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Behavior of the solutions of certain systems of non-integer differential equations : comparison and principle of the maximum

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Absrtact

This work is devoted to the generalization of certain theorems of comparisons known in the theory of ordinary differential equations,to systems of differential equations of fractional order.

Keywords : Fractional integral, Riemann-Liouville derivative, Caputo derivative, fractional differential system, differential inequality, integral inequality, comparison theorems.

Introduction

The object in this research work is the analysis of the behavior of solutions for certain systems of fractional differential equations. The behavior in question relates to the order of variation between the solutions when they are initially subjected as well as the respective source functions to a certain well determined order. So we are considerned by the study of comparisons between different solutions. We will be interested in differential inequalities as well as integral inequalities of the fractional type. Particular attention will be paid to fractional multi-order differential systems.

1 Fractional Differential Inequatilies

we consider the initial value probleme IVP for fractional differential équation given by

$$\begin{cases} {}^{C}D^{\alpha}u\left(t\right)=f\left(t,u\left(t\right)\right), & 0<t\leq T, \\ u\left(0\right)=u_{0}, \end{cases} \tag{1}$$

Where $f\in C\left(\left[0,T\right]\times\mathbb{R},\mathbb{R}\right), {}^{C}D^{\alpha}u$ is the fractional derivative of u and α is such that $0<\alpha<1$

since f is assumed continuous,the IVP is equivalent to the follonwing volterra fractional integral

$$u\left(t\right)=u_{0}+\frac{1}{\Gamma\left(\alpha\right)}\int_{0}^{t}\left(t-s\right)^{\alpha-1}f\left(s,u\left(s\right)\right)ds; \qquad 0\leq t\leq T, \tag{2}$$

That is , every solution of (2) is also a solution of (1) and vice versa.Here and elsewhere Γ denotes the gamma function.

Let us first recall one of the key results in the study of comparison theorems and differential or integral inequalities of the fractional type (see in [4]).

Lemme 1 *Let $m\in C^{1}\left(\left[0,T\right],\mathbb{R}\right)$, and for any $t_{1}\in\left[0,T\right]$ we have $m\left(t\right)\leq 0$ for $0\leq t\leq t_{1}$. Then it follows that, ${}^{C}D^{\alpha}m\left(t_{1}\right)\geq 0$.*

Théorème 1 *Let u, $v\in C\left(\left[0,T\right],\mathbb{R}\right), f\in C\left(\left[0,T\right]\times\mathbb{R},\mathbb{R}\right)$, $0<\alpha<1$ and*

$$^{c}D_{0^{+}}^{\alpha}u(t) \leq f\left(t,u(t)\right) \tag{3}$$

$$^{c}D_{0^{+}}^{\alpha}v(t) \geq f\left(t,v(t)\right),\ 0\leq t\leq T. \tag{4}$$

Assume that $\forall t$, $f\left(t,u\right)$ is non decreasing with respect to u and

$$u(0)<v(0). \tag{5}$$

Then :

$$u(t)<v(t),\ 0\leq t\leq T. \tag{6}$$

The objective of this work

The aim of this work is to study the comparison of solutions, in cases where the differential problem is in the form of a system of fractional differential equations. So, we are interested in the problems where the unknowns are vector functions : $u = (u_1, \ldots, u_n) \in \mathbb{R}^n$ and $v = (v_1, \ldots, v_n) \in \mathbb{R}^n$. Thus, we consider the systems of inequalities of the following form.

$$\begin{aligned} {}^cD_{0^+}^{\beta_i} u_i(t) &\leq f_i\left(t, u(t)\right) \\ {}^cD_{0^+}^{\alpha_i} v_i(t) &\geq f_i\left(t, v(t)\right), \ 0 \leq t \leq T, \end{aligned}$$

where $i = \overline{1,n}: \ 0 < \alpha_i \leq 1$ and $0 \leq \beta_i < 1$, and $f_i \in C\left([0,T] \times \mathbb{R}^n; \mathbb{R}\right)$
We will start with a first analysis on systems of order two (in $\mathbb{R}^2$).

Références

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