Relation between some summing operators

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Abstract

The space of *mid* p summing operators was introduced by Karn and Sinha in[4] and was later developed by Botelho, Compos and Santos in 2017 in their article [3]. In this work we give a representation of the dual and bidual of the space $\Pi_{p,mid}$ of *mid* p-summing operators. We give a new factorization theorem for the well-known concept of strongly p-summing operators and we terminate with some relationships between these classes.

Key words: p-summing operators, strongly p-summing operators, Cohen p-nuclear operators, factorization theorem.

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1 Introduction

Let X be a Banach space and X^* its dual. B_X denotes the closed unit ball of X. For $1 < p < \infty$, let p^* be its conjugate, i.e., $1/p + 1/p^* = 1$.
Let us define the new sequences spaces. We write the space of *mid* p-summable sequences $(x_i)_i$ in X with the norm

$$\|(x_i)_i\|_{p,mid} := \sup_{\|(x_n^*)_n\|_{p,w}=1} (\sum_i \sum_n |\langle x_n^*, x_i\rangle|^p)^{\frac{1}{p}}.$$

The concept of absolutely summing linear operators was mainly introduced by Crothendieck in the 1950's and was generalized by Pietsch in the 1967's [2]. A linear operator $T\colon X \longrightarrow Y$ is absolutely $p-$*summing* if sends weakly p -summable sequences to absolutely p -summable sequences. The concept of strongly p-summing linear operators was introduced by J. S. Cohen in 1973 [1] as a characterization of the conjugates of p^*-summing linear operators. An operator T between two Banach spaces X, Y is strongly p-summing if there is a positive constant C such that for any $(x_i)_i$ in $l_p(X)$ and $(y_i^*)_i$ in $l_p^w(Y^*)$ we have

$$\|(\langle T(x_i), y_i^*\rangle)_i\|_1 \leq C \|(x_i)_i\|_p \|(y_i^*)_i\|_{p^*,\omega} . \tag{1}$$

We denote by $D_p(X, Y)$ the class of all strongly p-summing operators from X into Y and $d_p(T)$ the smallest constant C such that the inequality (1).
The notion of Cohen p-nuclear operators was initiated by Cohen in 1973 [1]. An operator T between two Banach spaces X, Y is Cohen p-nuclear if there is a positive constant$C$ such that for any $(x_i)_i$ in $l_p^w(X)$ and $(y_i^*)_i$ in $l_p^w(Y^*)$ we have

$$\left|\sum_i \langle T(x_i), y_i^*\rangle\right| \leq C \|(x_i)_i\|_{p,\omega} \|(y_i^*)_i\|_{p^*,\omega} . \tag{2}$$

We denote by $N_p(X, Y)$ the class of all Cohen p-nuclear operators from X into Y and $n_p(T)$ the smallest constant C such that the inequality (2) .

2 Main results

In this section we give a representation of the dual and bidual of the space $\Pi_{p,mid}$ of *mid* p-summing operators. We give a new factorization theorem for the well-known concept of strongly p-summing operators, and we study some properties concerning this classes. A linear operator $T: X \longrightarrow Y$ is absolutely mid $p-$summing if sends *mid* p-summable sequences to absolutely p-summable sequences.

Definition 1. *An operator $T: X \to Y$ is strongly mid p-summing, if there is a positive constant C such that for any $(x_i)_i \in l_p(X)$, $(y_i^*)_i \in l_p^{mid}(Y^*)$ we have*

$$\sum_i |\langle T(x_i), y_i^*\rangle| \leq C \|(x_i)_i\|_p \|(x_i)_i\|_{p^*,mid} . \tag{3}$$

We denote by $D_{p,mid}(X, Y)$ the class of all strongly mid p-summing operators from X into Y and $d_{p,mid}(T)$ the smallest constant C such that the inequality (3) *.*

Theorem 2. *$(D_{p,mid}(X, Y), d_{p,mid}(.))$ is Banach operator ideal.*

Proposition 3. *Consider $1 < p \leq q < \infty$.and T linear operator If $T \in D_{q,mid}(X, Y)$ then $T \in D_{p,mid}(X, Y)$ and $d_{p,mid}(T) \leq d_{q,mid}(T)$.*

Definition 4. *[3] Let X and Y be a Banach spaces. An operator $T: X \to Y$ is weakly mid p-summing $(1 < p < \infty)$, if there is a positive constant C such that for any $(x_i)_i \in l_{p,\omega}(X)$*

$$\|(Tx_i)_i\|_{p,mid} \leq C \|(x_i)_i\|_{p,\omega} . \tag{4}$$

We denote by $\mathcal{W}_{p,mid}(X, Y)$ the class of all weakly mid p-summing operators from X into Y and $w_{p,mid}(T)$ the smallest constant C such that the inequality (4) *. we have $\mathcal{W}_p(X, Y) \subset \mathcal{W}_{p,mid}(X, Y)$ and $w_{p,mid}(T) \leq w_p(T)$.*

Proposition 5. *Let X and Y be a Banach spaces We have*
(1) $D_p(X,Y) \subseteq D_{p,mid}(X,Y)$ *and* $d_{p,mid}(T) \leq d_p(T)$.
(2) $\prod_p(X,Y) \subseteq \prod_{p,mid}(X,Y)$ *and* $\pi_{p,mid}(T) \leq \pi_p(T)$.
(3) $N_p(X,Y) \subseteq \mathcal{W}_{p,mid}(X,Y)$*and* $n_{p,mid}(T) \leq w_p(T)$.

Theorem 6. *Let X and Y be a Banach spaces. An operator $T\colon X \to Y$ is weakly mid p-summing $(1 < p < \infty)$ if and only if T^* is weakly mid p-summing i.e*

$$[\mathcal{W}_{p,mid}(X,Y)]^* = \mathcal{W}_{p,mid}(Y^*,X^*)$$

Theorem 7. *For $1 < p < \infty$, the following statements are equivalent*
1) *The operator T is absolutely mid p-summing*
2) *The operator T^* is strongly mid p-summing*
3) *The operator T^{**} is absolutely mid p-summing*

Theorem 8. *For $1 < p < \infty$, the following statements are equivalent*
1) *The operator T is strongly mid p-summing*
2) *The operator T^* is absolutely mid p-summing*
3) *The operator T^{**} is strongly mid p-summing*

Theorem 9. *Every strongly p-summing linear operator factors through weakly mid p-summing and strongly mid p-summing linear operators i.e*

$$D_p = \mathcal{W}_{p^*,mid} \circ D_{p,mid}$$

Theorem 10. *Let $(1 < p < \infty), X$ and Y be a Banach spaces. the following statements are equivalent:*
(1) $D_{p,mid}(X,Y)$ *or* $\prod_{p,mid}(X,Y)$ *is reflexive*
(2) X *and* Y *are reflexive*
(3) $D_p(X,Y)$ *or* $\prod_p(X,Y)$ *is reflexive*

3 Conclusion

Let X and Y be a Banach spaces, Let $(1 < p < \infty)$. We have

$$\Pi_p = \Pi_{p,mid} \circ \mathcal{W}_{p,mid}$$
$$D_p = \mathcal{W}_{p^*,mid} \circ D_{p,mid}$$

$$[\mathcal{W}_{p,mid}(X,Y)]^* = \mathcal{W}_{p,mid}(Y^*,X^*)$$
$$[\mathcal{W}_{p,mid}(X,Y)]^{**} = \mathcal{W}_{p,mid}(X^{**},Y^{**})$$

$$[\Pi_{p,mid}(X,Y)]^* = D_{p^*,mid}(Y^*,X^*)$$

$$[\Pi_{p,mid}(X,Y)]^{**} = \Pi_{p,mid}(X^{**},Y^{**})$$

$$[D_{p,mid}(X,Y)]^{*} = \Pi_{p^*,mid}(Y^{*},X^{*})$$
$$[D_{p,mid}(X,Y)]^{**} = D_{p,mid}(X^{**},Y^{**})$$

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