Statistical analysis of fatigue test in jute reinforced polyester composites using Weibull distribution

Djamel Djeghader
Civil and Hydraulic Engenieering department. Jijel University
Ouled Aissa City, Algeria
djameldjeghader@yahoo.fr

Redjel Bachir
Civil Engenieering department
Annaba University
Sidi Ammar City, Algeria
bredjel@yahoo.fr

***Abstract*—In this research, the unsaturated polyester resin (polylite 420-852) ise used in combination with bidirectional jute fiber to generate composites materials by a contact molding technique. The parameter which was carried out on the prepared samples was impact test to plot the S-N curve, which is based on Wöhler Equation. Two-parameter Weibull probability was used for analysis statistically the fatigue life results of composite jute/polyester samples. Weibull graphic was plotted for each loading level using fatigue data. Then, S–N curves were drawn for different reliability levels (Ps = 99%, Ps =90%, Ps =50%, Ps = 36.8% and Ps = 10%) using Weibull data. These S-N curves were introduced to the identification of the first failure time as reliability and safety limits of the material.**

Keywords—jute, polyester, fatigue life, Weibull distribution, reliability probability.

I. Introduction

The fast development of composite materials for several life applications leads researchers and investigators to deepen their studies in order to derive long-term forecast and lifetime of different structural elements. Indeed, parts and structural components in composite materials can suffer catastrophic damage due to different cyclic fatigue loads. Although applied loads are small and do not reach the yield strength, the damage is not significant and cracks are small, but with the no-discontinuity of cyclic operations, the cracks can develop over time until complete fracture of an element. To avoid this fatigue damage mechanism, several works have transmitted solutions that have allowed to locate and predict lifetime fatigue of composite material. Several publications have appeared in recent years documenting aims to give a comprehensive account of predicting the lifetime in fatigue of composite material, locating the cracks, expected damages and give the mechanical strength [1-3]. Fotouh et al. [4] have studied the cyclic fatigue of composite material based on hemp fibers by plotting S-N curve for different fibers fractions. The non-dimensional analysis is performed by Yang et al. [5] to establish endurance limit equation for wood reinforced composite materials.

In the last few years, there has been a growing interest in cellulosic fiber composites like jute which are used in many applications such as superstructure of ships and boats which may undergo cyclic, alternating and/or repeated loads. In order to achieve better design and use of these materials, which are characterized by their anisotropy and non-uniformity compared to conventional materials, it is very necessary to predict its response mechanical behavior at the time of loading by using statistical methods based survival probability of [6-8]. The most interesting approach to this issue has been proposed by Weibull, This approach is very useful for designing composite materials and to generate fatigue data of different characters [9]. In addition, the Weibull distribution function is a practical and versatile method to describe the properties of composite materials because the probability density function of the Weibull distribution has a wide variety of shapes [10]. This distribution is expressed in simple form functions with high precision which based on mathematical equations. Kassapoglou [11] has established a fatigue model based on the probability of failure to calculate the number of cycles needed for the residual strain that corresponds to experimentally applied value, the results obtained shows a strong accord with the experimental data. A new approach to the analysis of the fatigue test is established by Dai and Hahn [12] which is based on the determination of degradation parameters of the stressed material. These parameters are based on the fatigue stress, residual stress and number of applied cycles.

The main objective of this paper is to establish a graphical presentation of fatigue cyclic data in three points bending test of jute reinforced polyester composites with a probabilistic analysis based on Weibull approach.

II. EXPERIMENTAL WORK AND METHODS

A. *Materials*

The composite material used in this paper is bidirectional jute reinforced polyester composites manufactured by contact pressure molding method with values of fiber weight fractions 40%. The fabrication method provides rectangular plates (300x210 mm^2) with a mean thickness of 4±0.2 mm (Fig. 1). The bidirectional jute fiber reinforcement has an average density of 1300 kg/m3, an elastic modulus of 20-55 GPa and an elongation at break about 1,6% [13-16]. The polyester resin (Polylite 420-852) has 45 MPa tensile strength, 1.11 g.cm^{-3} density at ambient temperature and 20 dPa.s viscosity, the bending modulus is about 2,35 GPa and the breaking stress is 0,072 GPa.

Fig.1. Jute reinforced polyester composite plate

A three - point bending test characterization with constant cross-head speed of 2 mm/min according to ASTM D790 [17] is necessary in order to perform the cyclic fatigue test of composite material. Specimens for the bending strength static tests are in rectangular form of l = 80 mm length, h = 15 mm width and b = 4 mm thickness (Fig. 2).

Fig.2. Test specimens

The samples test of jute polyester composites material subjected to cyclic fatigue test using "ZWICK ROEL" machine test type ± 20 KN capacity, the error percentage of the test was found to be 1% (Fig. 3). The load variation is kept constant with 1.25 Hz frequency, a load ratio between the minimum and the maximum strength value equal to 0 at free air temperature. The loading type is sinusoidal with constant amplitude (Fig. 4). The fixed distance between the two supports is L = 64 mm; and the load is applied to the middle of the sample until the point of failure. The fatigue life was measured for different stress levels S ($S = \sigma/\sigma_r$; where σ is the maximum stress; and σ_r is the bending failure stress) of 80%, 70%, 60%, 55%, 45%, 35% and 25%. For each level load, a minimum of three test specimen was tested. The criterion adopted is the complete fracture of specimen.

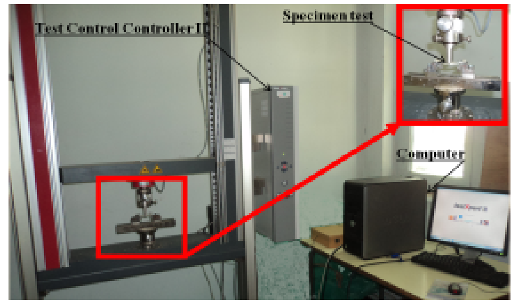

Fig.3. Fatigue test setup.

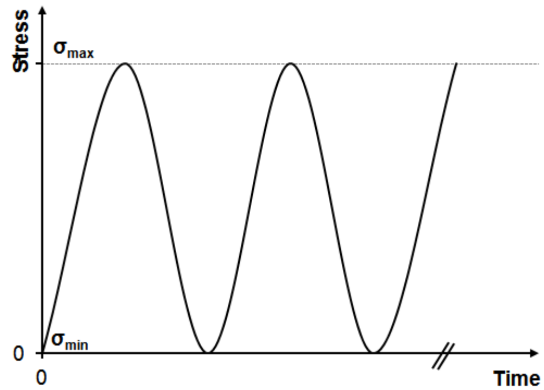

Fig.4. Constant-amplitude sinusoidal loading history used for flexural fatigue test.

B. *Weibull distribution*

The Most of experimental fatigue tests show a very dispersed results, in particular for composite materials. The Weibull statistical analysis allows as to better exploit the fatigue results and to evaluate the survival / reliability probability of various structural elements. The two-parameter Weibull distribution is commonly used to represent constant amplitude in fatigue life data [18-21]. It is used to model extreme values such as the lifetime in fatigue test and the failure time. Moreover, it estimates the dispersion of load and life which can be compared with the reported data

The probability density function (PDF) of two-parameter Weibull distribution given in the following “(1)”. This (PDF) Equation is the best known definition in reports [9, 22, 23 ,24]:

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \qquad (1)$$

Where α and β are the scale and shape parameters of the two-parameter Weibull distribution respectively. The two-parameter Weibull distribution can give some physical rules of damage and failure of tested element when the slope of the Weibull probability plots taken into account.

The cumulative density function (CDF) is obtained in “(2)” when the probability density function (PDF) Equation is integrated:

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$$P_f(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \quad (2)$$

$$P_s(x) = 1 - P_f(x) = e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \quad (3)$$

Where $P_f(x)$ is the failure probability, $P_s(x)$ is the survival/reliability probability, x is an independent variable that presents the number of fracture cycles (N_f) in fatigue analysis. The values of α and β are determined by rewriting "(2)" in the following form:

$$Ln\left(Ln\left(\frac{1}{1-P_f(x)}\right)\right) = \beta Ln(x) - \beta Ln(\alpha) \quad (4)$$

From "(4)" the relationship between Ln (Ln (1/1-Pf (x)) and Ln (x) is linear, the line slope presents the shape parameter: m = β, the scale parameter α can be obtained from the second term of "(4)".

The failure probability ($P_f(x)$) is calculated from the following Equation [9, 25, 26]:

$$P_f(x) = \frac{i - 0,3}{n + 0,4} \quad (5)$$

Where (n) is total number of samples and (i) is the failure serial number.

The mean life time to failure (MTTF), the standard deviation (SD) and the coefficient of variation (CV) of the two-parameter Weibull distribution were calculated from the following Equations [27-28]:

$$[MTTF] = \alpha\Gamma\left(1 + \frac{1}{\beta}\right) \quad (6)$$

$$[SD] = \alpha\sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)} \quad (7)$$

$$[CV] = \frac{[SD]}{[MTTF]} = \frac{\sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)}}{\Gamma\left(1+\frac{1}{\beta}\right)} \quad (8)$$

Where Γ is the gamma function

The coefficient of variation of "(8)" is a function of parameter β, according to Revol et al. [29]. This coefficient can be calculated by using "(9)" for values of β > 8.

$$[CV] = \frac{1,2}{\beta} \quad (9)$$

III. Results and Discussion

A. Static bending test

The bending stress versus strain deflection curve of jute reinforced composite material has the classical shape observed from the brittle composite materials (Fig. 5). This curve has three main regions: the first region present a linear increase in strain with the increase of the bending stress until reaching a specific stress level reflecting the elastic behavior of the material. The second region is characterized by non-linear behavior related to the evolution and the progressive degradation that occurs in the composite material. Finally, a sudden drop in the load beyond the maximum load announces the unstable collapse of the sample.

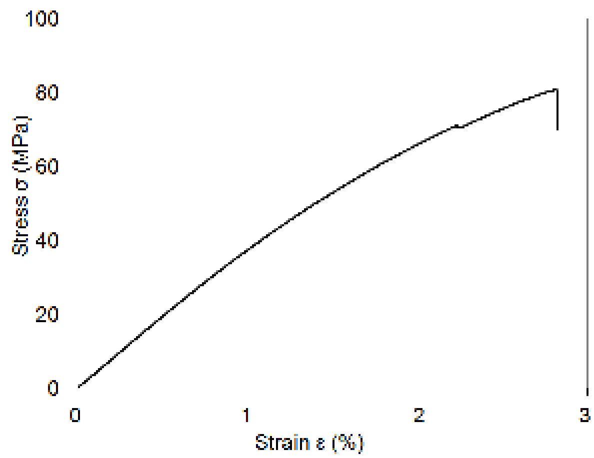

Fig. 5. Stress-strain curve of jute/polyester composite material (JP0)

The average values of ultimate flexural strength: σ_r = 83,5 ± 7,2 MPa, and Young's modulus: E_f = 4510 ± 563 MPa are in agreement what is habitually reported in technical and scientific open literature and measured on similar composite materials [1]. The result of σ_r is necessary to perform the cyclic fatigue test of composite material.

B. S-N Curve

The fatigue tests of the jute/polyester composite material were performed to plot the S-N curve (Fig. 6) using the Wöhler linear model [30] to get a relationship between the applied stress and fatigue life cycles with a straight line relationship. In Fig. 6, the (Y) axis is the maximum applied stress/three-point fracture strength (σ/σ_r), the (X) axis is the lifetime of tested specimens, and N_f is the number of fracture cycles. The arrowhead at 10^5 cycles indicates a 'run out' test which did not fail; A and B are parameters of material.

$$\sigma = A - B\log(N_f) \quad (10)$$

$$x = \log(N_f) \quad (11)$$

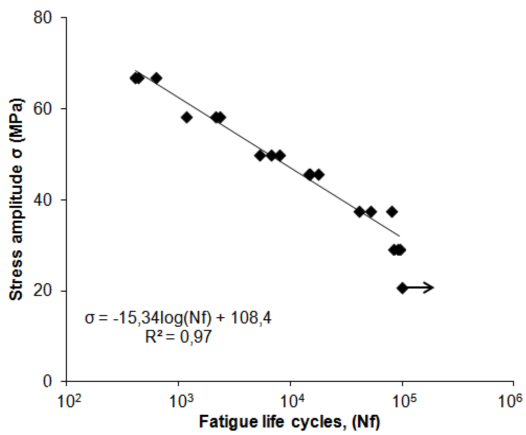

Fig. 6. S-N Curve of jute/polyester composite material.

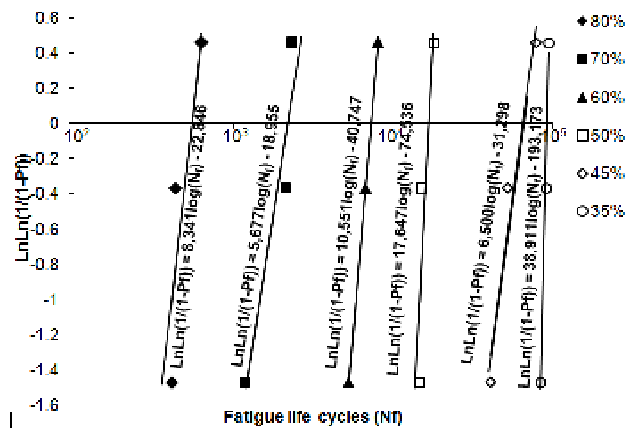

Fig. 7. Weibull plots for the tested load amplitudes.

The Wöhler curve shows a gradual decline in fatigue strength with an increasing number of fatigue cycles observed up to a stress level where there is no failure at 10^5 cycles, indicating the endurance limit of material which is estimated at 21 MPa. At high stress levels, the failure fiber occurs (interface or matrix failure) and the crack density stiffness is stable up to final failure [31]. For lower stress levels, breakage of fibers and matrix, does not lead quickly enough to fracture and damage of composite, but other mechanisms occur with the time loading such that interfacial shear and cracking of the matrix occurs. These mechanisms can then affect the overall state of damage in the material. The type of final damage observed in specimens tested for all level loads in fatigue test and static bending test was similar and failed in a brittle manner.

C. Statistical analysis of fatigue life data

The cumulative probability distribution of the fatigue test data was fitted in the least squares sense to compute the Weibull parameters. Fig. 7 shows the two-parameter Weibull curve fitting obtained for fatigue resistance of the samples tested using "(4)". However, the samples tested for the 25% loading level are not broken; this data cannot be used further in the analysis. In Fig. 7, the plot lines of linear regression lines of each loading level gives the slope which represents the value of β and the lifetime α (Table 1). All plots show good linearity with a reliability index ($R^2 > 0.75$), signifying a reasonable Weibull distribution fitting (Table 1).

TABLE I. WEIBULL PARAMETERS IN FATIGUE TEST

σ/σ_r (%)	**α**	**β**	**R^2**	**[CV]**	**Weibull mean life (N_0)**
80%	548	8.341	0.770	0.143	517
70%	2183	5.677	0.902	0.204	2019
60%	7270	10.551	0.999	0.114	6932
55%	16741	17.647	0.759	0.070	16244
45%	65315	6.500	0.948	0.180	60860
35%	92149	38.911	0.993	0.032	90841

The obtained values of α are greater than (1) which means that the damages were distributed throughout the stressed region by the fatigue load. This often reduces the stiffness of the composite material, although it does not always immediately reduce the strength. If there is a strength reduction, it might occur in the wear-out zone following small stress-induced, viscoelastic or creep deformations in the matrix [32]. The shape parameter (β) is related to the dispersion of the fatigue data (a larger β value indicates that the dispersion is small). As can be seen in Table 1, the Weibull module as a function of loading levels is between 5 and 18except for the lowest level loading (35%), which has a value of (β) at about 39, indicate that the dispersion result is less compared to the other level loadings of the fatigue test.

The mean life time to failure (MTTF) and the standard deviation (SD) for different loading levels are calculated using "(6)" and "(8)", and presented in Fig. 8. In this figure, it can be seen that the standard deviation (SD) has a minimum value for the level loading of 35%, indicating a low dispersion of results. However, a significant dispersion on the fatigue results for a lifetime of 2019 cycle which corresponds to a [CV] of 20.4% (Table 1). It is highly probable that the samples tested have structural manufacturing defects that negatively affect the mechanical response at high loading levels. It is very important to note that this tendency in the dispersion of fatigue life at different loading levels is extremely important to the designer and deserves much attention for the application of this type of composite materials [10].

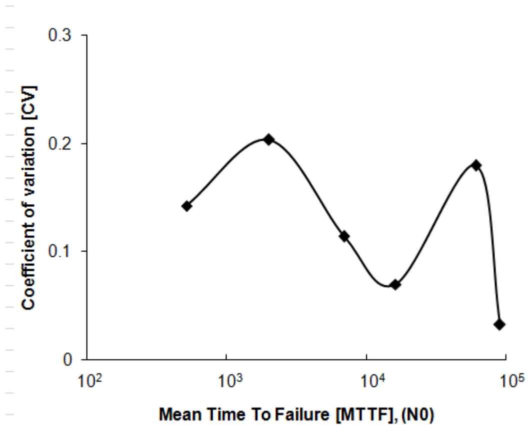

Fig. 8. Effect of mean fatigue life on the coefficient of variation [CV]

The reliability probability of cyclic fatigue samples can be assessed in terms of a cycle's number for a given applied load, as well as fatigue curves for different levels of reliability. To plot the survival probability curve as a function of the cyclic fatigue life of jute/polyester composite material (Fig. 9), we use "(4)", which has a linear function $\left(\mathrm{Ln}(x),\ \mathrm{Ln}\left(\mathrm{Ln}\left(\frac{1}{1-P_f(x)}\right)\right)\right)$.

Where: - $\beta \mathrm{Ln}(\alpha) = C$, so, "(12)" can be obtained:

$$\alpha = e^{\left(-\frac{C}{\beta}\right)} \qquad (12)$$

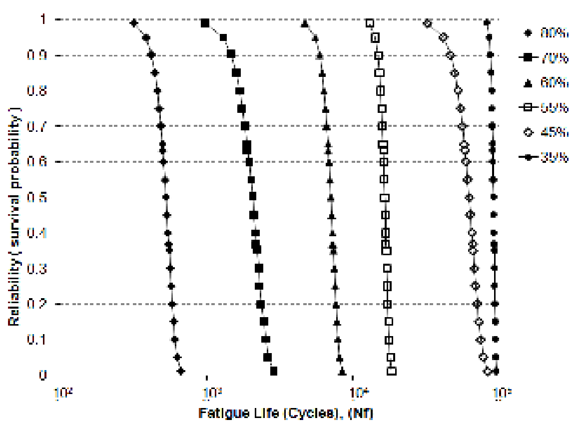

Fig. 9. Survival probability graphs for jute/polyester composite material.

The obtained reliability probability curves of Fig. 9 are of considerable value to the designer, where the cyclic fatigue life can be determined at any survival percent. The survival probability of 50% survival life can be determined from these curves by drawing a horizontal line that intersects with the distribution curves of the fatigue lifetime. The values at intersected points are lifetimes for a 50% survival probability at different loading levels. For example, for a 60% loading level, the intersection of horizontal line for a 10% survival probability gives a life cycle number of 7868 cycle. This cycle number value becomes a value of 94146 for 35% loading level.

In the case of ($P_s(x) = 0.368$), indicate that a part survives the characteristic life. This survival probability value can be determined from "(3)" by substituting ($x = \alpha$). Therefore, the probability of survival value:

$$P_s(x) = 1 - P_f(x) = e^{-\left(\frac{\alpha}{\alpha}\right)^{\beta}} = 0.368 \qquad (13)$$

According to this concept, the characteristic life ($x = \alpha$) is the number of cycles which 63.2% of the population is expected to fail.

Equation (14) can be obtained by "(3)" to plot the survival probability curves of all loading levels as a function of the lifetime for (Ps = 10%, 36.80%, 50%, 90% and 99%) (Fig. 10). These curves are characterized by the number of three-point cycles fatigue that jute/polyester composite material can support without any catastrophic damage. They allowed us to predict the fatigue lifetime required by the designer and the reliability limit or safety. The obtained results by "(14)" have been fitted by the power function [$y = Ax^B$], where A and B are constants of the material.

$$N_{P_x} = \alpha\left(\left(-\mathrm{Ln}(P_s(x)\right)\right) \qquad (14)$$

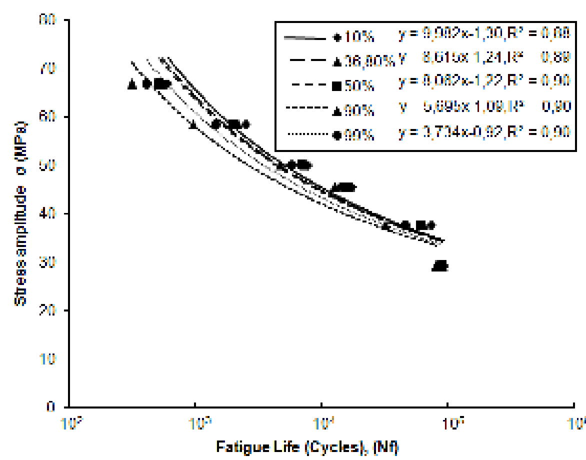

Fig. 10. The S–N curves for deferent reliable levels for jute/polyester composite material

It is clear that for higher levels of safety or reliability, the number of cycles at failure will be less. As well as the constant (A) decreases with the increase of level of survival probability level.

The curve in Fig. 10 allow to identify the first failure time in fatigue test, this step is considerably important to the designer because the safety limits on the practical applications depend on the reliability. If the samples contains a weaker or critical part, the safe lifetime should be calculated at a high confidence level, such as 99% [33].

CONCLUSION

The cyclic fatigue test was carried out in jute/polyester composite material in order to plot the Wöhler curve as well as to calculate the endurance limit of this material. The experimental results show a dispersion which is due to the anisotropy of material as well as the operation of cyclic fatigue test. Indeed, the two-parameter Weibull statistical analysis is important for exploiting and evaluating experimental fatigue data. The main conclusions are summarized as follows:

The jute / polyester composite material has noticeable strength under slow fatigue test.

The endurance limit of jute/ polyester bidirectional composite material can be estimated at a stress of 21 MPa with a failure cycle number at 10^5 cycles.

The scatter of cyclic fatigue results is mainly due to manufacturing defects and the processing method of composite materials.

The two-parameter Weibull cumulative density function allows predicting the survival/reliability probability of jute / polyester composites at different loading levels.

The S-N diagrams of survival probability at 99%, 90%, 50% and 10% and characteristic life of 36.8% according to maximum loading level are very important to designers to identify the first failure time.

The lifetime of jute / polyester composite material must be calculated at high safety or reliability level, particularly for structural composite that contain a critical section.

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