



N° d'ordre :

N° de série :

DEMOCRATIC AND POPULAR REPUBLIC OF ALGERIA  
MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

UNIVERSITY CENTER OF EL-OUED  
INSTITUTE OF SCIENCE AND TECHNOLOGY  
Department of Electronics

Dissertation Presented  
For Graduation of:

# ACADEMIC Master

Domain: **Sciences and Techniques**  
Sector: **Electronics**  
Specialty: **Telecommunications**

Presented by: **DADI Abdelghafour**

**Study of one and two-dimensional photonic crystals based on  
germanium (Ge) and lithium niobate (LiNbO<sub>3</sub>)**

Date of dissertation: 27 / 06 / 2012

In front of jury composed of:

|    |                            |       |           |
|----|----------------------------|-------|-----------|
| M. | <b>GHENDIR Saïd</b>        | M.A.B | President |
| M. | <b>REHOUMA Ferhat</b>      | Pr    | Examiner  |
| M. | <b>MAMANOU Abdellatif</b>  | M.A.A | Examiner  |
| M. | <b>BEGGAS Azziddine</b>    | M.A.B | Examiner  |
| M  | <b>MEDJOURI Abdelkader</b> | M.A.A | Reporter  |

2011-2012

# Contents

|                                                                                           |           |
|-------------------------------------------------------------------------------------------|-----------|
| <b>List of Figures</b> . . . . .                                                          | iii       |
| <b>List of Tables</b> . . . . .                                                           | vii       |
| <b>Acronyms</b> . . . . .                                                                 | viii      |
| <b>General Introduction</b> . . . . .                                                     | 1         |
| <b>1 The Photonic crystals</b>                                                            | <b>3</b>  |
| 1.1 Introduction . . . . .                                                                | 3         |
| 1.2 History of photonic crystals . . . . .                                                | 3         |
| 1.3 Photonic crystals . . . . .                                                           | 5         |
| 1.3.1 One-dimensional photonic crystals (Bragg grating) . . . . .                         | 5         |
| 1.3.2 Two-dimensional photonic crystals . . . . .                                         | 5         |
| 1.3.3 Three-dimensional photonic crystals . . . . .                                       | 6         |
| 1.4 Photonic crystal theory . . . . .                                                     | 6         |
| 1.4.1 Maxwell Equations . . . . .                                                         | 7         |
| 1.4.2 Bloch theorem . . . . .                                                             | 9         |
| 1.5 Conclusion . . . . .                                                                  | 10        |
| <b>2 Methods of modeling the Photonic Crystals</b>                                        | <b>11</b> |
| 2.1 Introduction . . . . .                                                                | 11        |
| 2.2 Modeling the Photonic Crystals with PWE method . . . . .                              | 12        |
| 2.2.1 Analysis and modeling of 1D Photonic Crystals . . . . .                             | 12        |
| 2.2.2 Analysis and modeling of 2D and 3D Photonic Crystals . . . . .                      | 25        |
| 2.3 Modeling the Photonic Crystals with FDTD method . . . . .                             | 38        |
| 2.4 Modeling the Photonic Crystals with TM method . . . . .                               | 38        |
| 2.5 conclusion . . . . .                                                                  | 39        |
| <b>3 Simulations of the photonic cryslas</b>                                              | <b>40</b> |
| 3.1 Introduction . . . . .                                                                | 40        |
| 3.2 Characteristics of the germanium ( <i>Ge</i> )material . . . . .                      | 41        |
| 3.3 One-dimensional photonic crystals simulation . . . . .                                | 42        |
| 3.3.1 Definition of the parameter . . . . .                                               | 42        |
| 3.3.2 Photonic band gap structures diagram . . . . .                                      | 42        |
| 3.3.3 Reflection and transmission diagrams . . . . .                                      | 43        |
| 3.4 Two-dimensional photonic crystals simulation . . . . .                                | 44        |
| 3.4.1 Air cylinders being placed in germanium ( <i>Ge</i> ) . . . . .                     | 44        |
| 3.4.2 Germanium ( <i>Ge</i> ) cylinders being placed in air . . . . .                     | 63        |
| 3.5 Characteristics of the Lithium niobate ( <i>LiNbO<sub>3</sub></i> )material . . . . . | 79        |
| 3.6 One-dimensional photonic crystals simulation . . . . .                                | 79        |

---

|                              |                                                         |     |
|------------------------------|---------------------------------------------------------|-----|
| 3.6.1                        | Definition of the parameter . . . . .                   | 79  |
| 3.6.2                        | Photonic band gap structures diagram . . . . .          | 80  |
| 3.6.3                        | Reflection and transmission diagrams . . . . .          | 81  |
| 3.7                          | Two-dimensional photonic crystals simulation . . . . .  | 81  |
| 3.7.1                        | Air cylinders being placed in Lithium niobate . . . . . | 81  |
| 3.7.2                        | Lithium niobate cylinders being placed in air . . . . . | 96  |
| 3.8                          | Conclusion . . . . .                                    | 109 |
| General Conclusion . . . . . |                                                         | 110 |
| Appendix A . . . . .         |                                                         | 113 |
| Appendix B . . . . .         |                                                         | 120 |
| Bibliography . . . . .       |                                                         | 123 |

# List of Figures

|      |                                                                                                                                                                                       |    |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 1.1  | Examples of 1D, 2D and 3D photonic crystals (a) 1D PhC, (b) 2D PhC, (c) 3D PhC . . . . .                                                                                              | 5  |
| 2.1  | Dispersion characteristics of the uniform medium versus periodic one .                                                                                                                | 14 |
| 2.2  | Process of the unit cell determination . . . . .                                                                                                                                      | 15 |
| 2.3  | Possible choice of primitive lattice vectors . . . . .                                                                                                                                | 16 |
| 2.4  | The scheme of the Brillouin zone selection for 2D PhC Brillouin zone .                                                                                                                | 19 |
| 2.5  | Typical band structure of 1D PhC . . . . .                                                                                                                                            | 22 |
| 2.6  | Off-axis radiation propagation inside 1D PhC . . . . .                                                                                                                                | 23 |
| 2.7  | Off-axis band structure of the 1D PhC. (a) <i>TM</i> polarization, (b) <i>TE</i> polarization . . . . .                                                                               | 24 |
| 2.8  | k-Path of the PhC with square (a) and hexagonal (b) lattices . . . . .                                                                                                                | 30 |
| 2.9  | k-Path of the Brillouin zone for the PhC with cubic lattice . . . . .                                                                                                                 | 30 |
| 2.10 | Definition of the set of reciprocal lattice vectors . . . . .                                                                                                                         | 31 |
| 2.11 | Examples of the band structure of rods arranged to square lattice with $r/a = 0.4$ computed for <i>TE</i> (a) and <i>TM</i> (b) polarizations . . . . .                               | 32 |
| 2.12 | Partial (a) and complete (b) photonic band gaps of the PhC . . . . .                                                                                                                  | 33 |
| 2.13 | Example of the PBG map formation . . . . .                                                                                                                                            | 33 |
| 2.14 | Off-plane radiation propagation inside the PhC . . . . .                                                                                                                              | 34 |
| 2.15 | The Brillouin zone for computation of the off-plane band structure . .                                                                                                                | 35 |
| 2.16 | In-plane (a) and off-plane (b) band structures . . . . .                                                                                                                              | 36 |
| 2.17 | Translated defect of the PhC, its unit cell and lattice vectors . . . . .                                                                                                             | 37 |
| 2.18 | The band structure of the PhC defect less structure (a) and with defect (b) . . . . .                                                                                                 | 38 |
| 3.1  | Index profile of 1D photonic crystals (a) with 3D (b) with 2D in 2 axes (x,z) . . . . .                                                                                               | 42 |
| 3.2  | Band diagram of a 1D photonic crystal air and germanium ( <i>Ge</i> ) with period $d = 1\mu m$ ; polarized <i>TE</i> and <i>TM</i> mode . . . . .                                     | 43 |
| 3.3  | diagram of transmission and reflection of the 1D photonic crystals with air and the germanium ( <i>Ge</i> ) with period $d = 1\mu m$ polarized <i>TE</i> and <i>TM</i> mode . . . . . | 43 |
| 3.4  | Index profile of 2D square photonic crystals with holes of air in germanium ( <i>Ge</i> ) ; (a) with 2D in 2 axes (x,z) (a) with 3D . . . . .                                         | 44 |

|      |                                                                                                                                                                                                                      |    |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 3.5  | Map of photonic band gap for both TE and TM polarizations. Case of a square PC of air holes ( $\varepsilon = 1$ ) in germanium ( $Ge$ ) . . . . .                                                                    | 45 |
| 3.6  | Band diagram of a photonic crystal square lattice holes of air in germanium ( $Ge$ ) polarized $TE$ mode for a filling factor $r/a = 0.49$ . . . . .                                                                 | 47 |
| 3.7  | Transmission and reflection through the square structures of air holes in the germanium ( $Ge$ ); TE mode polarized in $a=0.38595\mu m$ , $r=0.1891155\mu m$ . . . . .                                               | 49 |
| 3.8  | Band diagram of a photonic crystal square lattice holes of air in germanium ( $Ge$ ) polarized $TM$ mode for a filling factor $r/a = 0.468$ . . . . .                                                                | 51 |
| 3.9  | Band diagram of a photonic crystal square lattice holes of air in germanium ( $Ge$ ) polarized $TM$ and $TE$ mode for a filling factor $r/a = 0.468$ . . . . .                                                       | 52 |
| 3.10 | Transmission and reflection through the square structures of air holes in the germanium ( $Ge$ ); TM mode polarized in $a = 0.68975\mu m$ , $r = 0.33108\mu m$ . . . . .                                             | 53 |
| 3.11 | Index profile of 2D triangular photonic crystals with holes of air in germanium ( $Ge$ ); (a) with 2D in 2 axes (x,z) (a) with 3D . . . . .                                                                          | 54 |
| 3.12 | Map of photonic band gap for both TE and TM polarizations. Case of a triangular PC of air holes ( $\varepsilon = 1$ ) in germanium ( $Ge$ ) . . . . .                                                                | 55 |
| 3.13 | Band diagram of a photonic crystal triangular lattice holes of air in germanium ( $Ge$ ) polarized $TE$ mode for a filling factor $r/a = 0.44$ . . . . .                                                             | 57 |
| 3.14 | Transmission and reflection through the triangular structures of air holes in the germanium ( $Ge$ ) ; TE mode polarized in $a = 1.15041\mu m$ , $r = 0.5637\mu m$ . . . . .                                         | 58 |
| 3.15 | Band diagram of a photonic crystal triangular lattice holes of air in germanium ( $Ge$ ) polarized $TM$ mode for a filling factor $r/a = 0.46$ . . . . .                                                             | 59 |
| 3.16 | Band diagram of a photonic crystal square lattice holes of air in germanium ( $Ge$ ) polarized $TM$ and $TE$ mode for a filling factor $r/a = 0.46$ . . . . .                                                        | 61 |
| 3.17 | Transmission and reflection through the triangular structures of air holes in the in germanium ( $Ge$ ); TM mode polarized in $a = 0.5983\mu m$ , $r = 0.275218\mu m$ . . . . .                                      | 62 |
| 3.18 | Index profile of 2D square photonic crystals with square lattice of rods of germanium ( $Ge$ ) in air; (a) with 3D , (b) with 2D in 2 axes (x,z) . . . . .                                                           | 63 |
| 3.19 | Map of photonic band gap for both TE and TM polarizations. Case of 2D photonic crystals with square lattice of rods of germanium ( $Ge$ ) in air with radius of germanium cylinder changing from 0 to 0.5a . . . . . | 64 |
| 3.20 | Band diagram of a photonic crystal with square lattice of rods of germanium ( $Ge$ ) in air; polarized $TE$ mode for a filling factor $r/a = 0.16$ . . . . .                                                         | 66 |
| 3.21 | Transmission and reflection through the square lattice of rods of germanium ( $Ge$ ) in air ; polarized mode TE $a = 0.5604025\mu m$ , $r = 0.0896644\mu m$ . . . . .                                                | 67 |
| 3.22 | Band diagram of a photonic crystal with square lattice of rods of germanium ( $Ge$ ) in air; polarized $TM$ mode for a filling factor $r/a = 0.38$ . . . . .                                                         | 69 |
| 3.23 | Transmission and reflection through the square lattice of rods of germanium ( $Ge$ ) in air ; polarized mode TM $a = 0.73067\mu m$ , $r = 0.2776546\mu m$ . . . . .                                                  | 70 |
| 3.24 | Index profile of 2D triangular photonic crystals with rods of germanium ( $Ge$ ) in air; (a) with 2D in 2 axes (x,z) (a) with 3D . . . . .                                                                           | 71 |

|      |                                                                                                                                                                                                                        |    |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 3.25 | Map of photonic band gap for both TE and TM polarizations. Case of 2D photonic crystals with triangular lattice of rods of germanium ( $Ge$ ) in air with radius of germanium cylinder changing from 0 to $0.5a$ . . . | 72 |
| 3.26 | Band diagram of a photonic crystal with triangular lattice of rods of germanium ( $Ge$ ) in air; polarized $TE$ mode for a filling factor $r/a = 0.14$ . 74                                                            |    |
| 3.27 | Transmission and reflection through the triangular structures rods of germanium ( $Ge$ ) in the air; polarized mode TE $a = 0.6326170\mu m$ , $r = 0.08856638\mu m$ . . . . .                                          | 75 |
| 3.28 | Band diagram of a photonic crystal with triangular lattice of rods of germanium ( $Ge$ ) in air; polarized $TM$ mode for a filling factor $r/a = 0.14$ . 77                                                            |    |
| 3.29 | Transmission and reflection through the triangular structures rods of germanium ( $Ge$ ) in the air; polarized mode TM $a = 1.5252\mu m$ , $r = 0.213528\mu m$ . . . . .                                               | 78 |
| 3.30 | Index profile of 1D photonic crystals (a) with 3D (b) with 2D in 2 axes (x,z) . . . . .                                                                                                                                | 80 |
| 3.31 | Band diagram of a 1D photonic crystal air and lithium niobate ( $LiNbO_3$ ) with the period $d = 1\mu m$ ; a) polarized $TE$ mode, b) polarized $TM$ mode, c) polarized $TE$ and $TM$ mode . . . . .                   | 80 |
| 3.32 | diagram of ransmission and reflection of the 1D potonic crystals with air and the lithium niobate ( $LiNbO_3$ ) with the period $d = 1\mu m$ polarized $TE$ and $TM$ mode . . . . .                                    | 81 |
| 3.33 | Index profile of 2D square photonic crystals with holes of air in lithium niobate ( $LiNbO_3$ ); (a) with 2D in 2 axes (x,z) (a) with 3D . . . . .                                                                     | 82 |
| 3.34 | Map of photonic band gap for both TE and TM polarizations. Case of a square PC of air holes ( $\varepsilon = 1$ ) in $LiNbO_3$ . . . . .                                                                               | 83 |
| 3.35 | Band diagram of a photonic crystal square lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized $TE$ mode for a filling factor $r/a = 0.496$ . 84                                                            |    |
| 3.36 | Transmission and reflection through the square structures of air holes in the lithium niobate ( $LiNbO_3$ ); TE mode polarized in $a = 0.625425\mu m$ , $r = 0.31021080\mu m$ . . . . .                                | 85 |
| 3.37 | Band diagram of a photonic crystal square lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized $TM$ mode for a filling factor $r/a = 0.468$ . 87                                                            |    |
| 3.38 | Transmission and reflection through the square structures of air holes in the lithium niobate ( $LiNbO_3$ ); TM mode polarized in $a = 1.2586\mu m$ , $r = 0.5890248\mu m$ . . . . .                                   | 88 |
| 3.39 | Index profile of 2D triangular photonic crystals with holes of air in lithium niobate ( $LiNbO_3$ ); (a) with 2D in 2 axes (x,z) (a) with 3D . .                                                                       | 89 |
| 3.40 | Map of photonic band gap for both TE and TM polarizations. Case of a triangular PC of air holes ( $\varepsilon = 1$ ) in $LiNbO_3$ . . . . .                                                                           | 89 |
| 3.41 | Band diagram of a photonic crystal triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized $TE$ mode for a filling factor $r/a = 0.44$ . . . . .                                                    | 92 |
| 3.42 | Transmission and reflection through the triangular structures of air holes in the lithium niobate ( $LiNbO_3$ ); TE mode polarized in $a = 1.64145\mu m$ , $r = 0.722238\mu m$ . . . . .                               | 93 |

|      |                                                                                                                                                                                              |     |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 3.43 | Band diagram of a photonic crystal triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized $TM$ mode for a filling factor $r/a = 0.42$ . . . . .                          | 95  |
| 3.44 | Transmission and reflection through the triangular structures of air holes in the lithium niobate ( $LiNbO_3$ ); $TM$ mode polarized in $a = 0.689750\mu m$ , $r = 0.2896950\mu m$ . . . . . | 96  |
| 3.45 | Index profile of 2D square photonic crystals with square lattice of rods of lithium niobate ( $LiNbO_3$ ) in air; (a) with 3D, (b) with 2D in 2 axes (x,z) . . . . .                         | 97  |
| 3.46 | Map of photonic band gap for both TE and TM polarizations. Case of a square PC of rods $LiNbO_3$ in air . . . . .                                                                            | 97  |
| 3.47 | Band diagram of a photonic crystal square lattice with rods of ( $LiNbO_3$ ) in air; polarized $TE$ mode for a filling factor $r/a = 0.24$ . . . . .                                         | 99  |
| 3.48 | Transmission and reflection through the square structures of rods of lithium niobate ( $LiNbO_3$ ) in the air; TE mode polarized in $a = 0.651\mu m$ , $r = 0.15624\mu m$ . . . . .          | 100 |
| 3.49 | Index profile of 2D triangular photonic crystals with rods of lithium niobate ( $LiNbO_3$ ) in air; (a) with 2D in 2 axes (x,z) (a) with 3D . . .                                            | 101 |
| 3.50 | Map of photonic band gap for both TE and TM polarizations. Case of a triangular PC of rods $LiNbO_3$ in air . . . . .                                                                        | 101 |
| 3.51 | Band diagram of a photonic crystal triangular lattice with rods of ( $LiNbO_3$ ) in air; polarized $TE$ mode for a filling factor $r/a = 0.20$ . . .                                         | 104 |
| 3.52 | Transmission and reflection through the triangular structures rods of $LiNbO_3$ in the air; polarized mode TE $a = 0.7310575\mu m$ , $r = 0.14621150\mu m$                                   | 105 |
| 3.53 | Band diagram of a photonic crystal triangular lattice with rods of ( $LiNbO_3$ ) in air; polarized $TM$ mode for a filling factor $r/a = 0.22$ . .                                           | 107 |
| 3.54 | Transmission and reflection through the triangular structures rods of $LiNbO_3$ in the air; polarized mode TM $a = 1.54662875\mu m$ , $r = 0.34025833\mu m$                                  | 108 |

# List of Tables

|      |                                                                                                                                                |     |
|------|------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 1.1  | Analogy between electron and photon . . . . .                                                                                                  | 9   |
| 3.1  | Summary of TE band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the germanium (Ge) . . .        | 46  |
| 3.2  | Summary of TM band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the germanium (Ge) . . .        | 50  |
| 3.3  | Summary of TE band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the germanium (Ge) . . .    | 56  |
| 3.4  | Summary of TM band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the germanium (Ge) . . .    | 60  |
| 3.5  | Summary of TE band gaps of most interest to the case of a square photonic crystal of rods of germanium (Ge) in the air . . . . .               | 65  |
| 3.6  | Summary of TM band gaps of most interest to the case of a square photonic crystal of rods of germanium (Ge) in the air . . . . .               | 68  |
| 3.7  | Summary of TE band gaps of most interest to the case of a triangular photonic crystal of rods of germanium (Ge) in the air . . . . .           | 73  |
| 3.8  | Summary of TM band gaps of most interest to the case of a triangular photonic crystal of rods of germanium (Ge) in the air . . . . .           | 76  |
| 3.9  | Summary of TE band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the lithium niobate . . . .     | 84  |
| 3.10 | Summary of TM band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the lithium niobate . . . .     | 86  |
| 3.11 | Summary of TE band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the lithium niobate . . . . | 91  |
| 3.12 | Summary of TM band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the lithium niobate . . . . | 94  |
| 3.13 | Summary of TE band gaps of most interest to the case of a square photonic crystal of rods of lithium niobate in the air . . . . .              | 98  |
| 3.14 | Summary of TE band gaps of most interest to the case of a triangular photonic crystal of rods of lithium niobate in the air . . . . .          | 103 |
| 3.15 | Summary of TM band gaps of most interest to the case of a triangular photonic crystal of rods of lithium niobate in the air . . . . .          | 106 |

# Acronyms

|                        |                                      |
|------------------------|--------------------------------------|
| $1D$                   | One-dimensional photonic crystals.   |
| $2D$                   | Two-dimensional photonic crystals.   |
| $3D$                   | Three-dimensional photonic crystals. |
| $PBG$                  | Photonic Band Gap.                   |
| $BZ$                   | Brillouin Zone.                      |
| $KKR$                  | Korringa-Kohn-Rostoker.              |
| $PhC$                  | Photonic Crystal.                    |
| $PWE$                  | Plane Wave Expansion.                |
| $TE$                   | Transversal Electric.                |
| $TM$                   | Transversal Magnetic Mode.           |
| $Ge$                   | The germanium material.              |
| $LiNbO_3$              | The lithium niobate                  |
| $\lambda$              | Wavelength.                          |
| $\Gamma, X, M, \Gamma$ | High symmetry points.                |

---

|                        |                                                         |
|------------------------|---------------------------------------------------------|
| $\vec{E}(\vec{r}, t)$  | Vector of electric field.                               |
| $\vec{H}(\vec{r}, t)$  | Vector of magnetic field.                               |
| $\vec{B}(\vec{r}, t)$  | The magnetic field density or the magnetic induction.   |
| $\vec{D}(\vec{r}, t)$  | Electric displacement field or electric flux density.   |
| $\vec{J}(\vec{r}, t)$  | The current density.                                    |
| $\rho$                 | The charge density.                                     |
| $\varepsilon_0$        | Permittivity of free space or the electric constant.    |
| $\varepsilon(\vec{r})$ | The relative permittivity.                              |
| $C$                    | The speed of light in vacuum.                           |
| $\hat{\Theta}$         | The Hermitian operator.                                 |
| $\Psi$                 | Wave function.                                          |
| $a_1, a_2, a_3$        | Primitive lattice vectors.                              |
| $g(k)$                 | The dielectric function in wave vectors representation. |
| $k$                    | The wave vector.                                        |
| $G, G', G''$           | Reciprocal lattice vectors.                             |
| $\delta_{ij}$          | Kronecker delta.                                        |
| $b_1, b_2, b_3$        | Basis of primitive reciprocal lattice vectors.          |
| $h_{(k,n)}(x)$         | The periodic function with periodicity of lattice.      |
| $n$                    | The state numbe.                                        |

---

|                                 |                                                                                    |
|---------------------------------|------------------------------------------------------------------------------------|
| $h_{(k,n)}(G)$                  | Previously defined periodic function in representation of wave vectors.            |
| $\chi(G'')$                     | Fourier expansion coefficients of the inverted dielectric function.                |
| $\theta_B$                      | Brewster angle.                                                                    |
| $R$                             | The lattice vector.                                                                |
| $\chi(G)$                       | Fourier expansion coefficients of the inverted permittivity distribution function. |
| $\omega_{kn}^{(E)}$             | Eigen-frequency for electric field.                                                |
| $\omega_{kn}^{(H)}$             | Eigen-frequency for magnetic field.                                                |
| $\hat{\theta}_{G_x, G'_x}$      | Matrix element of $\hat{\theta}$ .                                                 |
| $\theta$                        | Angle.                                                                             |
| $\hat{\theta}$                  | Differential operator in matrix form.                                              |
| $r_{\parallel}$                 | In-plane coordinate vector.                                                        |
| $G_{\parallel}, G'_{\parallel}$ | 2D in-plane reciprocal lattice vector.                                             |
| $k_{\parallel}$                 | In-plane wave vector.                                                              |
| $\omega_{k_{\parallel}n}^{(E)}$ | Eigen-frequency for TM polarization.                                               |
| $\omega_{k_{\parallel}n}^{(H)}$ | Eigen-frequency for TE polarization.                                               |
| $\omega$                        | Angular frequency.                                                                 |
| $\Delta\omega$                  | Half-width of PBG, frequency difference.                                           |
| $\omega_0$                      | Center frequency of the photonic band gap.                                         |

# General Introduction

Since the discovery of the transistor, integrated electronics has progressed with ever increasing speed. The integration of components on microchips made of silicon today reached the dimensions of the order of microns.

The ability to transmit information at high speeds over hundreds or thousands of miles, through optical fibers based on silica, led to the rise of the conventional integrated optics and optoelectronics, as well as integration of many active or passive components. However, these components are millimeter size, leading to devices large and often insensitive.

It was in 1987 that were designed photonic crystal, this term hides a transposition of the study of "band-gap" to clean electronic solid state physics to the study of "band-gap" photonic optics.

These compounds, which include introducing a periodicity of the refractive index of a material, quickly emerged as a promising solution for the realization of photonic components, which allow acting directly on the light and handling.

This last decade optics has been enriched by the development of research in the field of photonic crystals. From these, it is possible to create waveguides with extremely small dimensions and can follow different trajectories. We can also produce resonant cavities very small that can serve as sources or filters. The set will enable medium term to replace the entire integrated optics and optoelectronics components by conventional smaller, cheaper, more efficient and ideal interconnection in microelectronics.

In this work, we chose a contribution to the study of one and two-dimensional photonic crystals on two different types of material: material type semiconductor (germanium (Ge)) and dielectric( lithium niobate (LiNbO3)). The objective is to combine the properties of photonic bandgap materials to those in order to achieve photonic band gap structures for applications in optical telecommunications  $\lambda = 1.55\mu m$ . To do this, two structures are proposed namely, the square structure and the triangular

structure and two different configurations for each structure, the configuration” holes” and” columns” configuration (or rods).

The choice of this theme was motivated, among other things, in that it constitutes a preliminary work to the extent that no previous study has been conducted at our university and this since it is new domain and is a broad. The principal motivate it because the only solution for an all-optical system for optical telecommunication

This work consists of three chapters:

In the first, the emphasis is on photonic crystals. It aims to give readers unfamiliar photonic crystal elements that will be needed to understand the sequence of memory. It will therefore be devoted to a literature review for the acquisition of some basic notions and concepts in optics, electromagnetic and photonic crystal types.

The second chapter describes the main methods used to describe the propagation of electromagnetic waves in photonic crystals. Among these methods, we will detail those used within the scope of this work:

- Plane Wave Expansion method (PWE)
- Finite Difference Time Domain method (FDTD)

The third chapter, divided into two parts, will be dedicated to the conception and design of two-dimensional periodic structures on two different substrates. In the first part we consider the case of substrate type dielectric niobate lithium ( $\text{LiNbO}_3$ ). In the second, it will issue a semiconductor substrate type (Ge). The objective is to achieve photonic band gap structures for applications in optical telecommunications  $\lambda = 1.55\mu\text{m}$ .

Initially, simulations (PWE) are conducted to determine the size and shape of the network (period, size of grounds ...). Once the structure produced with the correct dimensions, filtering it is characterized by transmission and reflection with FDTD simulations. in all this work will be use "The RSoft Photonics" software for the simulation parte.

# Chapter 1

## The Photonic crystals

### 1.1 Introduction

The study of electromagnetic wave propagation in periodic media has been a considerable revival in last years due to the introduction of the concept of photonic crystal (PhC). In this chapter, we will deal with the history of photonic crystals after that we will introduce the basics of photonic crystals. In addition to that we will give the concept of different types of photonic crystals, the equation of wave propagation in periodic media and finally we will use the Maxwell equations and the similarity between the photonic crystals and solid physics to use the Bloch theorem

### 1.2 History of photonic crystals

Although photonic crystals have been studied in one form or another since 1887, the term “photonic crystal” was first used introduce 100 years later, after Eli Yablonovitch and Sajeev John published two milestone papers on photonic crystals in 1987. Before 1987, one-dimensional photonic crystals in the form of periodic multi-layers dielectric stacks (such as the Bragg mirror) were studied extensively. Lord Rayleigh started his study in 1887, by showing that such systems have a one-dimensional photonic band-gap, a spectral range of large reflectivity, known as a stop-band. Today, such structures are used in a diverse range of applications; from reflective coatings to enhancing the efficiency of LEDs to highly reflective mirrors in certain laser cavities[1].

Purcell in 1946 indicated that, spontaneous emission of radio waves from nuclear spin levels could be controlled by a dispersion of small metallic particles in a nuclear-magnetic material, which would create a resonant oscillator. In 1972 Bykov considered that spontaneous emission of atoms at optical wavelengths could be reduced by placing them in a periodic lattice of dielectrics with pitches smaller than the radiation wave-

length, thus avoiding decay of excited states through the presence of opaque bands for the transition radiation and consequent generation of a dynamic state. Bykov also speculated what could happen if two- or three-dimensional periodic optical structures were used. However, these ideas were not taken off until the publication of two milestone papers in 1987 by Yablonovitch and John. Both papers concerned high dimensional periodic optical structures – photonic crystals. Yablonovitch’s main motivation was to engineer the photonic density of states, in order to control the spontaneous emission of materials embedded within the photonic crystal; John’s idea was to use photonic crystals to affect the localisation and control of light. Both works addressed the engineering of a structured material exhibiting ranges of frequencies at which the propagation of electromagnetic waves is not allowed, so called bandgaps, and their employment in the emission control of optically active materials[1].

After 1987, the number of research papers concerning photonic crystals began to grow exponentially. However, due to the difficulty of actually fabricating these structures at optical scales, early studies were either theoretical or in the microwave regime, where photonic crystals can be built on far more readily accessible centimeter scale. (This fact is due to a property of the electromagnetic fields known as scale invariance – in essence, the electromagnetic fields, as the solutions to Maxwell’s equations, has no natural length scale, and so solutions for centimeter scale structure at microwave frequencies are the same as for nanometer scale structures at optical frequencies.) By 1991, Yablonovitch had demonstrated the first three-dimensional photonic band-gap in the microwave regime. In 1996, Thomas Krauss made the first demonstration of a two-dimensional photonic crystal at optical wavelengths. This opened up the way for photonic crystals to be fabricated in semiconductor materials by borrowing the methods used in the semiconductor industry. Today, such techniques use photonic crystal slabs, which are two dimensional photonic crystals[32] “etched” into slabs of semiconductor; total internal reflection confines light to the slab, and allows photonic crystal effects, such as engineering the photonic dispersion to be used in the slab. Research is underway around the world to use photonic crystal slabs in integrated computer chips, in order to improve the optical processing of communications both on-chip and between chips. Although such techniques are still to mature into commercial applications, two-dimensional photonic crystals have found commercial use in the form of photonic crystal fibers (otherwise known as holey fibers, because of the air holes that run through them). Photonic crystal fibers were first developed by Philip Russell in 1998, and can be designed to possess enhanced properties over (normal) optical fibers.

### 1.3 Photonic crystals

Photonic crystals (PhCs) are novel class of optical media represented by natural or artificial structures with periodic modulation of the refractive index. Such optical media have some peculiar properties which gives an opportunity for a number of applications to be implemented on their basis. Depending on geometry of the structure, PhCs can be divided into three broad categories, namely one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) structures. Examples are shown in Figure 1.1

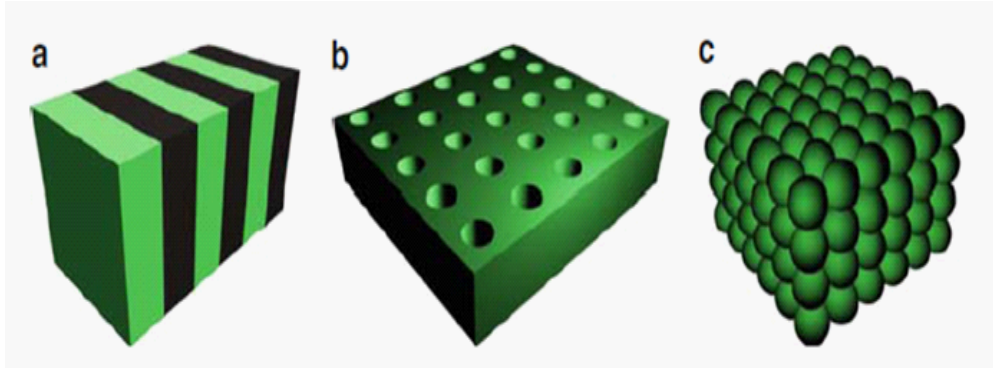


Figure 1.1: Examples of 1D, 2D and 3D photonic crystals (a) 1D PhC, (b) 2D PhC, (c) 3D PhC

#### 1.3.1 One-dimensional photonic crystals (Bragg grating)

In 1D PhCs, the periodic modulation of the permittivity occurs in one direction only, while in two other directions structure is uniform. an example of such a PhC can be given the Bragg grating which is widely used as a distributed reflector in vertical cavity surface emitting lasers. Besides, such structures are widely used as antireflecting coatings which allow decreasing dramatically the reflectance from the surface and are used to improve the quality of lenses, prisms and other optical components.

#### 1.3.2 Two-dimensional photonic crystals

2D PhC can have comparatively large variety of configurations, because it possesses periodicity of the permittivity along two directions, while in the third direction the medium is uniform. A good example of the 2D PhC is porous silicon with periodically arranged pores, which is represented by the silicon substrate with etched holes. Another example of 2D PhC is a periodically arranged system of dielectric rods in air.

2D PhC can also be found in nature. For instance, the pattern on the butterfly's wing and its rainbow play is caused by the light reflection from the microstructure on the wing.

### 1.3.3 Three-dimensional photonic crystals

3D PhC has permittivity modulation along all three directions. At that, the number of possible PhC configurations is much larger than in case of 1D or 2D PhC. Many works are dedicated to the design of new geometric configuration of 3D PhC, which discover new possibilities of their application. The most known naturally formed 3D PhC is valuable stone opal. This stone is known by its unique optical properties. When turned around, it plays different colors.

## 1.4 Photonic crystal theory

This theory is based on the similarities between PhC and solid-state physics that can be summarized in the following: Periodic modulation of the refractive index in a PhC forms a lattice similar to atomic lattice of solid-state; behavior of photons in a PhC is similar to electron and hole behavior in an atomic lattice; due to the lattice periodicity both PhC and solid-state provide band gap, the range of energies which particle cannot have inside the structure. From theoretical viewpoint, determination of the eigenfunctions in a PhC is very similar to calculation of the particle wave functions in the solid-state. This similarity is used to obtain photonic band structure.

However, along with strong similarity, there exist some essential differences. One of the main differences is the particle energy distribution. Electrons obey the Fermi–Dirac distribution while photons obey Bose–Einstein distribution. Besides, electrons are affected by intra crystalline field which leads to the necessity of taking it into account (it is necessary to note that the shape of intracrystalline field is unknown, and investigators have to use approximate methods such as  $k$ - $p$  method). Photons are not affected by intracrystalline field. Therefore, computation of the optical field distribution or the photonic band structure is essentially simplified.

The most important property which determines practical significance of the PhC is the presence of the photonic band gap[31]. The photonic band gap (PBG) refers to the energy or frequency range where the light propagation is prohibited inside the PhC. When the radiation with frequency inside the PBG incidents the structure, it appears to be completely reflected. However, if one introduces the defect to the strictly periodic structure, the effect of such a defect is the same as the defect introduction to the crystalline structure of a semiconductor. This means that new eigen-state appears

inside the PBG with energy corresponding to the eigen-frequency of the defect. Thus, the radiation within the defect frequency will propagate inside the structure or, in case of multiple defects radiation will be guided like in waveguide[6].

Thus, there exists quite strong analogy between PhC physics and solid-state physics both from the physical and mathematical points of view[32].

### 1.4.1 Maxwell Equations

Within the classic theory, all the electromagnetic phenomena are described by the system of Maxwell's equations which gives the time dependence of electric and magnetic fields and their interaction with optical medium.

$$\vec{\nabla} \cdot \vec{D}(\vec{r}, t) = \rho \quad (1.1)$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r}, t) = 0 \quad (1.2)$$

$$\vec{\nabla} \times \vec{E}(\vec{r}, t) = -\frac{\partial}{\partial t} \vec{B}(\vec{r}, t) \quad (1.3)$$

$$\vec{\nabla} \times \vec{H}(\vec{r}, t) = \vec{J}(\vec{r}, t) + \frac{\partial}{\partial t} \vec{D}(\vec{r}, t) \quad (1.4)$$

Where:

$t$  : denotes the time.

$\vec{r}$  : vector of position.

$\vec{\nabla}$  : is the nabla operator.

“.”, “ $\times$ ”: denotes scalar and vector products.

$\vec{E}(\vec{r}, t)$  : vector of electric field.

$\vec{H}(\vec{r}, t)$  : vector of magnetic field.

$\vec{B}(\vec{r}, t)$  : the magnetic field density or the magnetic induction.

$\vec{D}(\vec{r}, t)$  : electric displacement field or electric flux density.

$\vec{J}(\vec{r}, t)$  : the current density.

$\rho$  : the charge density.

In case of nonconductive medium like germanium or other material:

$$\rho = 0 \quad (1.5)$$

$$\vec{J}(\vec{r}, t) = 0 \quad (1.6)$$

With the relations between the vector of electric field and the electric displacement field the same time between the vector of magnetic field and the magnetic field density:

$$\vec{B}(\vec{r}, t) = \mu_0 \vec{H}(\vec{r}, t) \quad (1.7)$$

$$\vec{D}(\vec{r}, t) = \varepsilon(\vec{r}) \varepsilon_0 \vec{E}(\vec{r}, t) \quad (1.8)$$

Where:

$\mu_0$  : permeability of free space or the magnetic constant.

$\varepsilon_0$  : permittivity of free space or the electric constant.

$\varepsilon(\vec{r})$  : the relative permittivity electric ( $\varepsilon_r \varepsilon_0$ ).

Finally the Maxwell equations are written like this:

$$\vec{\nabla} \cdot (\varepsilon(\vec{r}) \vec{E}(\vec{r}, t)) = 0 \quad (1.9)$$

$$\vec{\nabla} \cdot \vec{H}(\vec{r}, t) = 0 \quad (1.10)$$

$$\vec{\nabla} \times \vec{E}(\vec{r}, t) = -\mu_r \mu_0 \frac{\partial}{\partial t} \vec{H}(\vec{r}, t) \quad (1.11)$$

$$\vec{\nabla} \times \vec{H}(\vec{r}, t) = \varepsilon(\vec{r}) \varepsilon_0 \frac{\partial}{\partial t} \vec{E}(\vec{r}, t) \quad (1.12)$$

By eliminating  $\vec{E}(\vec{r}, t)$  of equations, we obtain:

$$\vec{\nabla} \times \left( \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}, t) \right) = -\frac{1}{C^2} \frac{\partial^2}{\partial t^2} \vec{H}(\vec{r}, t) \quad (1.13)$$

where  $C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$  is the speed of light in vacuum.

When we seek solutions of the equation 1.13 in the form of harmonic waves:

$$\vec{H}(\vec{r}, t) = \vec{H}(\vec{r}) e^{(-j\omega t)} \quad (1.14)$$

we can write the eigenvalue equation as follows:

$$\vec{\nabla} \times \left( \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}, t) \right) = \frac{\omega^2}{C^2} \vec{H}(\vec{r}) \quad (1.15)$$

In this equation, the operator appearing in the left hand side is Hermitian (operator whose matrix elements are symmetric conjugate of a complex vector space)[64]. The Hermitian operator  $\hat{\Theta} : \hat{\Theta} = \vec{\nabla} \times \left( \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \right)$ , after that we can write the equation like this:

$$\hat{\Theta} \vec{H}(\vec{r}) = \frac{\omega^2}{C^2} \vec{H}(\vec{r}) \quad (1.16)$$

$\varepsilon(\vec{r})$  is periodic, equation 1.14 is formally analogous to the Schrödinger equation describing the motion of electrons in the periodic potential of a crystal (see Table 1.1)

By this analogy, the concepts of solid state physics, such as the notions of reciprocal lattice, Brillouin zone and the Bloch theorem is applicable for solving the wave equation. In this way there is a possibility to obtain photonic band gap (by analogy with electronic bandgaps). Table 1.1 provides a summary of this analogy to a one-dimensional system can be extended to two or three-dimensional crystals.

|                     | Electron (Schrödinger)                    | Photon (Maxwell)                                                                                       |
|---------------------|-------------------------------------------|--------------------------------------------------------------------------------------------------------|
| field               | $\Psi k(r) = uk(r)e^{(jkr)}$              | $\vec{H}(\vec{r}, t) = \vec{H}(\vec{r})e^{(-j\omega t)}$                                               |
| Hermitian operator  | $H = -\frac{\hbar^2 \nabla^2}{2m} + V(r)$ | $\hat{\Theta} = \vec{\nabla} \times \left( \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \right)$ |
| Eigenvalue equation | $H\Psi = E\Psi$                           | $\hat{\Theta}\vec{H}(\vec{r}) = \frac{\omega^2}{C^2}\vec{H}(\vec{r})$                                  |

Table 1.1: Analogy between electron and photon

### 1.4.2 Bloch theorem

The Bloch theorem states that in a periodic potential, all the solutions of the Schrödinger equation are called Bloch functions, that is to say that there exists a vector  $k$  for writing:

$$\Psi k(r) = uk(r)e^{(jkr)} \quad (1.17)$$

where  $uk(r)$  is a periodic function with the same periods as the potential. The wave functions of electrons in a perfect crystal (periodic, infinite ...) are simply the product between a plane wave and a periodic function.

The interest of this theorem is to show that we only need to know  $\Psi$  on the unit cell of the crystal, the values breeding in remaining. The vectors  $k$  are called the Bloch vector, and the wave functions are called Bloch functions.

In equation 1.15, by analogy with the Schrödinger equation, the eigenvalues of the dielectric constant  $\varepsilon$  play the role of mathematical potential.

In a photonic crystal, we know that this potential is by definition periodic in one or more dimensions.

We will take advantage of this discrete translational symmetry to solve the equation 1.15. Mathematically, the periodicity of  $\varepsilon$  can be expressed as follows:

$$\varepsilon(\vec{r}) = \varepsilon(\vec{r} + \vec{R}) \quad (1.18)$$

This function is invariant under translation by a lattice vectors,  $\varepsilon$  can be decomposed into Fourier series .

## 1.5 Conclusion

In this chapter we were given a brief introduction into the basics of photonic crystals which are a special class of optical media with periodic modulation of permittivity. We were given a general idea of what photonic crystals are and then describe different kinds of photonic crystal (One-dimensional photonic crystals (1D), two-dimensional photonic (2D) crystals and three-dimensional photonic crystals (3D)) and theory of photonic crystal. We used the Maxwell equations and analogy between the Schrödinger equation describing the motion of electrons in the periodic potential and the propagation wave (photons) in the photonic crystals and use the Bloch theorem to found the equation of the propagation wave (photons) in the different types of the photonic crystals

## Chapter 2

# Methods of modeling the Photonic Crystals

### 2.1 Introduction

To understand the behaviour of an electromagnetic wave in photonic crystals, we must calculate its band structure (dispersion curve of angular frequency  $\omega(k)$  as a function of wave vector  $\vec{k}$ ) and the map of photonic band gaps (*PBG*), remembering the analogy there was with the atomic crystal in chapter one. In this chapter we will give some mathematical tools used by most theorists to solve this problem. We will therefore give a brief overview of these methods like Transfer-matrix method (*TMM*), Finite Difference Time Domain method (*FDTD*) and we will focus on the Plane Wave Expansion method (*PWE*) to modeling all types of photonic crystals for that, we must give definition for parameters for structure like: unit cell, lattice vector, reciprocal lattice and reciprocal lattice vectors as well as the first Brillouin zone

We used this method because it has many advantages like:

It has high performance. For the simplest structure computation takes several seconds. At that, the system requirements are very unpretentious.

The *PWE* method allows computing the band structure not only for the simplest PhCs like infinite periodic structures but for more complicated structures such as point defects, PhC waveguides and PhC fibers.

Besides, the method allows computing the field distribution for each of eigen-states of the PhC which is extremely convenient when designing PhC-based active devices as well as investigating PhC fibres.

However, except the irrefutable advantages, it should be mentioned that the method has some essential disadvantages:

Inability to treat losses. The differential operator in Helmholtz equation is Hermitian only in case of lossless material. When one considers materials with losses, the imaginary part of the refractive index appears which makes the application of the method impossible.

As it was shown, the method consists of searching for the set of eigen frequencies at the specific value of the wave vector. In order for the algorithm to make sense, the permittivity should be the same at all frequencies within the computation range. Thus, without additional modifications, the method is unable to treat dispersive materials.

However, in spite of all the disadvantages, the area of the method applications is very large. The method gives the possibility to estimate the characteristics of linear

Inability to treat losses. The differential operator in Helmholtz equation is Hermitian only in case of lossless material. When one considers materials with losses, the imaginary part of the refractive index appears which makes the application of the method impossible.

As it was shown, the method consists of searching for the set of eigen frequencies at the specific value of the wave vector. In order for the algorithm to make sense, the permittivity should be the same at all frequencies within the computation range. Thus, without additional modifications, the method is unable to treat dispersive materials.

However, in spite of all the disadvantages, the area of the method applications is very large. The method gives the possibility to estimate the characteristics of linear and nonlinear devices. For more precise analysis, some other methods such as the finite differences time domain method can be used.

## 2.2 Modeling the Photonic Crystals with PWE method

In order to obtain a general idea of PhCs characteristics as well as to effectively design PhC-based devices, it is convenient to use band structures. Band structures give full information about radiation behaviour when propagating within the specific direction inside the PhC. Using the band structure, parameter values of the PhC can be accurately determined for each application. Moreover, band structure of such non-periodic structures as point, linear and bulk defects allow to find accurate values of resonant wavelengths or, in case of optical waveguide, to determine its modal and dispersive properties[48] [45].

PhC will be given. The plane wave expansion (*PWE*) method which is one of the most important for computation of the PhCs band structures will be described in details here.

### 2.2.1 Analysis and modeling of 1D Photonic Crystals

The problem of the band structure computation of a PhC is finding the dispersion relation, that is to say, the dependence of eigen-frequencies or resonant frequencies of the PhC on the wave vector of the radiation passing through it.

In order to obtain the dispersion relation, it is necessary to solve the eigen problem formulated for the Helmholtz equation inside infinite periodic structure.

The eigen-problem for the Helmholtz equation inside linear, non-dispersive medium can be written [54]:

$$\hat{\Theta}\vec{H}(\vec{r}) = \left(\frac{\omega^2}{C^2}\right)\vec{H}(\vec{r}), \hat{\Theta} = -\frac{\partial}{\partial x} \frac{1}{\varepsilon(x)} \frac{\partial}{\partial x} \quad (2.1)$$

The equation (2.1) should be read in the following manner: if  $H$  is an eigenfunction of some periodic structure which is defined by periodic dielectric function  $\varepsilon(x)$ , the influence of the operator  $\hat{\Theta}$  upon this function is equal to the multiplication of this function by the constant which is called the eigen-value. In our case, an eigen-value is represented by the expression  $\omega^2/c^2$  which includes frequency  $\omega$  of an eigen-function. Thus, the problem consists of searching for eigen-functions of a specific periodic structure and corresponding eigen-values.

First, let us find the dispersion relation in case of the radiation propagation inside the uniform medium, that is, in a medium without interfaces.

In order to solve the eigen-problem in case of an infinite periodic structure, the Bloch theorem is usually used which will be discussed later in this section. In detail, the Bloch theorem says that the eigen-function inside an infinite periodic medium can be represented in the form of a plane wave multiplied by a periodic function with periodicity of lattice, see in first chapter :

$$H(x) = h(x)e^{(jk_x)} \quad (2.2)$$

Where  $h(x) = h(x + T)$  is a periodic function with periodicity  $T$  of the lattice. However, if we consider the uniform medium where no modulation of the refractive index appears, period of the dielectric function is infinite. Thus, the eigen-function can be represented in the form of a plane wave multiplied by a constant. Substituting the wave eigen-function in this form according to the Helmholtz equation we can obtain the following

$$-\frac{\partial}{\partial x} \frac{1}{\varepsilon(x)} \frac{\partial}{\partial x} h(x)e^{(jk_x)} = \left( \frac{\omega^2}{C^2} \right) h(x) \cdot e^{(jk_x)} \quad (2.3)$$

Taking into account the fact that dielectric function is also constant in case of the uniform medium, we take the derivative and obtain the following relation:

$$\frac{1}{\varepsilon} h \cdot k^2 e^{(jk_x)} = \left( \frac{\omega^2}{C^2} \right) h \cdot e^{(jk_x)} \quad (2.4)$$

This leads to the following expression:

$$k = \frac{n\omega}{c} \quad (2.5)$$

Thus, the dispersion relation inside the uniform medium is represented by linear function. Let us consider the situation when a medium which is periodic in one dimension is taken instead of a uniform medium. The medium is assumed to be uniform within all other dimensions. As it was made previously, the field is represented in the form of Bloch functions (2.2). Substitution to the Helmholtz equation gives us the expression which is very similar to (2.4):

$$-\frac{\partial}{\partial x} \frac{1}{\varepsilon(x)} \frac{\partial}{\partial x} h(x) e^{jk_x} = \left( \frac{\omega^2}{C^2} \right) h(x) e^{jk_x} \quad (2.6)$$

In this expression, however, functions  $h(x)$  and  $\varepsilon(x)$  are not constants anymore, because of periodicity of the permittivity distribution. Therefore, solution of such kind of problems is much more complex than it was in the case of the uniform medium and, consequently, the dispersion relation will take a more complex form.

Typical form of the dispersion relation of 1D PhC in comparison to the dispersion relation of the uniform medium is given in Figure 2.1.

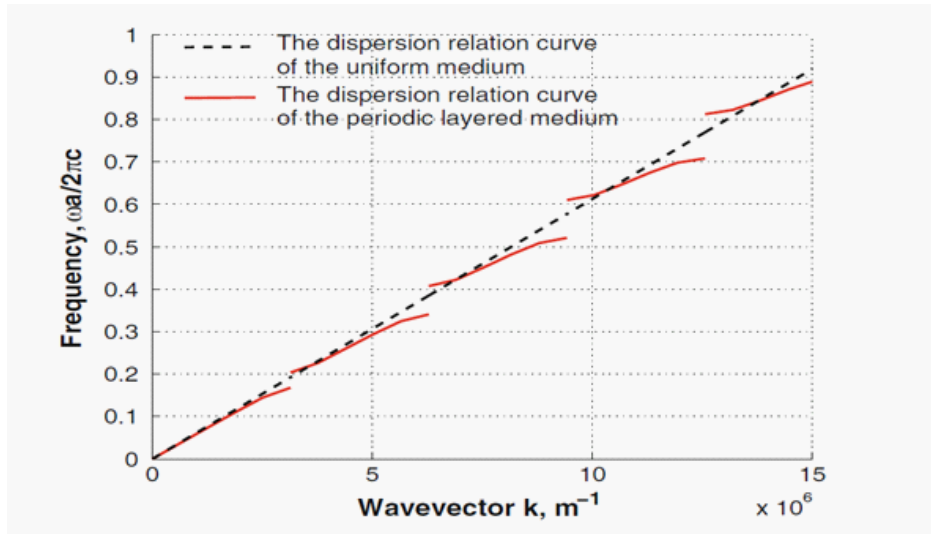


Figure 2.1: Dispersion characteristics of the uniform medium versus periodic one

Frequency gaps which appear within the dispersion relation of the periodic layered structure. Now we have to take into account the complicity of the band structure computation of the PhC.

### Unit Cell

First, let us remind that PhC is presented by an infinite periodic structure. If this is true, the information of a single period is enough to define the whole structure. A section of the PhC which, being translated over all possible dimensions fills a whole space is referred to as unit cell. For instance, if we have 2D PhC, we have two possible dimensions for the translation of the unit cell and, consequently, we are able to fill a whole 2D space with these translated cells.

Unit cell is a spatial domain where each point is closer to the selected base point than to any other neighbour base points. Unit cell of the PhC is defined by its shape and the permittivity or refractive index distribution inside the cell. The shape of the unit cell of a PhC defines the type of the PhC lattice. Determination of the unit cell can be made in several ways. Here, we consider the easiest one (see Figure ??).

The example is given for a case of 2D PhC, because of simple visualization.

However, the technique may be applied to the PhC of any dimensionality:

1. Selecting the base element of the PhC (see Figure 3.2(a)). As the base element, any peculiarity of the permittivity distribution can be used. For instance, if the PhC consists of rods in air, any rod can be selected as the base element.

2. Connecting the point of the base element with corresponding points of neighbouring elements by line segments (see Figure 3.2(b)). If we select the central point of the circular base element as the base point, we should connect it with central points of neighbouring elements.

3. Drawing straight lines (in 2D case) or planes (in 3D case) through the center of each of the previously drawn line segments and perpendicular to them (see Figure 3.2(c)).

4. The figure formed by these lines (in 2D case) or planes (in 3D case) is the unit cell of the PhC and is also known as Wigner–Zeits cell (see Figure 3.2(d)).

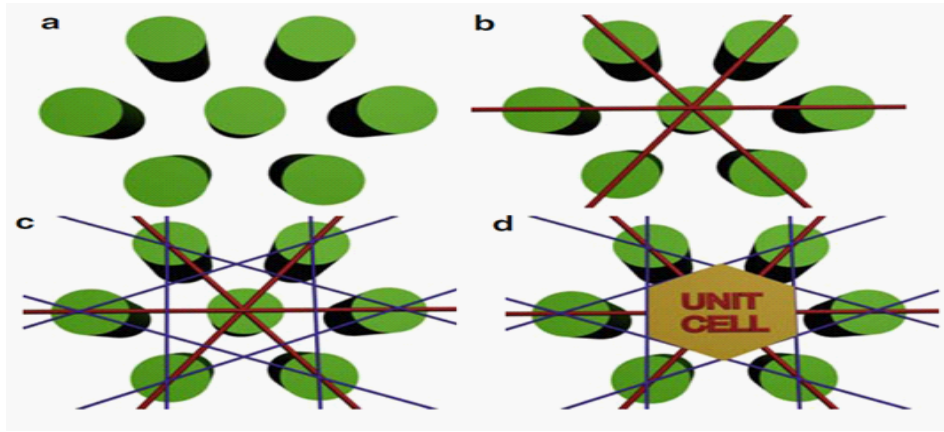


Figure 2.2: Process of the unit cell determination

As it was mentioned above, the method described is not the only possible one to determine the unit cell; thus, there are different possible shapes of the unit cell. The choice of the unit cell shape is stipulated by the simplicity of its mathematical definition[51].

The unit cell of 1D PhC is represented by the line segment with a length of a single

period of the PhC.

### Lattice Vectors

After the unit cell of the PhC is determined and its mathematical description is made, it is necessary to determine and describe the translation rules for this cell. The translation rules are defined by the so-called lattice vectors. The set of lattice vectors should be selected in a way to completely fill the space with the unit cell. On the other hand, we deal with an infinite structure; hence, it is impossible to define an infinite number of lattice vectors. And again we will use the periodicity of the structure to avoid an infinite set of the lattice vectors[40]. We define a set of basis vectors whose number is equal to the number of dimensions allowed. These vectors are referred to as primitive lattice vectors. The shift of the unit cell by a linear combination of primitive lattice vectors translates it to the similar one. Consequently, the lattice vector can be written in the general form :

$$R = la_1 + ma_2 + na_3 \quad (2.7)$$

Where  $l, m, n$  integers and  $a_1, a_2, a_3$  are primitive lattice vectors. The expression (2.7) is written in the most general form for 3D lattice. 2D and 1D PhC require two and one primitive lattice vectors, correspondingly[51].

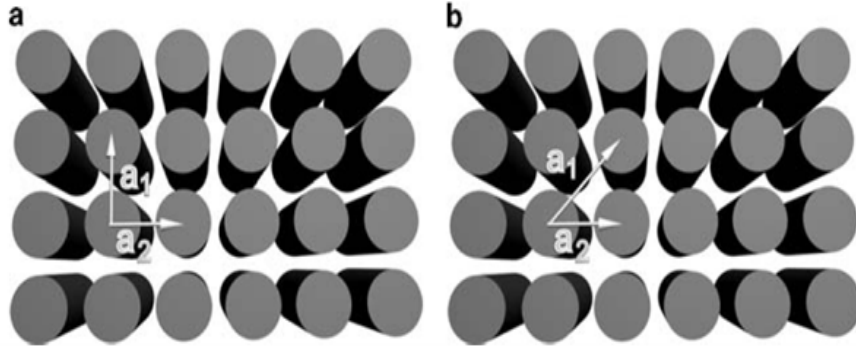


Figure 2.3: Possible choice of primitive lattice vectors

The choice of the lattice vectors as well as the choice of the unit cell is not standardized. The examples of different sets of primitive lattice vectors for the PhC with square lattice are shown in Figure 2.3. The only requirement is the possibility of the unit cell translation. Thus, we have the mathematical description of the PhC lattice, which can be used for computation of the PhC band structure.

### Reciprocal Lattice

In the theory of PhCs we can proceed in a similar way. When the work with dielectric function in coordinate representation (as a dependence of coordinate), the function appears to be an infinite one. However, when represent the function as a dependence of the wave vectors, that is, dealing with wave vector representation, we have some decaying function instead, which can be truncated at some point to obtain an approximate solution. Moreover, previously we defined the problem as searching for the dependence of eigen-values on the radiation wave vectors so that the wave vector representation will be very convenient to use[10].

Thus, in order to derive the dielectric function in wave vector representation, we will expand it over basis functions using the integral transform. In the most basic case, the basic functions take form of plane waves with frequencies which are proportional to the length of the wave vector:

$$\varepsilon(r) = \int g(k) e^{(j.k.r)} dk \quad (2.8)$$

Where  $g(k)$  is the dielectric function in wave vectors representation and  $k$  is the wave vector. Actually, such an expansion can be carried out for the basic functions of any form. However, in the simplest case the system of the basic functions is represented by a set of plane waves. Such an integral transform is referred to as Fourier transform. It is usually used to obtain the spectrum of some function dependent on time or coordinate. Now we write down the periodicity condition which says that the values of the function are repeated with some period  $R$ :

$$\varepsilon(r) = \varepsilon(r + R) \quad (2.9)$$

Substituting this condition into the expression (2.8) we obtains:

$$\varepsilon(r) = \int g(k) e^{(j.k.r)} dk \quad (2.10)$$

$$\varepsilon(r + R) = \int g(k) e^{(j.k.r)} . e^{(j.R.r)} dk \quad (2.11)$$

This condition is met when  $g(k) = 0$  or  $e^{(j.k.R)} = 1$  only. In other words,  $g(k) = 0$  everywhere except those values of  $k$  which satisfy condition  $e^{(j.k.R)} = 1$  for any lattice vector  $R$ [28][59].

Thus, when synthesizing the periodic function of the plane waves, it is necessary to use only those wave vectors which satisfy the condition  $e^{(j.k.R)} = 1$  for all lattice vectors  $R$ . There is a simple analogy in 1D case. If we synthesize the function with period  $\tau$

of harmonic functions, we have to use only fundamental ones in which period equals  $\tau, \tau/2, \tau/3$  and so on. Those wave vectors  $k$  which satisfy the condition  $e^{(j.k.R)} = 1$  or  $k.R = 2n\pi$ , where  $n$  are integers, are called reciprocal lattice vectors and usually designated as  $G$ . These vectors form their own periodic lattice. As far as we found that we have non-zero amplitudes of plane waves only for those ones which argument is multiple of a reciprocal lattice vector, we can write down the sum of plane waves over reciprocal lattice vector instead of the integral (2.8):

$$\varepsilon(r) = \sum_G g_G \cdot e^{(j.G.r)} \quad (2.12)$$

where  $g_G$  are coefficients at correspondent plane waves. Thus, instead of the Fourier transform we obtained the expansion to the Fourier series of the dielectric function.

### Reciprocal Lattice Vectors and Brillouin Zone

In the previous section we have defined conditions which determine exactly the set of reciprocal lattice vectors. It can be evaluated via the lattice vector of the PhC by means of the relation:

$$G.R = 2n\pi \quad (2.13)$$

Now, will be representing the reciprocal lattice vector in terms of basis vector as we made for lattice vectors:

$$G = lb_1 + mb_2 + nb_3 \quad (2.14)$$

Where  $b_1, b_2, b_3$  are basis of primitive reciprocal lattice vectors. Substituting (2.14) and (2.7) into (2.13) we obtain:

$$G.R = (l'b_1 + m'b_2 + n'b_3)(la_1 + ma_2 + na_3) = N.2\pi \quad (2.15)$$

Taking advantage of the rule of scalar multiplication, we obtain the following relation:

$$a_i.b_j = \delta_{ij}2\pi \quad (2.16)$$

Where  $\delta_{ij}$  is Kronecker delta. It is equal to 1 when  $i = j$  and 0 otherwise. Using the relation (2.15) we obtain the expressions for primitive reciprocal lattice vectors:

$$b_1 = 2\pi \frac{a_2 \times a_3}{a_1 \cdot a_2 \times a_3} \quad (2.17)$$

$$b_2 = 2\pi \frac{a_3 \times a_1}{a_1 \cdot a_2 \times a_3} \quad (2.18)$$

$$b_3 = 2\pi \frac{a_1 \times a_2}{a_1 \cdot a_2 \times a_3} \quad (2.19)$$

The process of the Brillouin zone determination consists of several steps:

1. Determination of the unit cell of the PhC.
2. Determination of primitive lattice vectors.
3. Computation of the primitive reciprocal lattice vectors by formulas (2.17, 2.18, 2.19).
4. Building several base points of reciprocal lattice from known reciprocal lattice vectors (see Figure 2.4(a)).
5. Selecting the base element of the PhC.
6. Connecting the point of the base element with corresponding points of neighbour elements by line segments (see Figure 2.4(b)).
7. Drawing straight lines or planes (in 3D case) through the center of each of the previously drawn line segments and perpendicular to them.
8. The figure formed by these lines or planes (in 3D case) is the unit cell of the reciprocal lattice is the Brillouin zone (see Figure 2.4(c)).

As you see, the scheme is very similar to the one we used to determine the unit cell. Actually, the Brillouin zone is the Wigner–Zeits cell in the reciprocal space. Figure 2.4, schematically depicts the determination process of the Brillouin zone of the 2D PhC with hexagonal lattice.

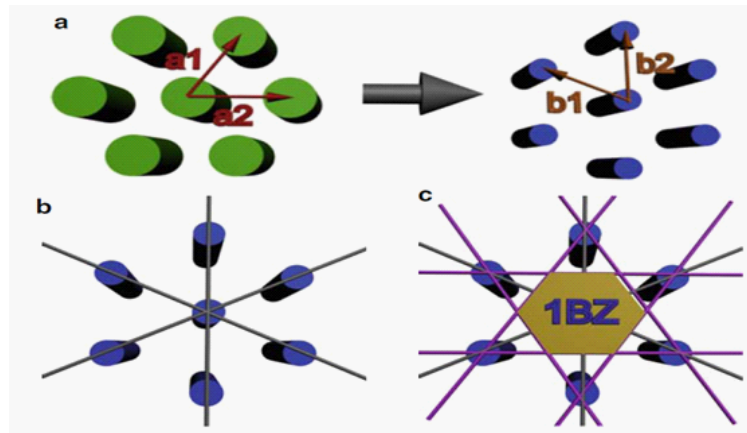


Figure 2.4: The scheme of the Brillouin zone selection for 2D PhC Brillouin zone

Reciprocal lattice and the Brillouin zone play an important role in the computation process. It allows obtaining the limits of the variation of the wave vectors where computation of the eigen-states will be carried out [65], [9].

### Band Structure Computation

In order to compute the band structure of 1D PhC, it is necessary to solve the eigen-problem for the 1D Helmholtz equation. The form of the Helmholtz equation for the electric field. However, computation of the band structure is more convenient to carry out using the equation for magnetic field component. The equation in this case takes the following form:

$$\frac{\partial}{\partial x} \frac{1}{\varepsilon(x)} \frac{\partial}{\partial x} h(x) e^{(ik_x)} + \left( \frac{\omega^2}{C^2} \right) h(x) e^{(jk_x)} = 0 \quad (2.20)$$

In order to solve the eigen-value equation (2.20), some approximate methods should be used which employ the periodicity of the permittivity distribution. For instance, we know that the eigen-function of an infinite periodic structure will also be infinite and periodic. That's why the Bloch theorem should be used for representation of the eigen-function of the PhC. The Bloch theorem says that eigen-function of an infinite periodic structure can be represented in the form of plane wave multiplied by some kind of periodic function with periodicity of lattice. Such a function has the following form :

$$H(x) = h_{(k,n)}(x) e^{(j.k.x)} \quad (2.21)$$

where  $h_{(k,n)}(x)$  is the periodic function with periodicity of lattice, which corresponds to wave vector  $k$  and the state number  $n$ .

As mentioned earlier, the computation of infinite functions inside the equations is impossible, so the function (2.21) to Fourier series by reciprocal lattice vectors must be expanded

$$H(x) = \sum_G h_{(k,n)}(G) e^{(j.(k+G).x)} \quad (2.22)$$

where  $h_{(k,n)}(G)$  is previously defined periodic function in representation of wave vectors. It is also the Fourier expansion coefficient.

The dielectric function is a periodic function as well, so it should also be expanded into Fourier series. However, for convenience of further operations, we will expand the Fourier series not permittivity but the inverted dielectric function as it is written down in (2.20):

$$\frac{1}{\varepsilon(x)} = \sum_{G'' \in G} \chi(G'') \cdot e^{(j \cdot G'' \cdot x)} \quad (2.23)$$

Where  $\chi(G'')$  are Fourier expansion coefficients of the inverted dielectric function.

After the expansion of all the infinite functions we substitute them in equation (2.20):

$$\sum_{G''} \frac{\partial}{\partial x} \chi(G'') \cdot e^{(j \cdot G'' \cdot x)} \frac{\partial}{\partial x} \sum_{G'} h_{(k,n)}(G') \cdot e^{(j \cdot (k+G') \cdot x)} + \left( \frac{\omega^2}{C^2} \right) \sum_G h_{(k,n)}(G) \cdot e^{(j \cdot (k+G) \cdot x)} = 0 \quad (2.24)$$

Taking into account that  $G = G' + G''$  [3], we reduce the first summand of the expression to a common summation index[3]:

$$\sum_G \sum_{G'} \frac{\partial}{\partial x} \chi(G-G') \cdot e^{(j \cdot (G-G') \cdot x)} h_{(k,n)}(G') \cdot \frac{\partial}{\partial x} e^{(j \cdot (k+G') \cdot x)} + \left( \frac{\omega^2}{C^2} \right) \sum_G h_{(k,n)}(G) \cdot e^{(j \cdot (k+G) \cdot x)} = 0 \quad (2.25)$$

Then, taking derivatives and combining the exponents we obtain:

$$\sum_G \sum_{G'} \chi(G-G') \cdot h_{(k,n)}(G') \cdot e^{(j \cdot (k+G) \cdot x)} \times \left( j \cdot (k+G') \cdot j \cdot (k+G) \right) + \left( \frac{\omega^2}{C^2} \right) \sum_G h_{(k,n)}(G) \cdot e^{(j \cdot (k+G) \cdot x)} = 0 \quad (2.26)$$

The projection to the basis  $e^{(j \cdot (k+G) \cdot x)}$  gives the equation:

$$-\sum_{G'} \chi(G-G') \cdot \left( (k+G') \cdot (k+G) \right) \cdot h_{(k,n)}(G') + \left( \frac{\omega^2}{C^2} \right) h_{(k,n)}(G) = 0 \quad (2.27)$$

which is known as “Master equation” for 1D PhC[51], [35].

Main peculiarity of the equation (2.27) is the absence of the coordinate dependence. All the variables depend on the reciprocal lattice vector only that is a consequence of the theory of representations which was given above. Moreover, it should be mentioned that instead of one differential equation containing infinite functions we have the system with an infinite number of linear equations and infinite number of unknowns. The information about the permittivity distribution inside the structure is now given in the form of Fourier expansion coefficients of corresponding function.

In spite of the fact that we have an infinite number of linear equations, it can be truncated to some quite low number of equations depending on the accuracy required. This is the crucial moment of the plane wave’s expansion method. The differential

operator in equation (2.27) is presented in the form of a matrix whose element can be found from the following expression:

$$\hat{\theta}_{G,G'} = \chi(G - G')((k + G') \cdot (k + G)) \quad (2.28)$$

### Computation Results

The result of the eigen-problem solution for 1D PhC is usually presented as the band structure like the one shown in Figure 2.5.

On the x-axis of the band structure of 1D PhC lies the wave vector. On the y-axis lies the so-called normalized frequency  $\omega a / 2\pi c$ . It is convenient to use normalized frequency. In the considered case, the frequency is normalized by the period of the PhC so it does not depend on the scale of the structure. The period of the PhC may be of order of either microns or millimetres, but the view of the band structure remains unchanged due to the employment of the normalized frequency while the layer thickness ratio and their permittivity's are kept constant.

Let us now consider the band structure in detail. The eigen-frequencies of the PhC start from zero frequency at a point  $k = 0$  except the case of so-called off-axis propagation when the angle between the radiation propagation direction and the medium interface differs from  $90^\circ$ . This case will be considered in detail in the final section of this part. Higher on the frequency axis the photonic band gap (PBG) appears. The PhC does not have eigen-states within the PBG. The bands and PBGs appear one-by-one on the frequency axis, except those cases when the PBG width is equal to zero. In the case of 1D PhCs, the PBG appears between almost each pair of the bands. In case of 2D and 3D PhCs the PBG appears much more rear.

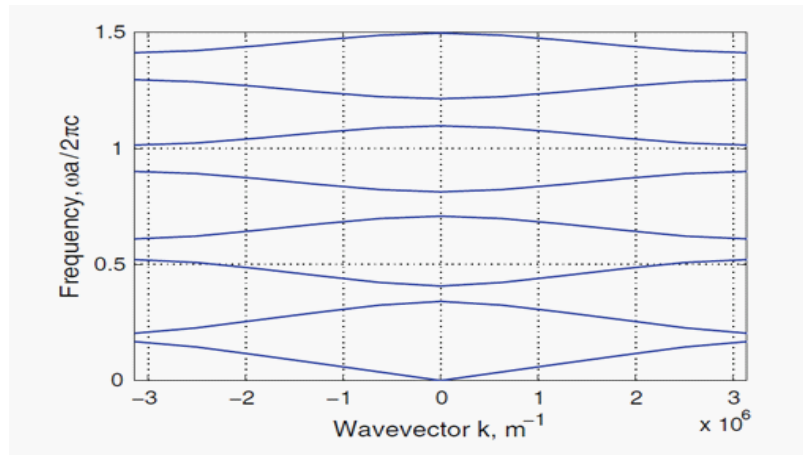


Figure 2.5: Typical band structure of 1D PhC

**Algorithm of PWE Method:**

Concluding the chapter dedicated to the band structure computation of 1D PhC, the whole computation process can be described as a consequence of operations:

1. Writing down the expression for the computation of Fourier expansion coefficients.
2. Limiting the variation of wave vector within the Brillouin zone  $k = (-\pi)/T \dots \pi/T$
3. Sets of  $G$  and  $G'$  vary within the limits of  $(-2\pi)/T \dots 2\pi/T$ , where  $2N + 1$  is the number of plane waves taken into account.
4. According to formulas (2.27) writing down the matrix differential operator for each wave vector within the selected range and computing the eigen-states of obtained matrix.

**Off-Axis Band Structure of 1D PhC**

Before this moment we considered only normal incidence of the radiation to medium interface, that is, pure 1D case. However, those situations where the radiation incidents the structure under some angle different from  $90^\circ$  may be interesting from the point of view of the devices design. The example of such propagation is shown in Figure. 2.6.

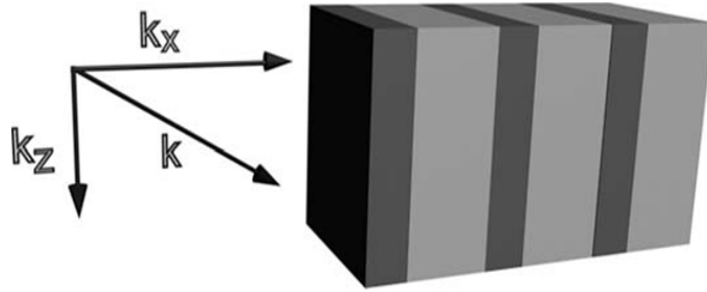


Figure 2.6: Off-axis radiation propagation inside 1D PhC

So what is the difference of such a case in comparison to pure 1D case ? It consists of additional component of the radiation wave vector which ought to be taken into account when composing matrix differential operator. Now the matrix element has the following form:

$$\hat{\theta}_{G_x, G'_x} = \chi(G_x - G'_x)((k_{x,z} + G'_x) \cdot (k_{x,z} + G_x)) \quad (2.29)$$

where  $k_{x,z} = k_x + k_z$  stands for total wave vector including in-axis and off-axis component while  $G_x$  and  $G'_x$  denote in-axis reciprocal lattice vector. Pay attention

that this time these variables are vectors in contrast to the Eq. 2.28 where we had pure 1D case and therefore scalar variables could be used.

This expression shows that in case of the off-axis propagation, when the component  $k_z$  is non-zero, total wave vector is non-zero as well independent on the value of  $k_x$ . Consequently, minimum eigen-frequency will no longer be zero as it was previously. This will lead to frequency shift of the whole band structure and thinning all the bands. At some quite large values of the off-axis wave vector component  $k_z$ , the variation of  $k_x$  within the limits of Brillouin zone will be negligible and therefore the bands will degenerate to a thin level and the band structure will take the form of a system of discrete levels. When computing the band structure for different values of the radiation incidence (which means just varying  $k_z$  component of wave vector) the reduced band structure will have the view presented in Figure 2.7.

Figure 2.7 (a) depicts the computation result of the reduced off-axis band structure of 1D PhC *TE* polarization and Figure 2.7 (b) the band structure for *TM* polarization. The process of the computation of reduced band structure consists of multiple computation of the band structure at different values of some parameter. Then, only the information about the complete PBG is taken and is drawn as a dependent on the parameter value. Here, the reduced band structure depicts presence of complete PBG at different angles of the radiation propagation. Here, dark areas correspond to the range of frequencies with eigen-states and light areas indicate the PBG[7].

One important fact should be mentioned. The behavior of the reduced off-axis band structure strongly depends on the radiation polarization. In case of *TE* polarization the growth of the off-axis wave vector component

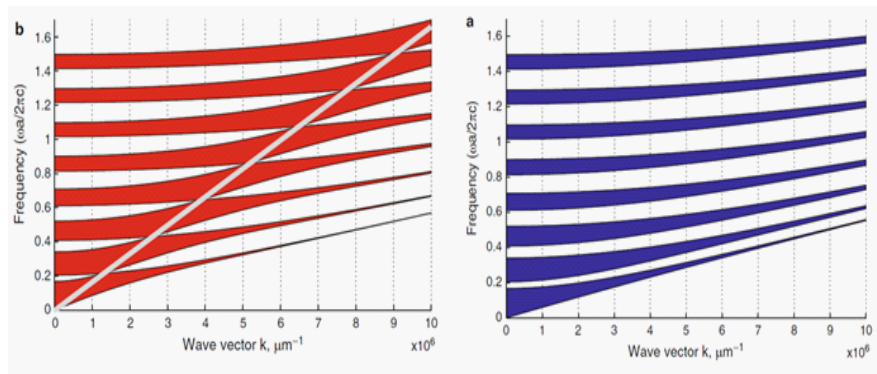


Figure 2.7: Off-axis band structure of the 1D PhC. (a) *TM* polarization, (b) *TE* polarization

results in growth of PBGs while in case of *TM* polarization there are some kind of waists at some specific values of frequency and wave vector. At that,  $k_z$  possesses the

value of:

$$k_z = \left(\frac{\omega}{c}\right) \cdot n_2 \cdot \sin(\theta_B) \quad (2.30)$$

where  $\theta_B$  is the Brewster angle. When the radiation of *TM* polarization incidents the interface under this angle, the Fresnel reflection is absent. Consequently, the radiation in the neighboring layers appears to be unbound so the reflection from the whole structure is absent as well. The straight line in Fig. 2.9(b) indicates values of frequency and off-axis component of the wave vector which satisfy the condition (2.45).

## 2.2.2 Analysis and modeling of 2D and 3D Photonic Crystals

### 3D Photonic Crystals

Since 3D PhCs possess periodicity in three directions, their band structure computation will be considered first and then the method for 2D PhCs will be discussed which allows some approaches and simplifications such as polarization separation so that it can be derived from one for 3D PhC[14].

Just as in the case of 1D photonic crystal, computation of the band structure requires the Helmholtz equations obtained from the Maxwell's equations. Then, it is necessary to determine the range of wave vectors variation to compute the Fourier expansion coefficients of the dielectric function[48].

The derivation of the Helmholtz 'equation, is the same as for 1D case. However, 3D case implies the permittivity variation in all possible directions. Thus, the Helmholtz equations can be written down in the following form:

$$\frac{1}{\varepsilon(r)} \nabla \times \{\nabla \times E(r)\} = \frac{\omega^2}{c^2} E(r) \quad (2.31)$$

$$\nabla \times \left\{ \frac{1}{\varepsilon(r)} \nabla \times H(r) \right\} = \frac{\omega^2}{c^2} H(r) \quad (2.32)$$

Where  $r$  is a 3D vector in coordinate space.

Since we are searching for eigen-states of infinite periodic structure, spatial distribution of the field components  $E$  and  $H$  may be represented in the form of Bloch functions, namely, the plane waves multiplied by periodic function with periodicity of lattice :

$$E(r) = E_{kn}(r) e^{j.k.r} \quad (2.33)$$

$$H(r) = H_{kn}(r) e^{j.k.r} \quad (2.34)$$

where  $E_{kn}$  and  $H_{kn}$  are periodic functions with periodicity of lattice. Indices indicate that the periodic function is different for each wave vector  $k$  and number of eigen-state  $n$ . Since the functions are periodic ones, they satisfy the following conditions:

$$E(r + R) = E_{kn}(r) e^{j.k.r} \quad (2.35)$$

$$H(r + R) = H_{kn}(r) e^{j.k.r} \quad (2.36)$$

where  $R$  is the lattice vector.

When apply the Bloch theorem to represent the wave functions, and constrain the shape of these functions within some limits. The shape is determined by the periodicity of the medium. Thus, instead of an arbitrary function shape, now there are some periodic functions with periodicity of the photonic crystal lattice. These functions which are represented in form of Bloch waves (2.35, 2.36), also depend on the wave vector  $k$  and on the number of eigen-state  $n$ .

As in case of 1D photonic crystal, direct solution of Helmholtz equation for infinite periodic structure is impossible. However, periodicity of wave functions (2.35, 2.36) leads to possibility of their Fourier expansion over reciprocal lattice vectors. Thus, now can be representing the wave function in wave vectors space instead of coordinate space:

$$E_{kn}(r) = \sum_G E'_{kn}(G) e^{j.(k+G).r} \quad (2.37)$$

$$H_{kn}(r) = \sum_G H'_{kn}(G) e^{j.(k+G).r} \quad (2.38)$$

where  $G$  is the reciprocal lattice vector.

By analogy with above mentioned transformation, the dielectric function can also be expanded to Fourier series due to the periodicity:

$$\frac{1}{\varepsilon(r)} = \sum_G \chi(G) e^{j.G.r} \quad (2.39)$$

where  $\chi(G)$  are Fourier expansion coefficients which depend on the reciprocal lattice vectors. Thus, employing the basic functions  $e^{j.G.r}$ , we made the transformation of the wave functions from the coordinate representation to the wave vectors representation[48]. The wave functions within wave vectors representation are written down as functions of the discrete argument  $G$  in the form of  $E'_{kn}(G)$  and  $H'_{kn}(G)$  for

electric and magnetic field components, respectively[49]. The wave vector representation of the dielectric function has also discrete argument and is written in the form of  $\chi(G)$ . Thus, instead of coordinate dependencies of the electromagnetic field distribution, we will use their Fourier expansion coefficients in wave vector representation. Substituting (2.37, 2.38) and (2.39) into (2.31, 2.32), and performing the simplification procedure described in first case (1D), we obtain the following eigen-value equations for the Fourier expansion coefficients of electric and magnetic fields :

$$-\sum_{G'} \chi(G - G') (k + G') \times \left\{ (k + G') \times E_{kn}(G') \right\} = \frac{\omega_{kn}^{(E)^2}}{c^2} E_{kn}(G) \quad (2.40)$$

$$-\sum_{G'} \chi(G - G') (k + G) \times \left\{ (k + G') \times H_{kn}(G') \right\} = \frac{\omega_{kn}^{(H)^2}}{c^2} H_{kn}(G) \quad (2.41)$$

These equations represent so called “*Master equations*” for 3D PhC. Their solutions are the eigen-states of the photonic crystal.

It is necessary to pay attention to the convergence of the PWE method for 3D case. The problem appears when one needs to compute the band structure for such a 3D photonic crystal because of its complexity. Computation of the Fourier expansion coefficients for 3D case the computation takes a lot of time.

The convergence of the method can be greatly improved when using the alternative systems of basis functions. For instance, the spherical waves basis can be used for 3D PhC represented by periodically placed spheres. This method is called the spherical wave expansion method or vectorial Korringa–Kohn–Rostoker (KKR) method. Though the application of such a method is limited by the spherical or cylindrical structures, the convergence of the KKR method is much better as compared to PWE method.

## 2D Photonic Crystals

When one investigates the light propagation in some 2D PhC which is formed by the system of rods with the generatrix along z-axis, two different forms of reduced Maxwell’s equations can be considered, which correspond to different radiation polarizations. At that, the radiation can be separated to *TE* and *TM*-polarizations (*H* and *E*, respectively) depending on which field vector has the component perpendicular to the plane of photonic crystal. The radiation has *TE* or *H*-polarization when it has nonzero  $H_z$ ,  $E_x$  and  $E_y$  components of the field. At that, the Maxwell’s equations can be reduced to the following form :

$$\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = -\frac{1}{c} \frac{\partial}{\partial t} B_z \quad (2.42)$$

$$\frac{\partial}{\partial y} H_z = \frac{1}{c} \frac{\partial}{\partial t} D_x \quad (2.43)$$

$$-\frac{\partial}{\partial x} H_z = \frac{1}{c} \frac{\partial}{\partial t} D_y \quad (2.44)$$

The radiation which has nonzero  $E_z$ ,  $H_x$  and  $H_y$  field components is referred to as  $TM$  or  $E$ -polarized radiation. The Maxwell's equations for  $TM$  polarization differ from the ones for  $TE$  polarization and take the following form:

$$\frac{\partial}{\partial y} E_z = -\frac{1}{c} \frac{\partial}{\partial t} B_x \quad (2.45)$$

$$-\frac{\partial}{\partial x} E_z = -\frac{1}{c} \frac{\partial}{\partial t} B_y \quad (2.46)$$

$$\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x = \frac{1}{c} \frac{\partial}{\partial t} D_z \quad (2.47)$$

The above given definition of  $TE$  and  $TM$  polarization is commonly used and is described in a lot of books dedicated to PhC physics. However, in some computation tools such as *RSoft*, the opposite  $TE$  and  $TM$  polarization definition is used in order to make the terms agree with other software modules.

In these cases, the  $TE$  polarization corresponds to the radiation with nonzero.

$H_z$ ,  $E_x$  and  $E_y$  components and the term  $TM$  polarization is used for the field with nonzero  $E_z$ ,  $H_x$  and  $H_y$ . Thus, in view of ambiguity of terminology, it is necessary to pay attention to definition of polarization used when reading publications or using computation tools[10], [4].

The Helmholtz equations obtained for  $TE$  and  $TM$  polarizations from Maxwell's equations (2.42, 2.43, 2.44) and (2.45, 2.46, 2.47) have some differences as compared to those for 3D PhC. Helmholtz equations for 2D case take the following form:

$$-\left\{ \frac{\partial}{\partial x} \frac{1}{\varepsilon(r_{\parallel})} \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \frac{1}{\varepsilon(r_{\parallel})} \frac{\partial}{\partial y} \right\} H_z(r_{\parallel}) = \frac{\omega^2}{c^2} H_z(r_{\parallel}) \quad (2.48)$$

$$-\frac{1}{\varepsilon(r_{\parallel})} \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right\} E_z(r_{\parallel}) = \frac{\omega^2}{c^2} E_z(r_{\parallel}) \quad (2.49)$$

Here  $r_{\parallel}$  is 2D vector in coordinate space which lies in plane of 2D PhC (in our case, in plane  $(X, Y)$ ). At that, we have in mind that permittivity variation of the PhC in  $Z$  direction as well as the variation of the electro-magnetic field is absent.

Subsequent band structure computation process is very similar to that one for 1D and 3D PhCs considered previously. First, we present the wave functions in the form of Bloch waves and expand them into the Fourier series over the reciprocal lattice vectors. Then we also expand the inversed dielectric function into Fourier series. Substituting these expressions into Helmholtz equations (2.48, 2.49) we obtain the eigen-value equations for the Fourier expansion coefficients:

$$\sum_{G_{\parallel}} \chi \left( G_{\parallel} - G'_{\parallel} \right) \left| k_{\parallel} + G'_{\parallel} \right|^2 E_{z, k_{\parallel} n} \left( G'_{\parallel} \right) = \frac{\omega_{k_{\parallel} n}^{(E)^2}}{c^2} E_{z, k_{\parallel} n} \left( G_{\parallel} \right) \quad (2.50)$$

$$\sum_{G_{\parallel}} \chi \left( G_{\parallel} - G'_{\parallel} \right) \left( k_{\parallel} + G_{\parallel} \right) \left( k_{\parallel} + G'_{\parallel} \right) \times H_{z, k_{\parallel} n} \left( G'_{\parallel} \right) = \frac{\omega_{k_{\parallel} n}^{(H)^2}}{c^2} H_{z, k_{\parallel} n} \left( G_{\parallel} \right) \quad (2.51)$$

which is “*Master equation*” for 2D PhC.

Here  $G_{\parallel}$  and  $G'_{\parallel}$  are in-plane reciprocal lattice vectors,  $k_{\parallel}$  is in-plane wave vector, and  $\omega_{k_{\parallel} n}^{(E)}$  and  $\omega_{k_{\parallel} n}^{(H)}$  are eigen-frequencies for *TM* and *TE* polarization, respectively (in case of 2D PhC the values of these frequencies are different)[49].

The complexity of subsequent computation process consists mainly in definition of the Brillouin zone, discovering the set of wave vectors the computation will be carried out for and expansion of dielectric function into Fourier series.

In the simplest cases of the PhC configuration, like periodically arranged dielectric rods in 2D PhC or dielectric spheres in 3D case, it is possible to obtain analytical expression for the Fourier expansion coefficients. However, if the structure slightly differs from the strictly periodic one, analytical expression of the Fourier coefficients are impossible to obtain. In such a case, numerical method should be involved.

### Wave Vectors Set

When considered the band structure computation procedure of 1D PhC, variation of the wave vector was limited by the first Brillouin zone. However, due to one-dimensionality of the Brillouin zone, the wave vectors set selection was not the problem. In case of 2D and 3D PhC, variation limits of the wave vectors are still the same and are determined by the first Brillouin zone. However, in contrast to 1D case, the computation of the band structure for all of points inside the Brillouin zone can take a lot of time and is hard to analyze. Instead of this, the high-symmetry points of the Brillouin zone are only considered, which are determined by the group theory. Briefly, the high-symmetry points are those points of the Brillouin zone, which are translated into themselves when rotating Brillouin by  $90^\circ$ ,  $180^\circ$ ,  $30^\circ$  or  $60^\circ$  depending

on the PhC lattice. It is supposed that the band structure is extreme in these points only and between these points it monotonically decreases or increases. Computation of the band structure usually begins from the center of the Brillouin zone. This point is designated by the Greek letter  $\Gamma$ . In this point, the wave vector equals to zero. Then, computation is carried out for all high-symmetry points and returns to the point  $\Gamma$ . The contour obtained by the connection of high-symmetry points is called  $k$ -path. As a rule, the PhCs with typical lattices have determined  $k$ -path.

The  $k$ -paths for the 2D PhC with square and hexagonal lattices are shown in Figure 2.8(a) and (b), respectively.

For 3D PhC there are a large number of PhC lattice types and the  $k$ -paths. Here, in Figure 2.9 we show the  $k$ -path for the PhC with cubic lattice only. As it is seen, the  $k$ -path has now a quite complicated shape. Like in 2D case it begins and finishes in  $\Gamma$  point.

To achieve a smoother band structure both in 2D and 3D cases, additional eigenstates may be computed for intermediate wave vectors on the  $k$ -path.

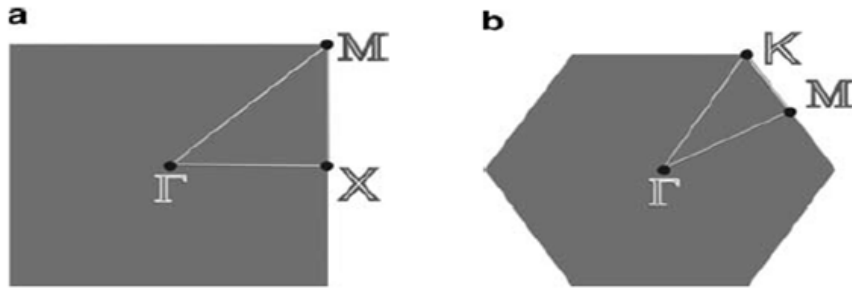


Figure 2.8:  $k$ -Path of the PhC with square (a) and hexagonal (b) lattices

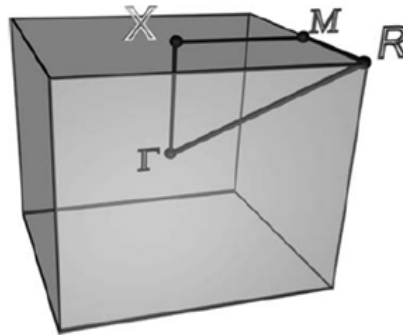


Figure 2.9:  $k$ -Path of the Brillouin zone for the PhC with cubic lattice

### Reciprocal Lattice Vectors Set

The set of the reciprocal lattice vectors should now be selected to provide correct Fourier expansion of the dielectric function and the Bloch functions. When we described the band structure computation of 1D PhC, we pointed out that reciprocal lattice vectors must be selected to form zero-symmetric set. In case of 2D and 3D PhCs, the selection procedure is the same. However, here we have 2D and 3D reciprocal lattice vectors. In this case, the number of the reciprocal lattice vectors in the set is discrete. For instance, for 2D PhC it can take values 1, 9, 25, 49, 81, 121 and so on. The example of the reciprocal lattice vectors set selection is shown in Figure 2.10. In the figure, the vectors which connect point  $(k_x = 0, k_y = 0)$  with corresponding points of neighbor cells of the reciprocal lattice are depicted. These vectors provide translation of the Brillouin zone. Since the number of the reciprocal lattice vectors has to be zero-symmetric, it cannot take an arbitrary number of plane waves such as 80 or 42. Otherwise, this leads to qualitative errors.

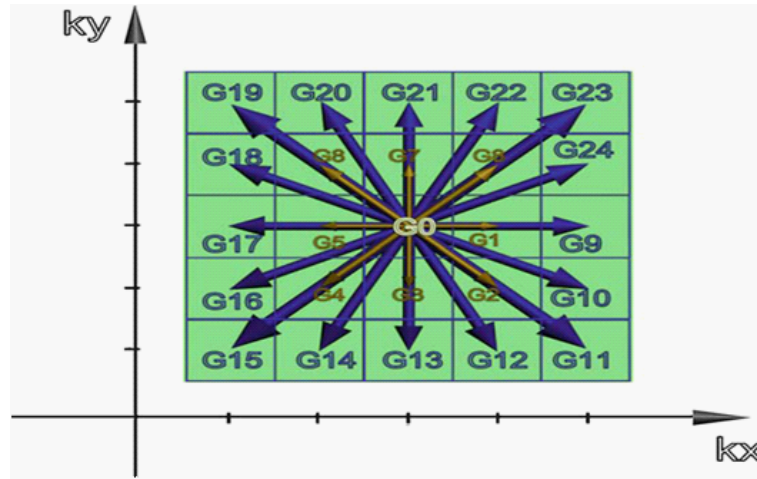


Figure 2.10: Definition of the set of reciprocal lattice vectors

### Results of the Band Structure Computation

It is necessary to pay attention to the method of the band structure plotting. If the band structures are plotted directly, the band structure for 2D PhC will be the surface in 3D coordinate system  $(k_x, k_y, \omega)$  and the band structure for 3D PhC will be represented in 4D coordinate system  $(k_x, k_y, k_z, \omega)$ . This means that we have to plot 3D or even 4D plots. But 3D plots are very hard to analyze and 4D plots are impossible to draw with standard methods.

In order to avoid such problems, the following can be done. In the plot, high symmetry points lie along horizontal axis while frequency lies along another one. In this case, we will have 2D plot independent of the dimensionality of the PhC. Such a plot is easy to analyze and draw. The example of such a plot is given in Figure 2.11.

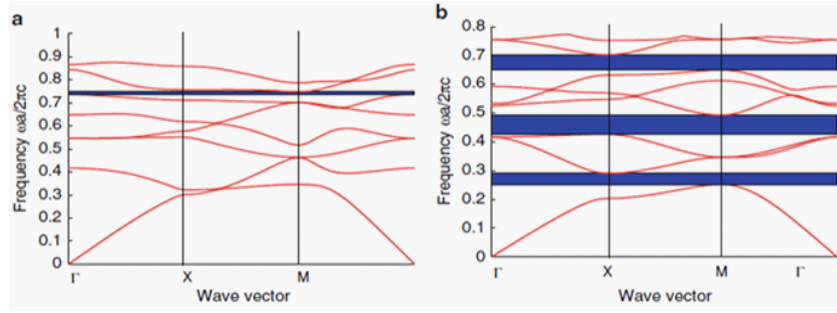


Figure 2.11: Examples of the band structure of rods arranged to square lattice with  $r/a = 0.4$  computed for  $TE$  (a) and  $TM$  (b) polarizations

### Analysis of the Band Structure

Because the wave vector space is connected with the coordinate space via the relations (2.17, 2.18, 2.19), it is quite easy to connect some points of the Brillouin zone with the radiation propagation direction inside the PhC. Thus, we can claim that for each of the directions inside PhC there exist band gap areas as it is shown in Figure 2.12(a). This means that radiation of the given wavelength cannot propagate inside the PhC under some specific angle. Such areas of wavelengths or frequencies are referred to as partial photonic band gaps.

At some specific frequencies, the band gaps are overlapped forming so called complete photonic band gap (*PBG*) (see Figure 2.12(b)). This means that radiation will not propagate inside the PhC under any angle. The PBG of the PhC can be designated in different ways. One of the ways is to specify frequencies where PBG appears. The second way is to specify percentage which is calculated by formula:

$$PBG = \frac{\Delta\omega}{\omega_0} 100\% \quad (2.52)$$

where  $\Delta\omega$  is the half width of the PBG and  $\omega_0$  is its center frequency. Besides, in investigating the band structure, it is possible to obtain the group velocity of the radiation inside the PhC. The group velocity is determined as the first wave vector derivative of the band structure. Consequently, the more flat the band of the PhC, the lower the group velocity at corresponding frequency [63].

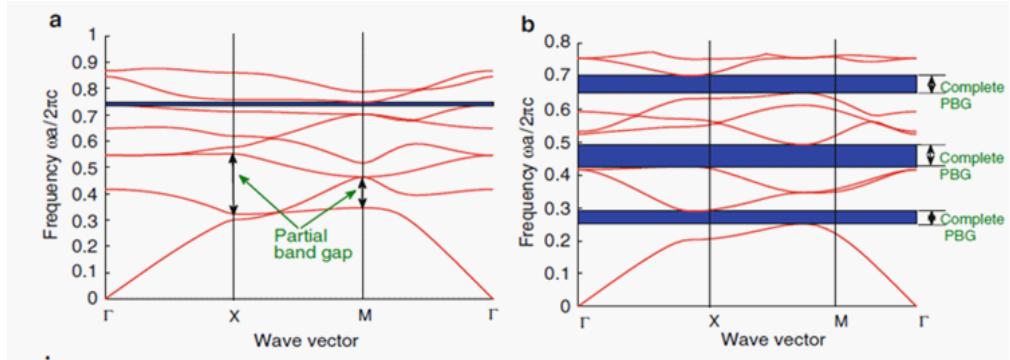


Figure 2.12: Partial (a) and complete (b) photonic band gaps of the PhC

Thus, the band structure of PhC gives the possibility to find and analyze the PBG which is the crucial moment for a number of practical applications and group velocities which allows predicting the radiation propagation conditions in different directions inside the PhC[53].

### Photonic Band Gap Map

When designing a PhC device, it is usually necessary to solve the problem, which aims not to obtain the band structure but to determine PhC parameters from known frequency characteristics. For instance, it is necessary to design a PhC with the PBG at the specific frequency. For this, we need to specify the permittivity's of the PhC materials, the size of elements and kind of the PhC lattice.

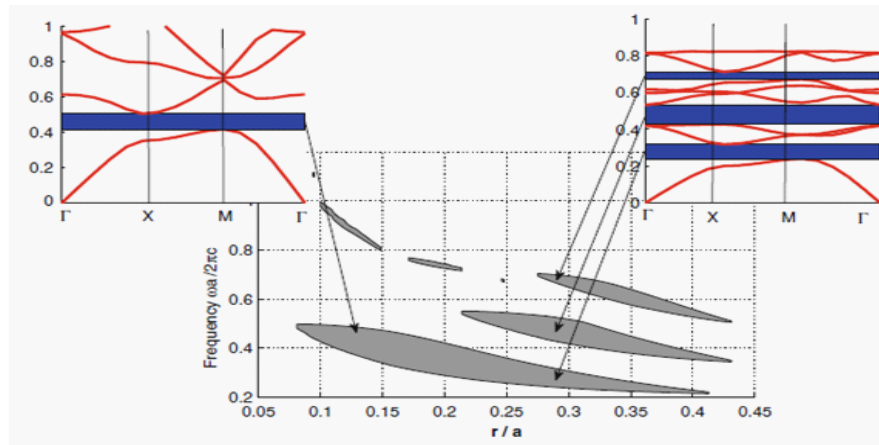


Figure 2.13: Example of the PBG map formation

In order to solve such kind of problems, it is convenient to use so-called PBG

maps also known as reduced band structures. The PBG maps are represented by the projection of the complete PBG obtained at different values of the PhC parameter (refractive index, PhC elements size, etc.). The example of such a PBG map is given in Figure 2.13.

In order to obtain PBG map, computation of the band structure at different values of the PhC parameter is carried out. Then one obtains the information about complete PBG and draws it upon a separate plot as a dependence of parameter's value. This is shown in Figure 2.13. In the figure, the PBG map of the 2D PhC of the rods arranged to square lattice and the initial band structures computed for two values of the PhC elements radius is also shown. Such band structures form the PBG map. In order to build the PBG map, only the information about complete PBG is taken from the band structure and is presented on the plot as a dependence on the radius. When investigating the PBG map in Figure 2.13, it is seen that such a PhC have more than one PBG at some frequency range (e.g., in the range of radius from 0.25 to 0.4). When increasing the permittivity's contrast, the number of PBGs as well as their width will grow.

The PBG maps can be applied to a broad class of problems when designing complex active and passive devices of the PhC basis. The information provided by these maps gives the possibility to determine PhC parameters at which the PhC has maximum reflectance or partially transmits radiation.

### Off-Plane Band Structure for 2D Photonic Crystal

In case of 2D PhC, situations are possible where the radiation propagates not in the plane of the PhC but under some angle (see Figure 2.14). The examples of devices utilizing such kind of propagation are the PhC fibres where the radiation propagates along the PhC elements. In case of the off-plane propagation, the  $k_z$ -component of the wave vector appears, which is perpendicular to the PhC plane. The band structure in this case can be dramatically modified[30].

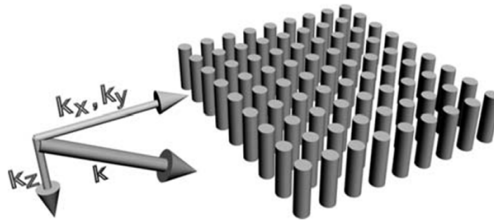


Figure 2.14: Off-plane radiation propagation inside the PhC

Lets us consider the variations of the band structure which appear with non-zero

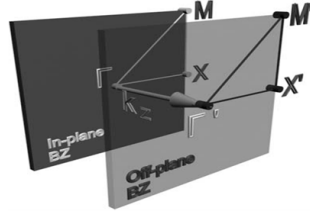


Figure 2.15: The Brillouin zone for computation of the off-plane band structure

$k_z$  component. The band structure computation is carried out over the  $k$ -path which includes high-symmetry points discussed in the last case. However, since the  $k_z$  component of the wave vector is no longer equal to zero, the Brillouin zone appears to be shifted in the  $z$ -direction to the value of  $k_z$  (see the Figure 2.15). At that, even in-point of the total wave vector length is no longer equal to zero. And if previously frequency of the first PhC eigen-state in-point was equal to zero, the situation is now changed so that minimum frequency of the first state now differs from zero. Besides, because  $k_z$  is now non-zero and for some cases its value can be quite large, for example, in PhC fibers, the variation of  $k_x$  and  $k_y$  components will make lower contribution to the variation of the total wave vector length. This leads to interesting effect. The variation of the frequency within a single band will be as lower as larger the  $k_z$  value. Consequently, the band will be more flat than at  $k_z = 0$ .

The computation of the off-plane band structure requires the modification of the Brillouin zone. The modification consists in parallel transition of the Brillouin zone to along the  $z$ -direction by the value of  $k_z$  wave vector component. At that, the shape of the Brillouin zone does not change (see figure 2.15 ). The modification of the Brillouin zone leads to the matrix differential operator modification. The modification consists of the additional component  $k_z$  of the wave vector:

$$\hat{H}_{G,G'} = \chi(G_{x,y} - G'_{x,y})((k_{z,x,y} + G_{x,y}) \cdot (k_{z,x,y} + G'_{x,y})) \quad (2.53)$$

where  $k_{z,x,y} = k_z + k_x + k_y$  is the 3D wave vector. In Figure 2.16 the results of in-plane and off-plane band structure computations are shown. The band structures were computed for the same PhC but for different  $k_z$  values. The obvious difference of obtained band structures is the fact that the off-plane band structure starts from non-zero value of the frequency. Moreover, if we consider bands with the same number (for instance, the first ones), it can be pointed out that the band of the in-plane band structure lies inside the range of frequencies from 0 to 0.24 while for the off-plane structure the limits are from 0.23 to 0.33, that is, it occupies the frequency range which is 2.4 times smaller for in-plane band structure[7].

As a result of the lowering of the frequency range occupied by the separate bands, at some high values of the  $k_z$  the PBGs can appear which are not present when the in-plane propagation is considered. Due to the existence of the PBG at high values of  $k_z$ , it becomes possible for the waveguiding effect appearance which is fundamentally different from the total internal refraction effect used in conventional step-index fiber[65].

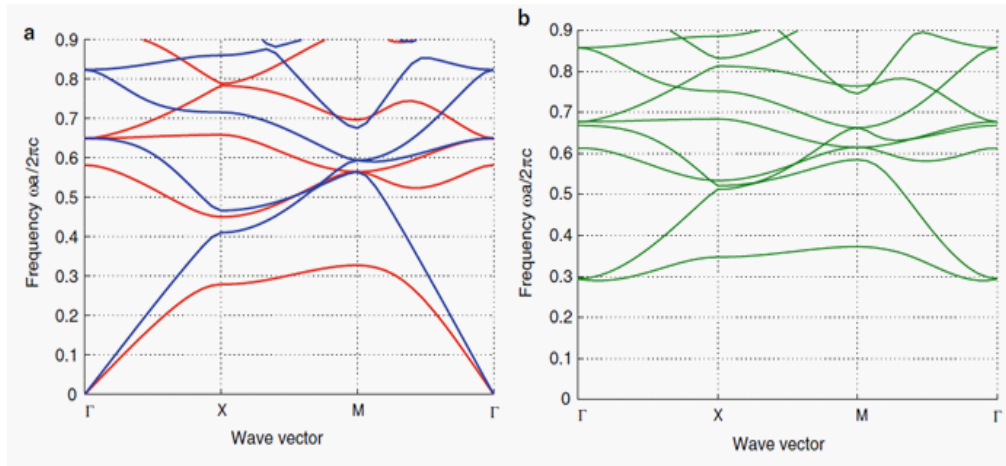


Figure 2.16: In-plane (a) and off-plane (b) band structures

### Band Structure Computation of the PhC with Defect

The defect of the periodic structure means some kind of violation of the PhC strict periodicity. This may be, for instance, missing rod or hole as well as elements displacement. In any case, the optical properties of such kind of structures differ from ones of pure PhC. In the introduction, we mentioned the similarities between solid-state crystal and photonic crystal. One of crucial similarities was connected with defect embedded to the structure. In both solid-state and photonic crystals, the eigen-states appear inside the forbidden energy range which allows the radiation or particles with such energies to exist inside the structure.

Now we will investigate this phenomenon in detail and consider the method for the band structure computation of the PhC with defect[48], [8], [7].

One of the most convenient methods used in this case is the plane wave expansion method, the PWE method deals with strictly periodic structures only. That is, why we have to use artificial method for the structure with defect. At first we must extract the area with defect inside the PhC. The area may be of 5 or 6 elements in each dimension. Then, translate this area over all directions and obtain a new structure

where the selected area forms new unit cell of the PhC. Corresponding lattice vectors are changed as well. After that, compute the band structure of the PhC with new unit cell.

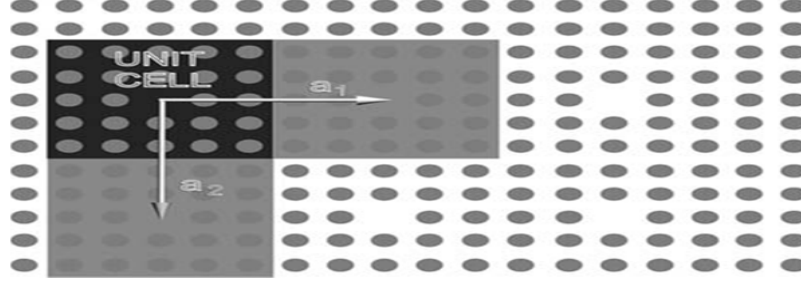


Figure 2.17: Translated defect of the PhC, its unit cell and lattice vectors

The computation procedure now is the following:

1. Determine the unit cell of the PhC containing defect. In our case, the unit cell should be large enough to insulate translated defects from interaction with each other. For this, we should take the unit cell including at least 5 or 6 PhC elements as shown in Figure 2.17. On the other hand, the unit cell should not be too large to avoid long computation time.
2. The range of the wave vectors is still limited by a Brillouin zone.
3. Define the set of the reciprocal lattice vectors. Since the PhC unit cell possesses complex refractive index profile and it has relatively small elements as compared with unit cell size, the number of reciprocal lattice vectors should be larger than that for strictly periodic structure. This allows taking into account more harmonics with higher frequencies to reproduce in detail the refractive index profile.
4. Form the matrix differential operator for each wave vector value within defined set and compute its eigen-values. This step is similar to the one for strictly periodic PhC.

Figure 2.18 shows the band structure of the PhC with defect and corresponding defect less structure. You can see that the eigen-state appears within forbidden frequency range. This means that the radiation possessing this frequency can propagate inside such a PhC.

There is another important detail the attention should be paid to. The eigen-frequency of the defect state does not depend on the radiation wave vector. This means that the defect eigen-frequency is the same for all propagation directions. This property of the defect is very important for modeling of such applications as PhC fibers or PhC waveguides since we may proceed with computation for a single value of the wave vector  $k$ .

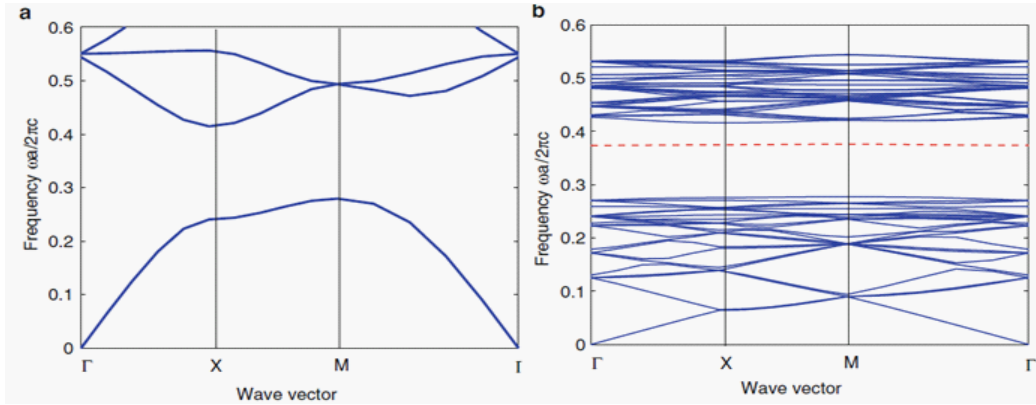


Figure 2.18: The band structure of the PhC defect less structure (a) and with defect (b)

### 2.3 Modeling the Photonic Crystals with FDTD method

Another method to solve Maxwell's equations on a discrete lattice of points in space is the finite-difference time-domain (FDTD) method. This method has a long history and has been used in a variety of applications which are reviewed in the book edited by Taflov. In 1995 Chan et al. were the first to apply this method to compute the band structure of photonic crystals and to prove its reliability in treating periodic structures of high complexity. The FDTD calculations are particularly useful for complicated structures because the memory and processing time requirements scale linearly with the number of grid-points included in the computation, allowing resolution of minute and intricate structures[53]. With the FDTD method an initial field is propagated applying the governing equations in a first-order differential form, both in space and time, at all points in the grid in a succession of time steps. Different types of boundary conditions can be applied including periodic ones, particularly useful for this type of system. The fields at selected points on the grid are finally Fourier-transformed from the time to the frequency domain such that observations on spectral content can be made[58].

### 2.4 Modeling the Photonic Crystals with TM method

The optical theory of thin-films was first presented in 1949 by Schuster. The following year, Abel'es extended it to multilayers and formalized the computing technique called the transfer matrix method (TMM). The basic formalism yields the amplitude of the electromagnetic field of monochromatic waves reflected by and transmitted through the mentioned structure. The solution is achieved through propagation of the

fields in the homogeneous layers, and the continuity of the tangential components of the electric and magnetic fields at the interfaces. Although the structure is one dimensional, propagation for non-normal incidence can be accounted for. The solutions for plane waves are in fact vectors of the three dimensional Euclidean space propagating in a plane. With the exception of approximations in the chosen model, i.e. simplified dimensionality and initial conditions, or neglect of material parameters, this analytical method is exact[4].

## 2.5 conclusion

This chapter is dedicated the detailed description of the plane wave expansion method used for computation of the band structure of 1D photonic crystals. We describe all steps of the band structure computation applied for simplest photonic crystal. After the problem formulation, terms such as unit cell, lattice vector, reciprocal lattice and reciprocal lattice vectors as well as the first Brillouin zone are introduced and their physical and mathematical essence is discussed in detail. Peculiarities of the band structure computation are considered and such crucial problems as Bloch theorem. The described model is also applied to off-axis radiation propagation. At the end of the chapter the example for band structure computation is given. After that we go to more complexity structure of other types of photonic crystals which is 2D and 3D photonic crystals here we used the same steps of the first case.

In the next chapter we will use this method to for simulations of different types of the photonic crystals with different structures and materials

## Chapter 3

# Simulations of the photonic crystals

### 3.1 Introduction

The photonic crystals offer new prospects for the propagation and confinement of light to the realization of new optical components. They offer the ability to filter (eliminate) the propagation of electromagnetic waves in certain frequency ranges (wavelengths). Furthermore, the use of nonlinear optical effects allows direct control of the response of the crystal photonic bandgap. Therefore, reconfigurable optoelectronic components can be developed for applications in optical telecommunications and the use of nonlinear material such as lithium niobate or material with high refractive index like germanium (Ge) can be of great interest for photonic technology.

This chapter is devoted to modeling of photonic bandgap structures in lithium niobate and germanium to determine the optimal parameters for a complete band gap or very wide band gap (for both modes of propagation transverse electric TE and transverse magnetic TM).

It will be presented the results of our simulations for three types of photonic crystals 1D and 2D for the second type we will simulate two types of the unit cell: square and triangular. The theoretical study of photonic band gap is achieved by the method of plane waves depending on the geometry and dimensions of the structure to find the hole diameter and period patterns by varying the ratio of the radius of the period of reasons ( $r/a$ ).

After that we calculated the transmission and reflection of electromagnetic field in our model structures with different dimensions to choose the optimal dimension

The aim of realizing photonic bandgap structures for applications in optical telecommunications  $\lambda = 1.55\mu m$ .

### 3.2 Characteristics of the germanium (*Ge*)material

The Germanium is a chemical element with the symbol *Ge* and atomic number 32. It is a lustrous, hard, grayish-white metalloid in the carbon group, chemically similar to its group neighbors tin and silicon. Purified germanium is a semiconductor, with an appearance most similar to elemental silicon. Like silicon, germanium naturally reacts and forms complexes with oxygen in nature. Unlike silicon, it is too reactive to be found naturally on Earth in the free (native) state[54].

Germanium is a semiconductor. Zone refining techniques have led to the production of crystalline germanium for semiconductors that has an impurity of only one part in 1010, making it one of the purest materials ever obtained[45].

Germanium "metal" (isolated germanium) is used as semiconductor in transistors and various other electronic devices. Historically the first decade of semiconductor electronics were entirely based on germanium, although its production for such use today is a small fraction (2%) of that of ultra-high purity silicon, which has largely replaced it. Germanium's major end uses in the present are fiber-optic systems and infrared optics. It is used in solar cell applications. Germanium compounds are used for polymerization catalysts[46].

The most notable physical characteristics of germania ( $GeO_2$ ) are its high index of refraction and its low optical dispersion. These make it especially useful for wide-angle camera lenses, microscopy, and for the core part of optical fibers. It also replaced titania as the silica dopant for silica fiber, eliminating the need for subsequent heat treatment, which made the fibers brittle. At the end of 2002 the fiber optics industry accounted for 60% of the annual germanium use in the United States, but this use accounts for less than 10% of world wide consumption. GeSbTe is a phase change material used for its optic properties, such as in rewritable DVDs.

Because germanium is transparent in the infrared it is a very important infrared optical material, that can be readily cut and polished into lenses and windows. It is especially used as the front optic in thermal imaging cameras working in the 8 to 14 micron wavelength range for passive thermal imaging and for hot-spot detection in military, night vision system in cars, and fire fighting applications. It is therefore used in infrared spectrometers and other optical equipment which require extremely sensitive infrared detectors. The material has a very high refractive index (4.0)[34].

### 3.3 One-dimensional photonic crystals simulation

As we said before, we begin with one dimensional photonic crystal because it is the simplest possible type of the photonic crystals to see the diagrams of characteristics of the photonic crystals (photonic band gap structures diagram, reflection and de transmission diagrams). We will use the Plane Wave Expansion (*PWE*) for this simulation.

#### 3.3.1 Definition of the parameter

In the first part of this work when we use the germanium ( $Ge : \varepsilon = 12; n = 4$ ) material and air ( $\varepsilon = 1; n = 1$ ), band gap and the coefficients of transmission and reflection has been computed for a 1D photonic crystal and. Both plane wave expansion (*PWE*) and *FDTD*, with two types of polarisation *TE* and *TM*; we took the direction of propagation on the axis  $X$ . The materials of the simulation in this case are the germanium (*Ge*) and air; layer of the germanium (*Ge*) after layer of air with period  $d = 1\mu m$  like we see in the figure 3.1

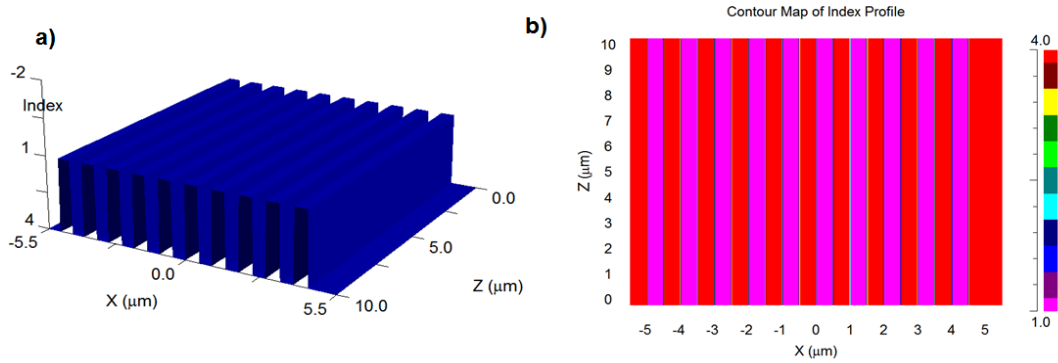


Figure 3.1: Index profile of 1D photonic crystals (a) with 3D (b) with 2D in 2 axes ( $x, z$ )

The last figure present the variation of the index profile of the structure for simulation in 3 dimension on three axes( $X, Z, index$ ) and in 2 dimension on two axes ( $X, Z$ ).like we see here the direction of the variation of the index profile on the axis  $X$

#### 3.3.2 Photonic band gap structures diagram

We have two types of the band diagram for *TE* polarisation and *TM* polarisation. but in this case this types of the polarizations it have the same band diagram ; in the result we have total bands gap see figure 3.2

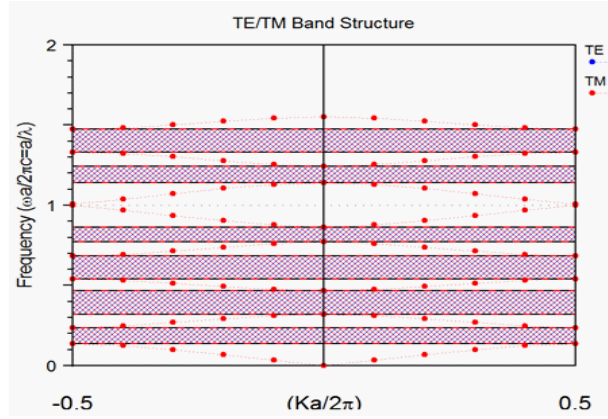


Figure 3.2: Band diagram of a 1D photonic crystal air and germanium ( $Ge$ ) with period  $d = 1\mu m$ ; polarized  $TE$  and  $TM$  mode

figure 3.2 present the photonic bands gaps diagram of 1D photonic crystal with 10 layers of air and 10 layers of the germanium ( $Ge$ ), with periodicity  $d = 1\mu m$  for that we obtain the results in figure. we have 7 PBG and we note they are total photonic band gape.

### 3.3.3 Reflection and transmission diagrams

In this part of the simulation, different wavelengths are sent over the structure with the aim to define the reflection and the transmission ratios:

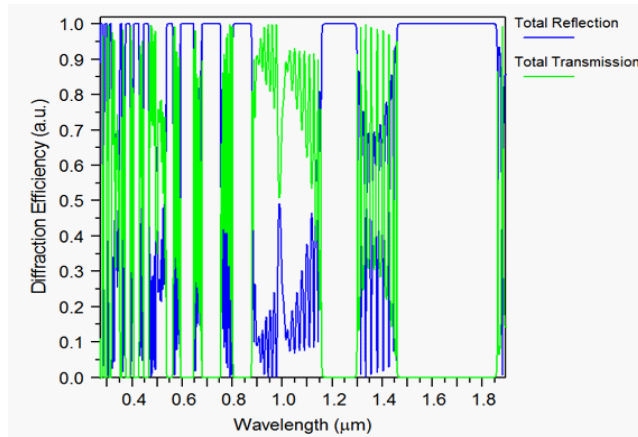


Figure 3.3: diagram of transmission and reflection of the 1D photonic crystals with air and the germanium ( $Ge$ ) with period  $d = 1\mu m$  polarized  $TE$  and  $TM$  mode

### 3.4 Two-dimensional photonic crystals simulation

In this part of simulations we will be used two types of structures (square, triangle) with two configurations: lettice holes of air inthe germanium ( $Ge$ ) and the rods of the germanium ( $Ge$ ) in air the same time any case with tow mode of polarisation  $TE$  and  $TM$

#### 3.4.1 Air cylinders being placed in germanium ( $Ge$ )

Numerical simulations are conducted for the two propagation modes  $TE$  and  $TM$  using the two geometries of structures (square, triangle). We consider the lattice holes of air ( $\varepsilon = 1; n = 1$ ) in lithium niobate ( $\varepsilon = 12, 8; n = 4$ )

#### Two-dimensional photonic crystals with square lattice

Such as what we have said before in the beginning of this part in this first case we have two-dimensional photonic crystals with square lattice holes of air in germanium ( $Ge$ )

**Definition of the parameter** for this case of two-dimensional photonic crystals with square lattice holes of air in germanium ( $Ge$ ) we have the index profile to see the variation of refractive index and the periodicity of the refractive index

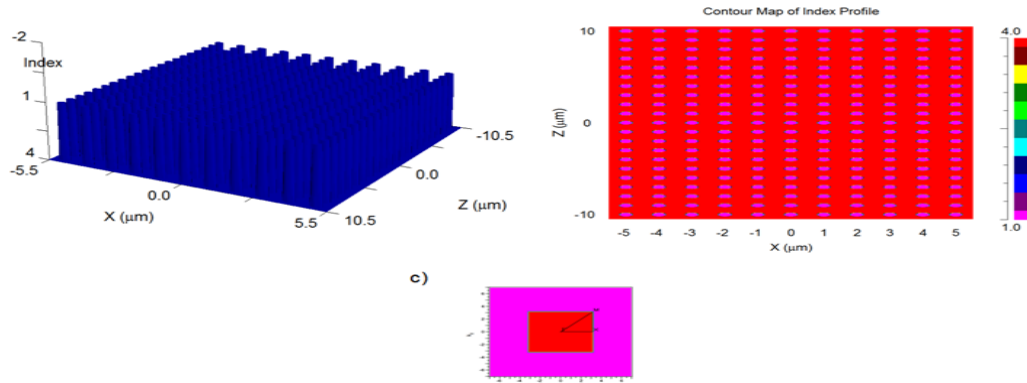


Figure 3.4: Index profile of 2D square photonic crystals with holes of air in germanium ( $Ge$ ) ; (a) with 2D in 2 axes (x,z) (a) with 3D

In the last figure ( figure 3.4 ) we see the index profile of the structure of simulation for first case for two-dimensional photonic crystals with germanium ( $Ge$ ) .this structure it is the reference to calculus the map of the photonic band gape. we took the direction of propagation on the axis  $X$

**Photonic band gap map diagram** The first scenario that we consider consists of square lattice holes of air in germanium ( $Ge$ ) substrate. The photonic band gap map for both  $TE$  and  $TM$  polarizations is illustrated by the following figure( figure 3.5).

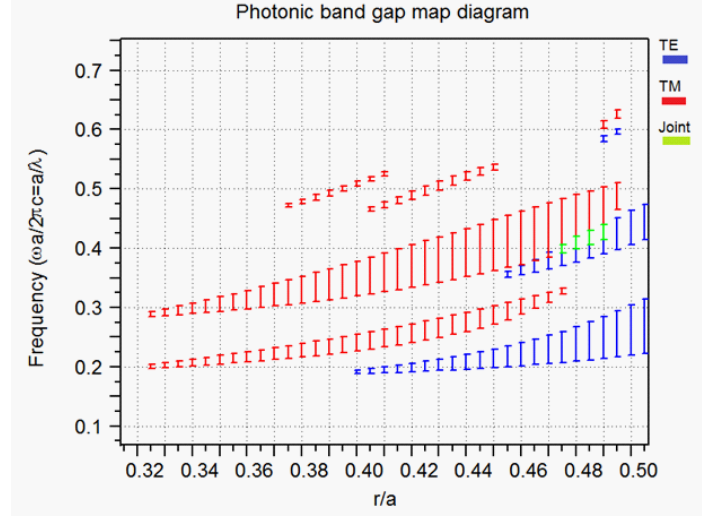


Figure 3.5: Map of photonic band gap for both  $TE$  and  $TM$  polarizations. Case of a square PC of air holes ( $\varepsilon = 1$ ) in germanium ( $Ge$ )

Figure 3.5 shows maps of photonic band gap for both polarizations  $TE$  and  $TM$  in the case of a photonic crystal of square lattice air holes ( $\varepsilon = 1$ ) in germanium ( $Ge$ ) substrate. For this structure we observe the appearance of some photonic band gap widths of  $TM$  and  $TE$  but also for very small filling factors very high.

We must achieve a ratio  $r/a = 0.40$  appear to see the bandgaps  $TE$  lowest energy and a ratio  $r/a = 0.33$  for the case of  $TM$  photonic bandgap lowest energy. we note there are overlap between the photonic bandgaps  $TE$  and  $TM$  and for that we observe there are complete photonic bandgaps, this complete photonic bandgaps begin with filling factors  $r/a = 0.47$  and it was finish with  $r/a = 0.49$ .

**TE case** To determine the photonic band gap that will operate in case of a square structure - holes configuration -  $TE$  polarization, an array containing the widths of the forbidden photonic bands most interesting was established (Table 3.1). From the following table below:

After this result we will be choose the The wide band gap photonic .

The wider bandgap photonic  $TE$  is observed for a filling factor  $r/a = 0.49$ , its width is 0.0704 and extends over the energy range  $a/\lambda \approx 0.2842 - 0.2138$  (Figure 3.44) with other two PBGs but they are small..

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$               | $r$                |
|-------------|----------------------|----------------------|---------------------|-------------------|-------------------|--------------------|
| 0.40        | 0.19400              | 0.188000             | 0.006000            | 0.191000          | 0.29605000        | 0.118420000        |
| 0.41        | 0.19950              | 0.189500             | 0.010000            | 0.194500          | 0.30147500        | 0.123604750        |
| 0.42        | 0.20550              | 0.191500             | 0.014000            | 0.198500          | 0.30767500        | 0.129223500        |
| 0.43        | 0.21300              | 0.193500             | 0.019500            | 0.203250          | 0.31503750        | 0.135466125        |
| 0.44        | 0.22000              | 0.196000             | 0.024000            | 0.208000          | 0.32240000        | 0.141856000        |
| 0.45        | 0.23000              | 0.198500             | 0.031500            | 0.214250          | 0.33208750        | 0.149439375        |
| 0.46        | 0.24040              | 0.201800             | 0.038600            | 0.221100          | 0.34270500        | 0.157644300        |
| 0.46        | 0.37000              | 0.355000             | 0.015000            | 0.362500          | 0.56187500        | 0.258462500        |
| 0.47        | 0.25300              | 0.205300             | 0.047700            | 0.229150          | 0.35518250        | 0.166935775        |
| 0.47        | 0.39300              | 0.365000             | 0.028000            | 0.379000          | 0.58745000        | 0.276101500        |
| 0.48        | 0.26725              | 0.209250             | 0.058000            | 0.238250          | 0.36928750        | 0.177258000        |
| 0.48        | 0.42000              | 0.376586             | 0.043414            | 0.398293          | 0.61735415        | 0.296329992        |
| <b>0.49</b> | <b>0.28420</b>       | <b>0.213800</b>      | <b>0.070400</b>     | <b>0.249000</b>   | <b>0.38595000</b> | <b>0.189115500</b> |
| 0.49        | 0.44000              | 0.390000             | 0.050000            | 0.415000          | 0.64325000        | 0.315192500        |
| 0.49        | 0.58850              | 0.580000             | 0.008500            | 0.584250          | 0.90558750        | 0.443737875        |
| 0.50        | 0.30450              | 0.219000             | 0.085500            | 0.261750          | 0.40571250        | 0.202856250        |

Table 3.1: Summary of TE band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the germanium (Ge)

**Photonic band gap structures diagram** The band structure in the case of the  $TE$  polarization and a fill factor  $r/a = 0.49$  is reported in figure 3.6. The calculation is done all along the axes of highest symmetry of the first Brillouin zone, the circuit  $\Gamma - X - M - \Gamma$ , using the  $PWE$  method. The energy bands is expressed as a function of the ratio  $a/\lambda$  (normalized frequency). For this case, the central frequency of this  $PBG$  :  $a/\lambda = 0.249$ . Note, there are other photonic bandgap there is not only one PBG, this case it called multiband. This characteristic it can give the possibility to make many devices work with multi wavelengths

Generally, the density of states of a photonic band gap structure is represented as a band structure which illustrates the mode frequencies permits based on the symmetry points of the Brillouin zone.

For each wave vector of the contour, there are several frequencies corresponding to different propagation modes that may propagate in the photonic crystal.

According to this calculation, we will determine the position of the bandgap of the material.

Considering all possible directions (defined by the entire contour of the first Brillouin zone), then we can have a photonic band gap when two modes are not intersecting. It is the frequency band that characterize the photonic bandgap materials. It is possible for a chosen network type to adjust the lattice parameter and the filling factor. These parameters directly affect the properties and appearance of the band diagram

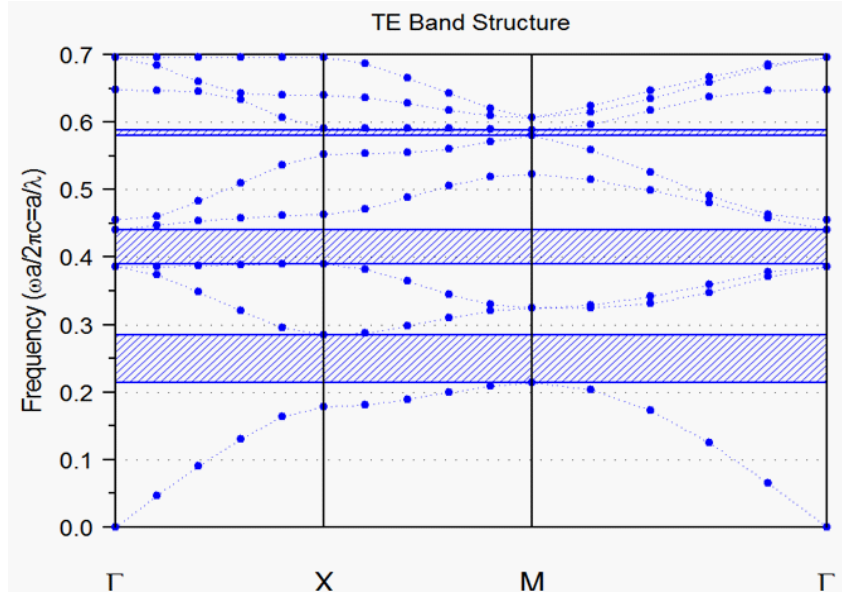


Figure 3.6: Band diagram of a photonic crystal square lattice holes of air in germanium (*Ge*) polarized *TE* mode for a filling factor  $r/a = 0.49$ .

associated with the photonic crystal because the photonic band gap are centered on normalized frequencies  $(a/\lambda)_0$ . But the days of the network depends on the value as we showed the following relationship:

$$a_{new} = (a/\lambda)_0 * \lambda_{new}$$

$a_{new}$  : the new period for new frequency.

$(a/\lambda)_0$  : normalized center frequency.

$\lambda_{new}$  : new wavelength.

Thus, the radius of holes or rods is:

$$r_{new} = a_{new} * x_0$$

Where  $x_0$  present value of the filling factor chosen from map of PBG

In this case of square lattice holes of air in germanium (*Ge*) with ratio  $r/a = 0.49$  for the *TE* polarization, there are 3 photonic band gap. our chose it is the first PBG because it the wider one, this last one it has central frequency  $a/\lambda = 0.249$

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.249 * 1.55 * 10^{-6} m = 0.38595\mu m.$$

This implies a pattern of radius:

$$r = 0.49 * 0.38595 * 10^{-6}m = 0.1891155\mu m.$$

**Reflection and transmission diagrams** The calculation of the transmission and reflection will enable us to determine the dimensions of the two-dimensional periodic structure of photonic band gap. The approach is initially to study the evolution of the transmission and reflection based on number of holes to determine the dimension of the structure

Figure 3.7 shows the transmission spectrum and reflection through structures of square lattice holes of air in germanium (*Ge*) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a *TE* polarization. Transmission measurements show a very strong disconnect between (1.2 microns, 2.1 microns) and that from the structure (b) reflection is at its maximum value. From the results obtained, we note that from the case (c) reflection remains unchanged .

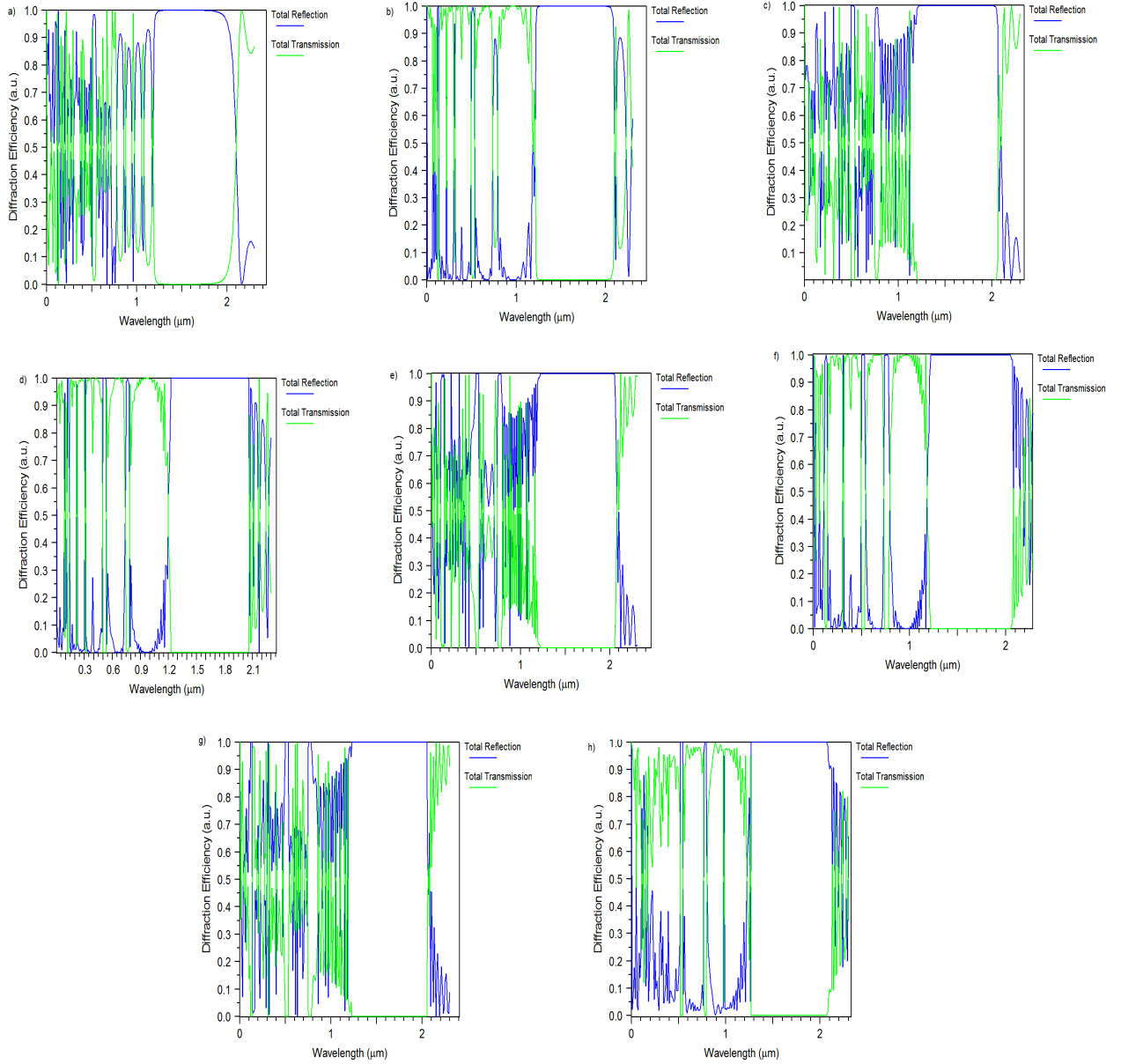


Figure 3.7: Transmission and reflection through the square structures of air holes in the germanium (Ge); TE mode polarized in  $a=0.38595\mu m$ ,  $r=0.1891155\mu m$ .

**TM case** Table 3.2 contains the photonic band gap, their widths, their center frequencies, the grating period and its range for each frequency (case of a square structure - holes configuration - *TM* polarization). From the table 3.2 :

The wider *PBG TM* is observed for a filling factor  $r/a = 0.48$ , its width is 0.09 and extends over the energy range  $a/\lambda \approx 0.40 - 0.49$  (Figure 3.8 ).

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$                |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|--------------------|
| 0.33        | 0.2070               | 0.1990               | 0.0080              | 0.20300           | 0.3146500        | 0.103834500        |
| 0.33        | 0.2970               | 0.2860               | 0.0110              | 0.29150           | 0.4518250        | 0.149102250        |
| 0.34        | 0.2130               | 0.2015               | 0.0115              | 0.20725           | 0.3212375        | 0.109220750        |
| 0.34        | 0.3070               | 0.2890               | 0.0180              | 0.29800           | 0.4619000        | 0.157046000        |
| 0.35        | 0.2190               | 0.2050               | 0.0140              | 0.21200           | 0.3286000        | 0.115010000        |
| 0.35        | 0.3175               | 0.2935               | 0.0240              | 0.30550           | 0.4735250        | 0.165733750        |
| 0.36        | 0.2250               | 0.2080               | 0.0170              | 0.21650           | 0.3355750        | 0.120807000        |
| 0.36        | 0.3280               | 0.2980               | 0.0300              | 0.31300           | 0.4851500        | 0.174654000        |
| 0.37        | 0.2320               | 0.2120               | 0.0200              | 0.22200           | 0.3441000        | 0.127317000        |
| 0.37        | 0.3400               | 0.3020               | 0.0380              | 0.32100           | 0.4975500        | 0.184093500        |
| 0.38        | 0.2390               | 0.2160               | 0.0230              | 0.22750           | 0.3526250        | 0.133997500        |
| 0.38        | 0.3520               | 0.3080               | 0.0440              | 0.33000           | 0.5115000        | 0.194370000        |
| 0.38        | 0.4825               | 0.4750               | 0.0075              | 0.47875           | 0.7420625        | 0.281983750        |
| 0.39        | 0.2470               | 0.2210               | 0.0260              | 0.23400           | 0.3627000        | 0.141453000        |
| 0.39        | 0.3650               | 0.3150               | 0.0500              | 0.34000           | 0.5270000        | 0.205530000        |
| 0.39        | 0.4975               | 0.4882               | 0.0093              | 0.49285           | 0.7639175        | 0.297927825        |
| 0.40        | 0.2550               | 0.2270               | 0.0280              | 0.24100           | 0.3735500        | 0.149420000        |
| 0.40        | 0.3780               | 0.3180               | 0.0600              | 0.34800           | 0.5394000        | 0.215760000        |
| 0.40        | 0.5130               | 0.5040               | 0.0090              | 0.50850           | 0.7881750        | 0.315270000        |
| 0.41        | 0.2632               | 0.2332               | 0.0300              | 0.24820           | 0.3847100        | 0.157731100        |
| 0.41        | 0.3925               | 0.3275               | 0.0650              | 0.36000           | 0.5580000        | 0.228780000        |
| 0.41        | 0.4780               | 0.4680               | 0.0100              | 0.47300           | 0.7331500        | 0.300591500        |
| 0.41        | 0.5290               | 0.5220               | 0.0070              | 0.52550           | 0.8145250        | 0.333955250        |
| 0.42        | 0.2720               | 0.2400               | 0.0320              | 0.25600           | 0.3968000        | 0.166656000        |
| 0.42        | 0.4100               | 0.3300               | 0.0800              | 0.37000           | 0.5735000        | 0.240870000        |
| 0.42        | 0.4960               | 0.4820               | 0.0140              | 0.48900           | 0.7579500        | 0.318339000        |
| 0.43        | 0.2820               | 0.2490               | 0.0330              | 0.26550           | 0.4115250        | 0.176955750        |
| 0.43        | 0.4200               | 0.3400               | 0.0800              | 0.38000           | 0.5890000        | 0.253270000        |
| 0.43        | 0.5130               | 0.4970               | 0.0160              | 0.50500           | 0.7827500        | 0.336582500        |
| 0.44        | 0.2920               | 0.2600               | 0.0320              | 0.27600           | 0.4278000        | 0.188232000        |
| 0.44        | 0.4320               | 0.3520               | 0.0800              | 0.39200           | 0.6076000        | 0.267344000        |
| 0.44        | 0.5280               | 0.5140               | 0.0140              | 0.52100           | 0.8075500        | 0.355322000        |
| 0.45        | 0.3020               | 0.2720               | 0.0300              | 0.28700           | 0.4448500        | 0.200182500        |
| 0.45        | 0.4475               | 0.3615               | 0.0860              | 0.40450           | 0.6269750        | 0.282138750        |
| 0.45        | 0.5420               | 0.5300               | 0.0120              | 0.53600           | 0.8308000        | 0.373860000        |
| 0.46        | 0.3140               | 0.2880               | 0.0260              | 0.30100           | 0.4665500        | 0.214613000        |
| 0.46        | 0.4620               | 0.3730               | 0.0890              | 0.41750           | 0.6471250        | 0.297677500        |
| 0.47        | 0.3260               | 0.3090               | 0.0170              | 0.31750           | 0.4921250        | 0.231298750        |
| 0.47        | 0.4750               | 0.3900               | 0.0850              | 0.43000           | 0.6665000        | 0.313255000        |
| <b>0.48</b> | <b>0.4900</b>        | <b>0.4000</b>        | <b>0.0900</b>       | <b>0.44500</b>    | <b>0.6897500</b> | <b>0.331080000</b> |
| 0.49        | 0.5032               | 0.4137               | 0.0895              | 0.45845           | 0.7105975        | 0.348192775        |
| 0.49        | 0.6150               | 0.6020               | 0.0130              | 0.60850           | 0.9431750        | 0.462155750        |

Table 3.2: Summary of TM band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the germanium (Ge)

**Photonic band gap structures diagram** In this case of square lattice holes of air in germanium (*Ge*) with ratio  $r/a = 0.48$  for the *TM* polarization, the wider PBG it has central frequency  $a/\lambda = 0.445$ .

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.445 * 1.55 * 10^{-6} m = 0.68975\mu m.$$

This implies a pattern of radius:

$$r = 0.48 * 0.68975 * 10^{-6} m = 0.33108\mu m.$$

Note in this case there are overlap between the photonic band gaps *TE* and *TM* and for that we observe there is the presence of complete photonic band gap see figure 3.9

the width of the band of the overlap between the photonic band gaps *TE* and *TM* it is so small, because that, we cannot use this band

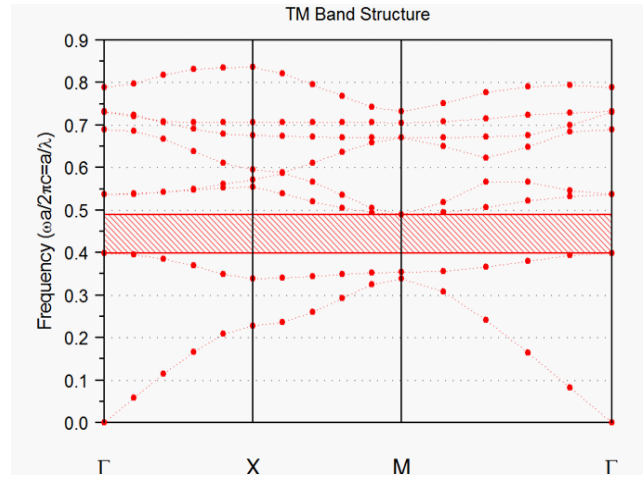


Figure 3.8: Band diagram of a photonic crystal square lattice holes of air in germanium (*Ge*) polarized *TM* mode for a filling factor  $r/a = 0.468$ .

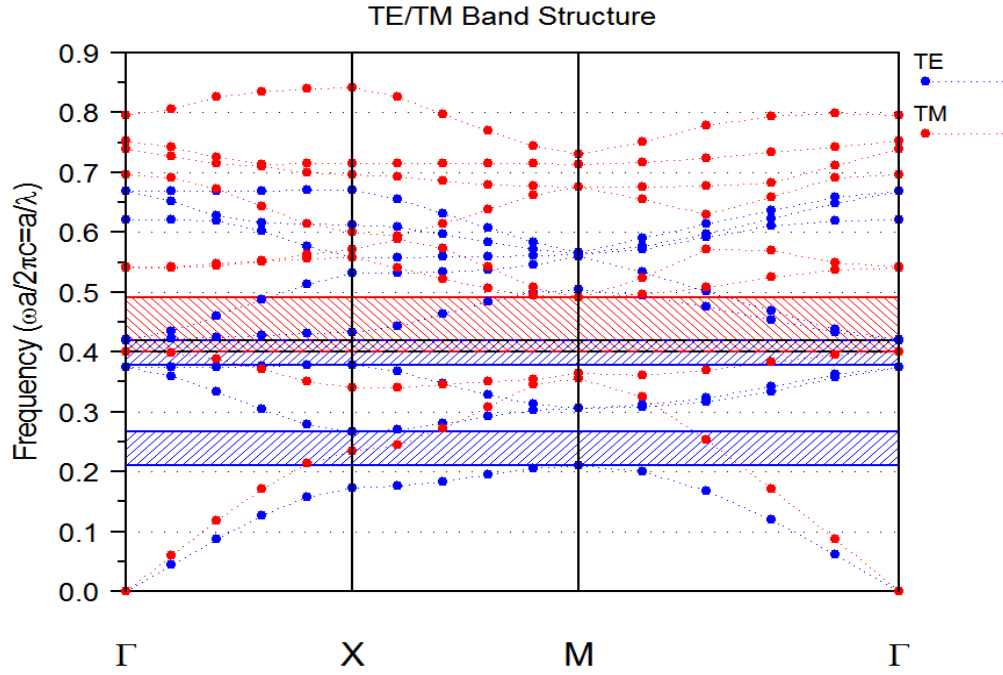


Figure 3.9: Band diagram of a photonic crystal square lattice holes of air in germanium (*Ge*) polarized *TM* and *TE* mode for a filling factor  $r/a = 0.468$ .

**Reflection and transmission diagrams** Figure 3.10 shows the transmission spectrum and reflection through structures of square lattice holes of air in germanium square lattice holes of air in germanium (*Ge*) of different sizes [(a)5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a *TM* show a very strong disconnect between (1.5 microns, 2.5 microns) and that from the structure (a) reflection is at its maximum value. From the results obtained, we note that from the case (b) reflection remains unchanged .

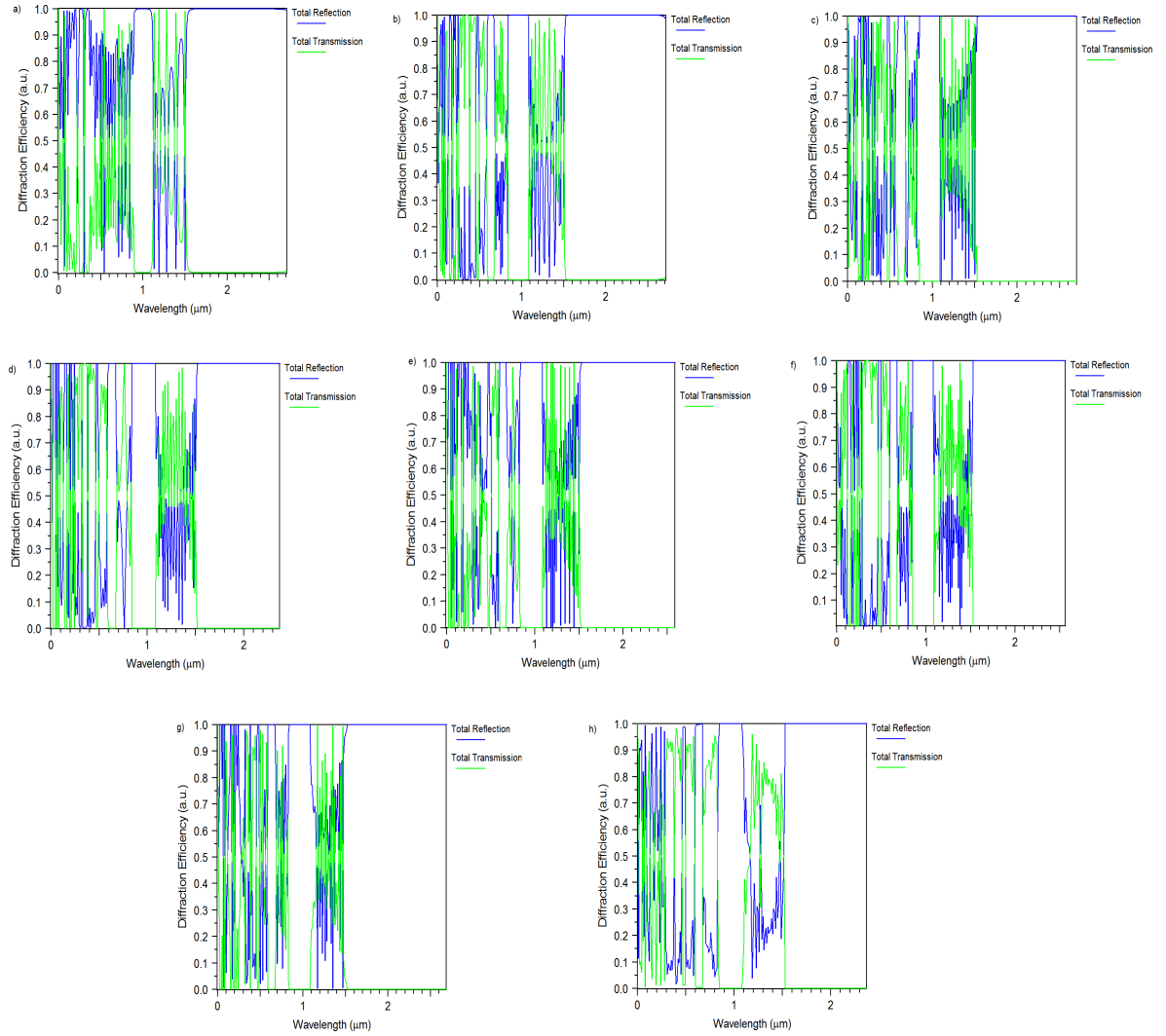


Figure 3.10: Transmission and reflection through the square structures of air holes in the germanium ( $Ge$ ); TM mode polarized in  $a = 0.68975\mu m$ ,  $r = 0.33108\mu m$

### Two-dimensional photonic crystals with triangular lattice

In the second case, we have two-dimensional photonic crystals with triangular lattice holes of air in germanium (*Ge*)

**Definition of the parameters** The variation of the wave vector (modulus and direction) when calculating the band diagrams for the triangular structure along high symmetry directions of the Brillouin zone,  $\Gamma \rightarrow M \rightarrow K \rightarrow \Gamma$  (Figure 3.11 (c)). and figure 3.11 shows the variation of refractive index and the periodicity of the refractive index of the our new structure .

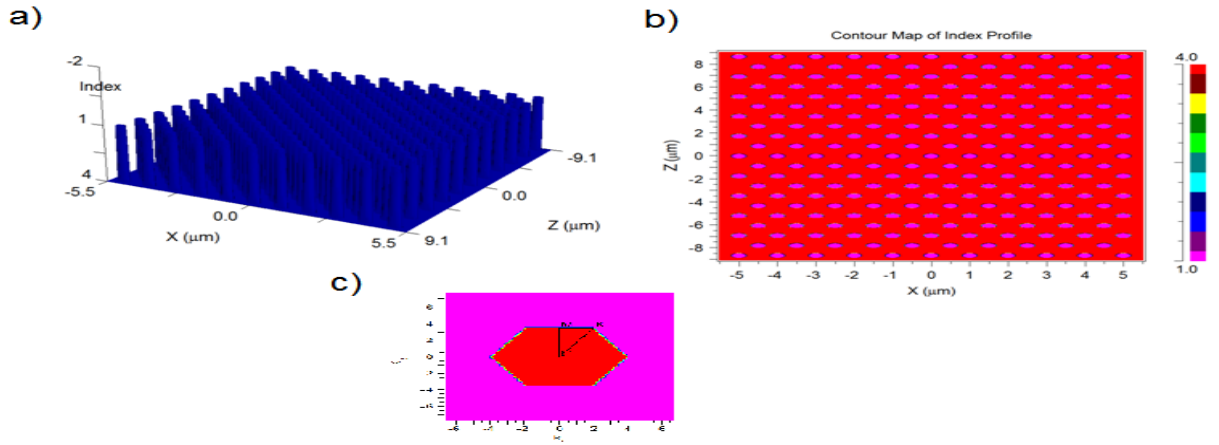


Figure 3.11: Index profile of 2D triangular photonic crystals with holes of air in germanium (*Ge*); (a) with 2D in 2 axes ( $x, z$ ) (a) with 3D

figure 3.11 shows the variation of refractive index on two axes ( $Z, X$ ) there are triangular lattice holes of air in germanium (*Ge*) but in the other direction (axis  $Y$ ) there is not any change

**Photonic band gap map diagram** Now consider the case of a triangular lattice of air holes in a substrate of germanium (*Ge*). The photonic band gap map for both *TE* and *TM* polarizations is shown in Figure 3.12.

Figure 3.12 shows maps of photonic band gap for both polarizations *TE* and *TM* in the case of a photonic crystal of triangular lattice air holes ( $\varepsilon = 1$ ) on substrate of germanium (*Ge*). For this structure we observe the appearance of some photonic band gap widths of *TM* and *TE* but also for very small filling factors very high.

We must achieve a ratio  $r/a = 0.37$  appear to see the bandgaps *TE* lowest energy and a ratio  $r/a = 0.18$  for the case of *TM* photonic bandgap so large. we note there are overlap between the photonic bandgaps *TE* and *TM* and for that we observe there

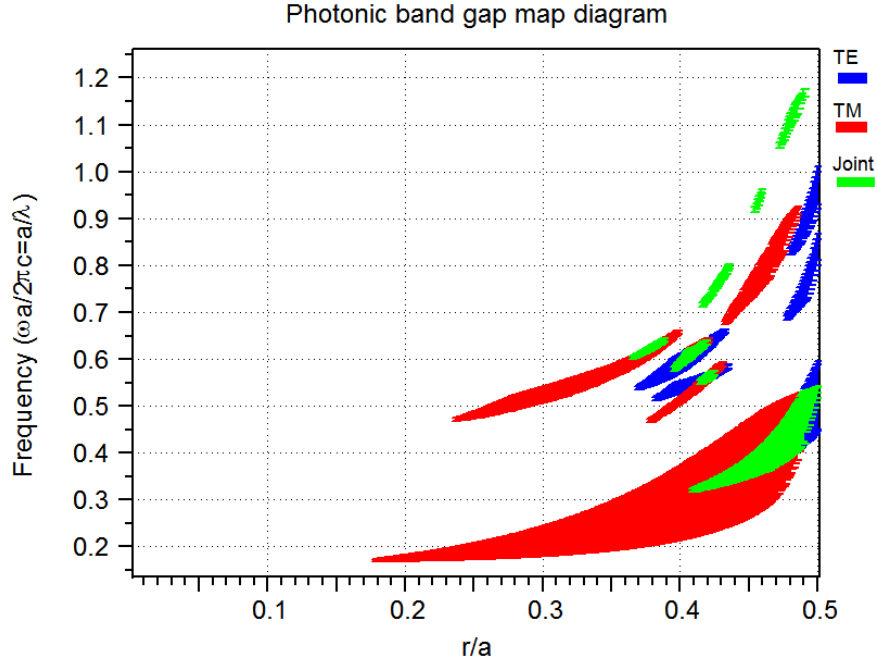


Figure 3.12: Map of photonic band gap for both TE and TM polarizations. Case of a triangular PC of air holes ( $\varepsilon = 1$ ) in germanium ( $Ge$ )

are complete photonic bandgaps, this complete photonic bandgaps begin with filling factors  $r/a = 0.36$  and it was finish with  $r/a = 0.49$ .

**TE case** For  $TE$  polarization, we note the existence  $TE$  photonic band gap. The photonic bandgap of greatest interest are grouped in the table 3.3

The wider bandgap photonic  $TE$  is observed for a filling factor  $r/a = 0.49$  its width is **0.0524** and extends over the energy range  $a/\lambda \approx 0.76840 - 0.71600$ (Figure 3.13) with other two PBGs but they are small..

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$                |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|--------------------|
| 0.37        | 0.542300             | 0.53550              | 0.0068              | 0.53890           | 0.8352950        | 0.309059150        |
| 0.38        | 0.514500             | 0.50800              | 0.0065              | 0.51125           | 0.7924375        | 0.301126250        |
| 0.38        | 0.563500             | 0.54520              | 0.0183              | 0.55435           | 0.8592425        | 0.326512150        |
| 0.39        | 0.53600              | 0.51700              | 0.0190              | 0.52650           | 0.8160750        | 0.318269250        |
| 0.39        | 0.58700              | 0.55700              | 0.0300              | 0.57200           | 0.8866000        | 0.345774000        |
| 0.40        | 0.55200              | 0.52800              | 0.0240              | 0.54000           | 0.8370000        | 0.334800000        |
| 0.40        | 0.61000              | 0.57350              | 0.0365              | 0.59175           | 0.9172125        | 0.366885000        |
| 0.41        | 0.32580              | 0.31750              | 0.0083              | 0.32165           | 0.4985575        | 0.204408575        |
| 0.41        | 0.55950              | 0.54000              | 0.0195              | 0.54975           | 0.8521125        | 0.349366125        |
| 0.41        | 0.62600              | 0.59400              | 0.0320              | 0.61000           | 0.9455000        | 0.387655000        |
| 0.42        | 0.33900              | 0.32300              | 0.0160              | 0.33100           | 0.5130500        | 0.215481000        |
| 0.42        | 0.56900              | 0.55300              | 0.0160              | 0.56100           | 0.8695500        | 0.365211000        |
| 0.42        | 0.63950              | 0.62050              | 0.0190              | 0.63000           | 0.9765000        | 0.410130000        |
| 0.43        | 0.35300              | 0.32940              | 0.0236              | 0.34120           | 0.5288600        | 0.227409800        |
| 0.43        | 0.58000              | 0.56900              | 0.0110              | 0.57450           | 0.8904750        | 0.382904250        |
| 0.44        | 0.37000              | 0.33720              | 0.0328              | 0.35360           | 0.5480800        | 0.241155200        |
| 0.45        | 0.39040              | 0.34650              | 0.0439              | 0.36845           | 0.5710975        | 0.256993875        |
| 0.46        | 0.41500              | 0.35800              | 0.0570              | 0.38650           | 0.5990750        | 0.275574500        |
| 0.47        | 0.44700              | 0.37200              | 0.0750              | 0.40950           | 0.6347250        | 0.298320750        |
| 0.48        | 0.48600              | 0.39100              | 0.0950              | 0.43850           | 0.6796750        | 0.326244000        |
| 0.48        | 0.70700              | 0.68550              | 0.0215              | 0.69625           | 1.0791875        | 0.518010000        |
| 0.49        | 0.53675              | 0.41345              | 0.1233              | 0.47510           | 0.7364050        | 0.360838450        |
| <b>0.49</b> | <b>0.76840</b>       | <b>0.71600</b>       | <b>0.0524</b>       | <b>0.74220</b>    | <b>1.1504100</b> | <b>0.563700900</b> |
| 0.49        | 0.90000              | 0.85000              | 0.0500              | 0.87500           | 1.3562500        | 0.664562500        |
| 0.5         | 0.59600              | 0.44300              | 0.1530              | 0.51950           | 0.8052250        | 0.402612500        |
| 0.5         | 0.85300              | 0.80000              | 0.0530              | 0.82650           | 1.2810750        | 0.640537500        |
| 0.5         | 0.99810              | 0.90700              | 0.0911              | 0.95255           | 1.4764525        | 0.738226250        |

Table 3.3: Summary of TE band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the germanium (Ge)

**Photonic band gap structures diagram** The band structure in the case of the  $TE$  polarization and a fill factor  $r/a = 0.49$  is reported in figure 3.13. The calculation is done all along the axes of highest symmetry of the first Brillouin zone, the circuit  $\Gamma - X - M - \Gamma$ ,

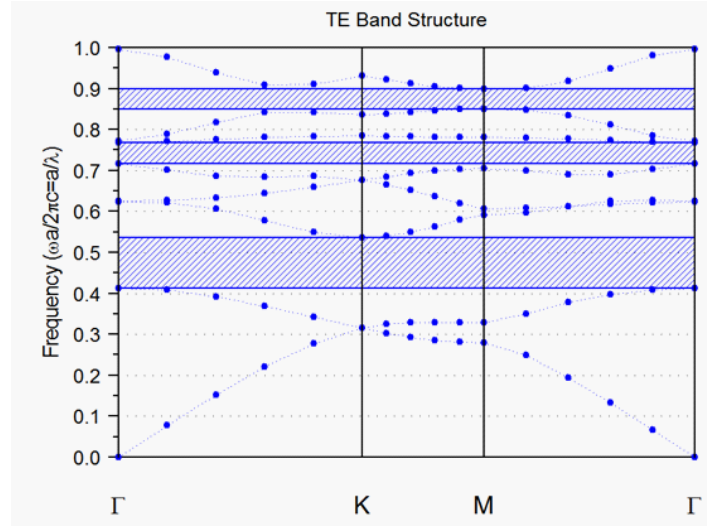


Figure 3.13: Band diagram of a photonic crystal triangular lattice holes of air in germanium ( $Ge$ ) polarized  $TE$  mode for a filling factor  $r/a = 0.44$ .

In this case of triangular lattice holes of air in germanium ( $Ge$ ) with ratio  $r/a = 0.49$  for the  $TE$  polarization, the photonic band gap used is centered on 0.7422.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.74220 * 1.55 * 10^{-6}m = 1.15041\mu m.$$

This implies a pattern of radius:

$$r = 0.44 * 1.15041 * 10^{-6}m = 0.50618\mu m$$

**Reflection and transmission diagrams** Figure 3.14 shows the transmission spectrum and reflection through structures of triangular lattice of air holes in a substrate of germanium ( $Ge$ ) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.4 microns, 1.6 microns) and that from the structure (e) reflection is at its maximum value. From the results obtained, we note that from the case (f) reflection remains unchanged.

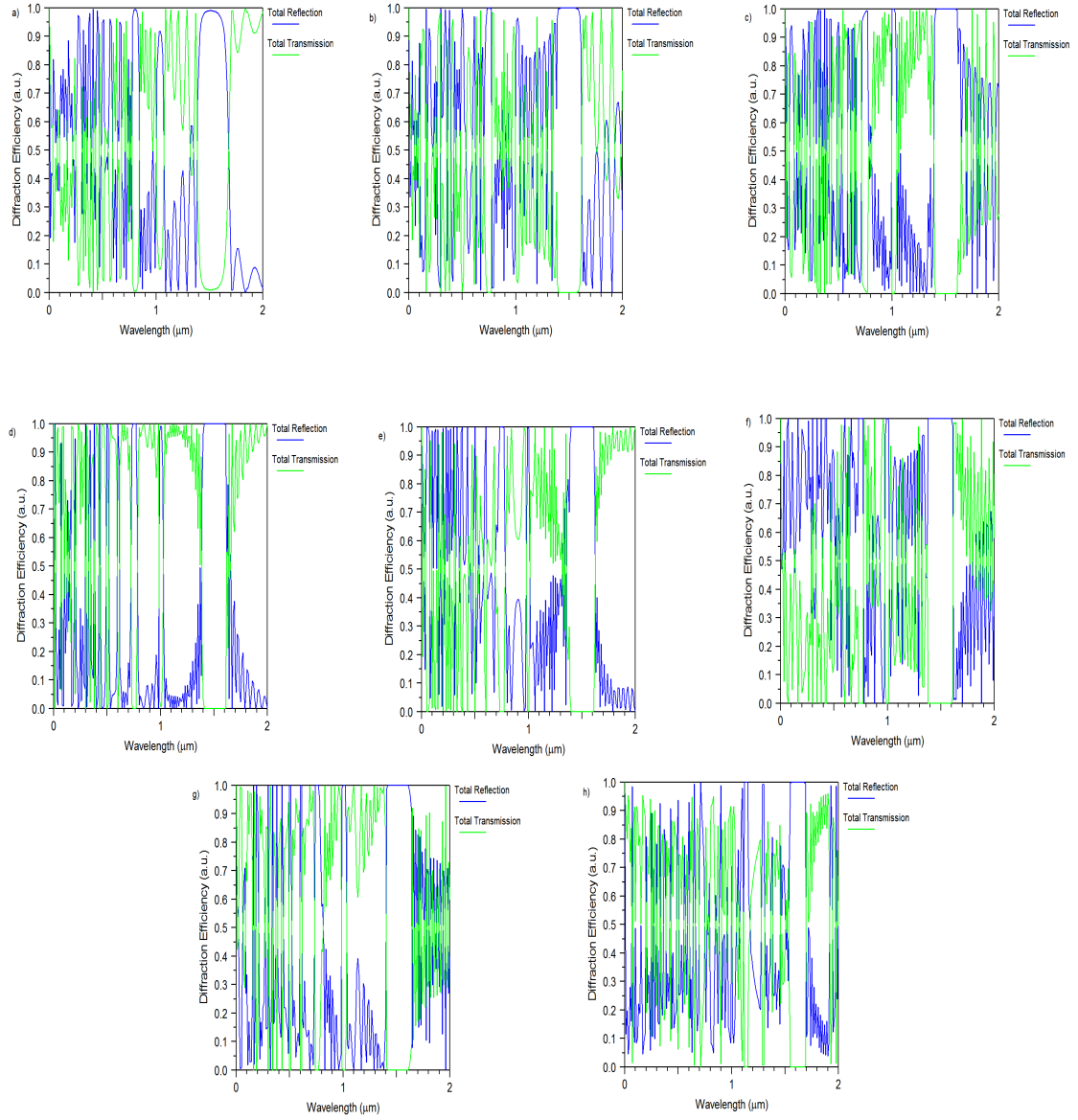


Figure 3.14: Transmission and reflection through the triangular structures of air holes in the germanium ( $Ge$ ) ; TE mode polarized in  $a = 1.15041 \mu m$ ,  $r = 0.5637 \mu m$

**TM case** For the case of  $TM$  polarization, we note the existence of several  $TM$  photonic band gap. The photonic bandgap of greatest interest are grouped in the table 3.4

The wider bandgap photonic  $TM$  is observed for a filling factor  $r/a = 0.46$  its width is 0.21 and extends over the energy range  $a/\lambda \approx 0.491 - 0.281$ . In this case we note the  $PBG$  so larger (Figure 3.15 ) with other two PBGs but, they are small..

**Photonic band gap structures diagram** In this case of triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) ratio  $r/a = 0.46$  for the  $TM$  polarization, the photonic band gap used is centered on 0.386.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.386 * 1.55 * 10^{-6}m = 0.5983\mu m$$

This implies a pattern of radius:

$$r = 0.46 * 0.5983 * 10^{-6}m = 0.275218\mu m$$

Like other case, there are overlap between the photonic band gaps  $TE$  and  $TM$  and for that we observe there is the presence of complete photonic band gap see figure 3.16 with filling factor  $r/a = 0.46$

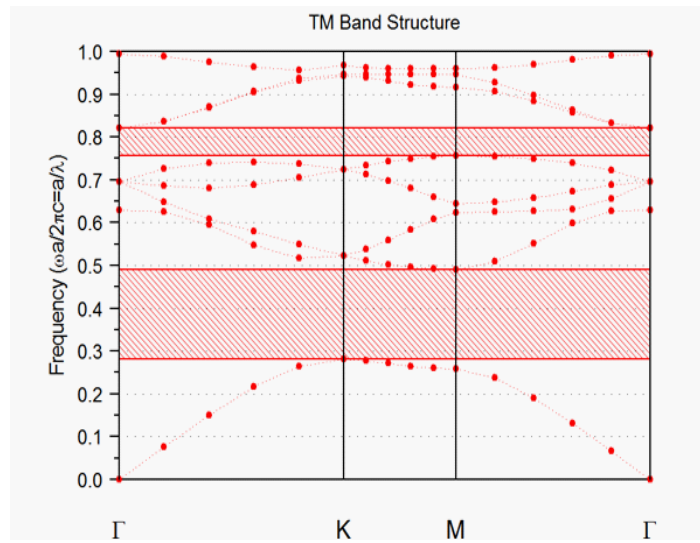


Figure 3.15: Band diagram of a photonic crystal triangular lattice holes of air in germanium ( $Ge$ ) polarized  $TM$  mode for a filling factor  $r/a = 0.46$ .

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|-------------------|
| 0.18        | 0.17450              | 0.1685               | 0.0060              | 0.17150           | 0.2658250        | 0.04784850        |
| 0.2         | 0.18210              | 0.1690               | 0.0131              | 0.17555           | 0.2721025        | 0.05442050        |
| 0.22        | 0.19100              | 0.1700               | 0.0210              | 0.18050           | 0.2797750        | 0.06155050        |
| 0.24        | 0.20200              | 0.1720               | 0.0300              | 0.18700           | 0.2898500        | 0.06956400        |
| 0.24        | 0.47700              | 0.4685               | 0.0085              | 0.47275           | 0.7327625        | 0.17586300        |
| 0.26        | 0.22750              | 0.1740               | 0.0535              | 0.20075           | 0.3111625        | 0.08090225        |
| 0.26        | 0.50000              | 0.4800               | 0.0200              | 0.49000           | 0.7595000        | 0.19747000        |
| 0.28        | 0.22750              | 0.1765               | 0.0510              | 0.20200           | 0.3131000        | 0.08766800        |
| 0.28        | 0.52400              | 0.4919               | 0.0321              | 0.50795           | 0.7873225        | 0.22045030        |
| 0.3         | 0.24340              | 0.1800               | 0.0634              | 0.21170           | 0.3281350        | 0.09844050        |
| 0.3         | 0.54050              | 0.5067               | 0.0338              | 0.52360           | 0.8115800        | 0.24347400        |
| 0.32        | 0.26200              | 0.1837               | 0.0783              | 0.22285           | 0.3454175        | 0.11053360        |
| 0.32        | 0.55750              | 0.5245               | 0.0330              | 0.54100           | 0.8385500        | 0.26833600        |
| 0.34        | 0.28370              | 0.1887               | 0.0950              | 0.23620           | 0.3661100        | 0.12447740        |
| 0.34        | 0.57650              | 0.5460               | 0.0305              | 0.56125           | 0.8699375        | 0.29577875        |
| 0.36        | 0.30900              | 0.1950               | 0.1140              | 0.25200           | 0.3906000        | 0.14061600        |
| 0.36        | 0.59880              | 0.5725               | 0.0263              | 0.58565           | 0.9077575        | 0.32679270        |
| 0.38        | 0.33870              | 0.2035               | 0.1352              | 0.27110           | 0.4202050        | 0.15967790        |
| 0.38        | 0.47500              | 0.4670               | 0.0080              | 0.47100           | 0.7300500        | 0.27741900        |
| 0.38        | 0.62740              | 0.6069               | 0.0205              | 0.61715           | 0.9565825        | 0.36350135        |
| 0.4         | 0.37260              | 0.2144               | 0.1582              | 0.29350           | 0.4549250        | 0.18197000        |
| 0.4         | 0.52010              | 0.5052               | 0.0149              | 0.51265           | 0.7946075        | 0.31784300        |
| 0.4         | 0.59800              | 0.5820               | 0.0160              | 0.59000           | 0.9145000        | 0.36580000        |
| 0.4         | 0.66400              | 0.6545               | 0.0095              | 0.65925           | 1.0218375        | 0.40873500        |
| 0.42        | 0.41100              | 0.2290               | 0.1820              | 0.32000           | 0.4960000        | 0.20832000        |
| 0.42        | 0.57000              | 0.5521               | 0.0179              | 0.56105           | 0.8696275        | 0.36524355        |
| 0.42        | 0.64400              | 0.6327               | 0.0113              | 0.63835           | 0.9894425        | 0.41556585        |
| 0.42        | 0.72800              | 0.7180               | 0.0100              | 0.72300           | 1.1206500        | 0.47067300        |
| 0.44        | 0.45000              | 0.2500               | 0.2000              | 0.35000           | 0.5425000        | 0.23870000        |
| 0.44        | 0.72100              | 0.6900               | 0.0310              | 0.70550           | 1.0935250        | 0.48115100        |
| 0.44        | 0.82320              | 0.8125               | 0.0107              | 0.81785           | 1.2676675        | 0.55777370        |
| <b>0.46</b> | <b>0.49100</b>       | <b>0.2810</b>        | <b>0.2100</b>       | <b>0.38600</b>    | <b>0.5983000</b> | <b>0.27521800</b> |
| 0.46        | 0.82150              | 0.7567               | 0.0648              | 0.78910           | 1.2231050        | 0.56262830        |
| 0.48        | 0.52250              | 0.3366               | 0.1859              | 0.42955           | 0.6658025        | 0.31958520        |
| 0.48        | 0.90500              | 0.8550               | 0.0500              | 0.88000           | 1.3640000        | 0.65472000        |
| 0.48        | 1.10600              | 1.0650               | 0.0410              | 1.08550           | 1.6825250        | 0.80761200        |
| 0.5         | 0.54500              | 0.5150               | 0.0300              | 0.53000           | 0.8215000        | 0.41075000        |

Table 3.4: Summary of TM band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the germanium (Ge)

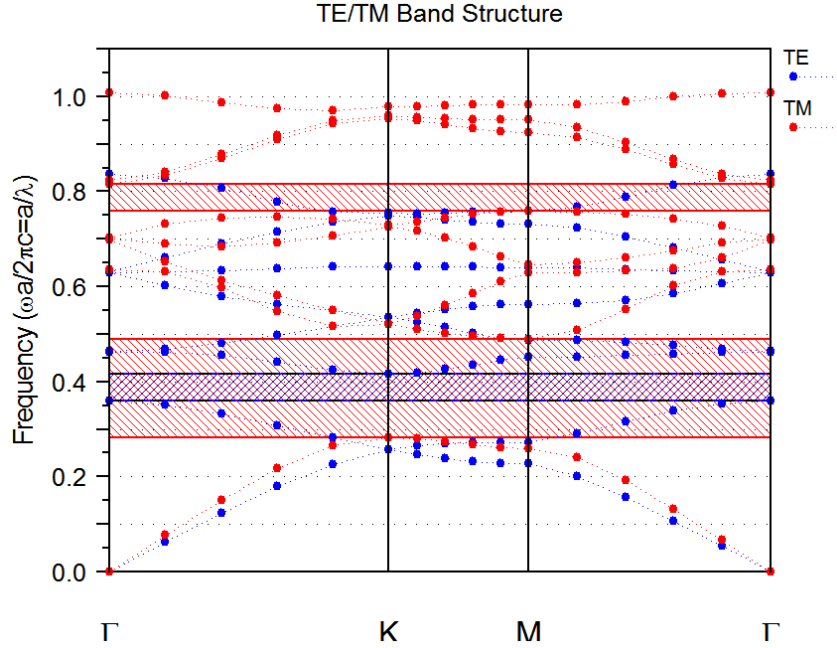


Figure 3.16: Band diagram of a photonic crystal square lattice holes of air in germanium (*Ge*) polarized *TM* and *TE* mode for a filling factor  $r/a = 0.46$ .

the width of the band of the overlap between the photonic band gaps *TE* and *TM* it is so large ( $\Delta(a/\lambda) = 0.0570$ ) see figure 3.16

**Reflection and transmission diagrams** Figure 3.17 shows the transmission spectrum and reflection through structures of triangular lattice of air holes in a substrate of germanium (*Ge*) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a *TM* polarization. Transmission measurements show a very strong disconnect between (1.3 microns, 1.6 microns) and that from the structure (c) reflection is at its maximum value. From the results obtained, we note that from the case (k) reflection remains unchanged .

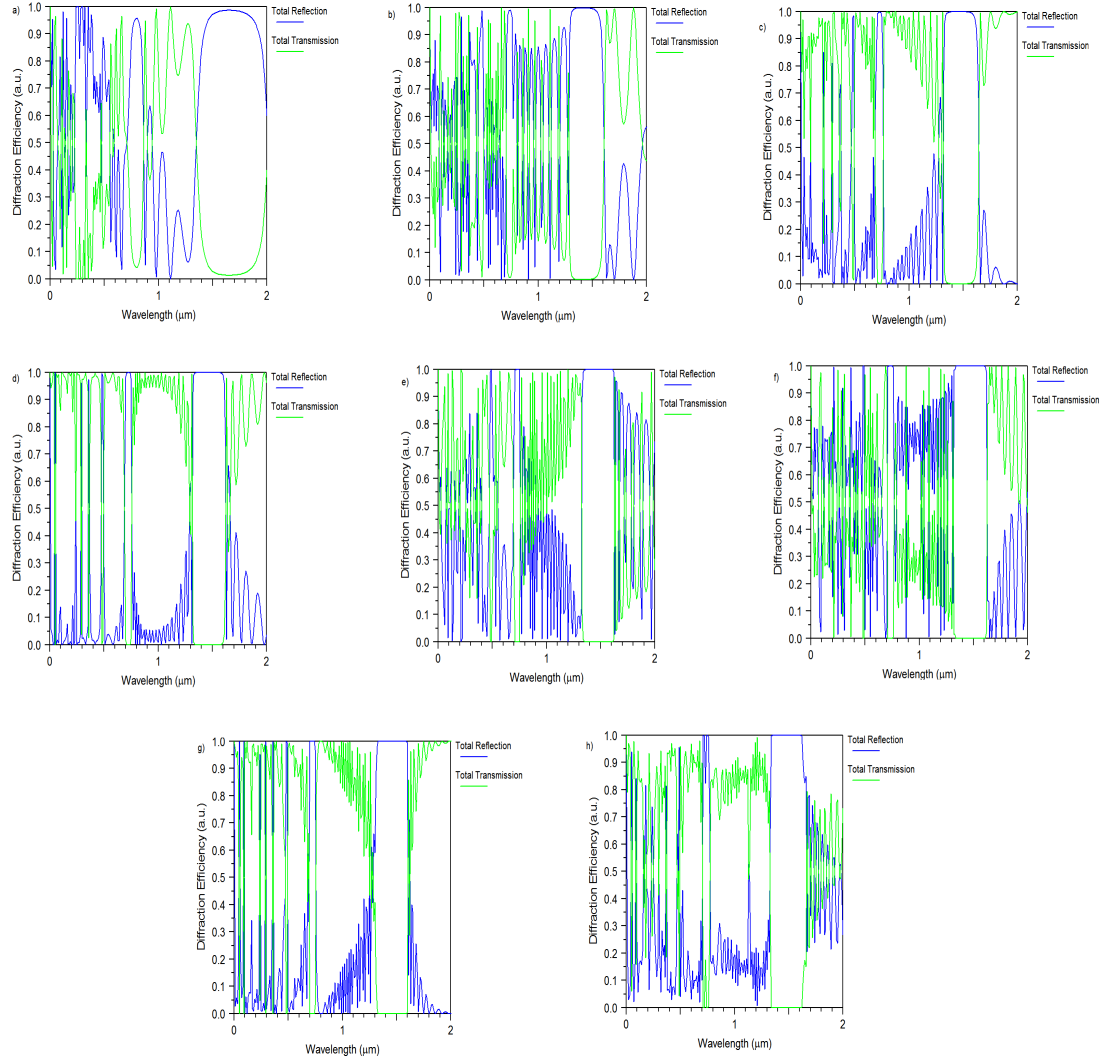


Figure 3.17: Transmission and reflection through the triangular structures of air holes in the in germanium ( $Ge$ ); TM mode polarized in  $a = 0.5983 \mu\text{m}$ .. ,  $r = 0.275218 \mu\text{m}$

### 3.4.2 Germanium ( $Ge$ ) cylinders being placed in air

In the latter case we will consider the reverse configuration, we will use the same structures, but with rods (columns) of germanium ( $Ge$ ) in the air.

#### Two-dimensional photonic crystals with square lattice

**Definition of the parameters** Figure 3.18 shows the variation of refractive index and the periodicity of the refractive index of our new structure (square lattice of rods of germanium ( $Ge$ ) in the air).

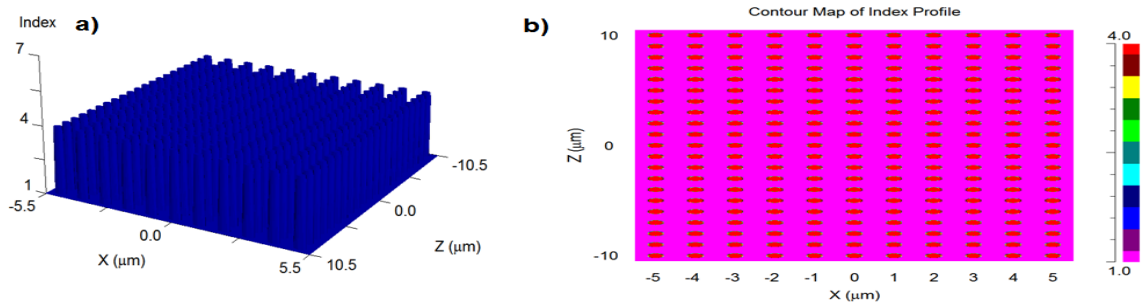


Figure 3.18: Index profile of 2D square photonic crystals with square lattice of rods of germanium ( $Ge$ ) in air; (a) with 3D, (b) with 2D in 2 axes ( $x, z$ )

the variation of refractive index on two axes ( $Z, X$ ) there are square lattice of rods of germanium ( $Ge$ ) in the air, but in the other direction (axis  $Y$ ) there is not any change.

**Photonic band gap map diagram** We note, from this figure, the presence of some Photonic band gap for  $TE$  and  $TM$  polarisation, and there is the presence of complete photonic band gap between ( $r/a \approx 0.33 - 0.44$ )

Figure 3.19 shows maps of photonic band gap for both polarizations  $TE$  and  $TM$  in the case of a photonic crystal with square lattice of rods of germanium ( $Ge$ ) in air ( $\epsilon = 1$ ). We must achieve a ratio  $r/a = 0.08$  appear to see the bandgaps  $TE$  lowest energy and a ratio  $r/a = 0.10$  for the case of  $TM$  photonic bandgap so large. we note there are overlap between the photonic bandgaps  $TE$  and  $TM$  and for that we observe there are complete photonic bandgaps, this complete photonic bandgaps begin with filling factors  $r/a = 0.33$  and it was finish with  $r/a = 0.44$ .

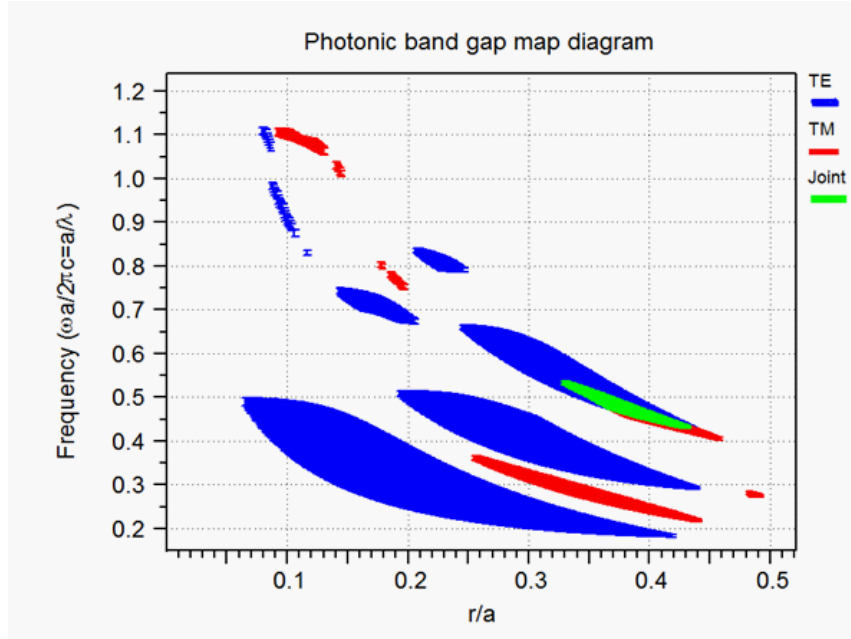


Figure 3.19: Map of photonic band gap for both TE and TM polarizations. Case of 2D photonic crystals with square lattice of rods of germanium ( $Ge$ ) in air with radius of germanium cylinder changing from 0 to  $0.5a$

**TE case** For  $TE$  polarization, we note the existence  $TE$  photonic band gap. The photonic bandgap of greatest interest are grouped in the table 3.5

The wider photonic bandgap  $TE$  is observed for a filling factor  $r/a = 0.16$ .

The ratio  $r/a = 0.16$  provides the ability to have a photonic band gap in  $TE$  mode of relatively large width [ $\Delta(a/\lambda) = 0.1651$ ] with maximum normalized frequency  $(a/\lambda)_{max} = 0.4441$  and the minimal normalized frequency  $(a/\lambda)_{min} = 0.2790$

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|-------------------|
| 0.08        | 0.4972               | 0.4264               | 0.0708              | 0.46180           | 0.7157900        | 0.57263200        |
| 0.10        | 0.4920               | 0.3730               | 0.1190              | 0.43250           | 0.6703750        | 0.06703750        |
| 0.10        | 0.9150               | 0.8930               | 0.0220              | 0.90400           | 1.4012000        | 0.14012000        |
| 0.12        | 0.4833               | 0.3330               | 0.1503              | 0.40815           | 0.6326325        | 0.07591590        |
| 0.14        | 0.4670               | 0.3026               | 0.1644              | 0.38480           | 0.5964400        | 0.08350160        |
| <b>0.16</b> | <b>0.4441</b>        | <b>0.2790</b>        | <b>0.1651</b>       | <b>0.36155</b>    | <b>0.5604025</b> | <b>0.08966440</b> |
| 0.16        | 0.7420               | 0.7000               | 0.0420              | 0.72100           | 1.1175500        | 0.17880800        |
| 0.18        | 0.4160               | 0.2604               | 0.1556              | 0.33820           | 0.5242100        | 0.09435780        |
| 0.18        | 0.7232               | 0.6857               | 0.0375              | 0.70445           | 1.0918975        | 0.19654155        |
| 0.20        | 0.3870               | 0.2450               | 0.1420              | 0.31600           | 0.4898000        | 0.09796000        |
| 0.20        | 0.5150               | 0.4889               | 0.0261              | 0.50195           | 0.7780225        | 0.15560450        |
| 0.20        | 0.6900               | 0.6700               | 0.0200              | 0.68000           | 1.0540000        | 0.21080000        |
| 0.22        | 0.3582               | 0.2327               | 0.1255              | 0.29545           | 0.4579475        | 0.10074845        |
| 0.22        | 0.5120               | 0.4514               | 0.0606              | 0.48170           | 0.7466350        | 0.16425970        |
| 0.22        | 0.8300               | 0.8000               | 0.0300              | 0.81500           | 1.2632500        | 0.27791500        |
| 0.24        | 0.3320               | 0.2222               | 0.1098              | 0.27710           | 0.4295050        | 0.10308120        |
| 0.24        | 0.5060               | 0.4200               | 0.0860              | 0.46300           | 0.7176500        | 0.17223600        |
| 0.24        | 0.8055               | 0.7859               | 0.0196              | 0.79570           | 1.2333350        | 0.29600040        |
| 0.26        | 0.3080               | 0.2140               | 0.0940              | 0.26100           | 0.4045500        | 0.10518300        |
| 0.26        | 0.4940               | 0.3950               | 0.0990              | 0.44450           | 0.6889750        | 0.17913350        |
| 0.26        | 0.6620               | 0.6229               | 0.0391              | 0.64245           | 0.9957975        | 0.25890735        |
| 0.28        | 0.2870               | 0.2060               | 0.0810              | 0.24650           | 0.3820750        | 0.10698100        |
| 0.28        | 0.4780               | 0.3730               | 0.1050              | 0.42550           | 0.6595250        | 0.18466700        |
| 0.28        | 0.6520               | 0.5850               | 0.0670              | 0.61850           | 0.9586750        | 0.26842900        |
| 0.30        | 0.2680               | 0.2000               | 0.0680              | 0.23400           | 0.3627000        | 0.10881000        |
| 0.30        | 0.4600               | 0.3550               | 0.1050              | 0.40750           | 0.6316250        | 0.18948750        |
| 0.30        | 0.6330               | 0.5530               | 0.0800              | 0.59300           | 0.9191500        | 0.27574500        |
| 0.32        | 0.2500               | 0.1950               | 0.0550              | 0.22250           | 0.3448750        | 0.11036000        |
| 0.32        | 0.4320               | 0.3400               | 0.0920              | 0.38600           | 0.5983000        | 0.19145600        |
| 0.32        | 0.6060               | 0.5250               | 0.0810              | 0.56550           | 0.8765250        | 0.28048800        |
| 0.34        | 0.2350               | 0.1910               | 0.0440              | 0.21300           | 0.3301500        | 0.11225100        |
| 0.34        | 0.4040               | 0.3270               | 0.0770              | 0.36550           | 0.5665250        | 0.19261850        |
| 0.34        | 0.5740               | 0.5010               | 0.0730              | 0.53750           | 0.8331250        | 0.28326250        |
| 0.36        | 0.2210               | 0.1874               | 0.0336              | 0.20420           | 0.3165100        | 0.11394360        |
| 0.36        | 0.3780               | 0.3160               | 0.0620              | 0.34700           | 0.5378500        | 0.19362600        |
| 0.36        | 0.5400               | 0.4800               | 0.0600              | 0.51000           | 0.7905000        | 0.28458000        |
| 0.38        | 0.2080               | 0.1846               | 0.0234              | 0.19630           | 0.3042650        | 0.11562070        |
| 0.38        | 0.3540               | 0.3070               | 0.0470              | 0.33050           | 0.5122750        | 0.19466450        |
| 0.38        | 0.5100               | 0.4620               | 0.0480              | 0.48600           | 0.7533000        | 0.28625400        |
| 0.40        | 0.1960               | 0.1823               | 0.0137              | 0.18915           | 0.2931825        | 0.11727300        |
| 0.40        | 0.3325               | 0.3000               | 0.0325              | 0.31625           | 0.4901875        | 0.19607500        |
| 0.40        | 0.4800               | 0.4468               | 0.0332              | 0.46340           | 0.7182700        | 0.28730800        |
| 0.42        | 0.3120               | 0.2937               | 0.0183              | 0.30285           | 0.4694175        | 0.19715535        |
| 0.42        | 0.4520               | 0.4340               | 0.0180              | 0.44300           | 0.6866500        | 0.28839300        |

Table 3.5: Summary of TE band gaps of most interest to the case of a square photonic crystal of rods of germanium (Ge) in the air

**Photonic band gap structures diagram** The band structure in the case of the  $TE$  polarization and a fill factor  $r/a = 0.16$  is reported in figure 3.20, we note there is other PBG

In this case of photonic crystal with square lattice of rods of germanium ( $Ge$ ) in air; polarized  $TE$  mode for a filling factor  $r/a = 0.16$ , the photonic band gap used is centered on 0.36155.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.36155 * 1.55 * 10^{-6}m = 0.5604025\mu m.$$

This implies a pattern of radius:

$$r = 0.16 * 0.5604025 * 10^{-6}m = 0.0896644\mu m.$$

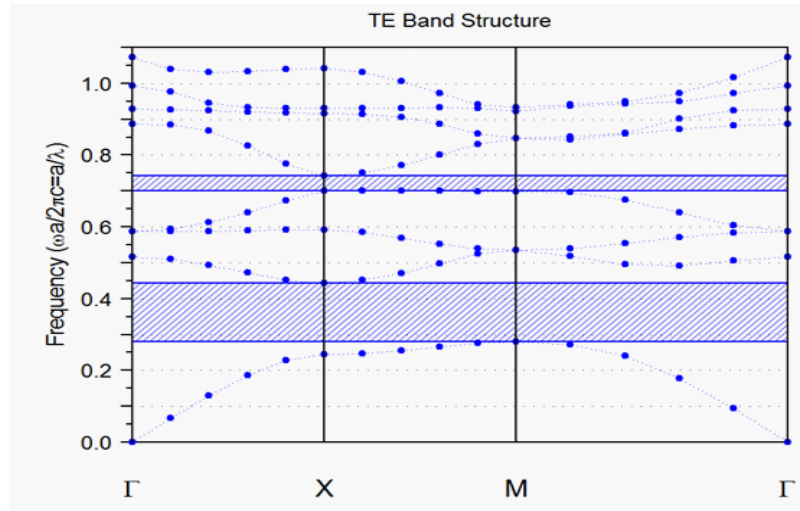


Figure 3.20: Band diagram of a photonic crystal with square lattice of rods of germanium ( $Ge$ ) in air; polarized  $TE$  mode for a filling factor  $r/a = 0.16$ .

**Reflection and transmission diagrams** Figure 3.21 shows the transmission spectrum and reflection through structures of the square lattice of rods of germanium ( $Ge$ ) in air with different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.3 microns, 2.05 microns) and that from the structure (c) reflection is at its maximum value. From the results obtained, we note that from the case (k) reflection remains unchanged.

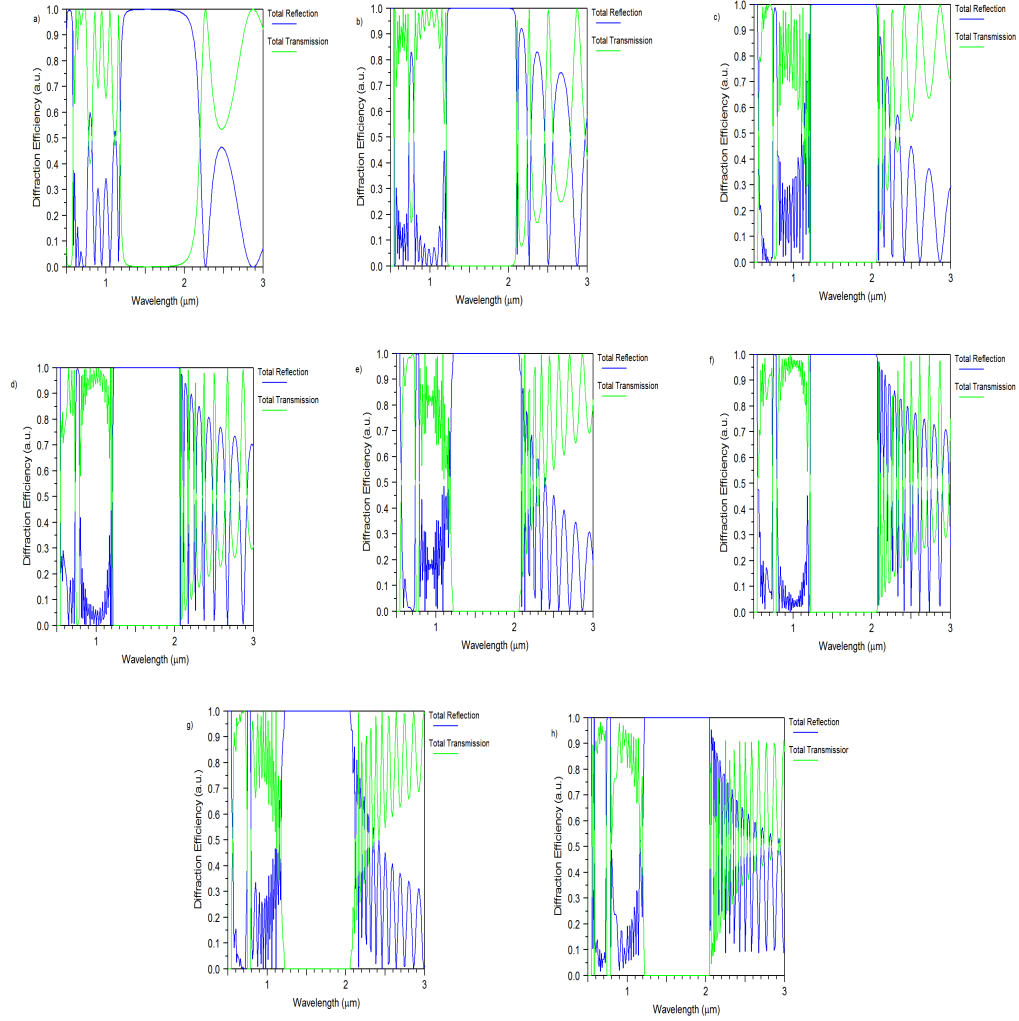


Figure 3.21: Transmission and reflection through the square lattice of rods of germanium ( $Ge$ ) in air ; polarized mode TE  $a = 0.5604025\mu m$ ,  $r = 0.0896644\mu m$ .

**TM case** For the case of  $TM$  polarization, we note the existence of several  $TM$  photonic band gap. The photonic bandgap of greatest interest are grouped in table 3.6:

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$                | $r$                |
|-------------|----------------------|----------------------|---------------------|-------------------|--------------------|--------------------|
| 0.10        | 1.1130000            | 1.09040              | 0.0226000           | 1.10170000        | 1.707635000        | 0.170763500        |
| 0.12        | 1.0914000            | 1.06900              | 0.0224000           | 1.08020000        | 1.674310000        | 0.200917200        |
| 0.14        | 1.0384000            | 1.02320              | 0.0152000           | 1.03080000        | 1.597740000        | 0.223683600        |
| 0.26        | 0.3628000            | 0.35077              | 0.0120300           | 0.35678500        | 0.553016750        | 0.143784355        |
| 0.28        | 0.3470000            | 0.32780              | 0.0192000           | 0.33740000        | 0.522970000        | 0.146431600        |
| 0.30        | 0.3312765            | 0.30735              | 0.0239265           | 0.31931325        | 0.494935538        | 0.148480661        |
| 0.32        | 0.3162100            | 0.28965              | 0.0265600           | 0.30293000        | 0.469541500        | 0.150253280        |
| 0.34        | 0.3005390            | 0.27365              | 0.0268890           | 0.28709450        | 0.444996475        | 0.151298802        |
| 0.34        | 0.5270000            | 0.51500              | 0.0120000           | 0.52100000        | 0.807550000        | 0.274567000        |
| 0.36        | 0.2846000            | 0.25930              | 0.0253000           | 0.27195000        | 0.421522500        | 0.151748100        |
| 0.36        | 0.5076000            | 0.48410              | 0.0235000           | 0.49585000        | 0.768567500        | 0.276684300        |
| 0.38        | 0.2695000            | 0.24650              | 0.0230000           | 0.25800000        | 0.399900000        | 0.151962000        |
| <b>0.38</b> | <b>0.4865000</b>     | <b>0.45630</b>       | <b>0.0302000</b>    | <b>0.47140000</b> | <b>0.730670000</b> | <b>0.277654600</b> |
| 0.40        | 0.2535500            | 0.23495              | 0.0186000           | 0.24425000        | 0.378587500        | 0.151435000        |
| 0.40        | 0.4655000            | 0.44000              | 0.0255000           | 0.45275000        | 0.701762500        | 0.280705000        |
| 0.42        | 0.2373300            | 0.22445              | 0.0128800           | 0.23089000        | 0.357879500        | 0.150309390        |
| 0.42        | 0.4450000            | 0.42660              | 0.0184000           | 0.43580000        | 0.675490000        | 0.283705800        |
| 0.44        | 0.2210000            | 0.21500              | 0.0060000           | 0.21800000        | 0.337900000        | 0.148676000        |
| 0.44        | 0.4254000            | 0.41380              | 0.0116000           | 0.41960000        | 0.650380000        | 0.286167200        |

Table 3.6: Summary of TM band gaps of most interest to the case of a square photonic crystal of rods of germanium (Ge) in the air

The wider photonic bandgap  $TE$  is observed for a filling factor  $r/a = 0.38$

The ratio  $r/a = 0.38$  provides the ability to have a photonic band gap in  $TE$  mode of relatively small width [ $\Delta(a/\lambda) = 0.0302$ ] with maximum normalized frequency  $(a/\lambda)_{\max} = 0.4865$  and the minimal normalized frequency  $(a/\lambda)_{\min} = 0.45630$

**Photonic band gap structures diagram** The band structure in the case of the  $TM$  polarization and a fill factor  $r/a = 0.38$  is reported in figure 3.22 ,we note there is other PBG

In this case of a photonic crystal with square lattice of rods of germanium ( $Ge$ ) in air with ratio  $r/a = 0.38$  for the  $TM$  polarization, the photonic band gap used is centered on 0.47140.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.4714 * 1.55 * 10^{-6} m = 0.73067\mu m.$$

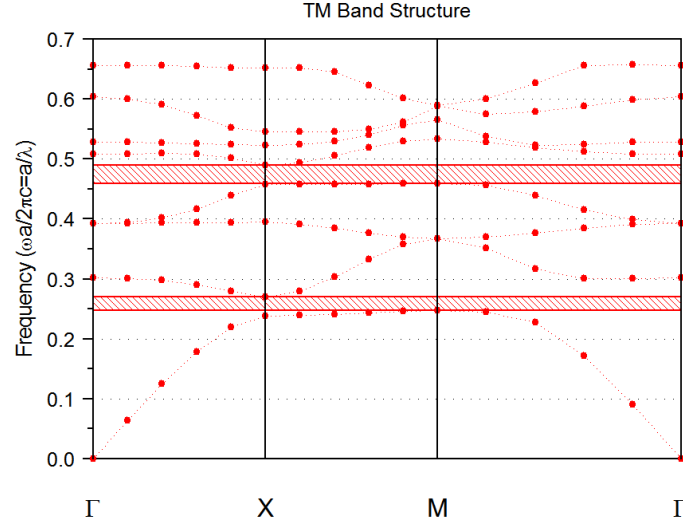


Figure 3.22: Band diagram of a photonic crystal with square lattice of rods of germanium (*Ge*) in air; polarized *TM* mode for a filling factor  $r/a = 0.38$ .

This implies a pattern of radius:

$$r = 0.38 * 0.73067 * 10^{-6} m = 0.2776546 \mu m.$$

**Reflection and transmission diagrams** Figure 3.23 shows the transmission spectrum and reflection through structures of the square lattice of rods of germanium (*Ge*) in air with different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a *TM* polarization. Transmission measurements show a very strong disconnect between (1.4 microns, 1.8 microns) and that from the structure (c) reflection is at its maximum value. From the results obtained, we note that from the case (k) reflection remains unchanged.

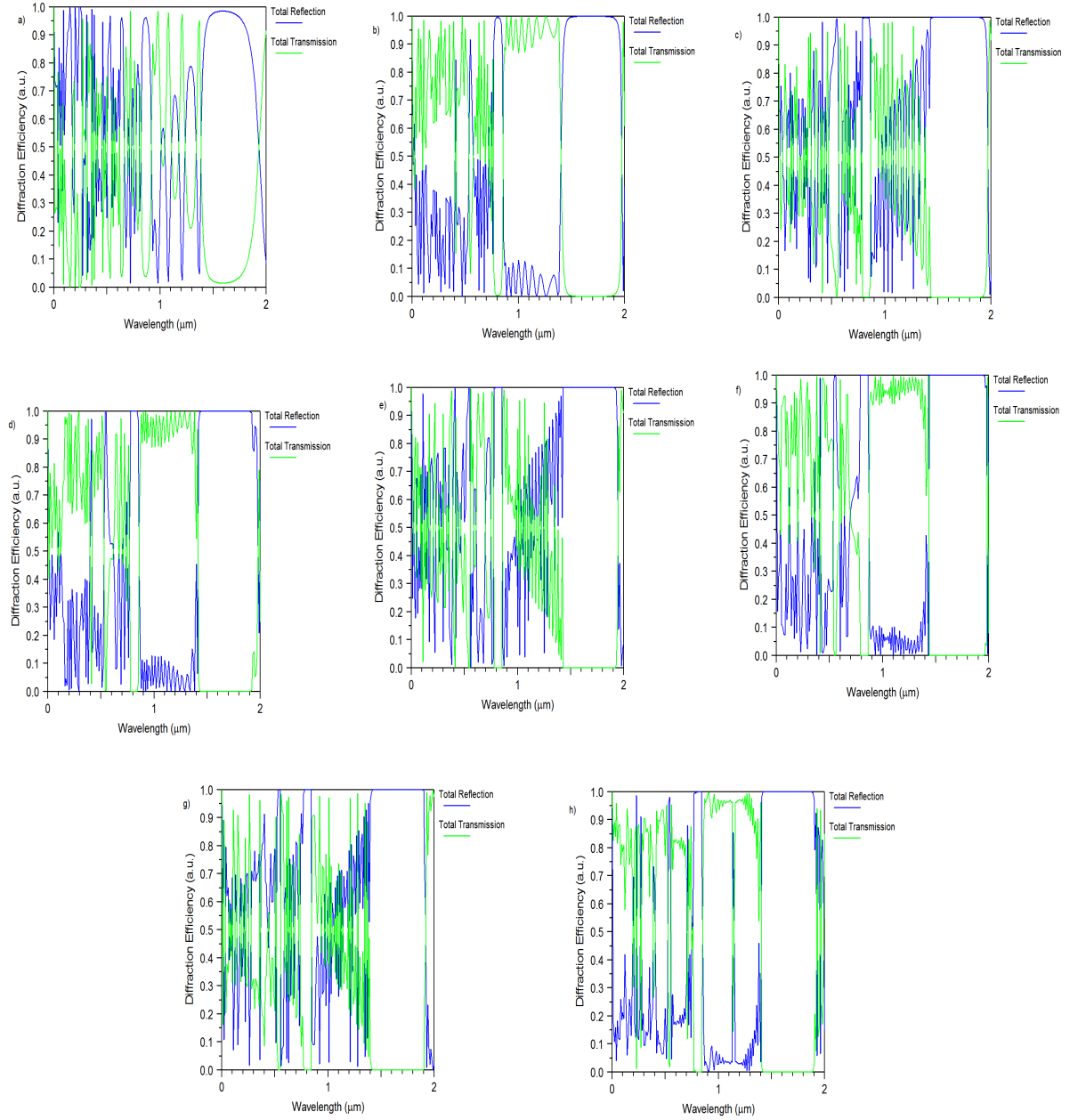


Figure 3.23: Transmission and reflection through the square lattice of rods of germanium ( $Ge$ ) in air ; polarized mode TM  $a = 0.73067\mu m$ ,  $r = 0.2776546\mu m$ .

### Two-dimensional photonic crystals with triangular lattice

**Definition of the parameters** Figure 3.24 shows the variation of refractive index and the periodicity of the refractive index of the second new structure (triangular lattice of rods of germanium (*Ge*) in the air).

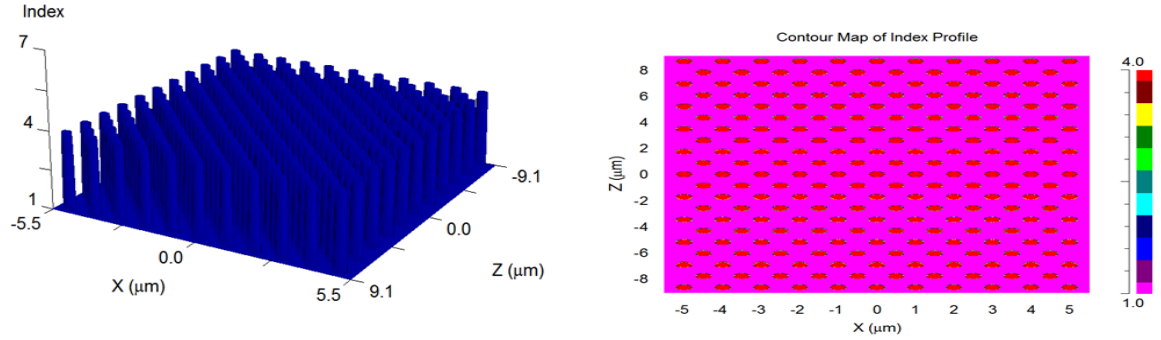


Figure 3.24: Index profile of 2D triangular photonic crystals with rods of germanium (*Ge*) in air; (a) with 2D in 2 axes ( $x, z$ ) (a) with 3D

the variation of refractive index on two axes ( $Z, X$ ) there are square lattice of rods of germanium (*Ge*) in the air, but in the other direction (axis  $Y$ ) there is not any change.

**Photonic band gap map diagram** From the figure 3.25 we note the presence of some Photonic band gap for *TE* and *TM* polarisation, and there is not any presence for the complete photonic band gap in this case.

Figure 3.25 shows maps of photonic band gap for both polarizations *TE* and *TM* in the case of a photonic crystal with triangular lattice of rods of germanium (*Ge*) in air ( $\epsilon = 1$ ). We must achieve a ratio  $r/a = 0.04$  appear to see the bandgaps *TE* and a ratio  $r/a = 0.8$  for the case of *TM* photonic bandgap so large. we note there is not the overlap between the photonic bandgaps *TE* and *TM*, for that we cannot see any complete photonic bandgaps.

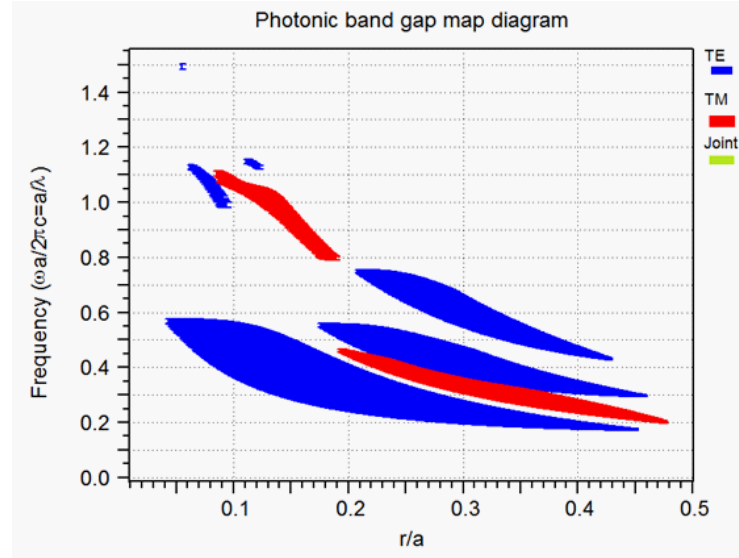


Figure 3.25: Map of photonic band gap for both TE and TM polarizations. Case of 2D photonic crystals with triangular lattice of rods of germanium ( $Ge$ ) in air with radius of germanium cylinder changing from 0 to  $0.5a$

**TE case** For  $TE$  polarization, we note the existence  $TE$  photonic band gap. The photonic bandgap of greatest interest are grouped in the following table:

The wider photonic bandgap  $TE$  is observed for a filling factor  $r/a = 0.14$

The ratio  $r/a = 0.14$  provides the ability to have a photonic band gap in  $TE$  mode with width  $[\Delta(a/\lambda) = 0.21972]$  and with maximum normalized frequency  $(a/\lambda)_{max} = 0.518$  and the minimal normalized frequency  $(a/\lambda)_{min} = 0.29828$

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|-------------------|
| 0.06        | 0.5757               | 0.48880              | 0.08690             | 0.53225           | 0.8249875        | 0.04949925        |
| 0.08        | 0.5718               | 0.41790              | 0.15390             | 0.49485           | 0.7670175        | 0.06136140        |
| 0.08        | 1.0847               | 1.02480              | 0.05990             | 1.05475           | 1.6348625        | 0.13078900        |
| 0.10        | 0.5634               | 0.36560              | 0.19780             | 0.46450           | 0.7199750        | 0.07199750        |
| 0.12        | 0.545                | 0.33000              | 0.21500             | 0.43750           | 0.6781250        | 0.08137500        |
| 0.12        | 1.137                | 1.12370              | 0.01330             | 1.13035           | 1.7520425        | 0.21024510        |
| <b>0.14</b> | <b>0.518</b>         | <b>0.29828</b>       | <b>0.21972</b>      | <b>0.40814</b>    | <b>0.6326170</b> | <b>0.08856638</b> |
| 0.16        | 0.471                | 0.27340              | 0.19760             | 0.37220           | 0.5769100        | 0.09230560        |
| 0.18        | 0.4413               | 0.25500              | 0.18630             | 0.34815           | 0.5396325        | 0.09713385        |
| 0.18        | 0.5588               | 0.53660              | 0.02220             | 0.54770           | 0.8489350        | 0.15280830        |
| 0.20        | 0.4062               | 0.24000              | 0.16620             | 0.32310           | 0.5008050        | 0.10016100        |
| 0.20        | 0.5588               | 0.53650              | 0.02230             | 0.54765           | 0.8488575        | 0.16977150        |
| 0.22        | 0.3735               | 0.22720              | 0.14630             | 0.30035           | 0.4655425        | 0.10241935        |
| 0.22        | 0.5488               | 0.45360              | 0.09520             | 0.50120           | 0.7768600        | 0.17090920        |
| 0.22        | 0.7533               | 0.71050              | 0.04280             | 0.73190           | 1.1344450        | 0.24957790        |
| 0.24        | 0.3445               | 0.21720              | 0.12730             | 0.28085           | 0.4353175        | 0.10447620        |
| 0.24        | 0.5356               | 0.42280              | 0.11280             | 0.47920           | 0.7427600        | 0.17826240        |
| 0.24        | 0.7455               | 0.66030              | 0.08520             | 0.70290           | 1.0894950        | 0.26147880        |
| 0.26        | 0.3444               | 0.21720              | 0.12720             | 0.28080           | 0.4352400        | 0.11316240        |
| 0.26        | 0.5168               | 0.39700              | 0.11980             | 0.45690           | 0.7081950        | 0.18413070        |
| 0.26        | 0.7270               | 0.61680              | 0.11020             | 0.67190           | 1.0414450        | 0.27077570        |
| 0.28        | 0.2960               | 0.20100              | 0.09500             | 0.24850           | 0.3851750        | 0.10784900        |
| 0.28        | 0.4944               | 0.37570              | 0.11870             | 0.43505           | 0.6743275        | 0.18881170        |
| 0.28        | 0.6982               | 0.57950              | 0.11870             | 0.63885           | 0.9902175        | 0.27726090        |
| 0.30        | 0.2761               | 0.19500              | 0.08110             | 0.23555           | 0.3651025        | 0.10953075        |
| 0.30        | 0.4716               | 0.35820              | 0.11340             | 0.41490           | 0.6430950        | 0.19292850        |
| 0.30        | 0.6590               | 0.54760              | 0.11140             | 0.60330           | 0.9351150        | 0.28053450        |
| 0.32        | 0.2582               | 0.18950              | 0.06870             | 0.22385           | 0.3469675        | 0.11102960        |
| 0.32        | 0.4437               | 0.34300              | 0.10070             | 0.39335           | 0.6096925        | 0.19510160        |
| 0.32        | 0.6143               | 0.52010              | 0.09420             | 0.56720           | 0.8791600        | 0.28133120        |
| 0.34        | 0.2420               | 0.18500              | 0.05700             | 0.21350           | 0.3309250        | 0.11251450        |
| 0.34        | 0.4440               | 0.34320              | 0.10080             | 0.39360           | 0.6100800        | 0.20742720        |
| 0.34        | 0.5740               | 0.49640              | 0.07760             | 0.53520           | 0.8295600        | 0.28205040        |
| 0.36        | 0.2272               | 0.18110              | 0.04610             | 0.20415           | 0.3164325        | 0.11391570        |
| 0.36        | 0.3911               | 0.32010              | 0.07100             | 0.35560           | 0.5511800        | 0.19842480        |
| 0.36        | 0.5377               | 0.47600              | 0.06170             | 0.50685           | 0.7856175        | 0.28282230        |
| 0.38        | 0.2140               | 0.17800              | 0.03600             | 0.19600           | 0.3038000        | 0.11544400        |
| 0.38        | 0.3683               | 0.31150              | 0.05680             | 0.33990           | 0.5268450        | 0.20020110        |
| 0.38        | 0.5040               | 0.45840              | 0.04560             | 0.48120           | 0.7458600        | 0.28342680        |
| 0.40        | 0.2018               | 0.17540              | 0.02640             | 0.18860           | 0.2923300        | 0.11693200        |
| 0.40        | 0.3479               | 0.30450              | 0.04340             | 0.32620           | 0.5056100        | 0.20224400        |
| 0.40        | 0.4726               | 0.44360              | 0.02900             | 0.45810           | 0.7100550        | 0.28402200        |
| 0.42        | 0.1906               | 0.17310              | 0.01750             | 0.18185           | 0.2818675        | 0.11838435        |
| 0.42        | 0.3290               | 0.29920              | 0.02980             | 0.31410           | 0.4868550        | 0.20447910        |
| 0.42        | 0.4429               | 0.43130              | 0.01160             | 0.43710           | 0.6775050        | 0.28455210        |
| 0.44        | 0.1804               | 0.17130              | 0.00910             | 0.17585           | 0.2725675        | 0.11992970        |
| 0.44        | 0.3120               | 0.29530              | 0.01670             | 0.30365           | 0.4706575        | 0.20708930        |

Table 3.7: Summary of TE band gaps of most interest to the case of a triangular photonic crystal of rods of germanium (Ge) in the air

**Photonic band gap structures diagram** The band structure in the case of the  $TE$  polarization and a fill factor  $r/a = 0.14$  is reported in figure 74, we note there is not any other PBG

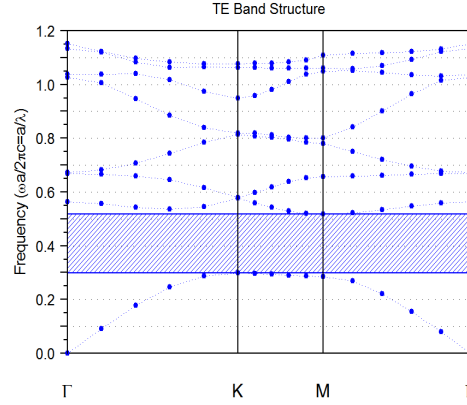


Figure 3.26: Band diagram of a photonic crystal with triangular lattice of rods of germanium ( $Ge$ ) in air; polarized  $TE$  mode for a filling factor  $r/a = 0.14$ .

In the case of a photonic crystal with triangular lattice of rods of germanium ( $Ge$ ) in air for the  $TE$  polarization, the photonic band gap used is centered on 0.40814.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.40814 * 1.55 \cdot 10^{-6} m = 0.6326170 \mu m.$$

This implies a pattern of radius:

$$r = 0.14 * 0.6326170 * 10^{-6} m = 0.08856638 \mu m.$$

**Reflection and transmission diagrams** Figure 3.27 shows the transmission spectrum and reflection through structures the triangular lattice rods of germanium ( $Ge$ ) in the air of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.15 microns, 1.95 microns) and that from the structure (d) reflection is at its maximum value. From the results obtained, we note that from the case (e) reflection remains unchanged .

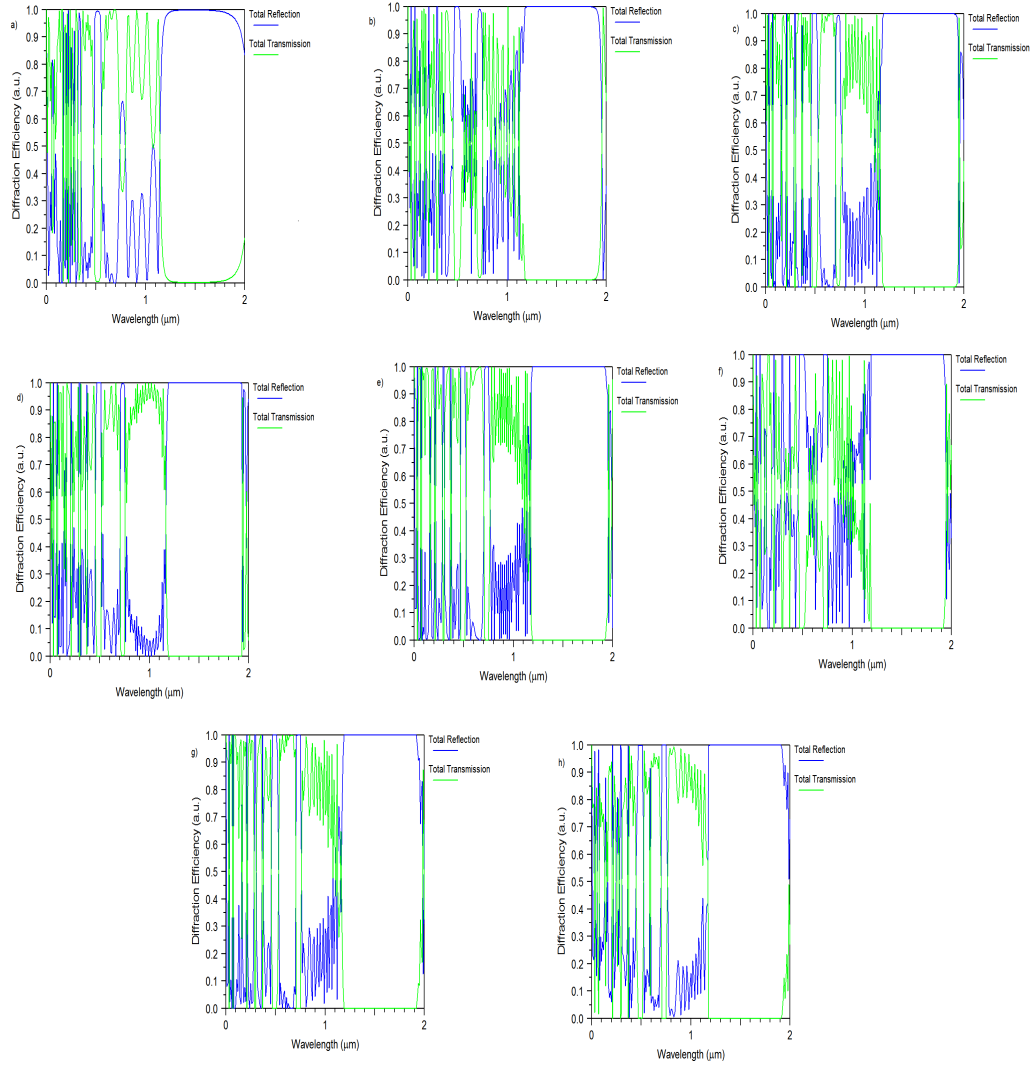


Figure 3.27: Transmission and reflection through the triangular structures rods of germanium ( $Ge$ ) in the air; polarized mode TE  $a = 0.6326170\mu m.$ ,  $r = 0.08856638\mu m.$

**TM case** For the case of  $TM$  polarization, we note the existence of several  $TM$  photonic band gap. The photonic bandgap of greatest interest are grouped in the following table:

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|-------------------|
| 0.10        | 1.0850               | 1.0485               | 0.0365              | 1.06675           | 1.6534625        | 0.16534625        |
| 0.12        | 1.0575               | 1.0025               | 0.0550              | 1.03000           | 1.5965000        | 0.19158000        |
| <b>0.14</b> | <b>1.0230</b>        | <b>0.945</b>         | <b>0.0780</b>       | <b>0.98400</b>    | <b>1.5252000</b> | <b>0.21352800</b> |
| 0.16        | 0.9227               | 0.8584               | 0.0643              | 0.89055           | 1.3803525        | 0.22085640        |
| 0.18        | 0.8350               | 0.7917               | 0.0433              | 0.81335           | 1.2606925        | 0.22692465        |
| 0.20        | 0.4597               | 0.4416               | 0.0181              | 0.45065           | 0.6985075        | 0.13970150        |
| 0.22        | 0.4420               | 0.406                | 0.0360              | 0.42400           | 0.6572000        | 0.14458400        |
| 0.24        | 0.4214               | 0.3754               | 0.0460              | 0.39840           | 0.6175200        | 0.14820480        |
| 0.26        | 0.3968               | 0.3491               | 0.0477              | 0.37295           | 0.5780725        | 0.15029885        |
| 0.28        | 0.3759               | 0.3262               | 0.0497              | 0.35105           | 0.5441275        | 0.15235570        |
| 0.30        | 0.3577               | 0.3062               | 0.0515              | 0.33195           | 0.5145225        | 0.15435675        |
| 0.32        | 0.3422               | 0.2884               | 0.0538              | 0.31530           | 0.4887150        | 0.15638880        |
| 0.34        | 0.3284               | 0.2723               | 0.0561              | 0.30035           | 0.4655425        | 0.15828445        |
| 0.36        | 0.3147               | 0.2582               | 0.0565              | 0.28645           | 0.4439975        | 0.15983910        |
| 0.38        | 0.2977               | 0.2451               | 0.0526              | 0.27140           | 0.4206700        | 0.15985460        |
| 0.40        | 0.2800               | 0.2338               | 0.0462              | 0.25690           | 0.3981950        | 0.15927800        |
| 0.42        | 0.2620               | 0.223                | 0.0390              | 0.24250           | 0.3758750        | 0.15786750        |
| 0.44        | 0.2430               | 0.2134               | 0.0296              | 0.22820           | 0.3537100        | 0.15563240        |
| 0.46        | 0.2218               | 0.2042               | 0.0176              | 0.21300           | 0.3301500        | 0.15186900        |

Table 3.8: Summary of TM band gaps of most interest to the case of a triangular photonic crystal of rods of germanium (Ge) in the air

The wider photonic bandgap  $TM$  is observed for a filling factor  $r/a = 0.14$

The ratio  $r/a = 0.14$  provides the ability to have a photonic band gap in  $TM$  mode of relatively small width [ $\Delta(a/\lambda) = 0.0780$ ] with maximum normalized frequency  $(a/\lambda)_{\max} = 1.0230$ , and the minimal normalized frequency  $(a/\lambda)_{\min} = 0.945$

**Photonic band gap structures diagram** The band structure in the case of the  $TM$  polarization and a fill factor  $r/a = 0.14$  is reported in figure 3.28 :

In the case of a photonic crystal with triangular lattice of rods of germanium (Ge) in air for the  $TM$  polarization, the photonic band gap used is centered on 0.984.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.984 * 1.55.10^{-6}m = 1.5252000\mu m.$$

$$a = 1.5252000\mu m.$$

This implies a pattern of radius:

$$r = 0.14 * 1.5252000 * 10^{-6}m = 0.21352800\mu m.$$

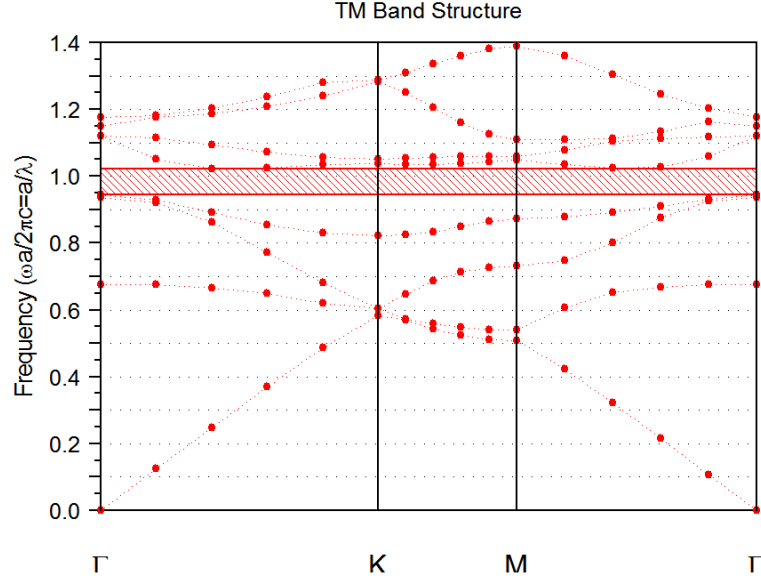


Figure 3.28: Band diagram of a photonic crystal with triangular lattice of rods of germanium ( $Ge$ ) in air; polarized  $TM$  mode for a filling factor  $r/a = 0.14$ .

**Reflection and transmission diagrams** Figure 3.29 shows the transmission spectrum and reflection through structures the triangular lattice rods of germanium ( $Ge$ ) in the air of different sizes [(a)5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TM$  polarization. Transmission measurements show a very strong disconnect between (1.49 microns, 1.59 microns).

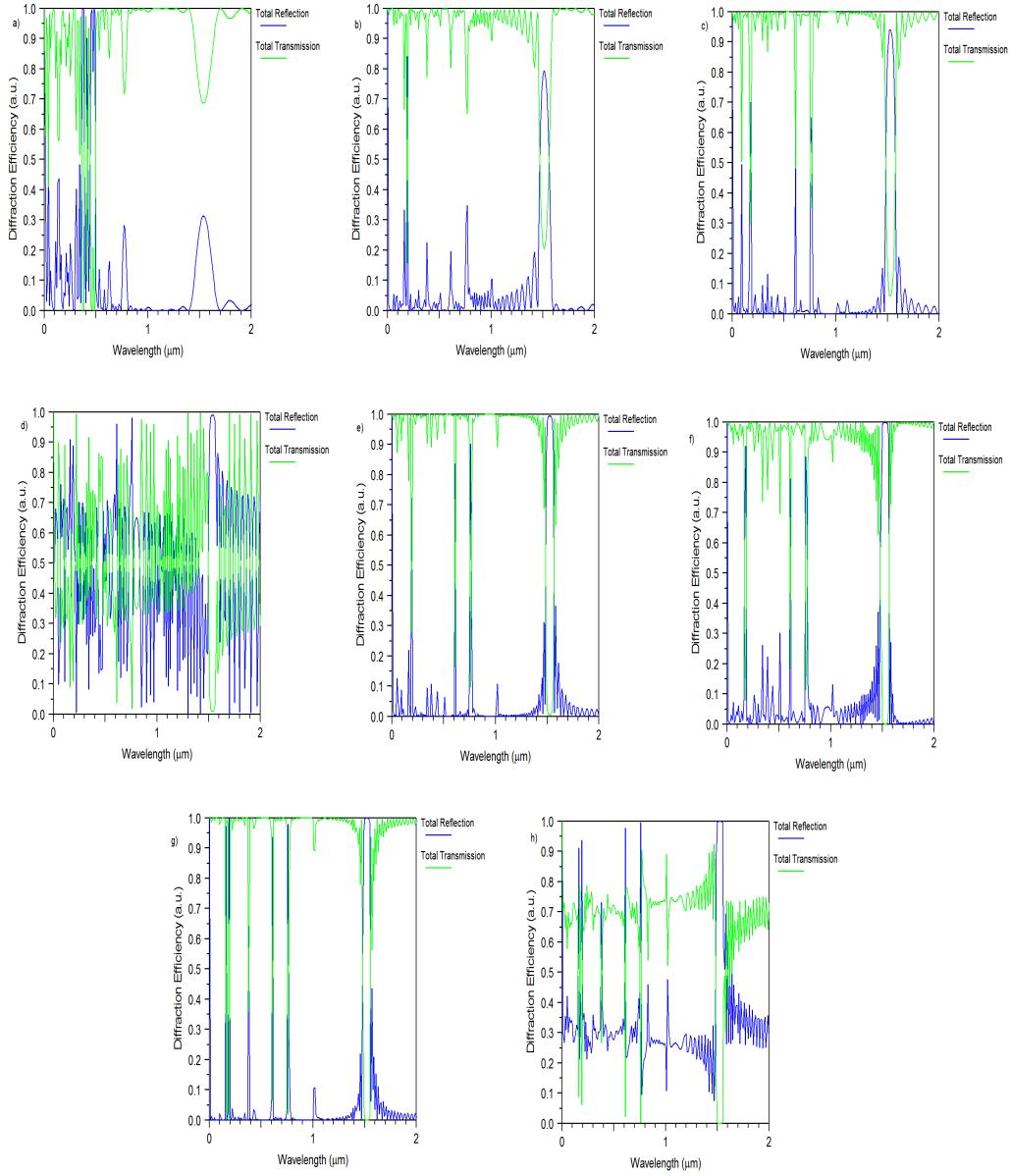


Figure 3.29: Transmission and reflection through the triangular structures rods of germanium ( $Ge$ ) in the air; polarized mode TM  $a = 1.5252\mu m, r = 0.213528\mu m$ .

### 3.5 Characteristics of the Lithium niobate ( $LiNbO_3$ ) material

Lithium niobate ( $LiNbO_3$ ) is an electro-optic material very interesting, now that, apart from the semiconductor, is a crystalline material used in integrated optical devices

Lithium niobate is a dielectric material, widely used in its optical properties. It has the following characteristics:

Transparency 0.4 - 4 microns;

high optical non-linearity;

Easy realization of optical waveguide which allows combining the performance of the guide structures with those of *PBG*.

Numerical simulations are conducted for the two propagation modes *TE* and *TM* using the two geometries of structures (square, triangle). We consider two types of lattice holes of air ( $\varepsilon = 1; n = 1$ ) in lithium niobate and the rods of the lithium niobate ( $\varepsilon = 4, 8; n = 2, 2$ )

### 3.6 One-dimensional photonic crystals simulation

As we said before, we are begin with one dimensional photonic crystal because it is the simplest possible type of the photonic crystals to see the diagrams of characteristics of the photonic crystals (photonic band gap structures diagram, reflection and de transmission diagrams). We will use the Plane Wave Expansion (*PWE*) for this simulation.

#### 3.6.1 Definition of the parameter

In the present work, band gap has been computed for a 1D photonic crystal and the diagram of the transmission and reflection . Both plane wave expansion (*PWE*) and *FDTD*, see chapter two, with tow types of polarisation *TE* and *TM* ; we took the direction of propagation on the axis *X* . Like we said before, the materials of the simulation in this case is the lithium niobate ( $LiNbO_3 : \varepsilon = 4, 8; n = 2, 2$ ) and air ( $\varepsilon = 1; n = 1$ ) layer of the the lithium niobate ( $LiNbO_3$ ) after layer of air with period of  $d = 1\mu m$  like we see in the figure 3.30

The last figure present the variation of the index profile of the structure for a simulation in 3 dimension on three axes( $X, Z, index$ ) and in 2 dimension on two axes ( $X, Z$ ).like we see here the direction of the variation of the index profile is on the axis *X*

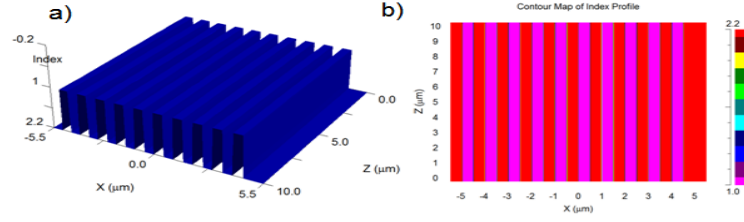


Figure 3.30: Index profile of 1D photonic crystals (a) with 3D (b) with 2D in 2 axes (x,z)

### 3.6.2 Photonic band gap structures diagram

We have two types of the band diagram, of  $TE$  polarisation and of  $TM$  polarisation because every type of polarisation has a special photonic band gaps see figure 3.31

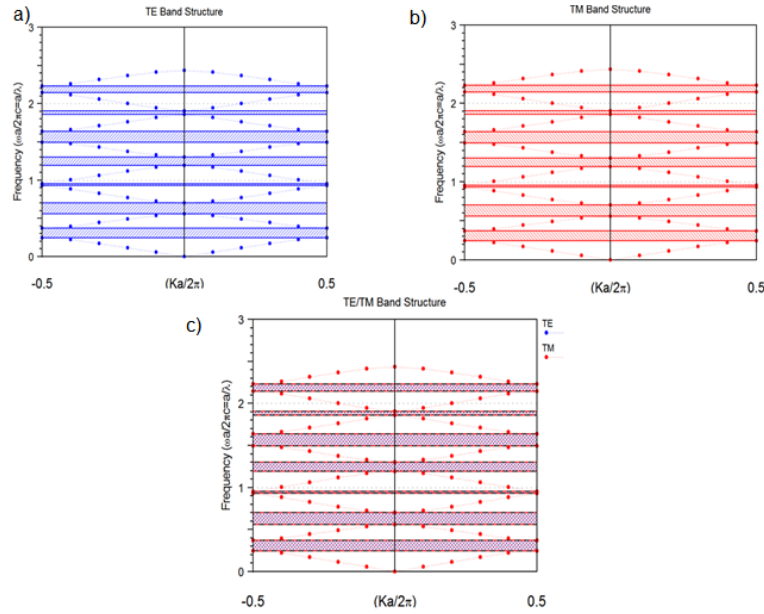


Figure 3.31: Band diagram of a 1D photonic crystal air and lithium niobate ( $LiNbO_3$ ) with the period  $d = 1\mu m$ ; a) polarized  $TE$  mode, b) polarized  $TM$  mode, c) polarized  $TE$  and  $TM$  mode

Figure 3.31 present the photonic band gaps diagram of 1D photonic crystal with 10 layers of air and 10 layers of the lithium niobate ( $LiNbO_3$ ), with periodicity of  $d = 1\mu m$ , for that we obtain the results in figure 3.31 (a) for  $TE$  polarization and (b) for  $TM$  polarization. For both polarisation modes, we have 7  $PBG$  and we note that there is a total photonic band gap.

### 3.6.3 Reflection and transmission diagrams

In this part of the simulation, different wavelengths are sent over the structure with the aim to define the reflection and the transmission ratios:

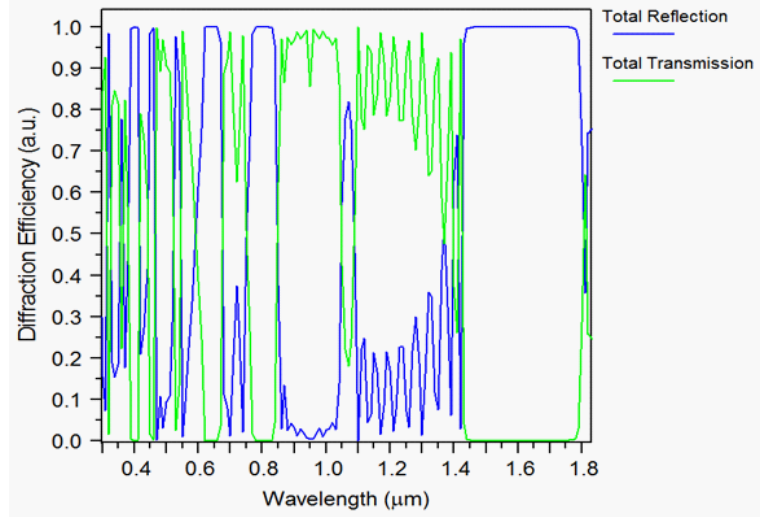


Figure 3.32: diagram of transmission and reflection of the 1D photonic crystals with air and the lithium niobate ( $LiNbO_3$ ) with the period  $d = 1\mu m$  polarized  $TE$  and  $TM$  mode

## 3.7 Two-dimensional photonic crystals simulation

In this part of simulations, we will use two structures (square, triangle) with two configurations: air holes lattice in lithium niobate medium, and the rods of the lithium niobate in air medium. The two modes of polarisation  $TE$  and  $TM$  will be defined.

### 3.7.1 Air cylinders being placed in Lithium niobate

Numerical simulations are conducted for the two propagation modes  $TE$  and  $TM$  using the two geometries of structures (square, triangle). We consider the lattice holes of air ( $\epsilon = 1; n = 1$ ) in lithium niobate medium ( $\epsilon = 4, 8; n = 2, 2$ )

Using the method of plane waves (presented in Chapter two), we studied the evolution of photonic bandgap. The calculations were performed by varying the wave vector (module and direction) along the directions of high symmetry of the first Brillouin zone.

### Two-dimensional photonic crystals with square lattice

As mentioned in the beginning of this part, we have two-dimensional photonic crystals with square lattice holes of air in lithium niobate

**Definition of the parameter:** For this case of two-dimensional photonic crystals with square lattice holes of air in lithium niobate we have the path of unit cell is :  $\Gamma \rightarrow X \rightarrow M \rightarrow \Gamma$  (Figure 3.33 (c)). and the index profile to see the variation of refractive index and the periodicity of the unit cell

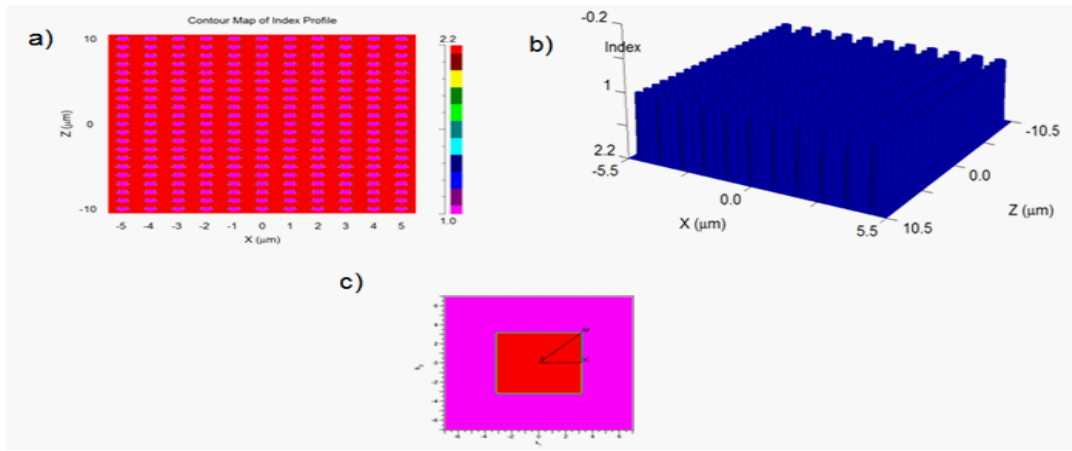


Figure 3.33: Index profile of 2D square photonic crystals with holes of air in lithium niobate ( $LiNbO_3$ ); (a) with 2D in 2 axes (x,z) (a) with 3D

From the last figure, we see the unit cell to make symmetry of the first Brillouin zone and the index profile of the structure of simulation for first case . This structure is the reference to calculate the map of the photonic band gap

**Photonic band gap map diagram:** A detailed study was conducted on the evolution of the photonic band gap and its width according to the ratio of the radius of the holes on the grounds of the period of the structure ( $r/a$ ). We considered both propagation modes  $TE$  and  $TM$ , and the calculation of  $PBG$  to seek all possible bandgaps.

Figure 3.34 shows maps of photonic band gap for both polarizations  $TE$  (ordinary index  $\epsilon_0 = 4,889$ ) and  $TM$  (extraordinary index  $\epsilon_e = 4,571$ ) in the case of a photonic crystal of square air holes ( $\epsilon = 1$ ) on substrate  $LiNbO_3$ . For this structure we observe the appearance of some photonic band gap widths of  $TM$  and  $TE$  but also for very small filling factors very high.

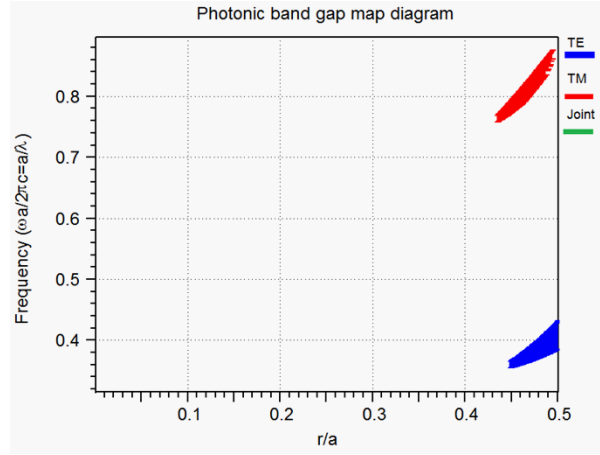


Figure 3.34: Map of photonic band gap for both TE and TM polarizations. Case of a square PC of air holes ( $\varepsilon = 1$ ) in  $LiNbO_3$

We must achieve a ratio of  $r/a = 0.452$  appear to see the bandgaps  $TE$  lowest energy and a ratio  $r/a = 0.436$  for the case of  $TM$  photonic bandgap lowest energy. No overlap between the photonic bandgap  $TE$  and  $TM$  is observed while no complete photonic bandgap.

**TE case :** To determine the photonic band gap that will operate in case of a square structure - holes configuration -  $TE$  polarization, an array containing the widths of the forbidden photonic bands most interesting was established from the table 3.9:

After this result we will choose the wide photonic band gap.

The wider bandgap photonic  $TE$  is observed for a filling factor  $r/a = 0.496$ , its width is 0.0430 and extends over the energy range  $a/\lambda \approx 0.382 - 0.425$  (figure 3.35).

| $r/a$        | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$               |
|--------------|----------------------|----------------------|---------------------|-------------------|------------------|-------------------|
| 0.452        | 0.369                | 0.356                | 0.0130              | 0.36250           | 0.5618750        | 0.25396750        |
| 0.456        | 0.373                | 0.358                | 0.0150              | 0.36550           | 0.5665250        | 0.25833540        |
| 0.460        | 0.377                | 0.355                | 0.0175              | 0.36875           | 0.5715625        | 0.26291875        |
| 0.464        | 0.382                | 0.362                | 0.0200              | 0.37200           | 0.5766000        | 0.26754240        |
| 0.468        | 0.386                | 0.364                | 0.0220              | 0.37500           | 0.5812500        | 0.27202500        |
| 0.472        | 0.392                | 0.366                | 0.0260              | 0.37900           | 0.5874500        | 0.27727640        |
| 0.476        | 0.396                | 0.368                | 0.0280              | 0.38250           | 0.5928750        | 0.28220850        |
| 0.480        | 0.402                | 0.371                | 0.0310              | 0.38650           | 0.5990750        | 0.28755600        |
| 0.484        | 0.408                | 0.373                | 0.0350              | 0.39050           | 0.6052750        | 0.29295310        |
| 0.488        | 0.413                | 0.376                | 0.0370              | 0.39450           | 0.6114750        | 0.29839980        |
| 0.492        | 0.418                | 0.379                | 0.0390              | 0.39850           | 0.6176750        | 0.30389610        |
| <b>0.496</b> | <b>0.425</b>         | <b>0.382</b>         | <b>0.0430</b>       | <b>0.40350</b>    | <b>0.6254250</b> | <b>0.31021080</b> |
| 0.500        | 0.432                | 0.385                | 0.0470              | 0.40850           | 0.6331750        | 0.31658750        |

Table 3.9: Summary of TE band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the lithium niobate

**Photonic band gap structures diagram:** The band structure in the case of the  $TE$  polarization and a fill factor  $r/a = 0.496$  is reported in figure 3.35. The calculation is done all along the axes of highest symmetry of the first Brillouin zone, the circuit  $\Gamma - X - M - \Gamma$ , using the  $PWE$  method. The energy bands is expressed as a function of the ratio  $a/\lambda$  (normalized frequency). For this case, a  $TE$  PBG appears around the value  $a/\lambda = 0.4035$ . Note, there is not any other photonic bandgap

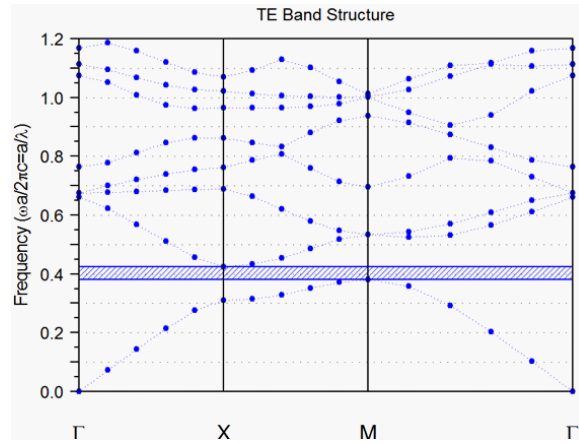


Figure 3.35: Band diagram of a photonic crystal square lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized  $TE$  mode for a filling factor  $r/a = 0.496$ .

In this case of square lattice holes of air in lithium niobate ( $LiNbO_3$ ) ratio  $r/a = 0.496$  for the  $TE$  polarization, the photonic band gap used is centered on 0.4035.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.4035 * 1.55 * 10^{-6}m = 0.625425\mu m.$$

This implies a pattern of radius:

$$r = 0.496 * 0.625425 * 10^{-6}m = 0.31021080\mu m.$$

**Reflection and transmission diagrams** We are interested in this study to control the propagation of light along a photonic bandgap structure.

The calculation of the transmission and reflection will permit to determine the dimensions of the two-dimensional periodic structure of photonic band gap. The approach is initially to study the evolution of the transmission and reflection based on number of holes to determine a dimension of the structure

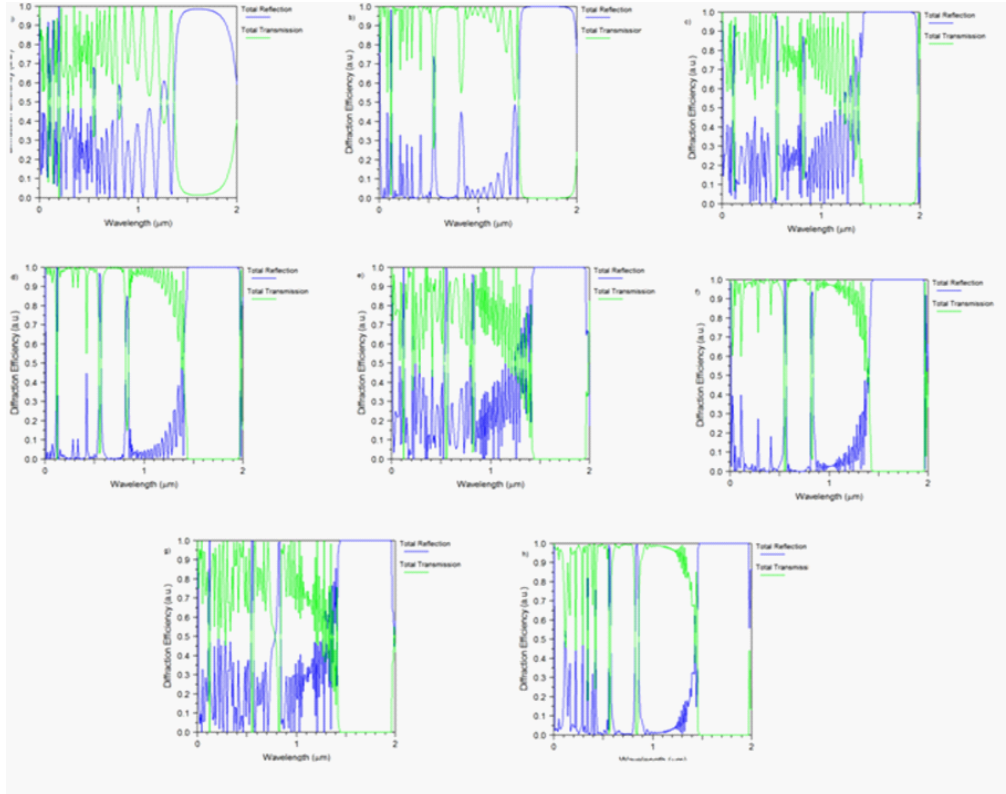


Figure 3.36: Transmission and reflection through the square structures of air holes in the lithium niobate ( $LiNbO_3$ ); TE mode polarized in  $a = 0.625425\mu m, r = 0.31021080\mu m$ .

Figure 3.36 shows the transmission spectrum and reflection through structures of square lattice holes of air in lithium niobate ( $LiNbO_3$ ) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.45 microns, 2 microns) and that from the structure (f) reflection is at its maximum value. From the obtained results , we note that from the case (g) reflection remains unchanged.

**TM case:** Table 3.10 contains the photonic band gaps, their widths, center frequencies, the grating period and the range for each frequency (case of a square structure - holes configuration -  $TM$  polarization). From the table below :

| $r/a$        | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$             | $r$              |
|--------------|----------------------|----------------------|---------------------|-------------------|-----------------|------------------|
| 0.436        | 0.771                | 0.759                | 0.012               | 0.7650            | 1.185750        | 0.5169870        |
| 0.440        | 0.777                | 0.763                | 0.014               | 0.7700            | 1.193500        | 0.5251400        |
| 0.444        | 0.784                | 0.767                | 0.017               | 0.7755            | 1.202025        | 0.5336991        |
| 0.448        | 0.791                | 0.771                | 0.02                | 0.7810            | 1.210550        | 0.5423264        |
| 0.452        | 0.797                | 0.775                | 0.022               | 0.7865            | 1.21907         | 0.5510219        |
| 0.456        | 0.805                | 0.780                | 0.025               | 0.7925            | 1.22837         | 0.5601390        |
| 0.460        | 0.812                | 0.785                | 0.027               | 0.798             | 1.237675        | 0.5693305        |
| 0.464        | 0.818                | 0.791                | 0.027               | 0.8045            | 1.246975        | 0.5785964        |
| <b>0.468</b> | <b>0.826</b>         | <b>0.798</b>         | <b>0.028</b>        | <b>0.8120</b>     | <b>1.258600</b> | <b>0.5890248</b> |
| 0.472        | 0.833                | 0.806                | 0.027               | 0.8195            | 1.270225        | 0.5995462        |
| 0.476        | 0.841                | 0.814                | 0.027               | 0.8275            | 1.282625        | 0.6105295        |
| 0.480        | 0.848                | 0.821                | 0.027               | 0.8345            | 1.293475        | 0.6208680        |
| 0.484        | 0.856                | 0.829                | 0.027               | 0.8425            | 1.305875        | 0.6320435        |
| 0.488        | 0.863                | 0.842                | 0.021               | 0.8525            | 1.321375        | 0.6448310        |
| 0.492        | 0.871                | 0.851                | 0.02                | 0.8610            | 1.334550        | 0.6565986        |

Table 3.10: Summary of TM band gaps of most interest to the case of a square photonic crystal of air holes in substrate of the lithium niobate

The wider  $PBG$   $TM$  is observed for a filling factor  $r/a = 0.468$ , its width is 0.028 and extends over the energy range  $a/\lambda \approx 0.798 - 0.826$  (Fig. III.31).

**Photonic band gap structures diagram** In this case of square lattice holes of air in lithium niobate ( $LiNbO_3$ ) ratio  $r/a = 0.468$  for the  $TM$  polarization, the used photonic band gap is centered on 0.8120.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.8120 * 1.55 * 10^{-6}m = 1.2586\mu m.$$

This implies a pattern of radius:

$$r = 0.468 * 1.2586 * 10^{-6}m = 0.5890248\mu m.$$

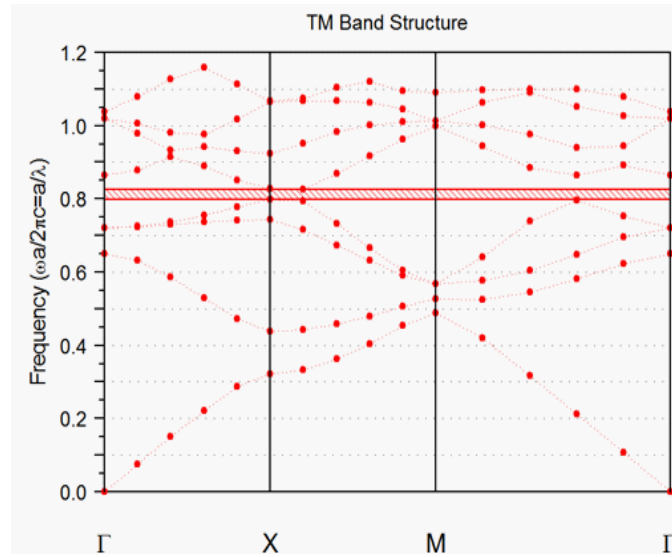


Figure 3.37: Band diagram of a photonic crystal square lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized  $TM$  mode for a filling factor  $r/a = 0.468$ .

**Reflection and transmission diagrams** Figure 3.38 shows the transmission spectrum and reflection through structures of square lattice holes of air in lithium niobate ( $LiNbO_3$ ) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.25 microns, 2.1 microns) and that from the structure (d) reflection is at its maximum value. From the results obtained, we note that from the case (f) reflection remains unchanged.

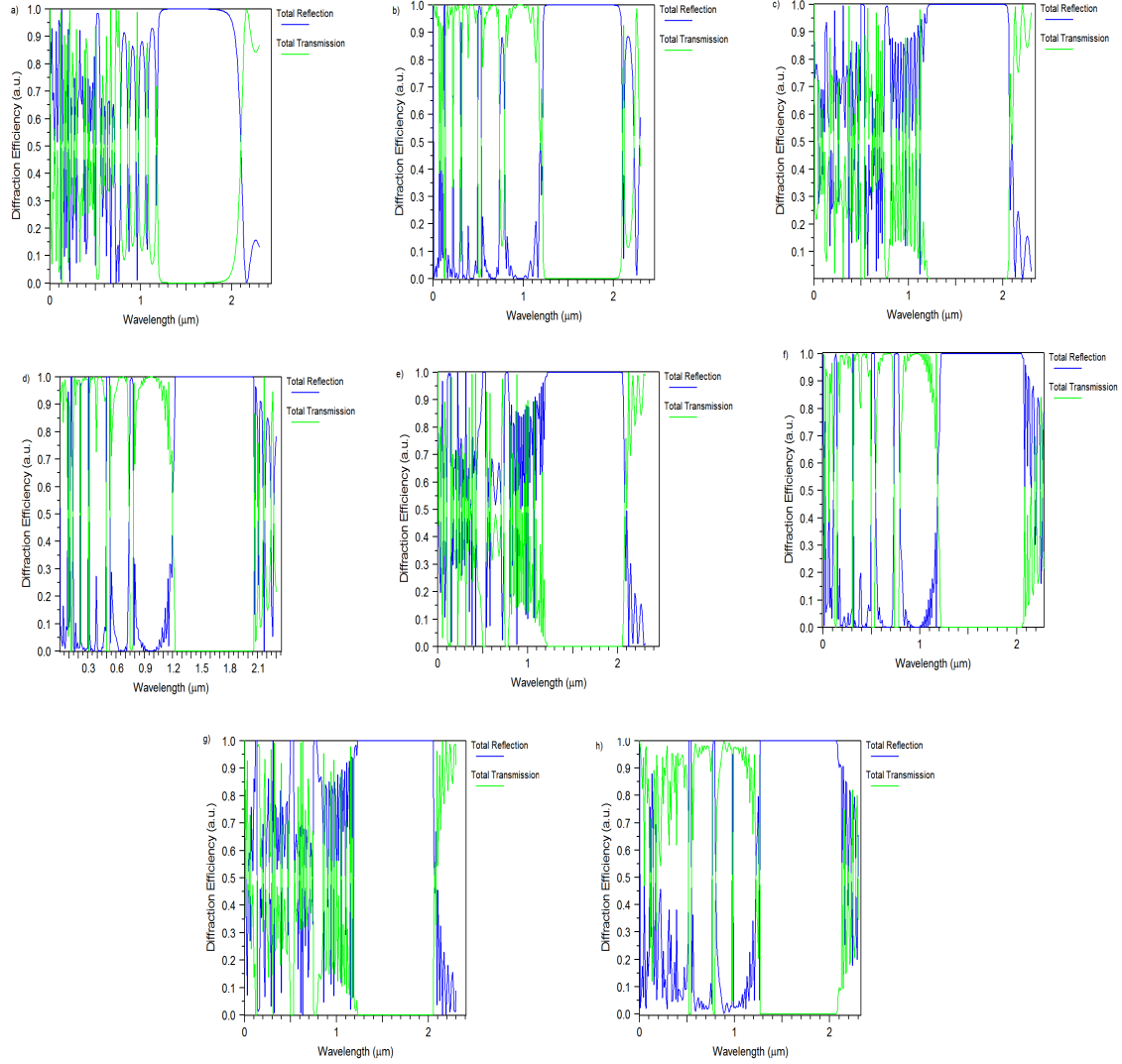


Figure 3.38: Transmission and reflection through the square structures of air holes in the lithium niobate ( $\text{LiNbO}_3$ ); TM mode polarized in  $a=1.2586\mu\text{m}$  ,  $r=0.5890248\mu\text{m}$ .

### Two-dimensional photonic crystals with triangular lattice

**Definition of the parameters:** The variation of the wave vector (modulus and direction) when calculating the band diagrams for the triangular structure along high symmetry directions of the Brillouin zone,  $\Gamma \rightarrow M \rightarrow K \rightarrow \Gamma$  (Figure 3.39).

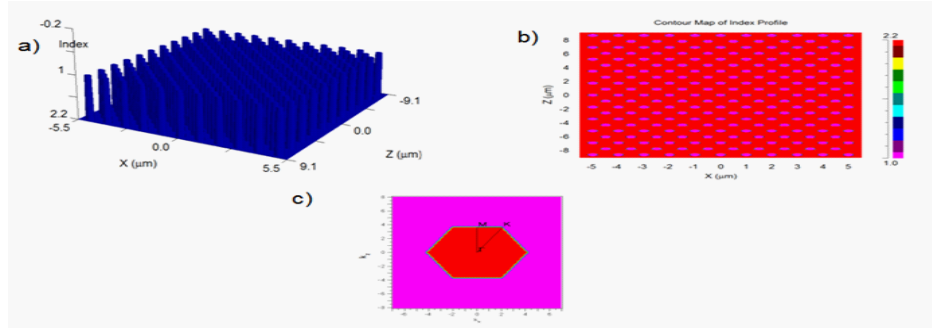


Figure 3.39: Index profile of 2D triangular photonic crystals with holes of air in lithium niobate ( $LiNbO_3$ ); (a) with 2D in 2 axes (x,z) (a) with 3D

**Photonic band gap map diagram:** Now we consider the case of a triangular lattice of air holes in a substrate of lithium niobate ( $LiNbO_3$ ). The photonic band gap map for both  $TE$  and  $TM$  polarizations is shown in Figure 3.40.

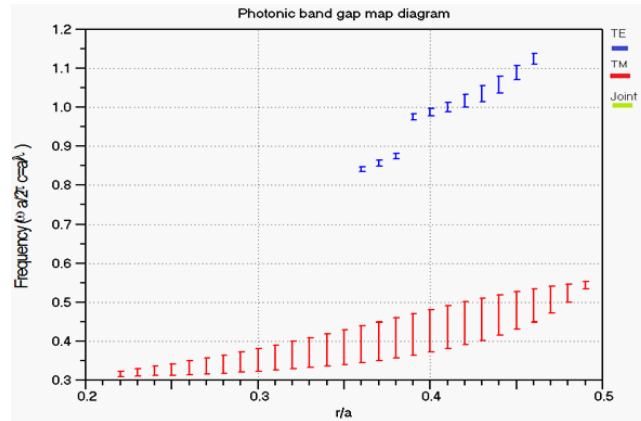


Figure 3.40: Map of photonic band gap for both  $TE$  and  $TM$  polarizations. Case of a triangular PC of air holes ( $\varepsilon = 1$ ) in  $LiNbO_3$

Figure 3.40 shows maps of photonic band gap for both polarizations  $TE$  (ordinary index  $\varepsilon_0 = 4,889$ ) and  $TM$  (extraordinary index  $\varepsilon_e = 4,571$ ) in the case of a photonic crystal of triangular lattice air holes ( $\varepsilon = 1$ ) on substrate  $LiNbO_3$ . For this structure

we observe the appearance of some photonic band gap widths of  $TM$  and  $TE$  but also for very small filling factors very high.

We must achieve a ratio of  $r/a = 0.36$  to see the bandgaps  $TE$  with lowest energy and a ratio  $r/a = 0.22$  for the case of  $TM$  photonic bandgap so large. No overlap between the photonic bandgap  $TE$  and  $TM$  is observed also no complete photonic bandgap.

**TE case:** For  $TE$  polarization, we note the existence  $TE$  photonic band gap. The photonic bandgap of greatest interest are grouped in the table 3.11. The wider bandgap photonic  $TE$  is observed for a filling factor  $r/a = 0.44$  its width is 0.042 and extends over the energy range  $a/\lambda \approx 1.08 - 1.038$  (Figure 3.41)

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$            | $r$             |
|-------------|----------------------|----------------------|---------------------|-------------------|----------------|-----------------|
| 0.36        | 0.847                | 0.834                | 0.013               | 0.841             | 1.30355        | 0.469278        |
| 0.37        | 0.864                | 0.848                | 0.016               | 0.856             | 1.3268         | 0.490916        |
| 0.38        | 0.882                | 0.868                | 0.014               | 0.875             | 1.35625        | 0.515375        |
| 0.39        | 0.983                | 0.967                | 0.016               | 0.975             | 1.51125        | 0.5893875       |
| 0.40        | 0.996                | 0.978                | 0.018               | 0.987             | 1.52985        | 0.61194         |
| 0.41        | 1.013                | 0.989                | 0.024               | 1.001             | 1.55155        | 0.6361355       |
| 0.42        | 1.033                | 1.000                | 0.033               | 1.0165            | 1.575575       | 0.6617415       |
| 0.43        | 1.055                | 1.015                | 0.040               | 1.035             | 1.60425        | 0.6898275       |
| <b>0.44</b> | <b>1.080</b>         | <b>1.038</b>         | <b>0.042</b>        | <b>1.059</b>      | <b>1.64145</b> | <b>0.722238</b> |
| 0.45        | 1.107                | 1.071                | 0.036               | 1.089             | 1.68795        | 0.7595775       |
| 0.46        | 1.138                | 1.112                | 0.026               | 1.125             | 1.74375        | 0.802125        |

Table 3.11: Summary of TE band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the lithium niobate

**Photonic band gap structures diagram:** In this case of triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) ratio  $r/a = 0.44$  for the  $TE$  polarization, the used photonic band gap is centered on 1.059.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 1.059 * 1.55 * 10^{-6}m = 1.64145\mu m.$$

This implies a pattern of radius:

$$r = 0.44 * 1.64145 * 10^{-6}m = 0.722238\mu m$$

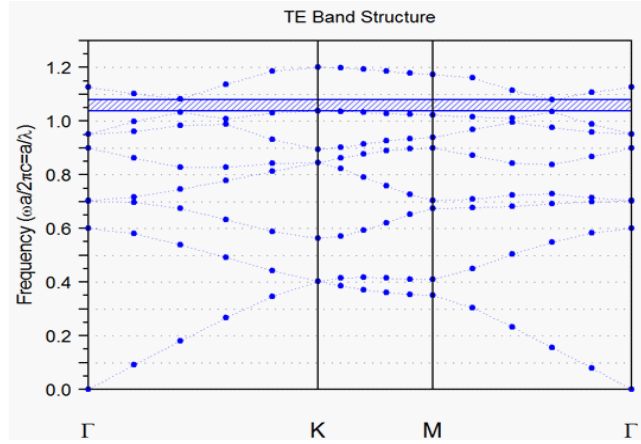


Figure 3.41: Band diagram of a photonic crystal triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized  $TE$  mode for a filling factor  $r/a = 0.44$ .

**Reflection and transmission diagrams** Figure 3.42 shows the transmission spectrum and reflection through structures of triangular lattice of air holes in a substrate of lithium niobate ( $LiNbO_3$ ) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.54 microns, 1.57 microns) and that from the structure (h) reflection is at its maximum

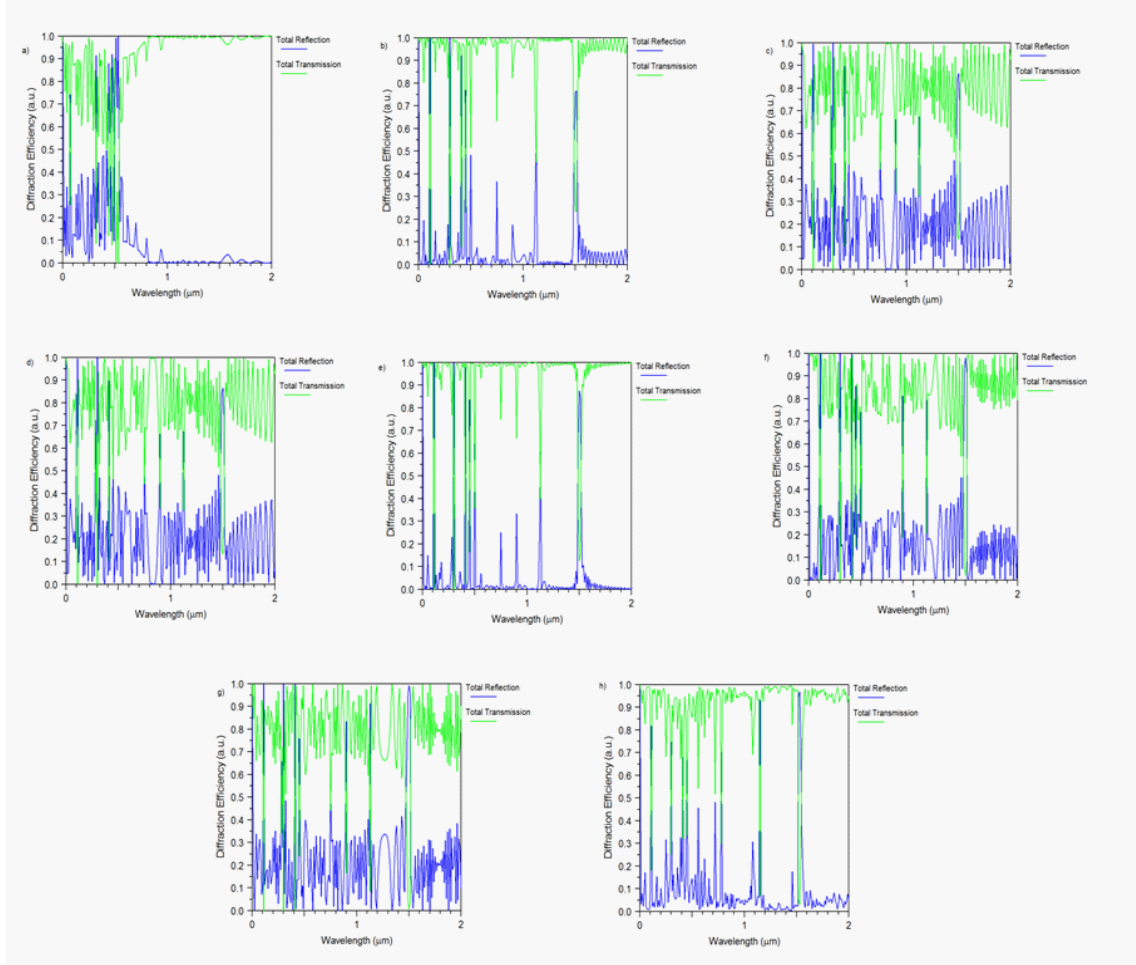


Figure 3.42: Transmission and reflection through the triangular structures of air holes in the lithium niobate ( $LiNbO_3$ ); TE mode polarized in  $a = 1.64145 \mu\text{m}$ ,  $r = 0.722238 \mu\text{m}$

**TM case** For the case of TM polarization, we note the existence of several TM photonic band gap. The photonic bandgap of greatest interest are grouped in the table 3.12:

The wider bandgap photonic *TM* is observed for a filling factor  $r/a = 0.42$  its width is 0.11 and extends over the energy range  $a/\lambda \approx 0.5 - 0.39$ . In this case we note the *PBG* is so larger (Figure 3.43).

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$             | $r$              |
|-------------|----------------------|----------------------|---------------------|-------------------|-----------------|------------------|
| 0.22        | 0.3230               | 0.3090               | 0.014               | 0.3160            | 0.489800        | 0.1077560        |
| 0.24        | 0.3350               | 0.3110               | 0.024               | 0.3230            | 0.500650        | 0.1201560        |
| 0.26        | 0.3490               | 0.3150               | 0.034               | 0.3320            | 0.514600        | 0.1337960        |
| 0.28        | 0.3640               | 0.3180               | 0.046               | 0.3410            | 0.528550        | 0.1479940        |
| 0.30        | 0.3805               | 0.3215               | 0.059               | 0.3510            | 0.544050        | 0.1632150        |
| 0.32        | 0.4000               | 0.3300               | 0.070               | 0.3650            | 0.565750        | 0.1810400        |
| 0.34        | 0.4200               | 0.3350               | 0.085               | 0.3775            | 0.585125        | 0.1989425        |
| 0.36        | 0.4400               | 0.3450               | 0.095               | 0.3925            | 0.608375        | 0.2190150        |
| 0.38        | 0.4600               | 0.3550               | 0.105               | 0.4075            | 0.631625        | 0.2400175        |
| 0.40        | 0.4800               | 0.3730               | 0.107               | 0.4265            | 0.661075        | 0.2644300        |
| <b>0.42</b> | <b>0.5000</b>        | <b>0.3900</b>        | <b>0.110</b>        | <b>0.4450</b>     | <b>0.689750</b> | <b>0.2896950</b> |
| 0.44        | 0.5150               | 0.4250               | 0.090               | 0.4700            | 0.728500        | 0.3205400        |
| 0.46        | 0.5350               | 0.4500               | 0.085               | 0.4925            | 0.763375        | 0.3511525        |
| 0.48        | 0.5460               | 0.5000               | 0.046               | 0.5230            | 0.810650        | 0.3891120        |

Table 3.12: Summary of TM band gaps of most interest to the case of a triangular photonic crystal of air holes in substrate of the lithium niobate

**Photonic band gap structures diagram:** In this case of triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) ratio  $r/a = 0.42$  for the  $TM$  polarization, the photonic band gap used is centered on 0.4450.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.4450 * 1.55 * 10^{-6}m = 0.689750\mu m$$

This implies a pattern of radius:

$$r = 0.42 * 0.689750 * 10^{-6}m = 0.2896950\mu m$$

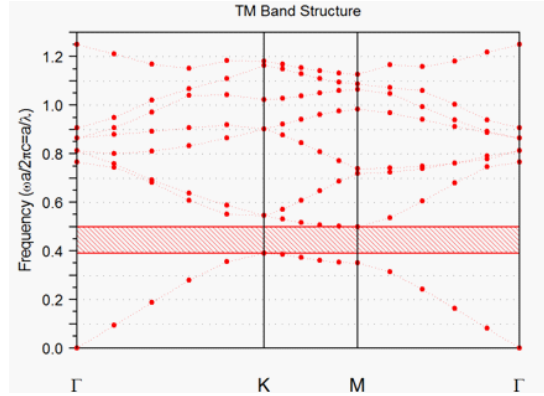


Figure 3.43: Band diagram of a photonic crystal triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) polarized  $TM$  mode for a filling factor  $r/a = 0.42$ .

**Reflection and transmission diagrams** Figure 3.44 shows the transmission spectrum and reflection through structures of triangular lattice of air holes in a substrate of lithium niobate ( $LiNbO_3$ ) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TM$  polarization. Transmission measurements show a very strong disconnect between (1.3 microns, 1.7 microns) and that from the structure (g) reflection is at its maximum value. From the results obtained, we note that from the case (h) reflection remains unchanged.

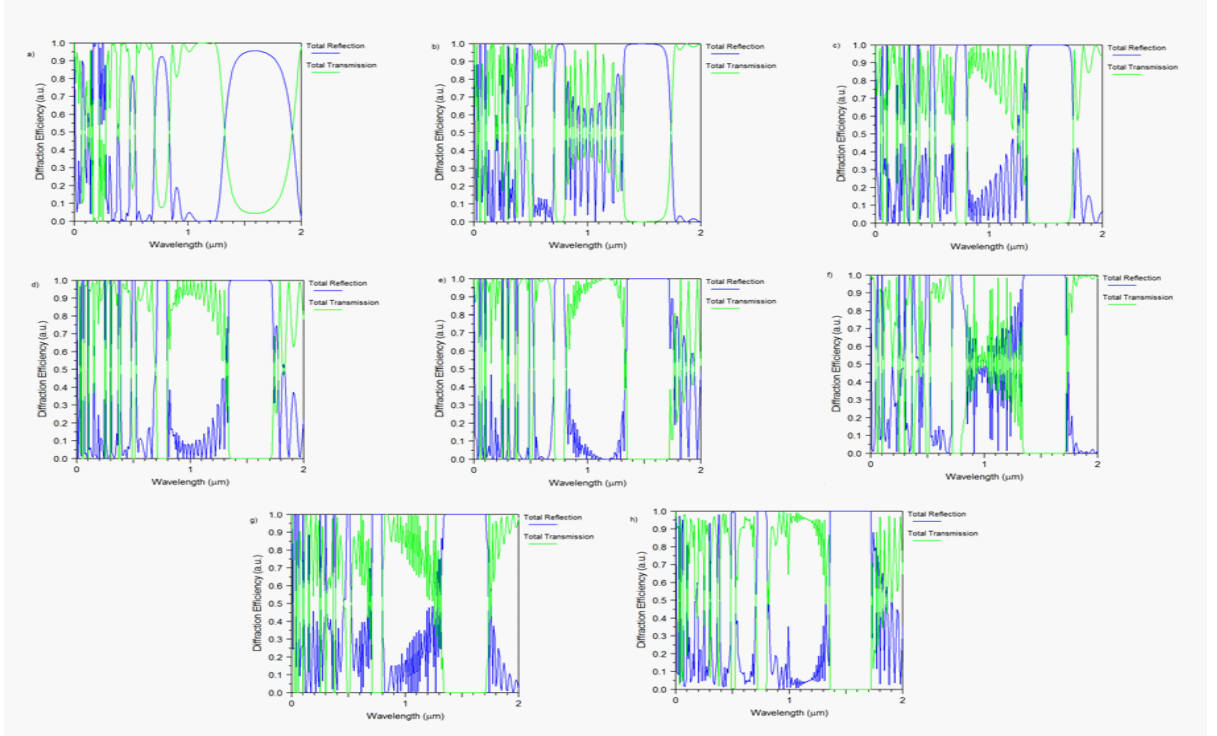


Figure 3.44: Transmission and reflection through the triangular structures of air holes in the lithium niobate ( $LiNbO_3$ ); TM mode polarized in  $a = 0.689750\mu m$ ,  $r = 0.2896950\mu m$

### 3.7.2 Lithium niobate cylinders being placed in air

In the latter case we will consider the reverse configuration, we will use the same structures, but with rods (columns) of  $LiNbO_3$  in the air ground.

#### Two-dimensional photonic crystals with square lattice

**Definition of the parameters:** Figure 3.45 shows the the variation of refractive index and the periodicity of the refractive index of the our new structure ( rods of lithium niobate ( $LiNbO_3$ ) in the air).

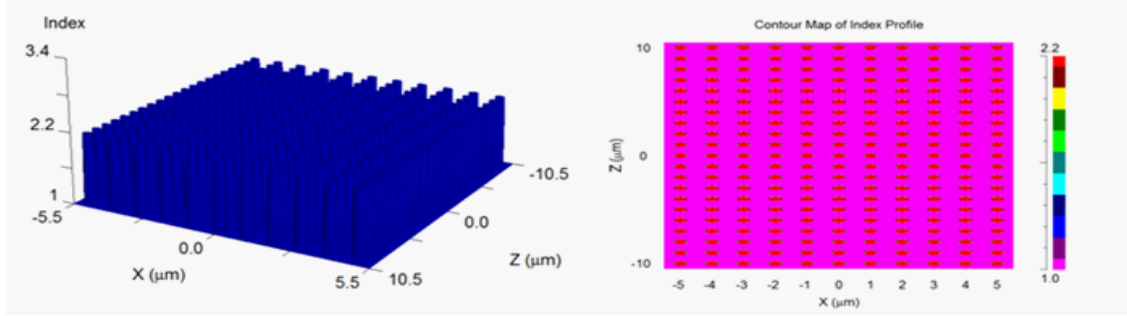


Figure 3.45: Index profile of 2D square photonic crystals with square lattice of rods of lithium niobate ( $LiNbO_3$ ) in air; (a) with 3D , (b) with 2D in 2 axes (x,z)

**Photonic band gap map diagram:** We note, from this figure, the presence of some Photonic band gap by  $TE$  against the  $TM$  Photonic band gap

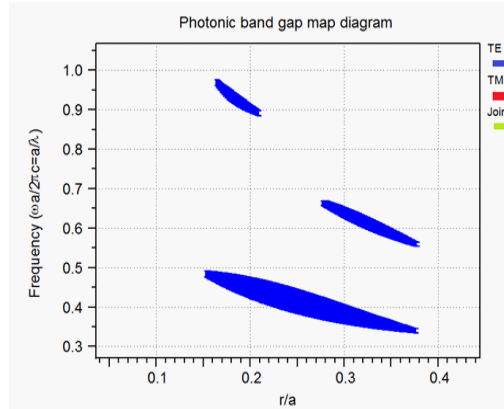


Figure 3.46: Map of photonic band gap for both TE and TM polarizations. Case of a square PC of rods  $LiNbO_3$  in air

#### TE case:

For the case of a square lattice of rods with  $LiNbO_3$  in the air and for  $TE$  polarization, the widths of the photonic bandgap of the most interesting, their center frequencies and the period and radius of each of these beeps are indicated in the following table:

The wider photonic bandgap  $TE$  is observed for a filling factor  $r/a = 0.24$

The ratio  $r/a = 0.24$  provides the ability to have a photonic band gap in  $TE$  mode of relatively small width  $[\Delta(a/\lambda) = 0.0600]$  with maximum normalized frequency  $(a/\lambda)_{max} = 0.450$  and the minimal normalized frequency  $(a/\lambda)_{min} = 0.3900$

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$             | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|-----------------|-------------------|
| 0.16        | 0.490                | 0.4660               | 0.0240              | 0.4780            | 0.740900        | 0.11854400        |
| 0.17        | 0.486                | 0.4540               | 0.0320              | 0.4700            | 0.728500        | 0.12384500        |
| 0.17        | 0.965                | 0.9440               | 0.0210              | 0.9545            | 1.479475        | 0.25151075        |
| 0.18        | 0.483                | 0.4420               | 0.0410              | 0.4625            | 0.716875        | 0.12903750        |
| 0.18        | 0.947                | 0.9200               | 0.0270              | 0.9335            | 1.446925        | 0.26044650        |
| 0.19        | 0.480                | 0.4320               | 0.0480              | 0.4560            | 0.706800        | 0.13429200        |
| 0.19        | 0.928                | 0.9050               | 0.0230              | 0.9165            | 1.420575        | 0.26990925        |
| 0.20        | 0.474                | 0.4220               | 0.0520              | 0.4480            | 0.694400        | 0.13888000        |
| 0.20        | 0.912                | 0.8930               | 0.0190              | 0.9025            | 1.398875        | 0.27977500        |
| 0.21        | 0.467                | 0.4130               | 0.0540              | 0.4400            | 0.682000        | 0.14322000        |
| 0.22        | 0.464                | 0.4050               | 0.0590              | 0.4345            | 0.673475        | 0.14816450        |
| 0.23        | 0.457                | 0.3972               | 0.0598              | 0.4271            | 0.662005        | 0.15226115        |
| <b>0.24</b> | <b>0.450</b>         | <b>0.3900</b>        | <b>0.0600</b>       | <b>0.4200</b>     | <b>0.651000</b> | <b>0.15624000</b> |
| 0.25        | 0.443                | 0.3840               | 0.0590              | 0.4135            | 0.640925        | 0.16023125        |
| 0.26        | 0.436                | 0.3770               | 0.0590              | 0.4065            | 0.630075        | 0.16381950        |
| 0.27        | 0.430                | 0.3740               | 0.0560              | 0.4020            | 0.623100        | 0.16823700        |
| 0.28        | 0.420                | 0.3670               | 0.0530              | 0.3935            | 0.609925        | 0.17077900        |
| 0.28        | 0.669                | 0.6525               | 0.0164              | 0.6607            | 1.024085        | 0.28674380        |
| 0.29        | 0.413                | 0.3620               | 0.0510              | 0.3875            | 0.600625        | 0.17418125        |
| 0.29        | 0.661                | 0.6390               | 0.0220              | 0.6497            | 1.007112        | 0.29206263        |
| 0.30        | 0.404                | 0.3580               | 0.0470              | 0.3810            | 0.590550        | 0.17716500        |
| 0.30        | 0.652                | 0.6250               | 0.0270              | 0.6385            | 0.989675        | 0.29690250        |
| 0.31        | 0.396                | 0.3540               | 0.0430              | 0.3750            | 0.581250        | 0.18018750        |
| 0.31        | 0.642                | 0.6140               | 0.0280              | 0.6277            | 0.973012        | 0.30163388        |
| 0.32        | 0.388                | 0.3500               | 0.0383              | 0.3691            | 0.572182        | 0.18309840        |
| 0.32        | 0.632                | 0.6020               | 0.0300              | 0.6170            | 0.956350        | 0.30603200        |
| 0.33        | 0.380                | 0.3460               | 0.0340              | 0.3630            | 0.562650        | 0.18567450        |
| 0.33        | 0.622                | 0.5920               | 0.0300              | 0.6070            | 0.940850        | 0.31048050        |
| 0.34        | 0.372                | 0.3430               | 0.0290              | 0.3575            | 0.554125        | 0.18840250        |
| 0.34        | 0.609                | 0.5823               | 0.0267              | 0.5956            | 0.923257        | 0.31390755        |
| 0.35        | 0.364                | 0.3405               | 0.0235              | 0.3522            | 0.545987        | 0.19109563        |
| 0.35        | 0.597                | 0.5736               | 0.0234              | 0.5853            | 0.907215        | 0.31752525        |
| 0.36        | 0.356                | 0.3380               | 0.0180              | 0.3470            | 0.537850        | 0.19362600        |
| 0.36        | 0.584                | 0.5660               | 0.0180              | 0.5747            | 0.890862        | 0.32071050        |
| 0.37        | 0.349                | 0.3360               | 0.0130              | 0.3425            | 0.530875        | 0.19642375        |
| 0.37        | 0.572                | 0.5580               | 0.0141              | 0.5650            | 0.875827        | 0.32405618        |

Table 3.13: Summary of TE band gaps of most interest to the case of a square photonic crystal of rods of lithium niobate in the air

**Photonic band gap structures diagram:** In this case of a square lattice with rods of  $LiNbO_3$  ratio  $r/a = 0.24$  for the  $TE$  polarization, the used photonic band gap is centered on 0.4200.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.4200 * 1.55 * 10^{-6} m = 0.651\mu m.$$

This implies a pattern of radius:

$$r = 0.24 * 0.651 * 10^{-6} m = 0.15624\mu m.$$

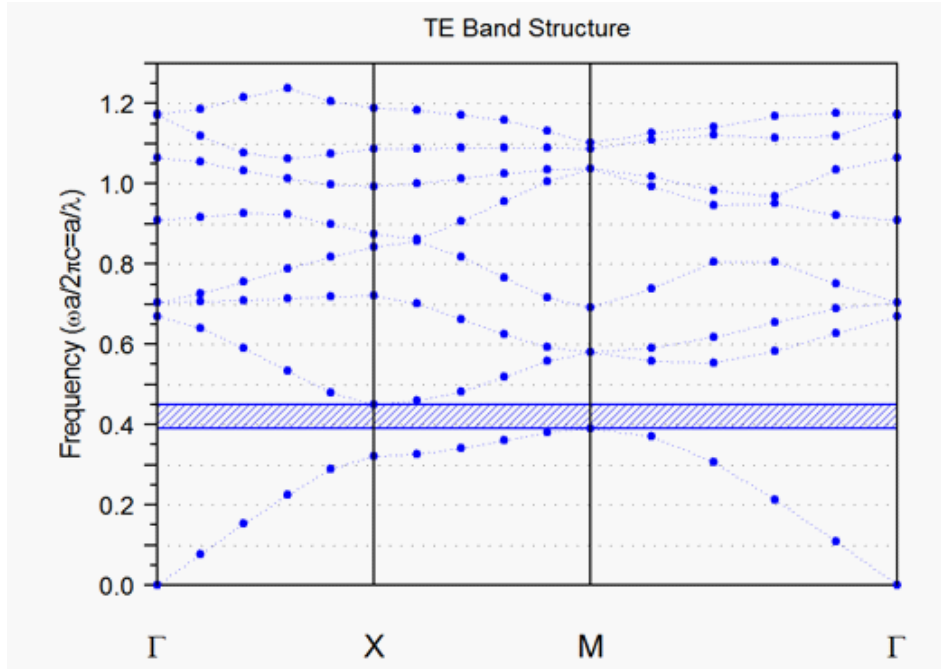


Figure 3.47: Band diagram of a photonic crystal square lattice with rods of ( $LiNbO_3$ ) in air; polarized  $TE$  mode for a filling factor  $r/a = 0.24$ .

**Reflection and transmission diagrams** Figure 3.48 shows the transmission spectrum and reflection through structures of square lattice rods  $LiNbO_3$  in the air of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.49 microns, 1.51 microns)

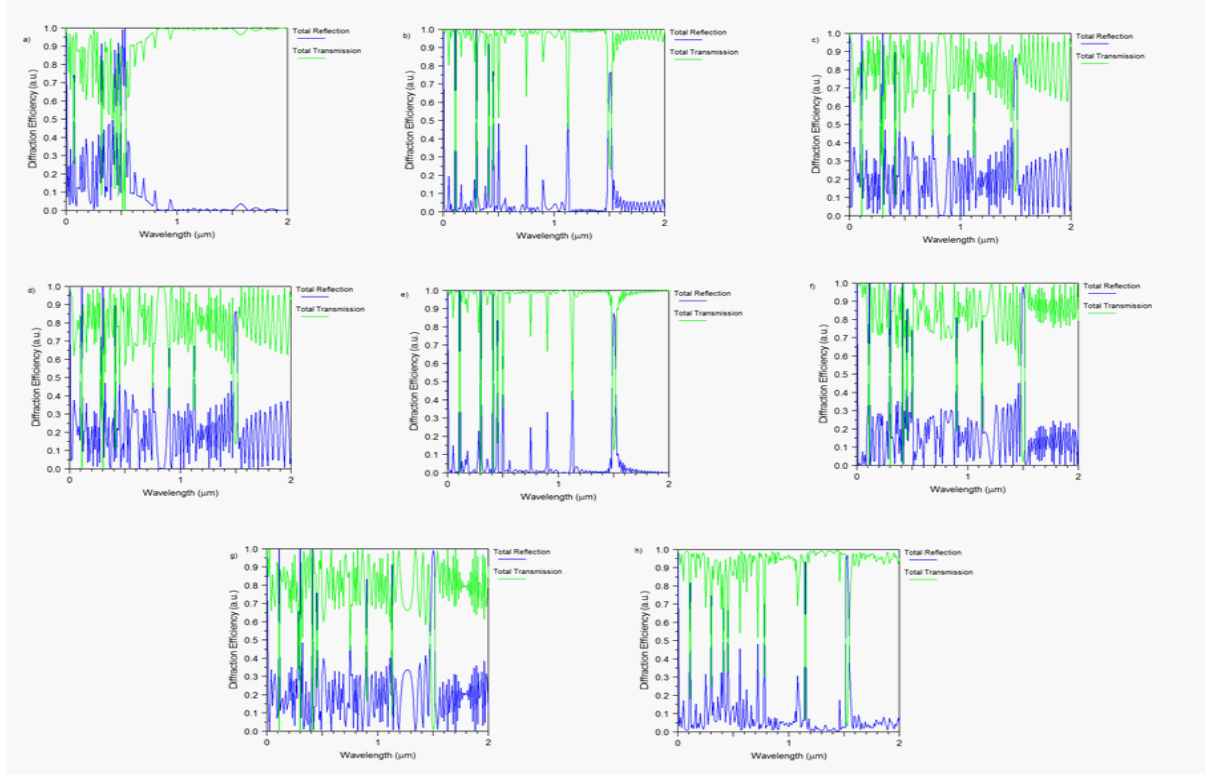


Figure 3.48: Transmission and reflection through the square structures of rods of lithium niobate ( $LiNbO_3$ ) in the air; TE mode polarized in  $a = 0.651\mu m$ ,  $r = 0.15624\mu m$

**TM case:** As mentioned above, the *TM* photonic band gaps are practically nonexistent.

### Two-dimensional photonic crystals with triangular lattice

**Definition of the parameters:** Figure 3.49 shows the the variation of refractive index and the periodicity of the refractive index of the our new structure ( rods of lithium niobate ( $LiNbO_3$ ) in the air).

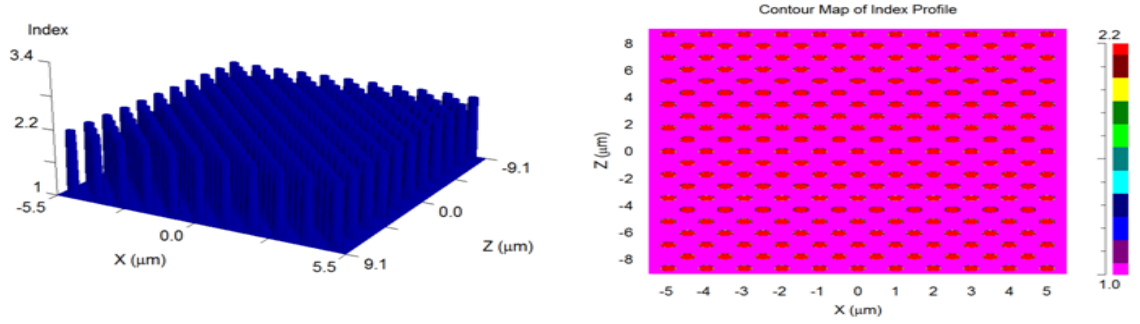


Figure 3.49: Index profile of 2D triangular photonic crystals with rods of lithium niobate ( $LiNbO_3$ ) in air; (a) with 2D in 2 axes (x,z) (a) with 3D

**Photonic band gap map diagram:** Now we consider the case of a triangular lattice of air holes in a substrate of lithium niobate ( $LiNbO_3$ ). The photonic band gap map for both  $TE$  and  $TM$  polarizations is shown in Figure III.16.

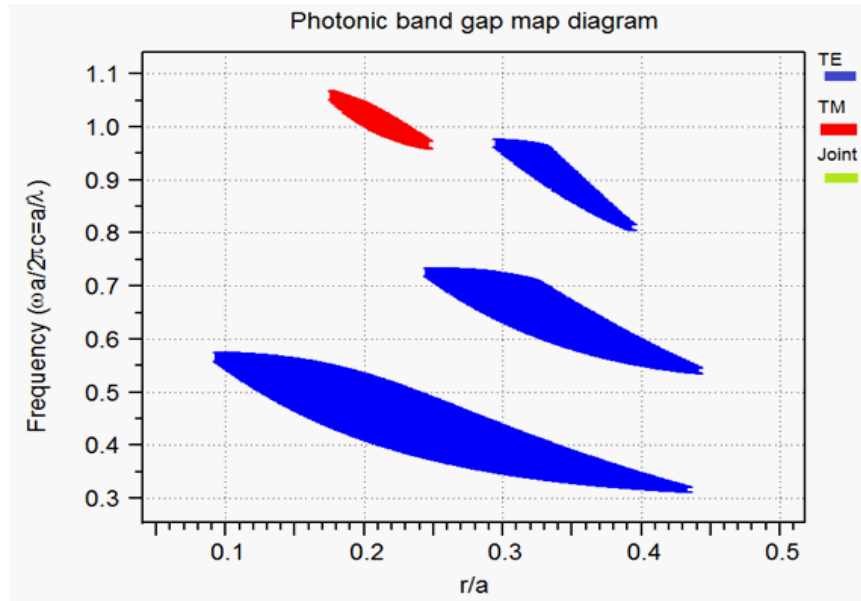


Figure 3.50: Map of photonic band gap for both  $TE$  and  $TM$  polarizations. Case of a triangular PC of rods  $LiNbO_3$  in air

Figure 3.50 shows maps of photonic band gap for both polarizations  $TE$  (ordinary index  $\varepsilon_0 = 4,889$ ) and  $TM$  (extraordinary index  $\varepsilon_e = 4,571$ ) in the case of a photonic crystal of triangular lattice air holes ( $\varepsilon = 1$ ) on substrate  $LiNbO_3$ . For this structure we observe the appearance of some photonic band gap widths of  $TM$  and  $TE$  but also for very small filling factors very high.

We must achieve a ratio  $r/a = 0.36$  to see the bandgaps  $TE$  with lowest energy and a ratio  $r/a = 0.22$  for the case of  $TM$  photonic bandgap so large. No overlap between the photonic bandgap  $TE$  and  $TM$  is observed also no complete photonic bandgap.

**TE case:**

The wider photonic bandgap  $TE$  is observed for a filling factor  $r/a = 0.20$

The ratio  $r/a = 0.20$  provides the ability to have a photonic band gap in  $TE$  mode of relatively small width  $[\Delta(a/\lambda) = 0.1233]$  with maximum normalized frequency  $(a/\lambda)_{max} = 0.5333$  and the minimal normalized frequency  $(a/\lambda)_{min} = 0.41$

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$              | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|------------------|-------------------|
| 0.10        | 0.5750               | 0.5450               | 0.0300              | 0.56000           | 0.8680000        | 0.08680000        |
| 0.12        | 0.5713               | 0.5103               | 0.0610              | 0.54080           | 0.8382400        | 0.10058880        |
| 0.14        | 0.5660               | 0.4800               | 0.0860              | 0.52300           | 0.8106500        | 0.11349100        |
| 0.16        | 0.5580               | 0.4530               | 0.1050              | 0.50550           | 0.7835250        | 0.12536400        |
| 0.18        | 0.5470               | 0.4300               | 0.1170              | 0.48850           | 0.7571750        | 0.13629150        |
| <b>0.20</b> | <b>0.5333</b>        | <b>0.4100</b>        | <b>0.1233</b>       | <b>0.47165</b>    | <b>0.7310575</b> | <b>0.14621150</b> |
| 0.22        | 0.5160               | 0.3935               | 0.1225              | 0.45475           | 0.7048625        | 0.15506975        |
| 0.24        | 0.4970               | 0.3790               | 0.1180              | 0.43800           | 0.6789000        | 0.16293600        |
| 0.26        | 0.4770               | 0.3660               | 0.1110              | 0.42150           | 0.6533250        | 0.16986450        |
| 0.26        | 0.7330               | 0.6910               | 0.0420              | 0.71200           | 1.1036000        | 0.28693600        |
| 0.28        | 0.4560               | 0.3554               | 0.1006              | 0.40570           | 0.6288350        | 0.17607380        |
| 0.28        | 0.7300               | 0.6600               | 0.0700              | 0.69500           | 1.0772500        | 0.30163000        |
| 0.30        | 0.4360               | 0.3460               | 0.0900              | 0.39100           | 0.6060500        | 0.18181500        |
| 0.30        | 0.7230               | 0.6330               | 0.0900              | 0.67800           | 1.0509000        | 0.31527000        |
| 0.30        | 0.9750               | 0.9520               | 0.0230              | 0.96350           | 1.4934250        | 0.44802750        |
| 0.32        | 0.4160               | 0.3380               | 0.0780              | 0.37700           | 0.5843500        | 0.18699200        |
| 0.32        | 0.7130               | 0.6100               | 0.1030              | 0.66150           | 1.0253250        | 0.32810400        |
| 0.32        | 0.9700               | 0.9130               | 0.0570              | 0.94150           | 1.4593250        | 0.46698400        |
| 0.34        | 0.3970               | 0.3315               | 0.0655              | 0.36425           | 0.5645875        | 0.19195975        |
| 0.34        | 0.6840               | 0.5900               | 0.0940              | 0.63700           | 0.9873500        | 0.33569900        |
| 0.34        | 0.9400               | 0.8770               | 0.0630              | 0.90850           | 1.4081750        | 0.47877950        |
| 0.36        | 0.3790               | 0.3256               | 0.0534              | 0.35230           | 0.5460650        | 0.19658340        |
| 0.36        | 0.6530               | 0.5730               | 0.0800              | 0.61300           | 0.9501500        | 0.34205400        |
| 0.36        | 0.8910               | 0.8460               | 0.0450              | 0.86850           | 1.3461750        | 0.48462300        |
| 0.38        | 0.3620               | 0.3200               | 0.0420              | 0.34100           | 0.5285500        | 0.20084900        |
| 0.38        | 0.6240               | 0.5606               | 0.0634              | 0.59230           | 0.9180650        | 0.34886470        |
| 0.38        | 0.8440               | 0.8190               | 0.0250              | 0.83150           | 1.2888250        | 0.48975350        |
| 0.40        | 0.3455               | 0.3166               | 0.0289              | 0.33105           | 0.5131275        | 0.20525100        |
| 0.40        | 0.5964               | 0.5500               | 0.0464              | 0.57320           | 0.8884600        | 0.35538400        |
| 0.42        | 0.5465               | 0.5350               | 0.0115              | 0.54075           | 0.8381625        | 0.35202825        |

Table 3.14: Summary of TE band gaps of most interest to the case of a triangular photonic crystal of rods of lithium niobate in the air

**Photonic band gap structures diagram:** In this case of triangular lattice holes of air in lithium niobate ( $LiNbO_3$ ) ratio  $r/a = 20$  for the  $TE$  polarization, the used photonic band gap is centered on **0.47165**.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.47165 * 1.55 * 10^{-6}m = 0.7310575\mu m.$$

This implies a pattern of radius:

$$r = 0.44 * 0.7310575 * 10^{-6}m = 0.14621150\mu m$$

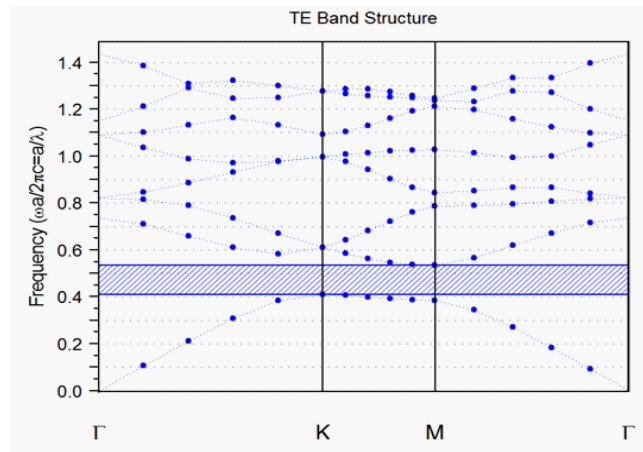


Figure 3.51: Band diagram of a photonic crystal triangular lattice with rods of ( $LiNbO_3$ ) in air; polarized  $TE$  mode for a filling factor  $r/a = 0.20$ .

**Reflection and transmission diagrams** Figure 3.52 shows the transmission spectrum and reflection through structures of triangular lattice of air holes in a substrate of lithium niobate ( $LiNbO_3$ ) of different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TE$  polarization. Transmission measurements show a very strong disconnect between (1.448 microns, 1.997 microns) and that from the structure (e) reflection is at its maximum value. From the results obtained, we note that from the case (g) reflection remains unchanged.

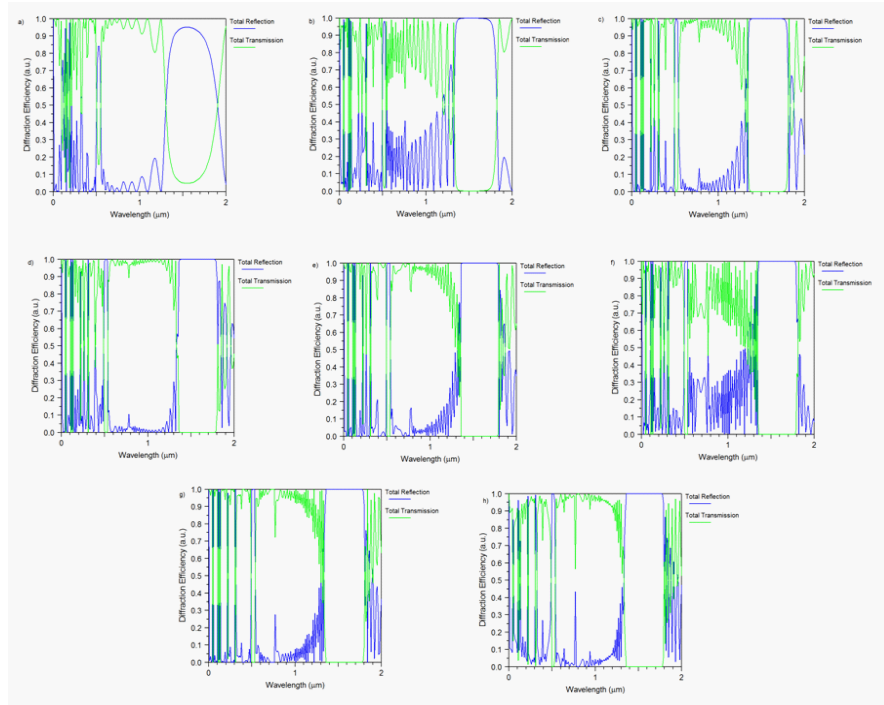


Figure 3.52: Transmission and reflection through the triangular structures rods of  $LiNbO_3$  in the air; polarized mode  $TE$   $a = 0.7310575\mu m$  ,  $r = 0.14621150\mu m$

### TM case:

For the case of  $TM$  polarization, we note the existence of several  $TM$  photonic band gap. The photonic bandgap of greatest interest are grouped in the table 3.15:

The wider photonic bandgap  $TM$  is observed for a filling factor  $r/a = 0.22$

The ratio  $r/a = 0.22$  provides the ability to have a photonic band gap in  $TM$  mode of relatively small width  $[\Delta(a/\lambda) = 0.03395]$  with maximum normalized frequency  $(a/\lambda)_{max} = 1.0148$  and the minimal normalized frequency  $(a/\lambda)_{min} = 0.98085$

| $r/a$       | $(a/\lambda)_{\max}$ | $(a/\lambda)_{\min}$ | $\Delta(a/\lambda)$ | central frequency | $a$               | $r$               |
|-------------|----------------------|----------------------|---------------------|-------------------|-------------------|-------------------|
| 0.18        | 1.0636               | 1.04140              | 0.02220             | 1.052500          | 1.63137500        | 0.29364750        |
| 0.19        | 1.0550               | 1.02000              | 0.03500             | 1.037500          | 1.60812500        | 0.30554375        |
| 0.20        | 1.0438               | 1.00570              | 0.03810             | 1.024750          | 1.58836250        | 0.31767250        |
| 0.21        | 1.0301               | 0.99190              | 0.03820             | 1.011000          | 1.56705000        | 0.32908050        |
| <b>0.22</b> | <b>1.0148</b>        | <b>0.98085</b>       | <b>0.03395</b>      | <b>0.997825</b>   | <b>1.54662875</b> | <b>0.34025833</b> |
| 0.23        | 0.9999               | 0.96840              | 0.03150             | 0.984150          | 1.52543250        | 0.35084948        |
| 0.24        | 0.9830               | 0.96170              | 0.02130             | 0.972350          | 1.50714250        | 0.36171420        |

Table 3.15: Summary of TM band gaps of most interest to the case of a triangular photonic crystal of rods of lithium niobate in the air

**Photonic band gap structures diagram:** In the case of a triangular lattice with rods of  $LiNbO_3$  ratio  $r/a = 0.22$  for the  $TM$  polarization, the used photonic band gap is centered on 0.997825.

For a band gap at  $\lambda = 1.55\mu m$ , the grating period must be:

$$a = 0.997825 * 1.55 * 10^{-6}m = 1.54662875\mu m.$$

This implies a pattern of radius:

$$r = 0.22 * 1.54662875 * 10^{-6}m = 0.34025833\mu m.$$

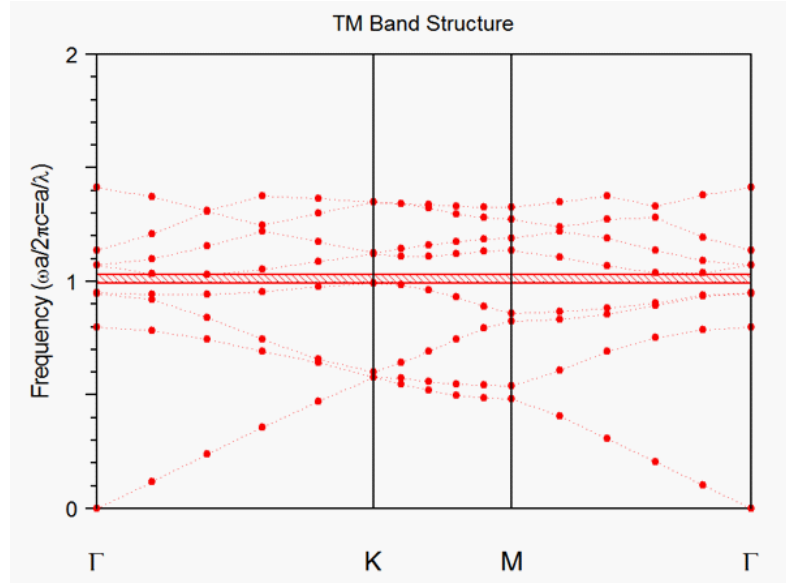


Figure 3.53: Band diagram of a photonic crystal triangular lattice with rods of ( $LiNbO_3$ ) in air; polarized  $TM$  mode for a filling factor  $r/a = 0.22$ .

**Reflection and transmission diagrams** Figure 3.54 shows the transmission spectrum and reflection through structures of triangular lattice rods  $LiNbO_3$  in the air with different sizes [(a) 5x5, (b) : 10x10, (c) : 15x15, (d) : 20x20, (e) : 25x25, (f) : 30x30, (g) : 35x35 and (h) : 40x40] for a  $TM$  polarization. Transmission measurements show a very strong disconnect between (1.448 microns, 1.537 microns)

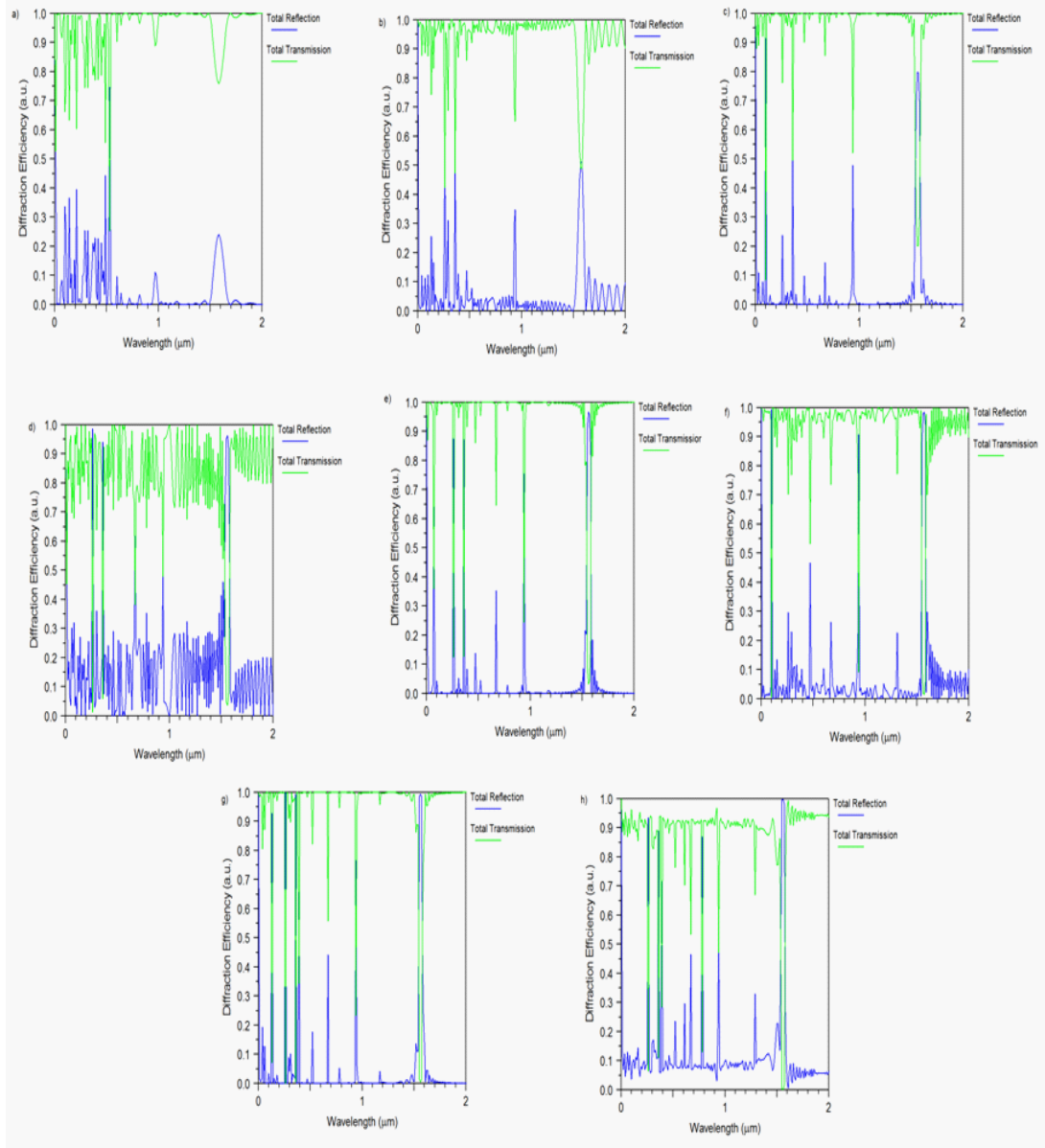


Figure 3.54: Transmission and reflection through the triangular structures rods of  $LiNbO_3$  in the air; polarized mode TM  $a = 1.54662875\mu m$ ,  $r = 0.34025833\mu m$

### 3.8 Conclusion

This last chapter was devoted to the simulation of light propagation in two different structures (square and triangular), with two different configurations (holes and columns) for two polarization modes. From this study deduced in the configurations "holes" and "columns" do not give equivalent results for both TE and TM polarizations:

Configuration "holes" preferably has a photonic bandgap for the TM polarization;

About the configuration "columns", it preferably has a PBG for the TE polarization.

For first material "Germanium (Ge)" we noted:

When germanium cylinders placed in air, the two kinds of PCs can generate TE and TM band gaps and TM band gap is dominating; as radius of germanium cylinder increases, band gap width increases initially and then decreases, central frequency decreases;

When air cylinders placed in germanium, 2D triangular-lattice PC can only generate TE band gap, 2D square-lattice PC cannot generate any band gaps; as radius of air cylinder increases, band gap width and central frequency increase.

For second material "Lithium niobate ( $\text{LiNbO}_3$ )":

When Lithium niobate cylinders placed in air, 2D triangular-lattice PC can only generate TE and TM band gap, 2D square-lattice PC cannot generate TM band gaps, only generate TE band gaps; as radius of Lithium niobate cylinder increases, band gap width increases initially and then decreases, central frequency decreases;

When air cylinders placed in Lithium niobate, the two kinds of PCs can generate TE and TM band gaps and TM band gap is dominating; as radius of Lithium niobate cylinder increases, band gap width increases initially and then decreases, central frequency decreases;

From first and second material we can say :

*The higher the index contrast, the greater the photonic band gap is larger. For a sufficiently high contrast, it is possible that the photonic band gap of the two polarizations overlap: we obtain a complete photonic bandgap. As a case of germanium.*

The unit cell for obtaining a complete photonic bandgap is the triangular mesh. For example, one can obtain a complete photonic band gap with air holes arranged in triangular mesh of high index contrast such as *Ge/air*. Indeed, for a periodic triangular lattice, the first Brillouin zone is the one with the most circular as possible, so most favorable to the opening of a bandgap.

# General Conclusion

This thesis has been devoted to the study of the photonic crystals .we took that in two parts .the first is the theoretical part and second is the simulation one.

The purpose of the first part is to provide a theoretical background for the propagation of the light wave in the photonic crystals and modeling methods. We have studied this in two parts:

We have studied in the first parts: History of photonic crystals and different types photonic crystals (One-dimensional photonic crystals (1D), two-dimensional photonic (2D) crystals and three-dimensional photonic crystals (3D)) and theory of photonic crystal for that we used the Maxwell equations and analogy between the Schrödinger equation describing the motion of electrons in the periodic potential and the propagation wave (photons) in the photonic crystals to using the Bloch theorem. We need this theorem to found the solutions of the propagation wave (photons) in the different types of the photonic crystals

In this part, we also provided a review of the different methods for modeling the photonic crystals (Plane Wave Expansion method (PWE), Finite Difference Time Domain method (FDTD) and Transfer-matrix method (TMM)) and we was focus on the Plane Wave Expansion method (PWE) method to modeling all types of photonic crystals for that, we given the definitions for parameters for structure like: unit cell, lattice vector, reciprocal lattice and reciprocal lattice vectors as well as the first Brillouin zone. In the first of this parte we took the first case it was the one-dimensional photonic crystals (1D) because it is the simplest after that the other types (two-dimensional photonic (2D) crystals and three-dimensional photonic crystals (3D)) on general cases.

We have shown in the third chapter how photonic crystals can be used to control the propagation of light, leading to effects that can be exploited in optical filtering or extraction and guiding light. Our work has been based on the 1D and 2D photonic crystals (at the wavelengths of the optical), for manufacturing easier than those of

3D structures. We considered two materials with different properties (germanium (Ge) and lithium niobate (LiNbO<sub>3</sub>)) which future applications can be micro-guides, micro-cavities and photonic crystal micro-lasers.

Initially, PWE simulations were performed to establish the mapping and the structures of photonic bandgap for two propagation modes (TE and TM). Secondly the size of the unit cell were calculated (radius and period) for each scenario. Once the dimensions of the structure are calculated, filtering it is characterized by transmission and reflection with FDTD simulations.

Simulations of the propagation of light through the two different structures (square and triangular) allowed us to verify the results found in the literature regarding the benefit of the triangular lattice with respect to the square lattice.

Regarding the two different configurations ("holes" and "columns"), simulation results have shown that a configuration "holes" preferably has a band gap for the TM polarization. By cons, a configuration "column" has a band gap for TE polarization.

A simple comparison between the case of the lithium niobate and germanium shows that if the refractive index so high we found other PBG and largest, like in the case of germanium, because that we note the PBG for TE and TM polarization so large and there are complete photonic bandgaps.

The configuration most suitable for obtaining a band gap complete (that is to say in TE and TM) is the triangular lattice of "holes" in a material of high refractive index such as germanium (Ge).

Finally, it is possible for a structure type chosen to adjust the lattice parameter and the filling factor. These parameters directly affect the properties and appearance of the band diagram associated with the photonic crystal made, in particular the width and position of the bandgap. Note that the energy band-edge shift to the high (low) energy when the filling factor increases for the case of a configuration "holes" ("columns").

To conclude, we can say that, like the solid crystals where the crystal potential requires periodic properties of conduction electrons, photonic crystals offer the prospect of controlling the flow of photons in dielectrics and semiconductors with their periodic structure. Photonic crystals are promising systems for controlling the light in submicron scales. These objects would be one of the basic building blocks of future photonic integrated circuits because of their shape and dimensions that allow their integration in many configurations, a bright future they are so booked.

About the prospects of this work, we consider it interesting to go to the modeling and realization of various optical functions need to introduce defects in a two-dimensional periodic structure, waveguide and cavity, it is also possible to combine

---

alization of optoelectronic components controllable (filter variable wavelength, cavity resonance wavelength variable ...) which would open the door to a new generation of optoelectronic components "active".

## Appendix A

# Computation of Transmission and Reflection coefficients

The transmission spectra of photonic crystals reflect their band structure directly. They are often used for the experimental characterization of real specimens. In this appendix, we will formulate the plane-wave expansion method for the numerical calculation of the transmittance and the Bragg reflectivity of 2D crystals. It will be demonstrated that the bandgaps and the uncoupled modes lead to opaque frequency ranges. It will also be shown that the small group velocity peculiar to the 2D and 3D crystals, that is, the group-velocity anomaly, is equivalent to a large effective refractive index. The refraction law on the surface of photonic crystals will be given.

Several methods are known for the numerical calculation of the transmission spectra of photonic crystals such as the spherical-wave expansion method [27, 30, 57, 58], the transfer matrix method [59], and the finite-difference time-domain (FDTD) method. Although the spherical-wave expansion method is only applicable to those crystals composed of dielectric spheres and cylinders, its convergence property is quite good. The transfer matrix method can be applied to any periodic structures and many excellent calculations have been performed. The FDTD method may be the most general one, which is based on the numerical time-integration of Maxwell's equation. Although it is time-consuming (i.e., the computational time is long) and needs a large memory capacity, fairly accurate calculation is possible thanks to the tremendous development of computing facilities.

On the other hand, the plane-wave expansion method [48, 61] is based on the Fourier expansion of the internal field and the dielectric function, and is applicable to any 2D photonic crystal as long as the wave vector of the incident plane wave lies in the 2D  $x - y$  plane. Although its convergence becomes poor and it is not practical when the contrast of the dielectric constant is, say, larger than 4:1, it is very intuitive and has an advantage that the Bragg reflectivity can be obtained at the same time. Complex dielectric constants can be dealt with as well as real dielectric constants. The low-threshold lasing due to the group-velocity anomaly in 2D photonic crystals was demonstrated by this method [62]. An example of the configuration for the calculation is shown in Figure A.1. The external plane wave in region 1 is incident on the left (front) surface of the specimen located in region 2, which consists of  $N$  layers of circular

rods with a radius  $r_a$  and the background dielectric. The dielectric constant of the rods and the background are denoted by  $\varepsilon_a$  and  $\varepsilon_b$ , respectively. The lattice constant is denoted by  $a_1$  in the  $x$  direction and  $a_2$  in the  $y$  direction, and the distance between the surface and the first layer of the rods is denoted by  $d$ . When we deal with a square lattice, we assume the same value for  $a_1$  and  $a_2$ . The dielectric constants of regions 1 and 3 are  $\varepsilon_1$  and  $\varepsilon_3$ , respectively. The total thickness of the specimen is:

$$L = (N - 1)a_2 + 2(R + d).$$

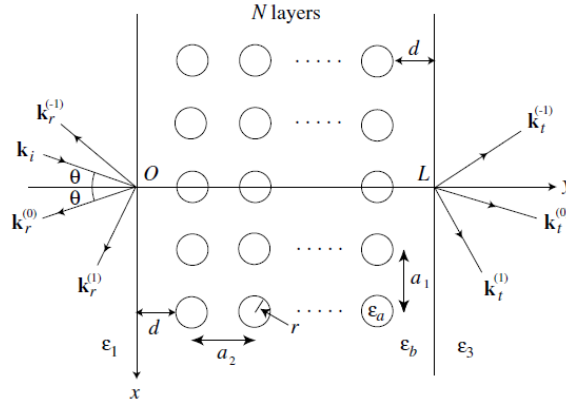


Figure A.1: Configuration for the calculation of the transmission and the Bragg reflection spectra (top view).

Since photonic crystals may be regarded as periodic gratings, the electromagnetic field in region 1 is the superposition of the incident plane wave and the reflected Bragg waves, whereas that in region 3 is composed of the transmitted Bragg waves. Here, we regard the specular reflection and the straight transmission as the reflected and the transmitted Bragg waves of the zeroth order. The 2D wave vector of the incident plane wave is denoted by  $k_{i\parallel} = (k_1 \sin \theta, k_1 \cos \theta) = (k_x, k_{1y})$ , where  $\theta$  is the angle of incidence.  $k_1 = \sqrt{\varepsilon_1} \omega / c$ , where  $\omega$  is the angular frequency of the incident field. The wave vectors of the reflected and the transmitted Bragg waves of order  $n$ ,  $k_{r\parallel}^{(n)}$  and  $k_{t\parallel}^{(n)}$ , are given by:

$$k_{r,x}^{(n)} = k_{x;t}^{(n)} = k_x^{(n)} = k_x + G_n \quad (\text{A.1})$$

$$G_n = 2\pi n/a_1 (n = 0 \pm 1, \pm 2, \dots), \quad (\text{A.2})$$

$$k_{r,y}^{(n)} = \begin{cases} -\sqrt{k_1^2 - (k_x^{(n)})^2} & \text{if } k_1 \geq |k_x^{(n)}| \\ -\sqrt{(k_x^{(n)})^2 - k_1^2} & \text{otherwise,} \end{cases} \quad (\text{A.3})$$

$$k_{t,y}^{(n)} = \begin{cases} -\sqrt{k_3^2 - (k_x^{(n)})^2} & \text{if } k_3 \geq |k_x^{(n)}| \\ -\sqrt{(k_x^{(n)})^2 - k_3^2} & \text{otherwise,} \end{cases} \quad (\text{A.4})$$

Here,  $k_3 = \sqrt{\varepsilon_3}\omega/c$ . Because of the spatial periodicity along the  $x$  axis, the reflected and the transmitted Bragg waves receive a momentum along the  $X$  axis which is a multiple of  $2\pi/a$  in (A.1) and (A.2).

As was explained in chapter 2, the vector wave equations derived from Maxwell's equations can be reduced to two independent scalar equations when the wave vector lies in the  $x - y$  plane, or in other words, when the fields are independent of the  $z$  coordinate. These two modes are called the  $E$  polarization, for which the electric field is parallel to the  $z$  axis, and the  $H$  polarization, for which the magnetic field is parallel to the  $z$  axis, respectively. Because of the continuity condition for the tangential component of the electric or the magnetic field at each boundary, the fields in regions 1 and 3 are also characterized by these polarizations. In the next two sections, the details of the plane-wave expansion method will be explained. Those readers who are not interested in the details of the calculation may skip them and proceed .

## A.1 Field Equations

### A.1.1 $E$ Polarization

First, we consider the  $E$  polarization. The electric field in region 2 ( $E_{2z}$ ):

$$\kappa_E E_{2z}(r_{\parallel}) \equiv \left\{ \frac{1}{\varepsilon(r_{\parallel})} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{\omega^2}{c^2} \right\} E_{2z}(r_{\parallel}) = 0 \quad (\text{A.5})$$

The differential operator  $\kappa_E$  is defined by the first equality in the above equation. The electric fields in region 1 ( $E_{1z}$ ) and region 3 ( $E_{3z}$ ) are given by

$$E_{1z}(r_{\parallel}) = E_0 \exp(ik_{i\parallel} \cdot r_{\parallel}) + \sum_{n=-\infty}^{+\infty} R_n^{(E)} \exp(ik_{r\parallel} \cdot r_{\parallel}), \quad (\text{A.6})$$

$$E_{3z}(r_{\parallel}) = \sum_{n=-\infty}^{+\infty} T_n^{(E)} \exp\{ik_{t\parallel}^{(n)}(r_{\parallel} - L_{\parallel})\} \quad (\text{A.7})$$

where  $E_0$ ,  $R_n^{(E)}$ , and  $T_n^{(E)}$  are the amplitudes of the electric field of the incident wave, the reflected Bragg waves, and the transmitted Bragg waves, respectively.  $L_{\parallel} = (0, L)$ . Now, we introduce a boundary-value function  $f_E(x, y)$  :

$$f_E(x, y) \equiv \frac{1}{L} \sum_{n=-\infty}^{\infty} \left\{ y T_n^{(E)} + (L - y) \left( \delta_{n0} E_0 + R_n^{(E)} \right) \right\} \exp \left( i k_x^{(n)} x \right) \quad (\text{A.8})$$

where  $\delta_{nm}$  is Kronecker's  $\delta$ . Then,  $f_E$  is equal to  $E_{1z}$  or  $E_{3z}$  on each surface of the specimen:

$$f_E(x, 0) = E_{1z}(x, 0) \quad \text{and} \quad f_E(x, L) = E_{3z}(x, L) \quad (\text{A.9})$$

Next, we define  $\psi_E$  by

$$\psi_E(x, y) \equiv E_{2z}(x, y) - f_E(x, y). \quad (\text{A.10})$$

Then

$$\kappa_E \psi_E = -\kappa_E f_E \quad (\text{A.11})$$

$$\psi_E(x, 0) = \psi_E(x, L) = 0 \quad (\text{A.12})$$

By means of this mathematical trick, we have converted the boundary conditions for  $E_{2z}$  into the field equation of  $\psi_E$  [63]. Then, we can relate the internal field in region 2 to  $E_0$  at the first stage of the calculation, thereby we can exclude the free oscillation and obtain the solution of the forced oscillation with the incident plane wave as the excitation source. The choice of  $f_E$  is arbitrary as long as (A.9) is satisfied.

Now, we expand  $\psi_E(x, y)$  and  $1/\varepsilon(x, y)$  in Fourier series:

$$\psi_E(x, y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} A_{nm}^{(E)} \exp \left( i k_x^{(n)} x \right) \sin \frac{m\pi}{l} y, \quad (\text{A.13})$$

$$\frac{1}{\varepsilon(x, y)} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} k_{nm} \exp \left\{ i \left( G_n x + \frac{m\pi}{L} y \right) \right\} \quad (\text{A.14})$$

Note that only sine functions appear in (A.13), since  $\psi_E(x, 0) = 0$ . Substituting (A.8), (A.13), and (A.14) into (A.11) and comparing the coefficients of the independent Fourier components, we obtain the following equation after a simple but lengthy calculation.

$$\frac{\omega^2}{c^2} A_{nm}^{(E)} + \sum_{n'=-\infty}^{\infty} \sum_{m'=-\infty}^{\infty} \left\{ \left( k_x \binom{n'}{n} \right)^2 + \left( \frac{m' \pi}{L} \right)^2 \right\} \quad (\text{A.15})$$

$$\times \left( k_{n-n', m+m'} - k_{n-n', |m+m'|} \right) A_{n'm'}^{(E)} \quad (\text{A.16})$$

$$= -\frac{2\omega^2}{\pi c^2} \frac{(-1)^{m-1} T_n^{(E)} + R_n^{(E)} + \delta_{n0} E_0}{m} \quad (\text{A.17})$$

$$+ \frac{2}{\pi} \sum_{n'=-\infty}^{\infty} \left( k_x \binom{n'}{n} \right) \sum_{m'=-\infty}^{\infty} \left( k_{n-n', |m+m'|} - k_{n-n', m+m'} \right) \quad (\text{A.18})$$

$$\times \frac{(-1)^{m'-1} T_{n'}^{(E)} + R_{n'}^{(E)} + \delta_{n0} E_0}{m'} \quad (\text{A.19})$$

When we derived (A.15), the following two relations were used.

$$y = \frac{2L}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m} \sin \frac{m\pi}{L} y \quad (\text{A.20})$$

$$1 = \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{1 - (-1)^{m-1}}{m} \sin \frac{m\pi}{L} y \quad (\text{A.21})$$

The remaining boundary conditions are the continuity of the  $x$  component of the magnetic field and the  $y$  component of the magnetic induction. The latter proves to be equivalent to the continuity of the  $z$  component of the electric field and does not yield an independent equation. Then, the former condition at both boundaries leads to

$$\pi \sum_{m=1}^{\infty} m A_{nm}^{(E)} = (iLk_{r,y} + 1) R_n^{(E)} - T_n^{(E)} + \delta_{n0} E_0 (iLk_{1,y} + 1) \quad (\text{A.22})$$

$$\pi \sum_{m=1}^{\infty} m (-1)^m A_{nm}^{(E)} = R_n^{(E)} + (iLk_{r,y} - 1) T_n^{(E)} + \delta_{n0} E_0 \quad (\text{A.23})$$

Equations (A.15), (A.18), and (A.19) determine the unknown coefficients,  $A_{nm}^{(E)}$ ,  $R_n^{(E)}$ ,  $T_n^{(E)}$ . In the actual numerical calculation, we restrict the number of terms that appear in the Fourier expansions as we did for the photonic band calculation in Chap. 2, and solve the linear equations.

### A.1.2 $H$ Polarization

The  $H$  polarization can be treated in a similar manner. The magnetic fields in region 1 ( $H_{1z}$ ) and in region 3 ( $H_{3z}$ ) are given as follows.

$$H_{1z}(r_{\parallel}) = H_0 \exp(ik_{i\parallel} \cdot r_{\parallel}) + \sum_{n=-\infty}^{+\infty} R_n^{(H)} \exp(ik_{r\parallel} \cdot r_{\parallel}), \quad (\text{A.24})$$

$$H_{3z}(r_{\parallel}) = \sum_{n=-\infty}^{+\infty} T_n^{(H)} \exp\left\{ik_{t\parallel}^{(n)}(r_{\parallel} - L_{\parallel})\right\} \quad (\text{A.25})$$

where  $H_0$ ,  $R_n^{(H)}$ , and  $T_n^{(H)}$  are the amplitudes of the magnetic field of the incident wave, the reflected Bragg waves, and the transmitted Bragg waves, respectively. The magnetic field in region 2 :

$$\kappa_H H_{2z}(r_{\parallel}) \equiv \left\{ \frac{\partial}{\partial x} \frac{1}{\varepsilon(r_{\parallel})} \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \frac{1}{\varepsilon(r_{\parallel})} \frac{\partial}{\partial y} + \frac{\omega^2}{c^2} \right\} H_{2z}(r_{\parallel}) = 0 \quad (\text{A.26})$$

where the differential operator  $\kappa_H$  is defined by the first equality in the above equation. Now, we introduce another boundary-value function  $f_H(x, y)$ :

$$f_H(x, y) \equiv \frac{1}{L} \sum_{n=-\infty}^{\infty} \left\{ y T_n^{(E)} + (L - y) \left( \delta_{n0} H_0 + R_n^{(H)} \right) \right\} \exp\left(ik_x^{(n)} x\right) \quad (\text{A.27})$$

Then we have:

$$f_H(x, 0) = H_{1z}(x, 0) \quad \text{and} \quad f_H(x, L) = H_{3z}(x, L) \quad (\text{A.28})$$

Next, we define  $\psi_H$  by:

$$\psi_H(x, y) \equiv H_{2z}(x, y) - f_H(x, y). \quad (\text{A.29})$$

Then:

$$\kappa_H \psi_H = -\kappa_H f_H \quad (\text{A.30})$$

$$\psi_E(x, 0) = \psi_E(x, L) = 0 \quad (\text{A.31})$$

Now, we expand  $\psi_H(x, y)$  in Fourier series as before:

$$\psi_H(x, y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} A_{nm}^{(H)} \exp\left(ik_x^{(n)} x\right) \sin \frac{m\pi}{l} y, \quad (\text{A.32})$$

The remaining boundary condition is the continuity of the x component of the electric field, whereas the continuity of the y component of the electric displacement is equivalent to that of the z component of the magnetic field. The former boundary condition at  $y = 0$  and  $y = L$  leads to:

$$\begin{aligned}
\pi \varepsilon_1 \sum_{m=1}^{\infty} m A_{nm}^{(H)} &= \left( i \varepsilon_2 L k_{r,y}^{(n)} + \varepsilon_1 \right) R_n^{(H)} - \varepsilon_1 T_n^{(H)} + \delta_{n_0 H_0} \left( i \varepsilon_2 L k_{1,y}^{(n)} + \varepsilon_1 \right) \quad (A.33) \\
\pi \varepsilon_3 \sum_{m=1}^{\infty} m (-1) A_{nm}^{(H)} &= \varepsilon_3 R_n^{(H)} + \left( i \varepsilon_2 L k_{r,y}^{(n)} + \varepsilon_1 \right) T_n^{(H)} + \delta_{n_0 H_0} \varepsilon_3 \quad (A.34)
\end{aligned}$$

Equations (A.32), (A.33), and (A.34) determine the unknown coefficients,  $A_{nm}^{(H)}$ ,  $R_n^{(H)}$  and  $T_n^{(H)}$ .

## Appendix B

# Fourier Transform of the Dielectric Function for 2D

### B.1 Square Lattice

the Fourier coefficients of the dielectric function,  $\kappa_{nm}$ . First, we calculate them for the configuration shown in Figure A.1. In order to do this, we extend  $\varepsilon^{-1}(x, y)$  symmetrically to the region of negative  $y$  ( $-L \leq y \leq 0$ ). This extension, of course, does not make changes in  $\varepsilon^{-1}(x, y)$  in the positive  $y$  region ( $0 \leq y \leq L$ ). Whether we extend it symmetrically or antisymmetrically to the negative  $y$  region is not critical. It is thus expressed as:

$$\frac{1}{\varepsilon(x, y)} = \frac{1}{\varepsilon_b} + \left( \frac{1}{\varepsilon_a} + \frac{1}{\varepsilon_b} \right) \sum_{j=1}^2 \sum_{l=-\infty}^{\infty} \sum_{l'=1}^{N-1} S(r \parallel -u_{j\parallel}(l, l')) \quad (\text{B.1})$$

where  $u_{j\parallel}(l, l')$  is given by:

$$u_{1\parallel}(l, l') = (a_1 l, a_2 l' + r_a + d) \quad (\text{B.2})$$

$$u_{2\parallel}(l, l') = (a_1 l, -a_2 l' - r_a - d) \quad (\text{B.3})$$

and:

$$S(r \parallel) \left\{ \begin{array}{ll} 1 & \left( |r_{\parallel}| \leq r_a \right) \\ 0 & \left( |r_{\parallel}| \geq r_a \right) \end{array} \right\} \quad (\text{B.4})$$

the Fourier coefficients are simplified. For this case,  $\kappa_{nm}$  are given by:

$$\kappa_{nm} = \left\{ \begin{array}{ll} \frac{f}{\varepsilon_a} + \frac{1-f}{\varepsilon_b} & (n=0, m=0) \\ 2f \left( \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right) \frac{J_1(G_{n,m} r_a)}{G_{n,m} r_a} & (n=0, m=0) \\ 2f \left( \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right) (-1)^j \frac{J_1(G_{n,m} r_a)}{G_{n,m} r_a} & (m=2jN) \\ 0 & (otherwise) \end{array} \right\} \quad (\text{B.5})$$

where  $j$  denotes a positive integer.

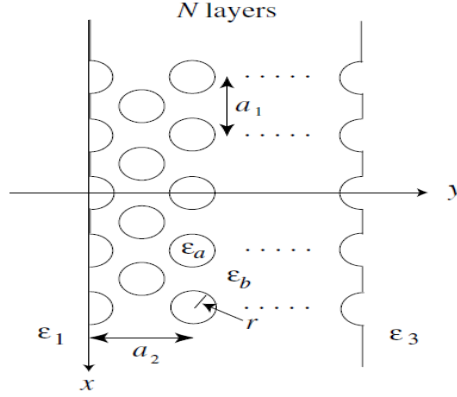


Figure B.1: Top view of the geometry of the 2D crystal for the numerical calculation of the transmission and the Bragg reflection spectra by means of the plane-wave expansion method

## B.2 Hexagonal Lattice

When we calculate the transmittance of the hexagonal lattice or that of the square lattice in the  $\Gamma$  to  $M$  direction, we have to deal with a structure such as that shown in Figure B.1. For this structure, we assume that each surface crosses the center of the rods. Let us calculate the Fourier coefficients of the dielectric function of this structure.  $\varepsilon^{-1}$  is now given by:

$$\frac{1}{\varepsilon(x, y)} = \frac{1}{\varepsilon_b} + \left( \frac{1}{\varepsilon_a} + \frac{1}{\varepsilon_b} \right) \sum_{j=1}^2 \left[ \sum_{l=-\infty}^{\infty} S(r \parallel -u_{1\parallel}(l, l')) + \sum_{l'=1}^{N-1} S(r \parallel -u_{2\parallel}(l, l')) \right] \quad (\text{B.6})$$

where  $u_{j\parallel}(l, l')$  is given by:

$$u_{1\parallel}(l, l') = (a_1 l, a_2 l') \quad (\text{B.7})$$

$$u_{2\parallel}(l, l') = (a_1(l + 1/2), -a_2(l' + 1/2)) \quad (\text{B.8})$$

The Fourier coefficients are simplified. For this case:

$$\kappa_{nm} = \left\{ \begin{array}{ll} \frac{f}{\varepsilon_a} + \frac{1-f}{\varepsilon_b} & (n=0, m=0) \\ 2f \left( \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right) \frac{J_1(G_{n,m} r_a)}{G_{n,m} r_a} & (n=0, m=0) \\ 2f \left( \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right) \left\{ 1 + (-1)^{n+j} \right\} \frac{J_1(G_{n,m} r_a)}{G_{n,m} r_a} & (m=2jN \text{ and } n \neq 0, m \neq 0) \\ 0 & (\text{otherwise}) \end{array} \right\} \quad (\text{B.9})$$

$$f = \frac{2\pi r_z^2}{a_1 a_2}$$

is the filling factor for the present structure, and  $G_{nm}$  is given by:

$$G_{nm} = \sqrt{\left(\frac{2n\pi}{a}\right)^2 + \left(\frac{m\pi}{L}\right)^2}$$

# Bibliography

- [1] D.Gerard, “Etude en champ proche et en champ lointain de composants périodiquement nanostructurées : cristaux photoniques et tamis à photons”, thèse doctorat, université de Bourgogne, (2004).
- [2] V. Mizeikis, S. Juodkazis, A. Marcinkevicius, S. Matsuo, H. Misawa, “Tailoring and characterization of photonic crystals”, *Journal of Photochemistry and Photobiology C: Photochemistry Reviews*, 2, p.35-69, (2001).
- [3] M.Loncar, T.Doll, J.Vuckovic, and A.Scherer, “Design and Fabrication of Silicon Photonic Crystal Optical Waveguides ”, *Journal of Lightwave Technology*, Vol. 18, 10, p. 1402-1411, (2000).
- [4] C.Ciminelli, F.Peluso, and M.N. Armenise, “Modeling and Design of Two-Dimensional Guided-Wave Photonic Band-Gap Devices”, *Journal of Lightwave Technology*, Vol. 23, Issue 2, p. 886-901, (2005).
- [5] F.Raineri, “Optique non linéaire dans les cristaux photoniques en semi-conducteur III-V”, thèse doctorat, université Paris XI Orsay, (2004).
- [6] E.Yablonovitch, T.J. Gmitter et K.M.Leung, “Photonic band structure: the face-centred-cubic case employing nonspherical atoms”, *Physical Review Letters*, vol.67, n°.17, p. 2295-2298, (1991).
- [7] K.Busch, “Photonic band structure theory: assessment and perspectives”, *C.R.Physique* 3, p.53-66, (2002).
- [8] J.D. Joannopoulos, R.D. Meade, and J.N. Winn, “Photonic Crystals: Molding the Flow of Light” (Second Edition). (Princeton University Press, Princeton 1995)
- [9] K. Sakoda, “Optical Properties of Photonic Crystals” (First Edition). (Springer, Heidelberg, 2001)
- [10] J.-M. Lourtioz, H. Benisty, V. Berger, et al., “Photonic Crystals, Towards Nanoscale Photonic Devices”. (Springer, Heidelberg, 2005)

- 
- [11] K. Yitzhak, "An introduction to Harmonic Analysis", 2nd corrected ed. (Dover, NY, 1976)
  - [12] W. Kohn, N. Rostoker, "Solution of Schrödinger equation in periodic lattice with an application to metallic lithium". *Phys. Rev.* 94, 1111 (1954)
  - [13] V. N. Astratov, D. M. Whittaker, I. S. Culshaw, et al., "Photonic band-structure effects in the reflectivity of periodically patterned waveguides". *Phys. Rev. B* 60(24), 255–258 (1999)
  - [14] S. Asimakis, G. Meloni, J.Y.Y. Leong, et al., "Low walk-off Kerr-shutter using a dispersion-shifted lead silicate holey fiber". *IEEE Photon. Technol. Lett.* 19(15), 1112–1114 (2007)
  - [15] T. Matsui, K. Nakajima, I. Sankawa, "Dispersion compensation over all the telecommunication bands with double-cladding photonic-crystal fiber". *IEEE J. Lightwave. Technol.* 25(3), 757–762 (2007)
  - [16] H. Xiao, D. Yao, "Optical limitation in two-dimensional nonlinear photonic crystal with triangular lattice". *Phys. Lett. A* 359(6), 723–727 (2006)
  - [17] I.V. Guryev, J.A.A. Lucio, O.G.I. Manzano, et al., "Optical power limiter on the basis of 2D photonic crystal. *Int. Conf. Advanced Optoelectronics and Lasers, CAOL 2008, Crimea, Ukraine*, 180–182 (2008)
  - [18] Pendry J.B., "Photonic band structures". *Journal of Modern Optics* 41, p. 209 (1994).
  - [19] Leuenberger D. "Experimental and numeric investigation of two-dimensional photonic crystals for application in intergrated optics". PhD thesis, Ecole Polytechnique Fédérale de Lausanne, 2004.
  - [20] Sharkawy, S. Y. Shi, D. W. Prather, and R. A. Soef, "Electro-optical switching using coupled photonic crystal waveguides", *Optics Express*, vol. 10, no. 20, pp. 1048–1059, 2002.
  - [21] Martinez, F. Cuesta, and J. Marti, "Ultrashort 2-D photonic crystal directional couplers", *IEEE Photonics Technol. Lett.*, vol. 15, no. 5, pp. 694–696, 2003.
  - [22] R.C. Alferness, R.V. Schmidt, and E. H. Turner, "Characteristics of Ti diffused LiNbO<sub>3</sub> optical directional couplers", *Appl. Opt.*, vol. 18, pp. 4012, 1979.

- 
- [23] S. Boscolo, M. Midrio, and C. G. Someda, "Coupling and decoupling of electromagnetic waves in parallel 2-D photonic crystal waveguides", *Ieee Journal of Quantum Electronics*, vol. 38, no. 1, pp. 47{53, 2002.
- [24] T. F. Krauss, T. Baba, and editors, "Feature section on Photonic Crystal Structures and Applications", *IEEE J. Quantum Electron.*, vol. 38, no. 7, 2002.
- [25] U. Peschel, A. L. Reynolds, B. Arredondo, F. Lederer, P. J. Roberts, T. F. Krauss, and P. J. I. de Maagt, "Transmission and reflection analysis of functional coupled cavity components", *IEEE J. Quantum Electron.*, vol. 38, no. 7, pp. 830{836, 2002.
- [26] O. J. Painter, *Optical Nanocavities in Two-Dimensional Photonic Crystal Planar Waveguides*, Ph.D. thesis, California Institute of Technology, 2001.
- [27] K. Sakoda, "Transmittance and Bragg reflectivity of two-dimensional photonic lattices.", *Phys. Rev. B*, vol. 51, no. 12, pp. 8992{9002, 1995.
- [28] K. Sakoda, "Optical transmittance of two-dimensional triangular photonic lattice", *Phys. Rev. B*, vol. 51, no. 7, pp. 4672{4675, 1995.
- [29] L. Li, "Use of Fourier series in the analysis of discontinuous periodic structures", *J. Opt. Soc. Am. A*, vol. 13, no. 9, pp. 1870{1876, 1996.
- [30] M. Plihal and A. A. Maradudin, "Photonic band structure of two-dimensional systems: The triangular lattice.", *Phys. Rev. B*, vol. 44, no. 16, pp. 8565{8571, 1991.
- [31] Tokushima M. and Yamada H., "Photonic crystal line defect waveguide directional coupler", *Electronics Letter* 37, p. 1454 (2001).
- [32] Olivier S., Smith C., Rattier M., Benisty H., Weisbuch C., Krauss T., Houdré R. and Oesterle U., "Miniband transmission in a photonic crystal coupled-resonator optical waveguide". *Optics Letter* 26, p. 1019 (2001).
- [33] Tokushima M., Hosaka H., Tomita A. and Yamada H., "Lightwave propagation through a 120° sharply bent single-line-defect photonic crystal waveguide". *Applied Physics Letters* 76, p. 952 (2000).
- [34] Labilloy D., Benisty H., Weisbuch C., Krauss T. F., Bardinal V. and Oesterle U., "Demonstration of cavity modes between two-dimensional photonic-crystal mirrors". *Electronics Letter* 33, p. 1978 (1997).

- [35] Zelsmann M., “Cristaux photoniques en silicium sur isolant pour le guidage, le filtrage, l’émission et l’extraction de lumière”. Thèse de doctorat, Université Joseph Fourier Grenoble 1 (2003).
- [36] Pendry J.B., “Photonic band structures”. *Journal of Modern Optics* 41, p. 209 (1994).
- [37] N. Yokoi, T. Fujisawa, K. Saitoh, M. Koshiba, "Apodized Photonic Crystal Waveguide Gratings", *Opt. Express* 14, 4459-4468 (2006)
- [38] E. Yablonovitch, T. J. Gmitter, K. M. Leung, "Photonic Band Structure: The Face-Centered-Cubic Case Employing Nonspherical Atoms," *Phys. Rev. Lett.* 67, 2295-2298 (1991)
- [39] R. Zengerle, "Photonic Crystal Waveguides: Dispersion, Anomalous Refraction and Application," *Advances in Solid State Physics* 44, Springer, (2004)
- [40] L.Arizmendi, “Photonic applications of lithium niobate crystals”, *Phys. Stat. Sol (a)*, 201, N°2, p.253-283, (2004).
- [41] A.Barra, D.Cassagne, and C.Jouanin, “Existence of two-dimensional absolute photonic band gaps in the visible”, *Applied Physics Letters*, Vol 72, N°6, p.627-629, (1998).
- [42] M.L.A.Dedood, “Design and optimisation of 2D photonic crystal waveguides based on silicon”, *Opt. Quant. Electron.* 34, p.145–159, (2002).
- [43] D.Cassagne, C.Jouanin, and D.Bertho, “Photonic band gaps in a two-dimensional graphite structure”, *Physical Review B*, Vol 52, N° 4, p.2217-2220, (1995).
- [44] J.Yu Ye, V.Mizeikis, Y.Xu, S.Matsuo, and H.Misawa, “Fabrication and optical characteristics of silicon-based two-dimensional photonic crystals with honeycomb lattice”, *Optics Communications* 211, p.205-213, (2002).
- [45] D.Cassagne, C.Jouanin, and D.Bertho, “Optical properties of two-dimensional photonic crystals with graphite structure”, *Appl.Phys.lett.*70, (3), p.289-291 (1997).
- [46] J.Yu Ye, S.Matsuo, V.Mizeikis, and H.Misawa, “Silicon-based honeycomb photonic crystal structures with complete photonic band gap at 1.5  $\mu\text{m}$  wavelength”, *Journal of Applied Physics*, Vol 96, N°11, p.6934-6936 (2004).
- [47] T.I.Weng, and G.Y.Guo, “Band structure of honeycomb photonic crystal slabs”, *Journal of Applied Physics*, 99, p. 093102 (2006).

- 
- [48] M.Roussey, M.P.Bernal, N.Courjal, F.Baida et D.Van Labeke, "Cristaux photoniques en niobate de lithium actifs", Journées nationales de l'optique guidée, Paris 2004.
- [49] A.Boudrioua, P.Moretti, J.C.Loulergue, "Fabrication, characterization and electro-optic performances of proton-implanted waveguides in LiNbO<sub>3</sub>", Journal of non Crystalline Solids 187, p.443-447, (1995).
- [50] M.Qiu, B. Jaskorzynska, M. Swillo, and H. Benisty, "Time-domain 2D modeling of slab-waveguide based photonic-crystal devices in the presence of out-of-plane radiation losses", Microwave Opt. Techn. Lett., Vol. 34, No. 5, p.387-393, (2002).
- [51] M.Qiu, "Effective index method in heterostructure slab waveguide based two-dimensional photonic crystals", Appl. Phys. Lett., Vol. 81, No. 7, p.1163-1165, (2002).
- [52] D.Cassagne, C.Jouanin, and D.Bertho, "Hexagonal photonic-band-gap structures", Physical review B, Vol 53, N°11, p.7134-7142, (1996).
- [53] H. Benisty, C. Weisbuch, D. Labilloy, M. Rattier, C. J. M. Smith, T. F. Krauss, Richard M. De La Rue, R. Houdré, U. Oesterle, C. Jouanin, and D. Cassagne,"Optical and Confinement Properties of Two-Dimensional Photonic Crystals", Journal of Lightwave Technology, Vol. 17, N°. 11, p.2063-2077, (1999).
- [54] B.Wild "Etude expérimentale des propriétés optiques des cristaux photoniques bidimensionnels et leur accordabilité" thèse doctorat, école polytechnique fédérale de Lausanne, (2006).
- [55] U.Grüning, V. Lehman, and C.M.Engelhardt, "Two dimensional infrared photonic band gap on porous", Applied physics letters, vol 66, n° 24, p 3254-3256, (1995).
- [56] M.-P. Bernal , N. Courjal, J. Amet, M. Roussey, and C.H. Hou, "Lithium niobate photonic crystal waveguides: Far field and near field characterisation", Optics Communications vol. 265, no1, p.180-186, (2006).
- [57] C.Lin, H.Chen, W.Chao, C.Hsieh, and W.Chang, "Optical characterization of two – dimensional photonic crystals based on spectroscopic ellipsometry with rigorous coupled – wave analysis", Microelectronic Engineering 83, p.1798-1804, (2006).
- [58] R. Costa, A. Melloniand, M. Martinelli. "Bandpass resonant filters in photonic-crystal waveguides". IEEE Photonics Technology Letters, 15(3), 2003: 401–403.

- 
- [59] L. Oyhenart, “Modélisation, réalisation et caractérisation de cristaux photoniques tridimensionnels en vue d’applications à la compatibilité électromagnétique”. Thèse de doctorat, N°3114, 2005.
  - [60] H. Takeda, K. Yoshino. “Tunable light propagation in Y-shaped waveguides in two-dimensional photonic crystals utilizing liquid crystals as linear defects”. Phys Rev B, Vol.67, 073106, 2003.
  - [61] G. Tayeb, D. Maystre. “Rigorous theoretical study of finite-size two dimensional photonic crystals doped by microcavities”. J. Opt. Soc. Am. A. – Vol. 14, N°12, 1997.
  - [62] K.M. Ho, C. T. Chan, C. M. Soukoulis. “Existence of a photonic gap in periodic dielectric structures”. Phys. Rev. Lett., Vol.65, 1990: 3152 - 3155.
  - [63] E. Yablonovitch, “Inhibited Spontaneous Emission in Solid-State Physics and Electronics”. Phys. Rev. Lett., 58, 2059, 1987.
  - [64] W. L. Bragg, “The diffraction of short electromagnetic waves by a crystal”. Proc. Cambridge Phil. Soc., 17, 43, 1913.
  - [65] M.R. Beghou, R.Kremer, A.Boudrioua and S. Latreche, “Theoretical Study of Photonic Band Gap Structures in LiNbO<sub>3</sub> ”, Colloque international “Lithium Niobate, from material to device, from device to system”, Metz (France), May 23-25 (2005).

## Abstract

This work is a contribution to the study of photonic crystals in one and two dimensions (1D and 2D) photonic crystals on two different streams of materials: semi-conductors (Ge) and dielectric (LiNbO<sub>3</sub>). It is therefore a matter of studying different structures and different configurations to choose an optimal solution. Initially, calculations based on the plane wave expansion method are presented whose purpose is to determine the exact dimensions of the structures. Once the geometrical parameters are correctly chosen, the filtering of these is characterized by transmission and reflection with FDTD simulations.

*Key words:* Photonic crystals (1D, 2D and 3D), photonic band gap, Plane Wave Expansion method (PWE).

## Résumé

Ce travail est une contribution à l'étude des cristaux photoniques en une et deux dimensions (1D et 2D) des cristaux photoniques sur deux différents courants de matériaux: les semi-conducteurs (Ge) et diélectriques (LiNbO<sub>3</sub>). Il est donc une question d'étudier les différentes structures et différentes configurations de choisir une solution optimale. Initialement, les calculs basés sur la méthode de décomposition en ondes planes sont présentés dont le but est de déterminer les dimensions exactes des structures. Une fois les paramètres géométriques sont correctement choisis, le filtrage de ceux-ci est caractérisé par la transmission et de réflexion avec des simulations FDTD.

*Les mots clés:* Cristaux photoniques (1D, 2D and 3D), bande interdite photonique, la méthode de décomposition en ondes planes.

## الملخص

يتمحور هذا العمل حول دراسة البلورات الضوئية (البلورات الضوئية أحادية البعد وثنائية) وذلك باستعمال مادتين من مجموعتين مختلفتين المواد: أشباه الموصلات (الجرمانيوم Ge) والعوازل (نيوبات الليثيوم LiNbO<sub>3</sub>). ولذلك تطرقنا من خلالها إلى دراسة عدة أشكال وبنيات بهدف الوصول إلى الاختيار الأمثل، في البداية، اعتمدنا طريقة تحليل الأمواج المستوية لأجل تحديد أبعاد البنيات ثم طريقة FDTD التي سمحت لنا باستخراج معامل الانعكاس

كلمات المفتاحية: البلورات الضوئية، ذات فجوة الحزمة الضوئية، التحلل الموجات الطائفة.

