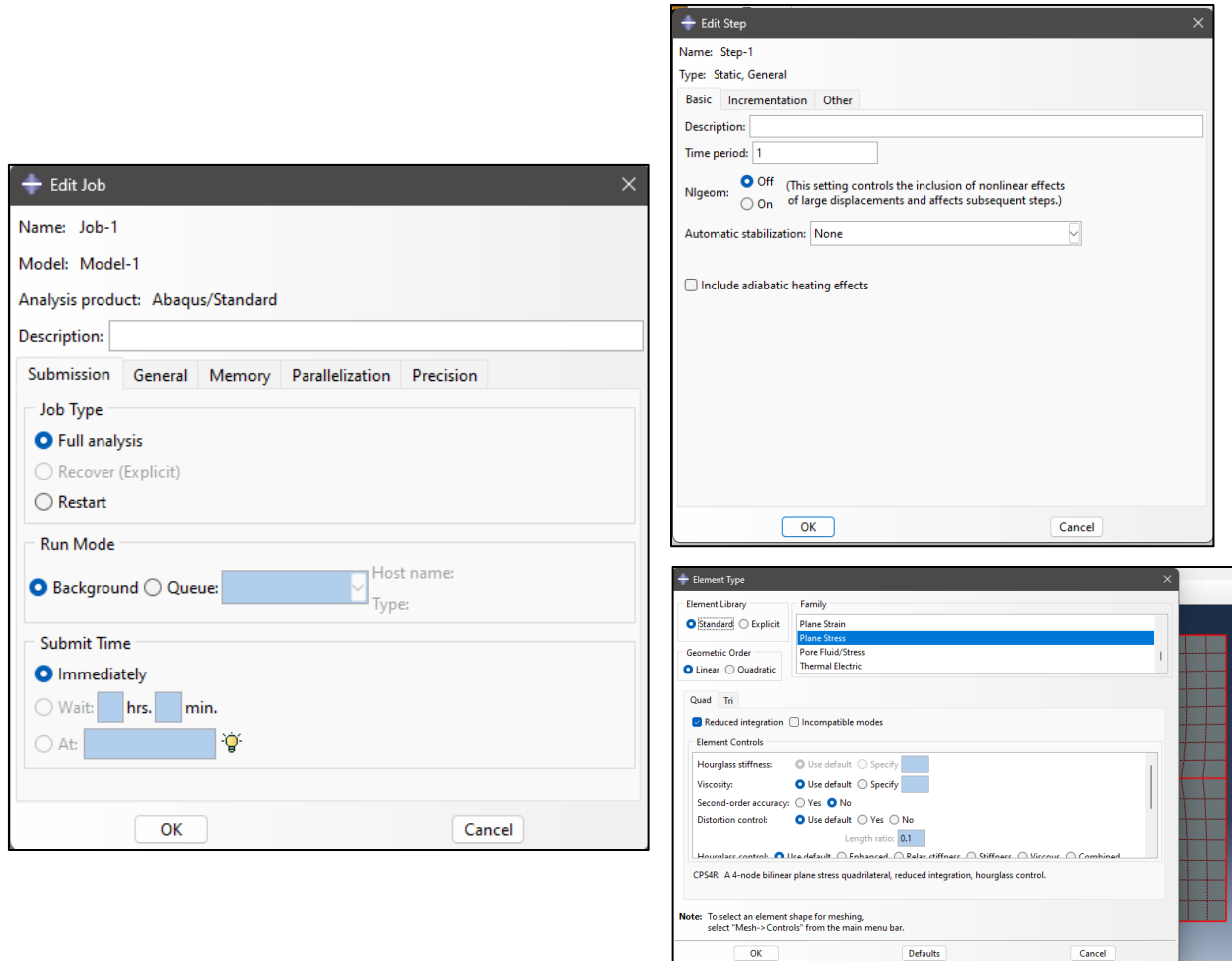


Figure III.9 calculation process

➤ **Results:**

After applying the job, we get to the results which is basically a way for the software to show every solution and visualize it.

In essence, ABAQUS generates a data file containing the calculated solution, and this data is then interpreted and displayed visually through the CAE interface to show things like **stresses, strains, displacements, and how the model deforms.**

Also, the pressure path, the magnitude, the reaction force, and the creep.

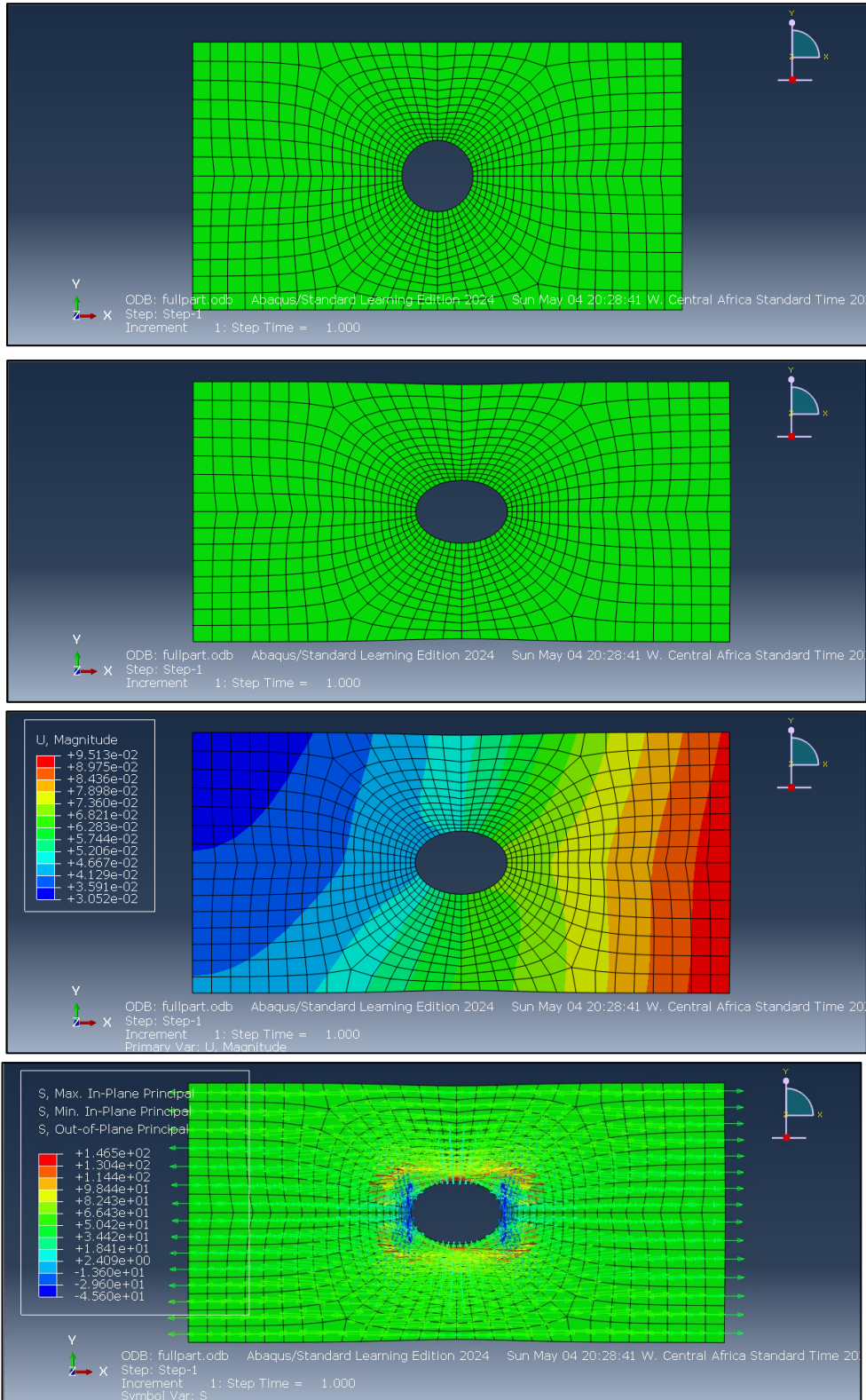


Figure III.9 RESULTS AND VISUALASATION

III.4.2 Simulation and comparison on 3D ASTM D638 Tensile Specimen:

Dimensions and design of the piece:

The subject of the test was a 165mm long specimen, the curved section width is 13mm, a width of 19mm in the outer line and thickness of 3mm of the full section. Each hole has a radius of 3mm.

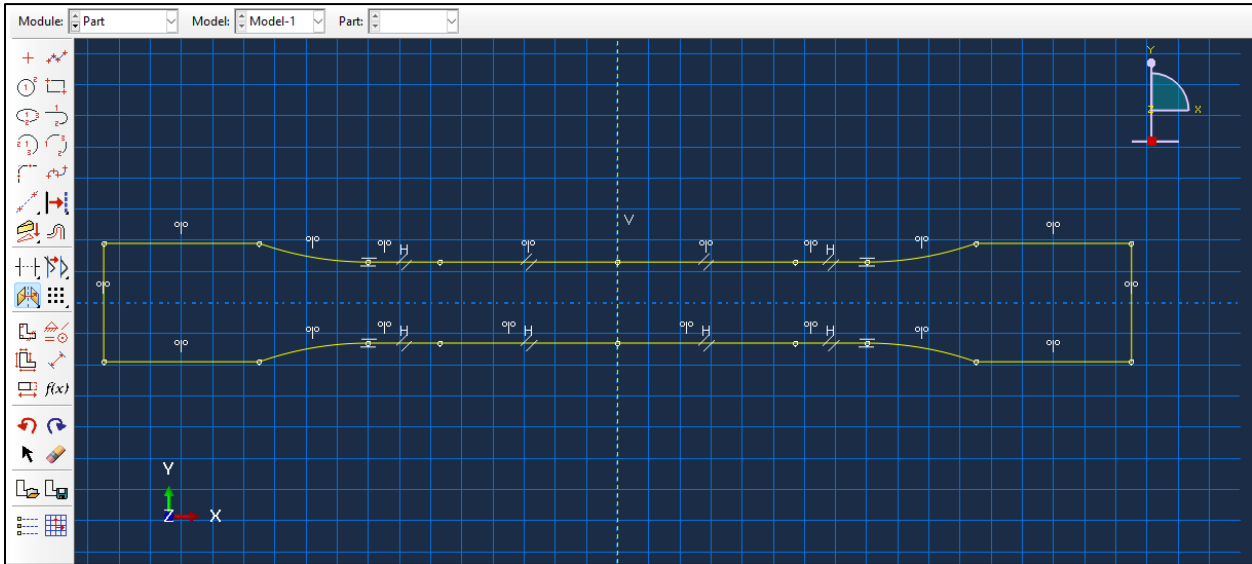


Figure III.10 ASTM D638 designing in ABAQUS

DATA:

The same applied force was used in every testing piece to get the specific results we need to compare between the pieces, as well as the same material, the same force direction, the boundary condition, the magnitude, the uniform, the section selection, and meshing method.

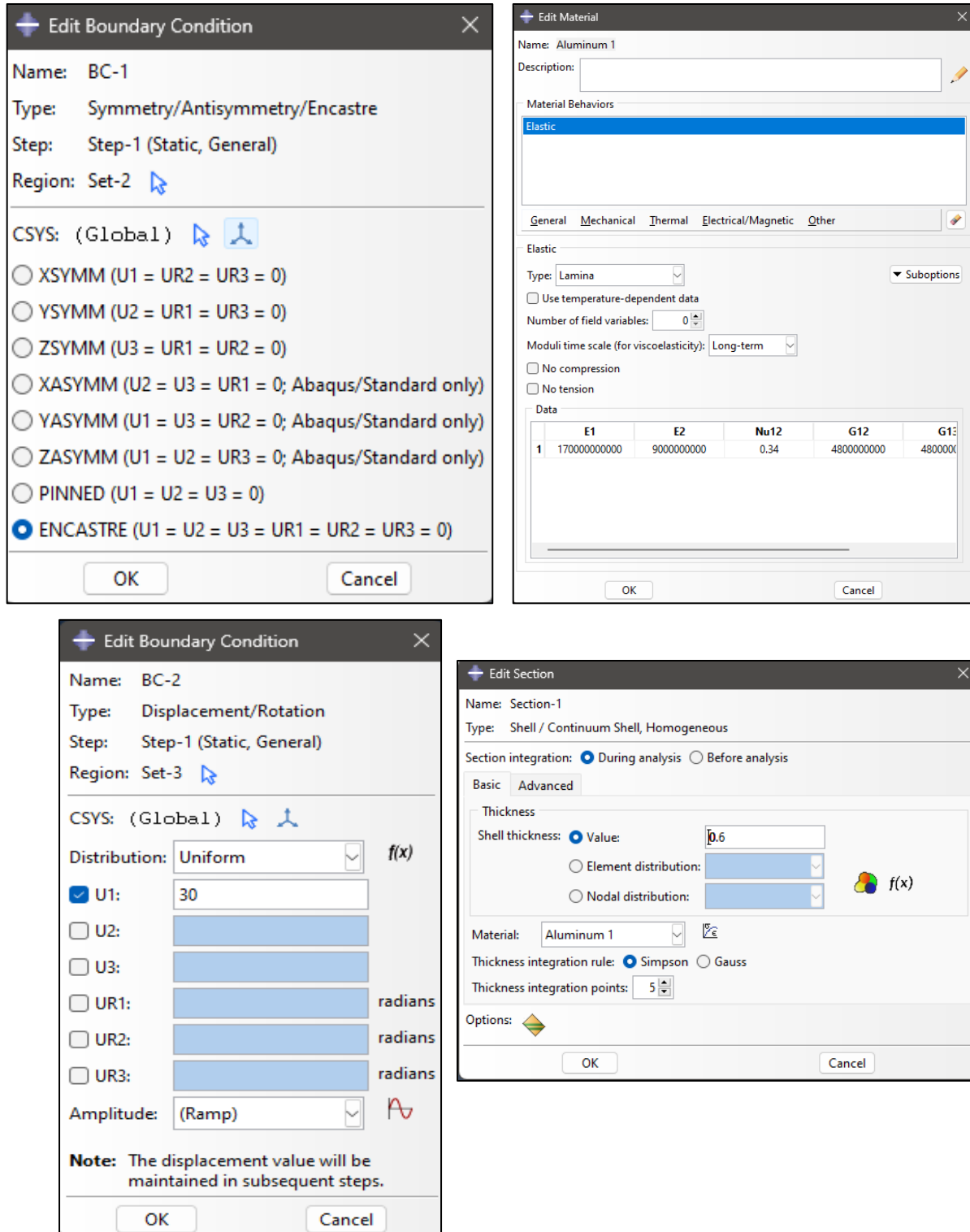


Figure III.11 material selection, editing section and boundary conditions

a) The first testing piece with one hole:

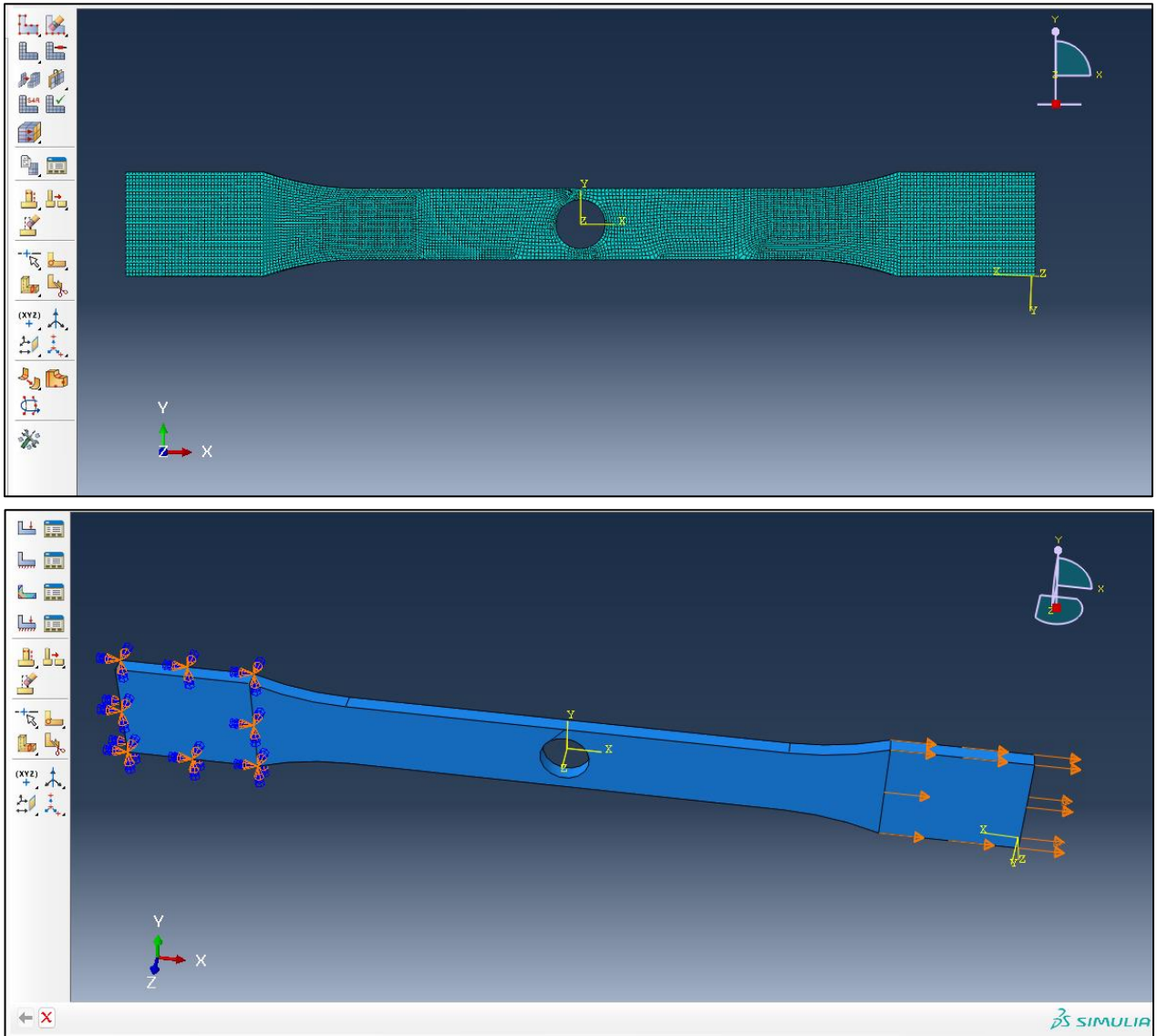


Figure III.12 One holed specimen mesh and loaded modules

a.1 The results we got:

One hole the deformed shape and the magnitude and stress vectors

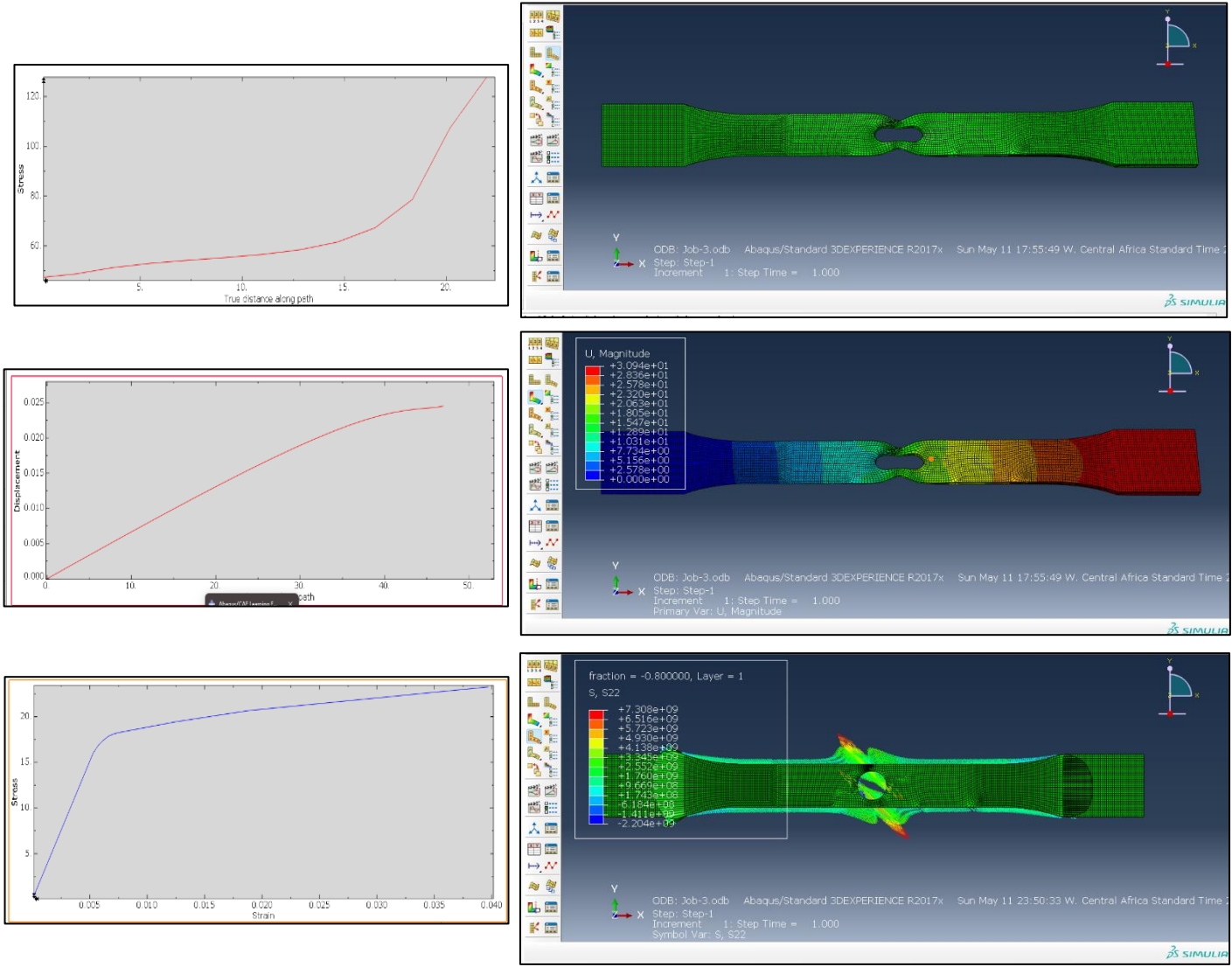


Figure III.13 1_Visualizations of U, E and Stress along with curves of deformation stress misses and stress-strain

b) The second piece with two holes:

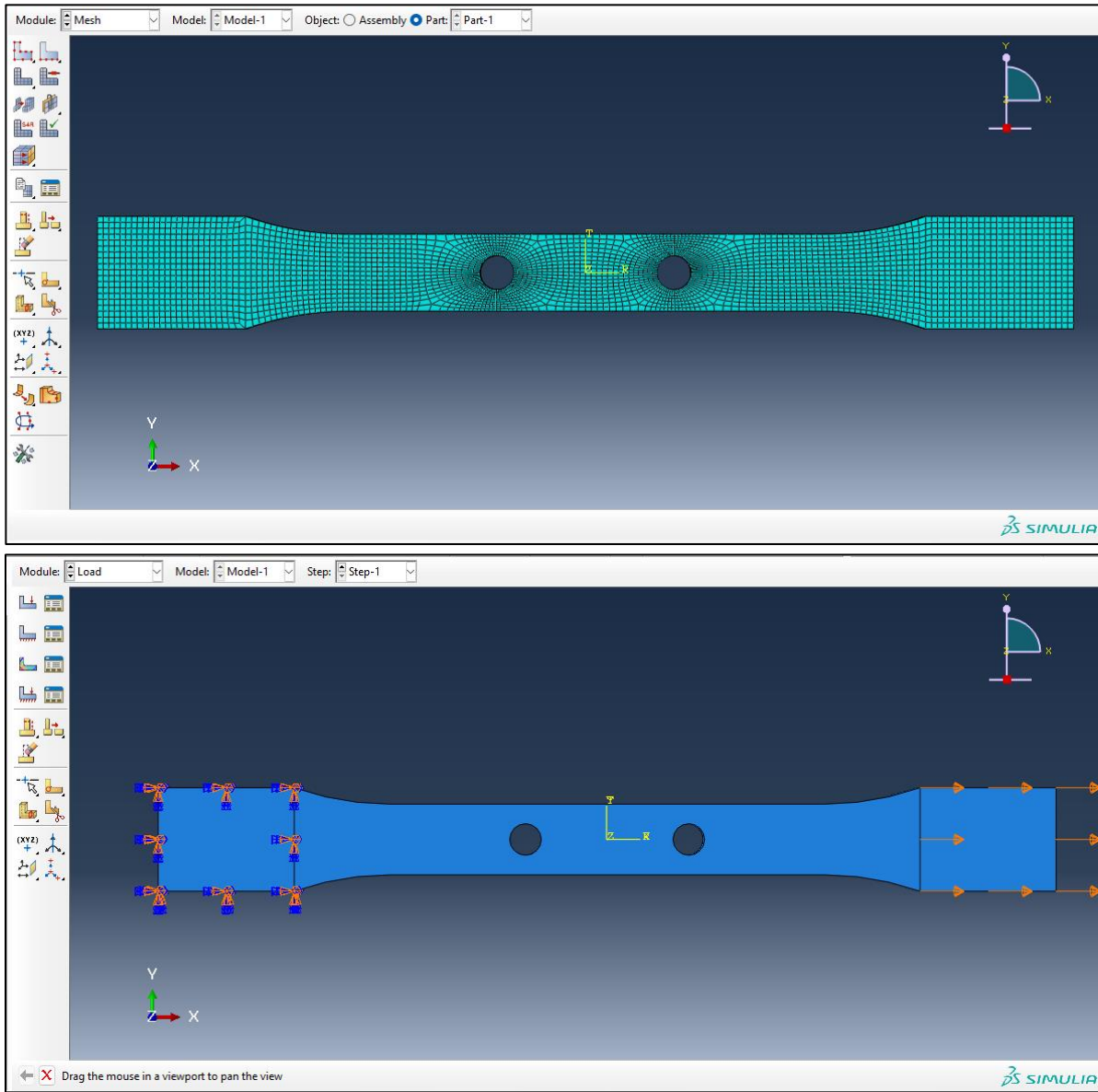


Figure III.14 Two holed specimen mesh and loaded modules

b.1 The results:

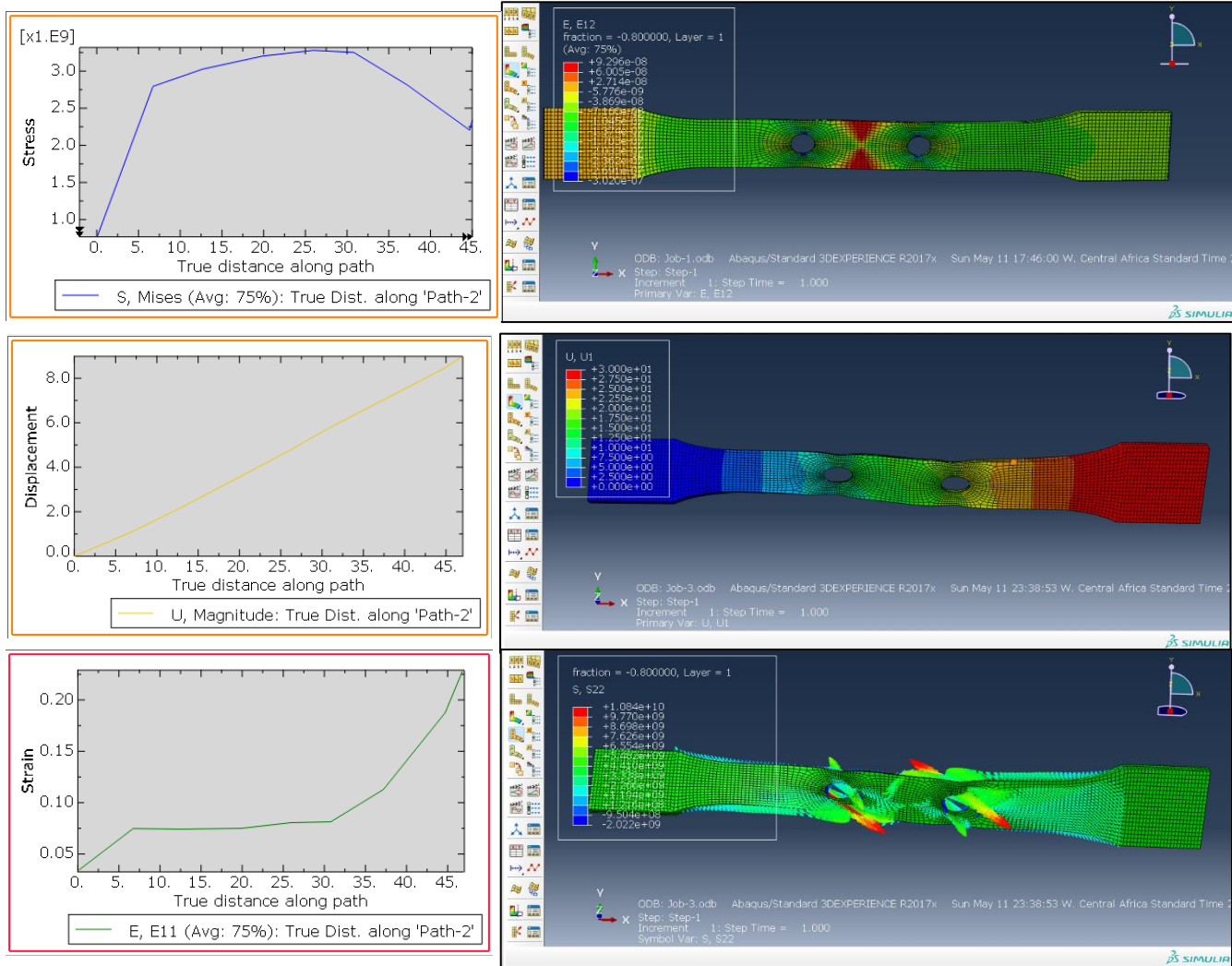


Figure III.15 2_ Visualizations of U, E and Stress along with curves of deformation stress misses

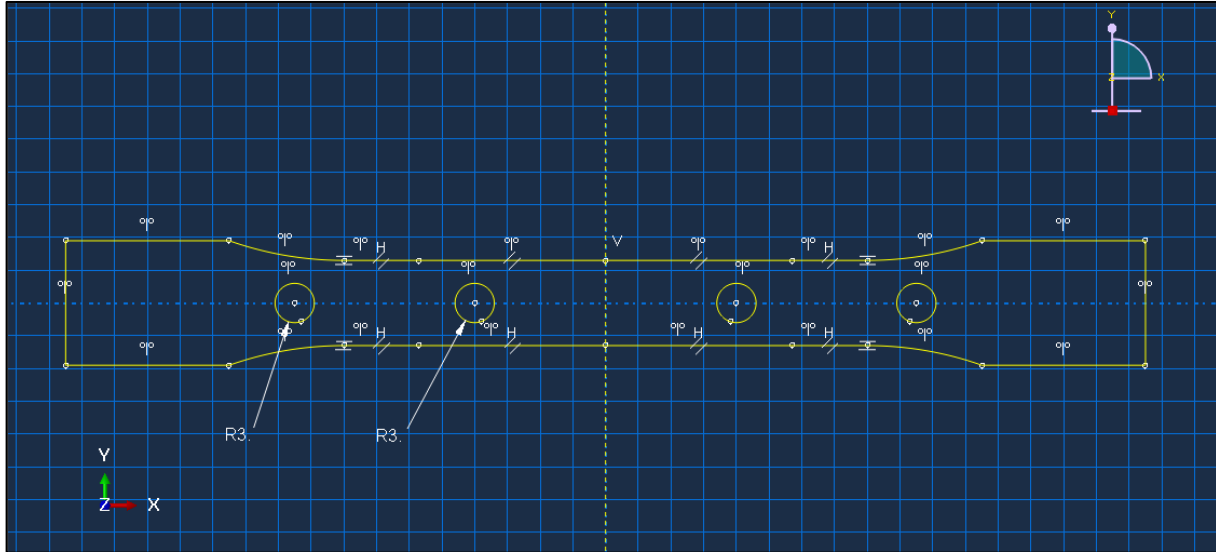
Contour plots of stress magnitude and stress vectors are presented. High stress concentration is observed around the edges of both holes.

the stress contours appear to show increased peak stress and interaction between the stress fields of the two holes in the material region separating them. So, it seems like when holes are closely spaced

This overlap of stress fields implies a more **complex and potentially higher localized strain pattern** in the material section or "ligament" situated between the two holes compared to the strain around a single isolated hole.

c) The third piece with four holes:

For this piece we used a more precise and accurate dimensions for the design we made sure that every hole is in the exact same place in the other half of the piece let beside the same diameter for all four holes, we needed to be sure that the shape would be as perfect as can be.



We even used the mirror method for drawing which let us draw a half of the piece before selecting a line to mirror of off, then the software automatically draws the same design with each dimension given in the mirrored section with best precision possible.

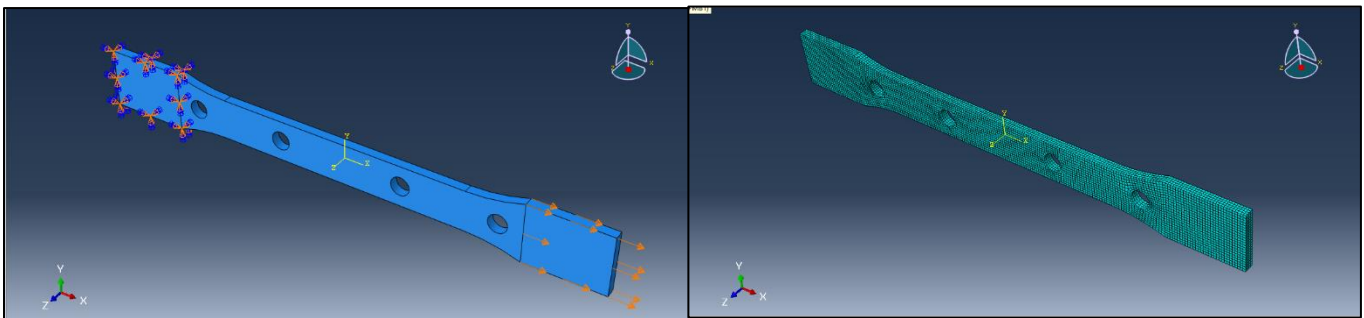


Figure 2 Four holed specimen mesh and loaded modules

c.1 The results:

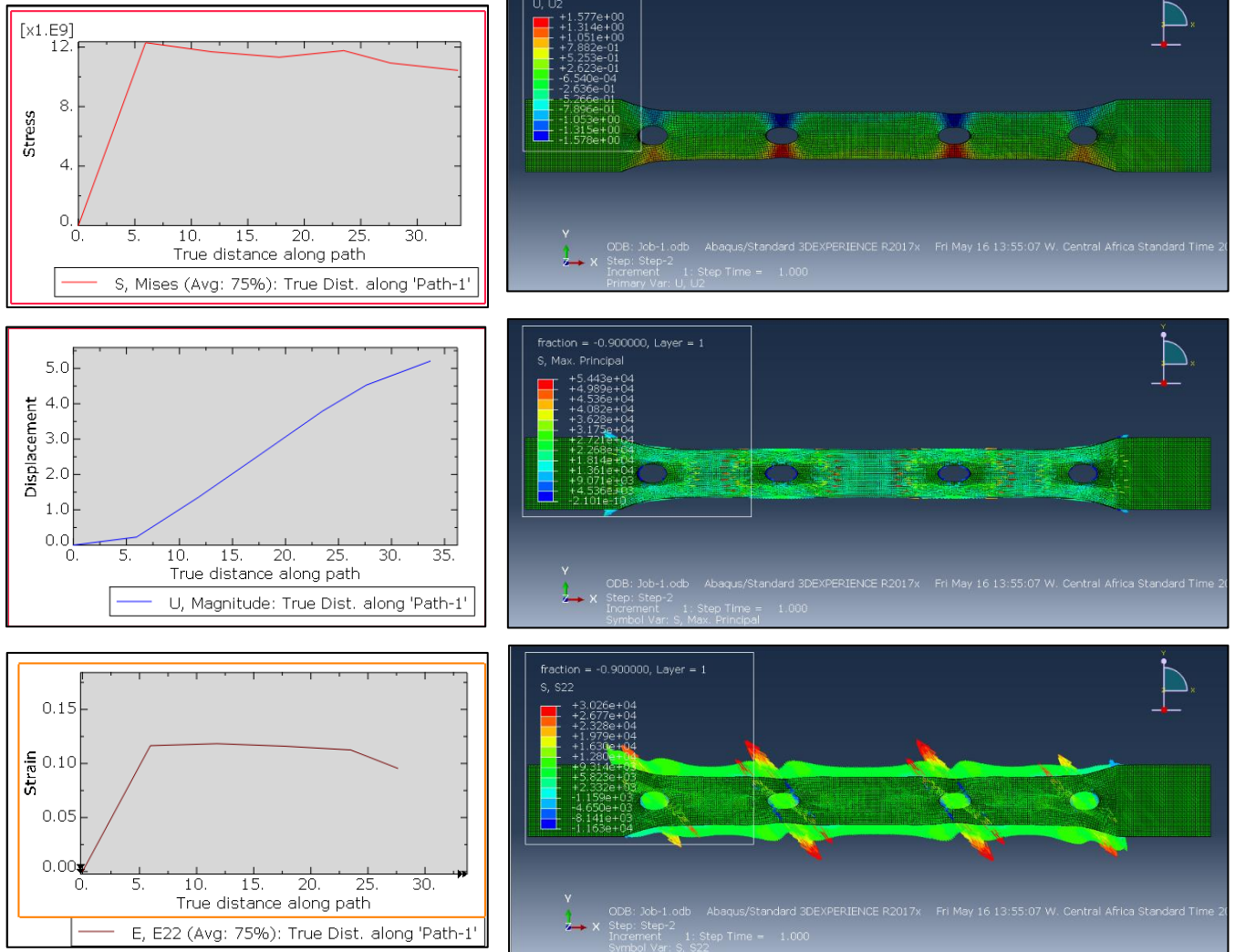


Figure III.17 4_ Visualizations of U, E and Stress along with curves of deformation stress misses

The contour plots of stress magnitude and stress vectors for the four-holed, clearly showing significant stress concentration around all four holes. A key observation from the visual results of this simulation is the demonstration of complex interaction and overlap between the stress concentration regions of the multiple closely spaced holes.

upon the results from the four-holed piece, alongside the others, reinforcing the principle that increasing the number of holes leads to increased and more complex stress concentration around them as well as the strain as shown.

PS: in all the simulations we used the same path for plotting the curves

Edit XY Data

Name: _temp_1

	X	Y
1	0	0
2	5.94817	0.116468
3	11.7823	0.118302
4	17.7863	0.115943
5	23.498	0.112488
6	27.6272	0.0954269
7	33.6896	-0.0661709
8		
9		
10		

Quantity Types
 X: True distance Y: Strain

OK Cancel

III.5 Simulation conclusion:

In this simulation we aimed to study the effects of varying the **number of holes** on the mechanical behavior of a standard tensile specimen. Here is what we concluded:

***a)* One Hole Piece:**

- Contour plots show the distribution of stress magnitude and stress vectors. A region of high stress concentration is clearly visible around the edge of the single hole. This aligns with the general principle that holes act as stress concentrators.
- A curve showing stress along a specific path indicates a peak stress value in the region of the hole.
- A curve showing displacement along the same path indicates how the material displaces under the load, with greater displacement observed further from the fixed boundary.

***b)* Two Holes Piece:**

- Contour plots of stress magnitude and stress vectors are presented. High stress concentration is observed around the edges of both holes.
- Crucially, the stress contours appear to show increased peak stress and interaction between the stress fields of the two holes in the material region separating them. So, it seems like when holes are closely spaced, stress concentration regions can interact and overlap, producing even higher peak stresses in the material between them. The visual results for the two-hole case seem to support this, showing significant stress build-up in the ligament between the holes.

***c)* Four Holes Piece:**

Since we are using a more precise design process for this piece to ensure accuracy in hole placement and size.

- Contour plots of stress magnitude and stress vectors are presented. Significant stress concentration is observed around all four holes.
- The visual results likely demonstrate complex interaction and overlap between the stress concentration regions of the multiple closely spaced holes, leading to high stress gradients and potentially even higher peak stresses in the material ligaments between them compared to the one or two-hole cases [implied by comparing stress contours in 170, 171, and 172, and supported by the principle in 121].

Conclusion from the Simulation:

Based on the visual results of the simulations presented in this Chapter, the study concludes that increasing the number of holes in the tensile specimen, while keeping the applied load and other parameters constant, leads to increased and more complex stress concentration around the holes like we saw in the visual shapes.

Specifically, the simulations visually demonstrate the principle that:

- Holes act as stress concentrators, causing stresses to be much higher around their edges than the nominal stresses elsewhere in the part.
- When multiple holes are present, particularly when closely spaced, the stress concentration regions around each hole interact and overlap.

This interaction leads to potentially higher peak stresses in the material ligaments between the, compared to the stress around a single isolated hole.

Therefore, the simulation results support the understanding that an increased number of holes detrimentally affects the stress distribution and overall structural integrity of the component by creating multiple areas of high stress concentration and complex interacting stress fields. This has implications for the material's load-carrying capacity, fatigue life, and fracture toughness, as discussed elsewhere in the sources. The visualization of stress contours is the primary way the software shows the solution and allows for the analysis of these effects.

III.6 Simulation for Determination of state of stress by the finite element method:

The focus of the test calculations was the stress distribution in a plate featuring multiple circular holes with the following dimensions: length $L = 80$ mm, width $B = 50$ mm, thickness $g = 3$ mm, and hole diameter $d = 4$ mm, similar to case A. The plate, composed of aluminum sheet, was subjected to biaxial loading, specifically tension along the z-axis and compression along the y-axis, with forces $P_z = 550$ kN and $P_y = -550$ kN, as observed in cases B and C.

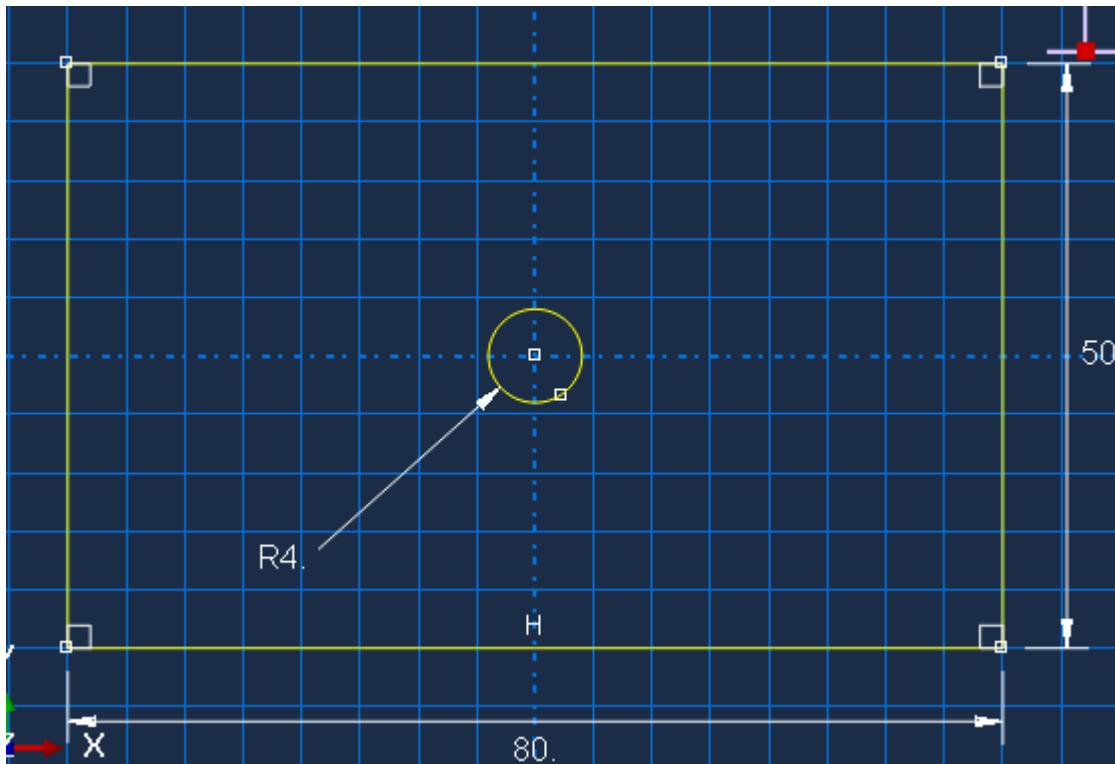


Figure 18 design sketch in ABAQUS for one holed plate

The purpose of the test calculations was to show which commercially available finite element method program is more effective for the design of plate structures by engineers. The comparison of the obtained calculation results is to help engineers in choosing the appropriate calculation tool, which is currently the commonly used finite element method. The result of this comparison was to show that the ABAQUS program is more effective because it enables a triaxial analysis of the state of stress of plates with holes, which is very important when searching for the location of stress concentration in the area of the plate hole. [28]

Table III.2 Strength properties of aluminum 6.777J [26]

Yield strength R_e [MPa]	Tensile strength R_m [MPa]	Youngs modulus E [MPa]	Poisson ratio ν [-]
30	47-70	70000	0.34

The objective of these test calculations was to ascertain the most effective commercially available **finite element method (FEM) program** for the design of plate structures by engineers. A comparison of the resulting calculations aimed to guide engineers in selecting an appropriate computational tool. The findings indicated that the **ABAQUS** program is more effective, primarily due to its capacity for **triaxial analysis of the stress state** in holed plates, a critical feature for identifying stress concentration locations within the vicinity of the plate hole.

Figure below illustrates the loading method and corresponding reduced stress distributions (σ_{red}) for plates featuring a central circular hole. These stress values, given in MPa, were calculated based on the Mises hypothesis.

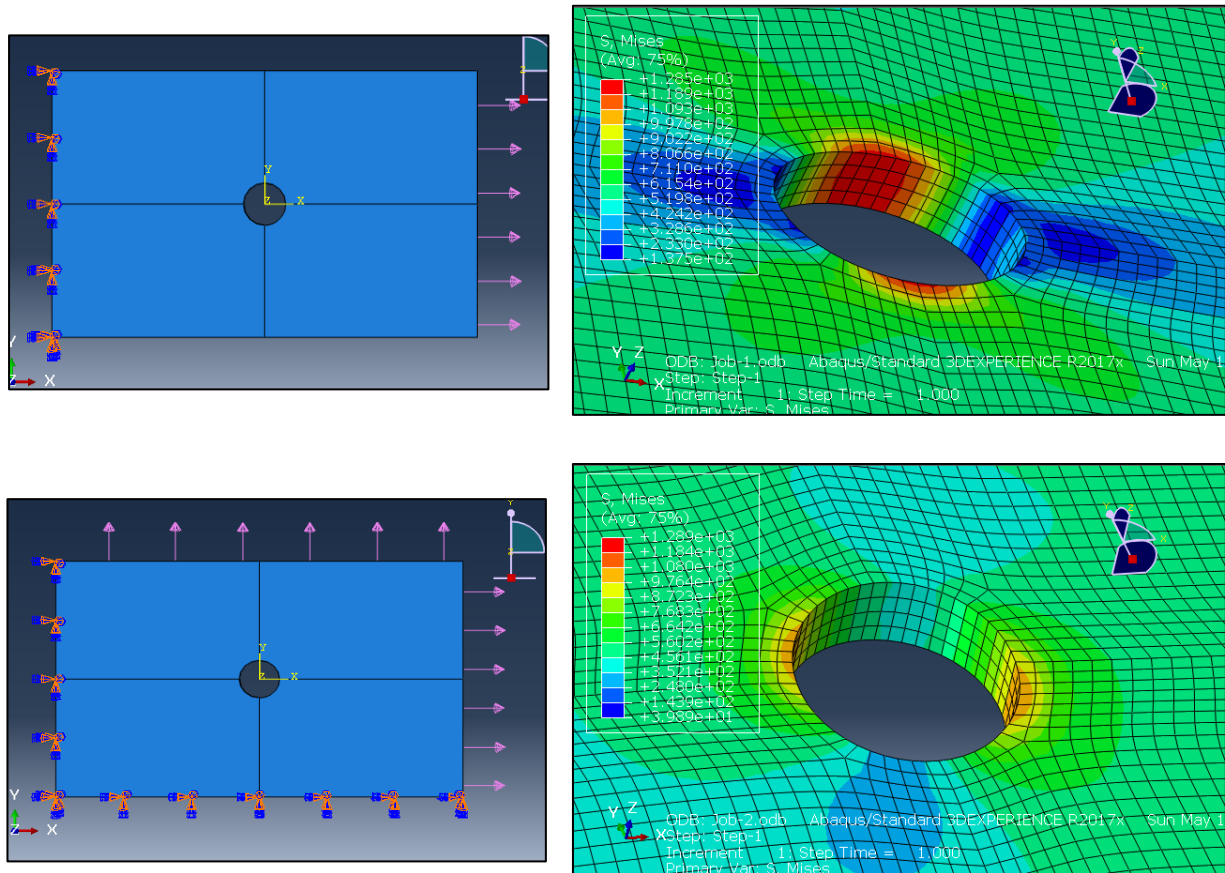


Figure III.119 loading the plate one with circular hole: a) $P_x = -550$ kN, $P_y = 0$ kN; $\sigma_{red \max} = 1200$ MPa; b) $P_x = -550$ kN, $P_y = -550$ kN; $\sigma_{red \max} = 821.07$ MPa

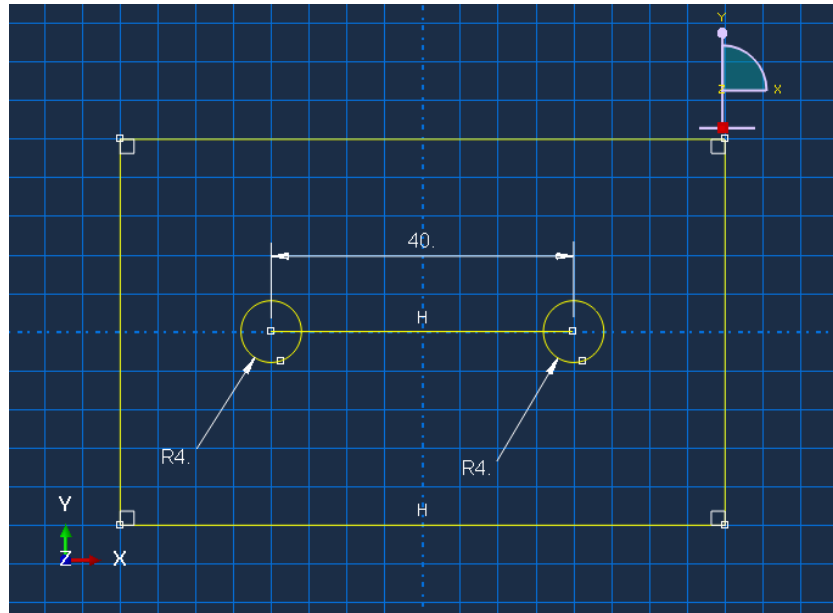
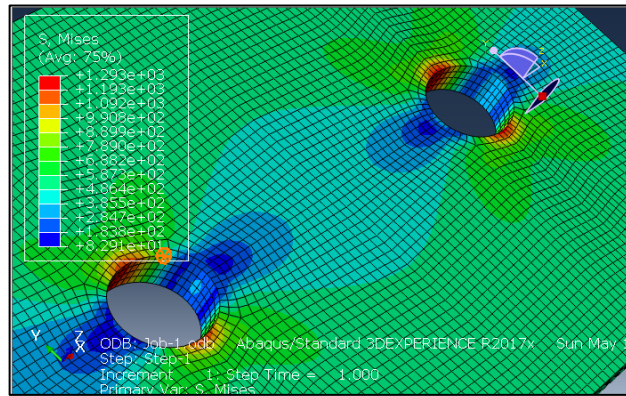
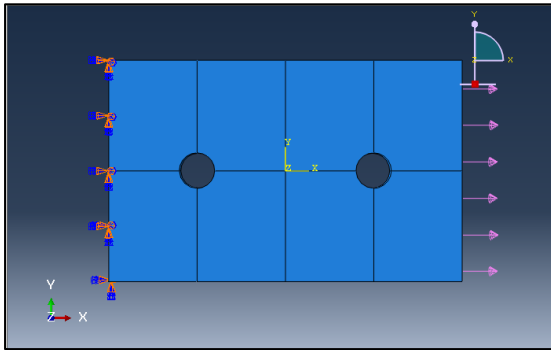


Figure 20 design sketch in ABAQUS for two holed plate

Figure below illustrates the loading method and corresponding reduced stress distributions (σ_{red}) for plates featuring two circular holes. These stress values, given in MPa, were calculated based on the Mises hypothesis.

a)



b)

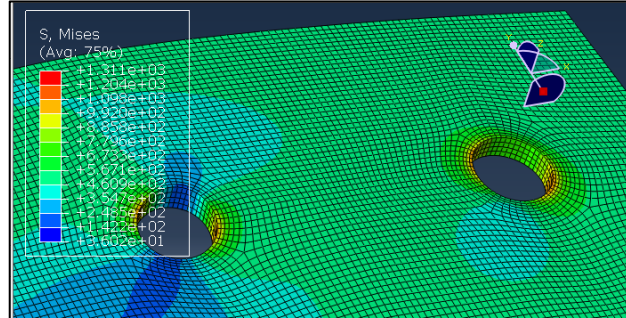
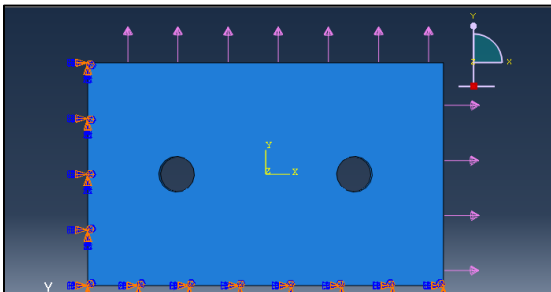


Figure III.21 loading the plate with two circular holes: a) $P_z = -550$ kN, $P_y = 0$ kN; b) $\sigma_{red \max} = 1405.85$ MPa; c) $P_z = -550$ kN, $P_y = -550$ kN; d) $\sigma_{red \max} = 1128.08$

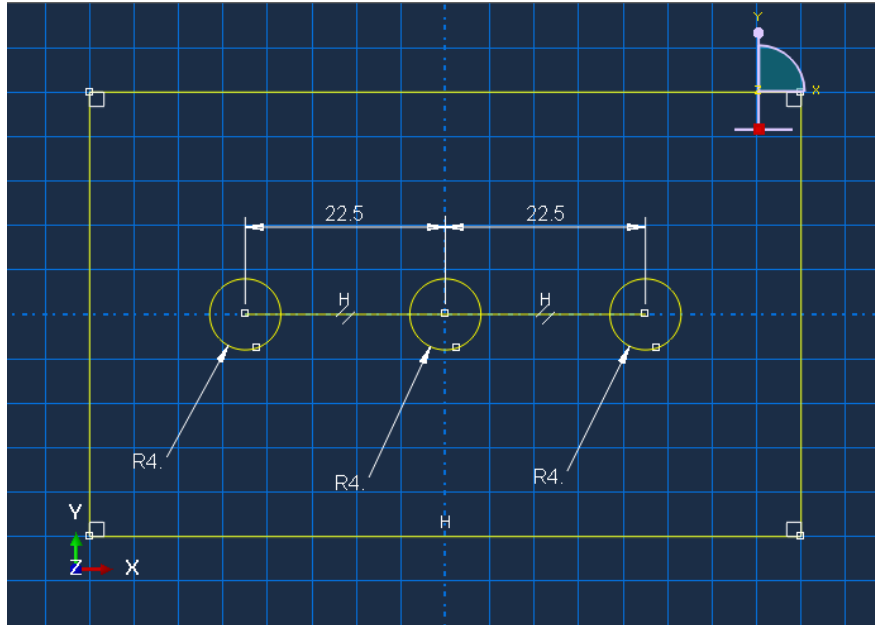


Figure 22 design sketch in ABAQUS for three holed plate

Figure below illustrates the loading method and corresponding reduced stress distributions (σ_{red}) for plates featuring three circular holes. These stress values, given in MPa, were calculated based on the Mises hypothesis.

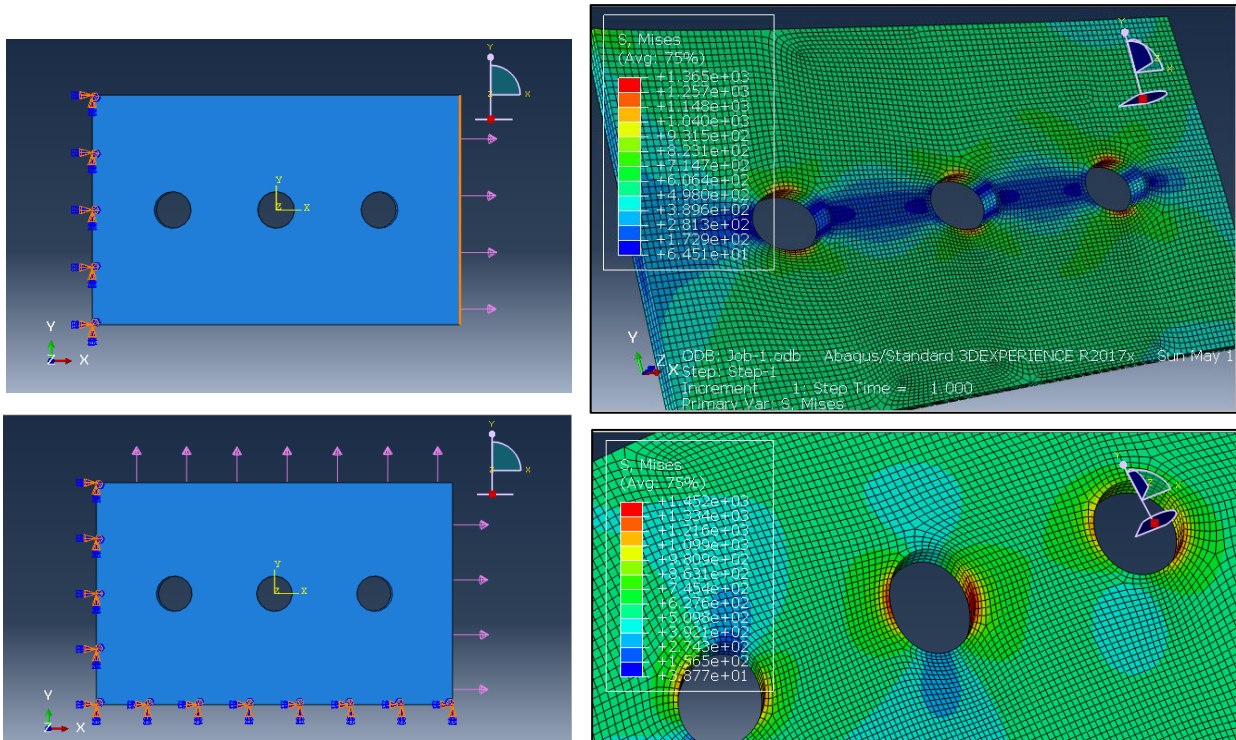


Figure III.23 loading the plate with three circular holes: a) $P_z = -550 \text{ kN}$, $P_y = 0 \text{ kN}$; b) $\sigma_{red \text{ max}} = 1512.75 \text{ MPa}$; c) $P_z = -550 \text{ kN}$, $P_y = -550 \text{ kN}$; d) $\sigma_{red \text{ max}} = 1406.62$

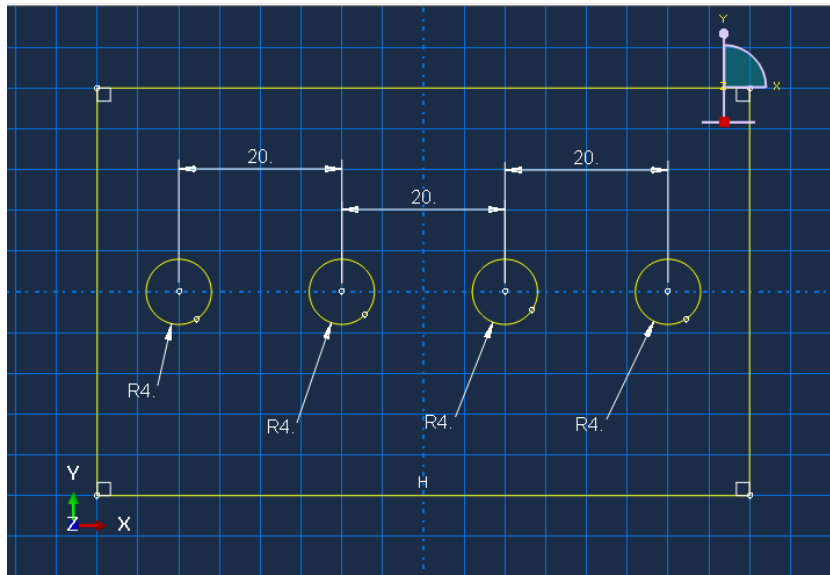


Figure 24 design sketch in ABAQUS for four holed plate

Figure below illustrates the loading method and corresponding reduced stress distributions (σ_{red}) for plates featuring four circular holes. These stress values, given in MPa, were calculated based on the Mises hypothesis.

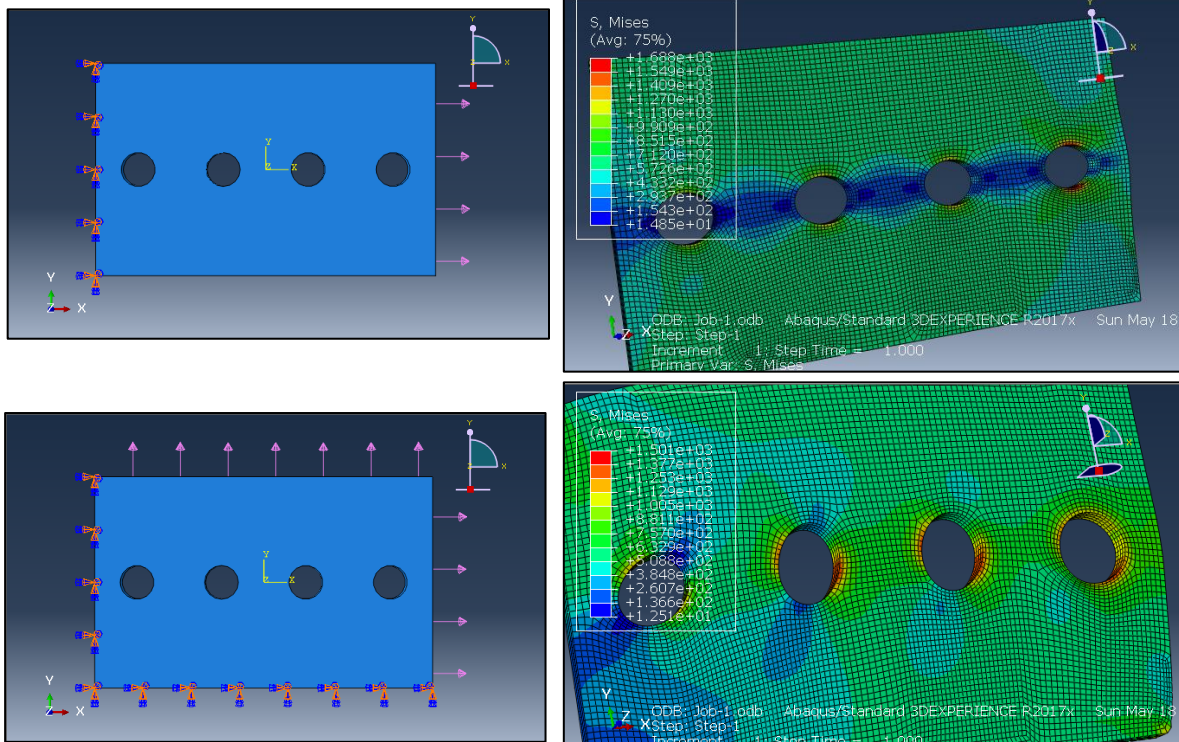


Figure III.25 loading the plate with four circular holes: a) $P_z = -550$ kN, $P_y = 0$ kN; b) $\sigma_{red\ max} = 1690.59$ MPa; c) $P_z = -550$ kN, $P_y = -550$ kN; d) $\sigma_{red\ max} = 1458.32$

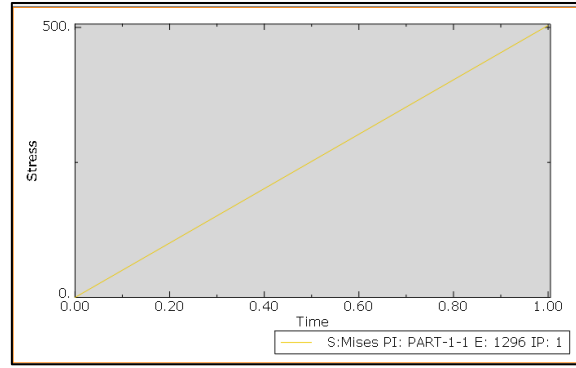
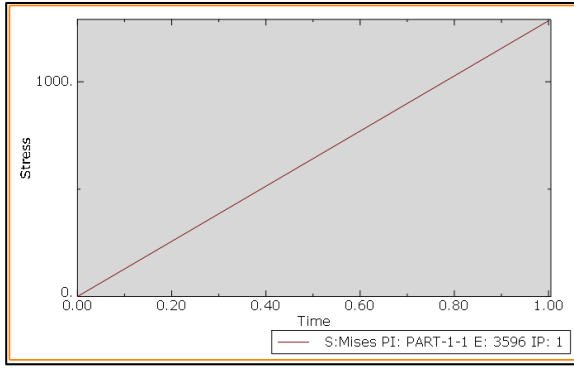


Figure III.26 a) max stress curve in one hole on load 1. b) max stress curve in one hole on load 2

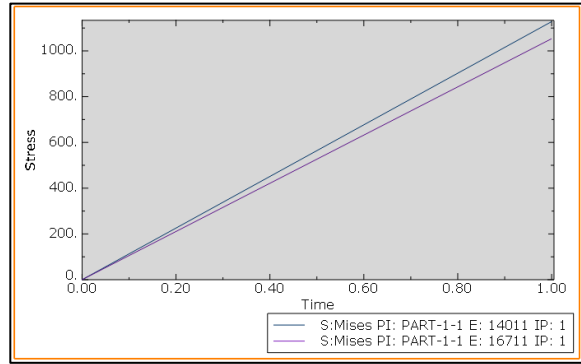
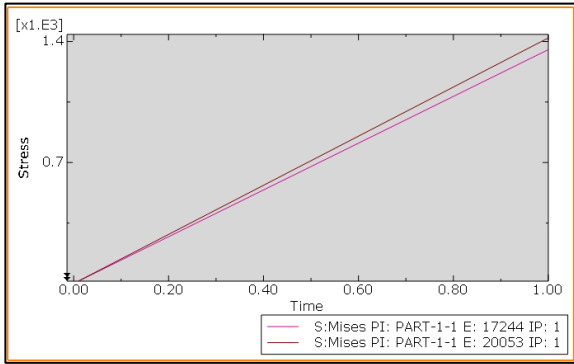


Figure III.27 c) max stress curve in two hole on load 1. d) max stress curve in one two on load 2

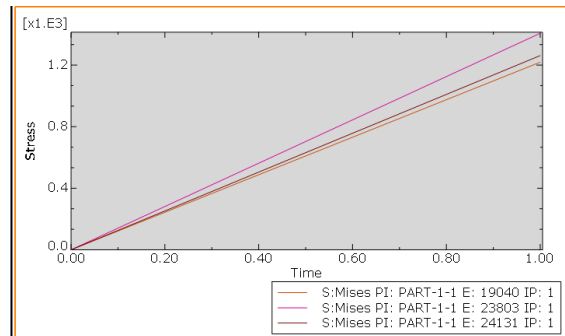
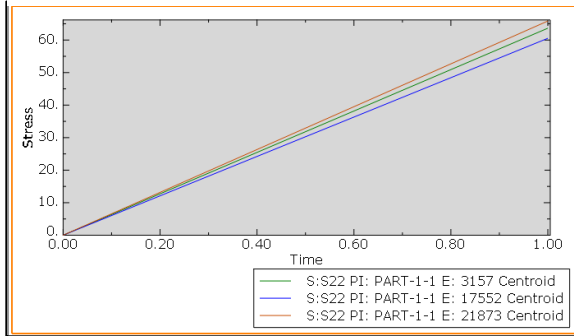


Figure III.28 e) max stress curve in three hole on load 1. f) max stress curve in one three on load 2

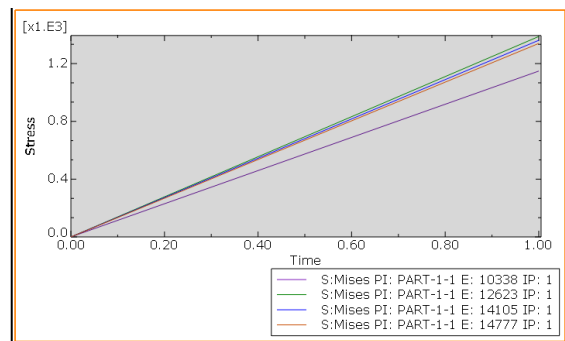
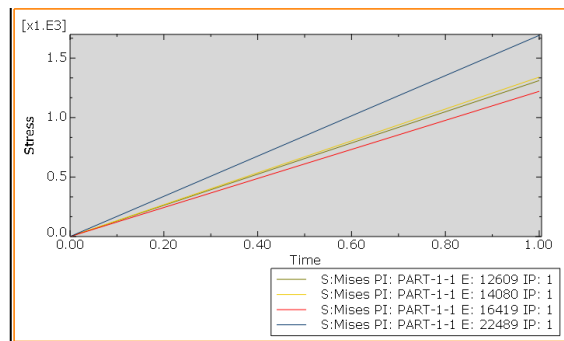


Figure III.29 g) max stress curve in three hole on load 1. h) max stress curve in one three on load 2

Stress concentration occurs when high stress areas form in the components because of their irregularity and abrupt changes in cross-section [32].

When looking at how stress concentration affects structure, we use the concept of a stress concentration factor as discussed in (chapter I page 46). It is referred to by k [28]. Stress concentration factor is the relationship between the maximum stress found at the cut-out edge and the nominal stress. The effect of circular holes in this case is measured by a stress concentration factor k , found by using the formula given in [28,32].

$$K = \frac{\sigma_{redmax}}{\sigma_{rednom}} \quad (III.1)$$

where: k denotes the stress concentration factor ; $\sigma_{red\ max}$ represents the maximum reduced stress [MPa]; $\sigma_{red\ nom}$ signifies the nominal decreased stress in the plate devoid of a hole, computed in accordance with equation [MPa]: [28].

$$\sigma_{red} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \quad (III.2)$$

where σ_{red} represents the reduced stress [MPa]; σ_x and σ_y denote the stress along the x-axis and y-axis, respectively [MPa]; τ_{xy} signifies the tangential stress. [MPa].

In order to illustrate the percentage increase in reduced stress in the plate when holes are made in it for structural or technological reasons, the stress concentration factor k is applied. It is assumed that the plate's holes are smooth and devoid of notches. It is practical to use a dimensionless value of reduced stress by adding a stress concentration factor k in order to conduct a comparative analysis of the state of stress in a plate that has been weakened by holes.

III.6.1 RESULTS:

Table gives a summary of the results of the maximum reduced stress $\sigma_{red\ max}$ according to Mises shown in curves in (page 96). and the stress concentration factor k in the plate with a circular hole subjected to a biaxial load.

Table III.3 Results

Load		1 hole		2 holes		3 holes		4 holes	
Px[kN]	Py[kN]	$\sigma_{red\ max}$ [MPa]	k	$\sigma_{red\ max}$ [MPa]	k	$\sigma_{red\ max}$ [MPa]	k	$\sigma_{red\ max}$ [MPa]	k
-550	0	1200	7.34	1405.58	7.98	1512.75	8.38	1690.52	9.11
-550	-550	821.07	9.41	1128.08	10.18	1406.62	11.04	1458.32	11.54
		$\sigma_{nom}=163.48$		$\sigma_{nom}=176.14$		$\sigma_{nom}=180.$		$\sigma_{nom}=185.57$	

III.6.2 Results conclusion:

The study shows that a higher number of holes results in increased stress when the soil is tested under uniaxial compression ($P_x = -550$ kN, $P_y = 0$ kN) and biaxial compression ($P_x = -550$ kN, $P_y = -550$ kN).

We found the following conclusions based on the results table:

- In most cases, a greater number of circular holes in the plate means that $\sigma_{red\ max}$ and k will both rise. Using the first case, the strength at maximum load ($\sigma_{red\ max}$) increases from 1200 MPa for a single hole to 1690.52 MPa for four holes and the value of k changes from 7.34 to 9.11. Just as before, in the second loading case, $\sigma_{red\ max}$ rises from 821.07 MPa to 1458.32 MPa as the number of holes increases from one to four. Meanwhile, k increases from 9.41 to 11.54.
- When a hole in the shape of a circle exists, it creates stress concentration expressed by the factor k . An increase in the number of holes shows that the reduction in stress becomes higher as there are more openings.
- Maximum reduced stress and stress concentration factor are both affected by the loading condition. While comparing the two situations, it becomes obvious that the biaxial compression case usually results in larger stress concentration factors (k), despite there being cases where it has lower maximum reduced stress. The reason for this is that the nominal stress values of each configuration are not the same.

Using ABAQUS for finite element modeling proves that the program is capable of accurately testing computational methods for plate structures and can effectively identify areas close to holes that suffer higher stresses. The study highlights the importance of considering the number of holes and the specific loading conditions when determining the state of stress and stress concentration in plate structure

Conclusion

General conclusion

I. Summary of the Most Important Theoretical Results:

We laid a strong theoretical foundation for understanding how materials behave under load, particularly in the presence of geometric discontinuities like notches (such as holes).

In Chapter III: Numerical study and simulation in ABAQUS:

we detailed a Finite Element Analysis (FEA) simulation study conducted using the ABAQUS software. This chapter defines ABAQUS and its capabilities, explaining the FEM and specifically the XFEM method used. In this chapter we made the simulation and comparison of the tensile behavior of a 3D ASTM D638 specimen with one, two, and four holes. The simulations used the same material, applied force, and boundary conditions. And another aluminum plate to determine the state of stress and stress concentration factor.

The simulation results (primarily visualized through stress contours) for the one-hole piece show clear stress concentration around the hole. The results for the two-hole piece visually suggest increased peak stress and interaction between the stress fields of the holes. The results for the four-hole piece, designed with precision, demonstrate significant stress concentration around all holes and complex interaction/overlap of stress regions, leading to potentially even higher peak stresses in the material between the holes compared to the one or two-hole cases.

The conclusion we reached from the simulation is that increasing the number of holes leads to increased and more complex stress concentration. The simulations visually support the theoretical principle that multiple, closely spaced holes cause interacting and overlapping stress fields, resulting in potentially higher peak stresses in the ligaments between them compared to a single isolated hole. This reinforces the understanding that a greater number of holes detrimentally affects stress distribution, state and concentration, and structural integrity, with implications for load capacity, fatigue life, and fracture toughness.

II. Recommendations to Improve Material Resistance to Cracks:

- **Designing to Minimize Stress Concentration:**

Designing structures with appropriate notch geometries and aiming for low stress concentration is important. Circular holes generally have lower stress concentration than those with sharp corners or elliptical shapes.

Rounding corners or using fillets on edges can help reduce stress concentration.

When designing components with geometric discontinuities, material selection is an important consideration. Selecting materials with low notch sensitivity is beneficial.

- **Enhancing Material Properties or State Around Discontinuities:**

One experimental study discussed in our work explored applying transverse normal pressure around the boundary of a hole in a composite material. This method was found to mitigate stress concentration, increase the notched tensile strength, and enhance fatigue endurance compared to unclamped specimens. [22]

Another study focused on cold expansion of holes in aluminum alloy. Cold expansion introduces compressive residual stress and forms a dense grain structure (a crystal refinement layer) around the hole, which can significantly enhance fatigue life. The study emphasized that determining the optimal reaming size after cold expansion is crucial, as excessive reaming can remove this beneficial layer and reduce the residual stress field, thereby weakening the material's ability to withstand fatigue. This suggests introducing beneficial residual stresses and preserving favorable microstructures near potential crack initiation sites can improve resistance. [22]

- **Considering Fracture Toughness and Performing Analysis:**

Selecting materials with high fracture toughness (K_{IC}) is key, as fracture toughness represents a material's resistance to propagation of a crack. Fracture occurs when the stress intensity factor (K_I) at the crack tip reaches the material's critical fracture toughness value (K_{IC}).

When designing critical structures, especially those with multiple holes where the stress fields around discontinuities can interact, a thorough fracture mechanics analysis is necessary to assess the potential for crack initiation and propagation from these stress concentration points.

III. Future Research Directions in This Field:

Using new materials:

Innovate a plate using composite materials or modern alloys that withstand stresses better, thus extending the plate's lifespan and reducing its weight.

Applications in various industries:

Aviation: Improving aircraft wings that contain holes for rivet fastening, which increases safety and reduces weight. Drones wind resistance and speed enhancements.

Automotive: Designing lighter and stronger structures for electric vehicles.

Medicine: Developing perforated metal plates used in bone implants (such as surgical fixation plates) with balanced stress distribution.

Developing new manufacturing techniques:

Innovate methods for manufacturing plates (such as 3D printing) such that the holes are integrated in a way that reduces stresses from the beginning.

Integrating smart technology: Adding integrated sensors into the plate to monitor stresses in real-time, allowing for failure prediction before it occurs (such as using the Internet of Things - IoT).

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