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**Study of Sensorless Control for Five-phase permanent magnet
synchronous Motors under Load Variations**

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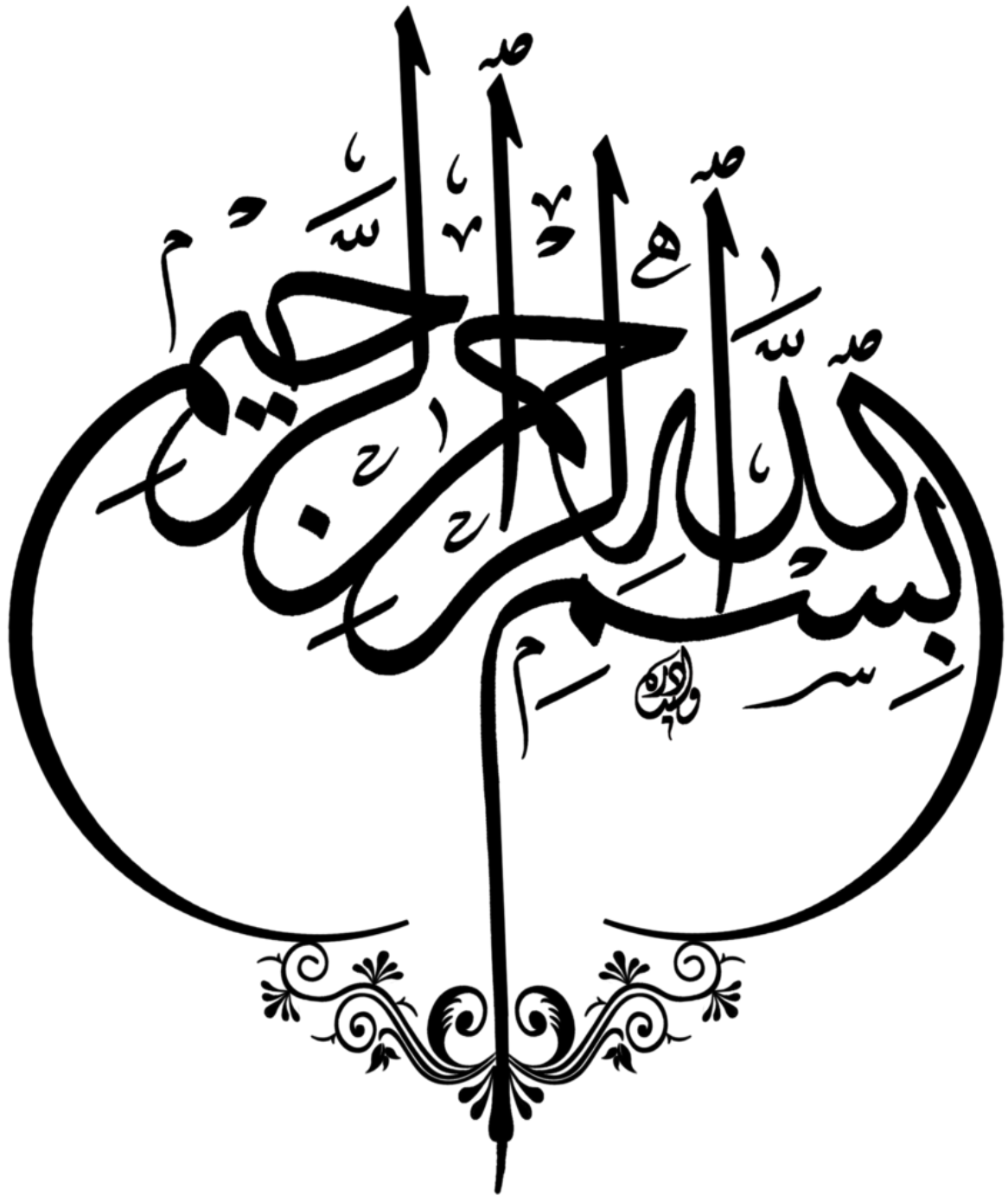
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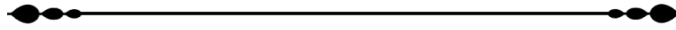
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Thank God. My university studies have come to an end after exhaustion and hardship. And here I am, finishing my thesis with renewed vigor and energy. I dedicate my diploma and my joy to my father, my mother, and all my family members. I must not forget my friends, my professors, and all those who have supported and encouraged me throughout my life and given me a helping hand.

Thank you



Summary



Summary

This study investigates the behavior of a Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) under Load Variations, it applies a sensorless control approach that combines backstepping control with a Model Reference Adaptive System (MRAS) observer. Using measurable electrical quantities, including stator currents and voltages, the proposed method accurately estimates both rotor position and speed. The design of the backstepping controller follows a structured procedure to achieve global or semi-global stability, making it suitable for applications that demand reliable operation under changing conditions. Lyapunov theory is used to analyze and verify the stability of the combined observer and controller. The main objective of the research is to maintain motor performance and stable operation in the presence of load variation. The Simulations conducted in MATLAB/Simulink compare normal motor performance with changed load mode to evaluate the effectiveness of the control strategy. The findings show that the backstepping controller, supported by the MRAS observer, enhances the control reliability. The simulation results confirm that the proposed approach sustains motor operation with minimal decline in performance, offering a dependable solution for industrial systems requiring resilience to faults.

5P-PMSM: Five-Phase Permanent Magnet Synchronous Motor

Backstepping : Contrôle des pas en arrière

MRAS: Model Reference Adaptive System

Résumé

Cette étude examine le comportement d'un moteur Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) sous des variations de charge (*Load Variations*). Elle applique une approche de commande sans capteur (*sensorless control*) combinant la commande backstepping avec un observateur Model Reference Adaptive System (MRAS). En utilisant des grandeurs électriques mesurables, notamment les courants et tensions statoriques, la méthode proposée permet d'estimer avec précision la position et la vitesse du rotor. La conception du contrôleur backstepping suit une procédure structurée afin d'assurer une stabilité globale ou semi-globale, ce qui le rend adapté aux applications nécessitant un fonctionnement fiable dans des conditions variables. La théorie de Lyapunov est utilisée pour analyser et vérifier la stabilité de l'ensemble observateur–contrôleur. L'objectif principal de cette recherche est de maintenir les performances du moteur ainsi qu'un fonctionnement stable en présence de variations de charge. Les simulations réalisées sous MATLAB/Simulink comparent le fonctionnement normal du moteur avec un mode de charge variable afin d'évaluer l'efficacité de la stratégie de commande. Les résultats montrent que le contrôleur backstepping, soutenu par l'observateur MRAS, améliore la fiabilité de la commande. Les résultats de simulation confirment que l'approche proposée maintient le fonctionnement du moteur avec une dégradation minimale des performances, offrant ainsi une solution fiable pour les systèmes industriels nécessitant une forte robustesse face aux défauts.

5P-PMSM: Five-Phase Permanent Magnet Synchronous Motor

Backstepping : Contrôle des pas en arrière

MRAS: Model Reference Adaptive System

ملخص

تدرس هذه الدراسة سلوك محرك متزامن خماسي الأطوار ذي مغناطيس دائم (5P-PMSM) تحت تأثير تغيرات الحمل، وتطبق نهج تحكم بدون مستشعرات يجمع بين التحكم بالخطوة الخلفية **Backstepping control** ومراقب نظام مرجعي تكيفي (MRAS). باستخدام كميات كهربائية قابلة للقياس، بما في ذلك تيارات وفولتيات الجزء الثابت، تُقدّر الطريقة المقترحة بدقة كلاً من موضع وسرعة الدوار. يتبع تصميم وحدة التحكم بالخطوة الخلفية إجراءً منظماً لتحقيق استقرار شامل أو شبه شامل، مما يجعلها مناسبة للتطبيقات التي تتطلب تشغيلاً موثوقاً به في ظل ظروف متغيرة. تُستخدم نظرية لياپونوف لتحليل استقرار المراقب ووحدة التحكم المدمجين والتحقق منه. الهدف الرئيسي من البحث هو الحفاظ على أداء المحرك وتشغيله المستقر في وجود تغيرات الحمل. تُقارن عمليات المحاكاة التي أُجريت في MATLAB/Simulink بأداء المحرك الطبيعي مع وضع الحمل المتغير لتقييم فعالية استراتيجية التحكم. تُظهر النتائج أن وحدة التحكم بالخطوة الخلفية، المدعومة بمراقب MRAS، تُحسن المتانة وموثوقية التحكم. تؤكد نتائج المحاكاة أن النهج المقترح يحافظ على تشغيل المحرك مع الحد الأدنى من الانخفاض في الأداء، مما يوفر حلاً موثوقاً به للأنظمة الصناعية التي تتطلب القدرة على تحمل الأعطال.

5P-PMSM : محرك متزامن خماسي الأطوار ذي مغناطيس دائم

MRAS : مراقب نظام مرجعي تكيفي

Backstepping control : التحكم بالخطوة الخلفية

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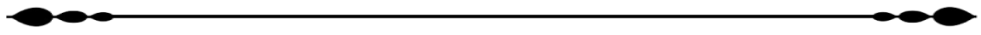
Abbreviations list

- PMSMs: permanent magnet synchronous Motors
- 5-phase PMSM, 5P- PMSM: five Phase Permanent Magnet Synchronous Motor
- MRAS: model reference adaptive system.
- SMO: Sliding Mode Observer
- AI: Artificial Intelligence
- EMF: Electromotive force
- MMF: Magnetomotive force
- HF: High Frequency
- DSP: Digital signal processor
- DC machine: Direct Current machine
- AC machine: Alternative Current machine
- VSI: Voltage Source Inverter
- CSI: Current Source Inverter
- PWM: Pulse-Width Modulation
- IGBT: Insulated gate bipolar transistors
- MOSFET: metal-oxide-semiconductor field-effect transistor
- DCMI: Diode Clamped Inverter.
- FCMI: Flying Capacitor Inverter
- PI: proportional integral control
- PID: proportional integral Derivative control.
- DTC: direct torque control
- FOC: field-oriented control
- RFOC: rotor-flux oriented control
- SMC: sliding mode control
- MPC: model predictive control
- I_q : q-axis current
- I_d : d- current
- $X(t)$: state variable or state vector
- $S(x)$: Sliding control Surface
- ρ : is a positive constant
- r : degree relative of sliding surface
- $e(t)$: state error relative
- A : is the state matrix of the observer
- B : is a control input matrix
- C : is output matrix
- V_s : is input vector of Luenberger observer
- G : gain matrix of Luenberger observer
- \dot{X} : the derivative of state variable
- \hat{X} : the estimated value of the state variable
- Y : the output system
- L_d : direct inductance
- L_q : Quadratic inductance
- L_l : leakage inductance
- PWM: Pulse width modulation
- $(\alpha-\beta)$ ($\alpha_1, \beta_1, \alpha_2, \beta_2$): stationary frame or the stator reference frame
- $(d-q)$ (d_1, q_1, d_2, q_2): rotational frame or rotor reference frame
- $(U V)$: Synchronous reference frame
- (a, b, c, d, e) : natural frame for 5P-PMSM representation
- GA: Genetic Algorithms
- $[V_s]$: stator voltages
- $[I_s]$: stator currents
- $[\Phi_s]$: stator Flux
- $[\varphi_m]$: Matrix Vector of the flux created by the permanent

- $[L_{s0}]$: is the proper inductances matrix of stator windings,
- $[L_{s1}]$: The mutual inductances matrix of stator windings
- L_m : is the main stator inductance
- L_s : is linkage inductance.
- P : is pair of pol number.
- Θ : rotor angular position
- Ω : mechanical speed
- w : is the electrical speed
- T_{em} : electro-magnetic torque
- T_L : eternal load torque
- J : is inertia moment of the motor and load combined.
- F : is the friction coefficient
- Ω : is the mechanical energy delivered by rotor
- $[I_s]^T$ is the transposed matrix of stator current
- θ_{co} is the angle between the a-axis of referential frame and the d1-axis (coordinate angle)
- w_{co} : is the speed coordinate
- w_s : is the synchronous speed.
- K : is a constant factor depend of the power transformation from original frame to new reference frame
- $[T]$: Park matrix transformation
- $[I_{\alpha 1}, I_{\beta 1}, I_{\alpha 2}, I_{\beta 2}]$: Stator currents in stationary frame
- $[V_{\alpha 1}, V_{\beta 1}, V_{\alpha 2}, V_{\beta 2}]$: Stator voltages in stationary frame
- $[\varphi_{\alpha 1}, \varphi_{\beta 1}, \varphi_{\alpha 2}, \varphi_{\beta 2}]$: Stator voltages in stationary frame
- $[I_{q1}, I_{d1}, I_{q2}, I_{d2}]$: Stator currents in rotational frame
- $[V_{q1}, V_{d1}, V_{q2}, V_{d2}]$: Stator voltages in rotational frame
- magnets
- $[R_s]$: stator resistances
- $[\varphi_{q1}, \varphi_{d1}, \varphi_{q2}, \varphi_{d2}]$: Stator voltages in rotational frame
- L_{d1}, L_{q1} : are the direct and quadratic inductances of principal frame
- L_{d3}, L_{q3} : are the direct and quadratic inductances of secondar frame
- $[A]$: is the system (or state) matrix
- $[B]$: is the input matrix
- $[C]$: represent the output matrix,
- $[E]$: represent the disturbance input matrix
- D : is the disturbance input of system
- V_{dc} : is the DC-link source
- $f(x, u)$: system function
- $V_1(x)$: first Lyapunov function
- $\dot{V}_1(x)$: derivative of first Lyapunov function
- $f(x_1, x_2)$: system of the second order
- e_1 : the tracking error
- \dot{e}_1 : Derivative of the tracking error
- \ddot{e}_1 : Derivative of the tracking error
- \dot{X} : is the derivative of desired output.
- x_1^* is the desired reference output
- $V_2(x)$: second Lyapunov function
- $\dot{V}_2(x)$: derivative of second Lyapunov function

-
- X_{c2} : virtual command of control
 - u : the global control
 - \ddot{x}^* : second derivative of variable.
 - K_1^2 : square of gain.
 - T_{em} : electromagnet torque
 - T_L : Load charge torque.
 - J : motor inertia
 - B : viscous friction
 - Φ_m : Permanent Electromagnet
 - w^* : reference speed
 - I_{d1ref}, V_{d1ref} : reference current and voltage in rotational frame
 - k_1, k_2 : positives constants of Lyapunov functions
 - $H1, H2, H3, H4$: are the backstepping control gains
 - $e1, e2, e3, e4$: are the backstepping control error
 - L_s : Principal frame inductance
 - L_{Ls} : secondaire frame inductance
 - n : random numbers
 - $\Delta\omega$: the relative errors of speed
 - Δi : errors of current
 - $\text{sgn}(s)$: signum function

General Introduction



Introduction General

The continuous advancement of modern industrial technologies has greatly increased the demand for high-performance electric drive systems capable of providing high efficiency, reliability, robustness, and precise control. Among the different electrical machines employed in advanced industrial applications, Permanent Magnet Synchronous Motors (PMSMs) have gained significant importance because of their high-power density, rapid dynamic response, reduced maintenance requirements, and superior efficiency compared with conventional machines. The operating principle of PMSMs is based on the interaction between the rotating magnetic field generated by the stator windings and the magnetic field produced by permanent magnets located on or inside the rotor. Depending on the rotor configuration, PMSMs are commonly categorized into Surface Permanent Magnet Synchronous Motors (SPMSMs) and Interior Permanent Magnet Synchronous Motors (IPMSMs), each presenting specific advantages according to the intended application [11].

Recently, multiphase electrical machines, especially Five-Phase Permanent Magnet Synchronous Motors (5P-PMSMs), have become increasingly attractive for high-performance applications due to their enhanced fault tolerance capability, reduced torque ripple, lower harmonic distortion, and improved power sharing compared with conventional three-phase systems. Furthermore, five-phase machines ensure continued operation even under phase fault conditions, which makes them particularly suitable for critical applications such as electric vehicles, aerospace systems, marine propulsion, robotics, and high-power industrial processes. As a result, the study of the structure, operating principles, and performance of multiphase PMSMs has become an important research area in modern electric drive systems [12].

For effective analysis and high-performance control of the 5P-PMSM, accurate mathematical modeling is essential to describe the electrical and mechanical dynamics of the machine under different operating conditions. The modeling procedure generally starts with several simplifying assumptions followed by the derivation of voltage equations, flux linkage equations, and mechanical equations in the natural reference frame. To facilitate system analysis and controller design, Clarke and Park transformations are commonly applied to transform the five-phase variables into stationary and rotating reference frames. This transformation simplifies the mathematical representation of the machine and enables the development of dynamic models in stationary (α,β) and rotating (d-q) frames, which are well suited for simulation and advanced control implementation. In addition, the operation of the five-phase PMSM requires a five-phase voltage source inverter combined with Pulse Width

Modulation (PWM) techniques to generate controlled voltages with improved dynamic behavior and reduced harmonic distortion. Because of the nonlinear characteristics and increasing complexity of multiphase PMSM drive systems, advanced nonlinear control techniques have become necessary to ensure stability and superior dynamic performance[4].

Among these approaches, Backstepping Control is considered one of the most efficient nonlinear control methods thanks to its recursive design methodology and its capability to guarantee system stability through Lyapunov theory. This strategy divides the nonlinear system into interconnected subsystems and progressively designs both virtual and actual control laws to stabilize the complete system. For the 5P-PMSM drive, Backstepping Control provides precise speed tracking, effective current regulation, low overshoot, and strong robustness against disturbances and parameter variations through the generation of appropriate reference currents and voltages [21].

Alongside advanced control methods, sensorless control techniques have attracted considerable attention in modern electric drives because they eliminate the need for mechanical sensors used to measure rotor position and speed. Removing these sensors reduces the overall system cost, increases reliability, decreases maintenance requirements, and improves robustness, particularly in harsh industrial environments. Among the available estimation methods, the Model Reference Adaptive System (MRAS) is widely regarded as a reliable and efficient sensorless estimation technique due to its simple structure, fast response, and accurate estimation capability over a broad operating range [8]. The MRAS technique relies on comparing a reference model with an adaptive model, where the estimation error is continuously minimized through an adaptive mechanism in order to estimate the rotor speed and position accurately. The combination of the MRAS observer with the Backstepping controller results in an advanced sensorless control architecture for the Five-Phase PMSM capable of achieving high-performance operation under both normal and disturbed conditions. The effectiveness of this integrated approach can be validated using MATLAB/Simulink simulations under normal operating conditions as well as load variation scenarios [7]. Simulation results generally confirm excellent speed tracking performance, accurate estimation of rotor variables, reduced tracking errors, and high robustness against sudden load disturbances. Therefore, integrating multiphase PMSM technology with nonlinear Backstepping Control and MRAS-based sensorless estimation represents a highly effective and promising solution for modern industrial electric drive applications that require high precision, efficiency, reliability, and robustness [18].

Chapter I: General Overview and Theoretical Background

I-1- Introduction:

Permanent Magnet Synchronous Motors (PMSMs) are widely used in modern industrial applications due to their high efficiency, high power density, fast dynamic response, and low maintenance requirements. They operate through the interaction between the stator magnetic field and the permanent magnets mounted on or embedded inside the rotor. PMSMs are mainly classified into Surface PMSM (SPMSM) and Interior PMSM (IPMSM). Recently, five-phase PMSMs have gained increasing interest because they provide better reliability, reduced torque ripple, improved fault tolerance, and lower harmonic distortion compared to conventional three-phase machines. These advantages make them suitable for critical applications such as electric vehicles, aerospace, and marine propulsion. To improve drive performance, sensorless control methods are used to estimate rotor speed and position without mechanical sensors, reducing cost and increasing reliability. Among these methods, the Model Reference Adaptive System (MRAS) is widely preferred for its simplicity, robustness, and accurate estimation capability. In addition, Backstepping Control has become an effective nonlinear control technique due to its ability to guarantee system stability using Lyapunov theory while ensuring accurate tracking performance and strong robustness against disturbances and parameter variations.

I.2 Permanent Magnet Synchronous Motors (PMSM)

I.2.1. Definition of PMSM

A permanent magnet synchronous motor (PMSM) is defined as a type of synchronous motor in which permanent magnets are integrated into the rotor to generate a constant magnetic field, while the stator consists of three-phase windings fed with alternating current to produce a rotating magnetic field. As a result, the rotor rotates at a speed synchronized with the stator's magnetic field, and torque generation depends on the interaction between the rotor and stator magnetic fields. [1] .

I.2.2. Operating principle

A Permanent Magnet Synchronous Motor (PMSM) operates through the interaction between the rotating magnetic field generated by the stator windings and the fixed magnetic field produced by permanent magnets mounted on the rotor. When a three-phase AC supply is applied to the stator, it creates a rotating magnetic field within the motor air gap. The rotor magnets continuously follow this rotating field, resulting in rotor motion at the same speed as the stator field, which ensures synchronous operation without slip. The developed electromagnetic torque arises from the magnetic coupling between the stator and rotor fields.

In contrast to induction motors, PMSMs do not need an external rotor excitation current, leading to lower copper losses and higher efficiency. These advantages provide improved power density, fast dynamic response, and superior performance, making PMSMs highly suitable for applications such as electric vehicles, robotic systems, aerospace equipment, and advanced industrial drives.

I .2.3.Types (SPMSM / IPMSM)

Permanent magnet synchronous motors are classified into two main types based on the position of the permanent magnets within the rotor: surface magnet motors and internal magnet motors. This difference directly affects the electromagnetic characteristics and motor performance.

In the first type, the surface magnet motor (SPMSM), the permanent magnets are mounted on the outer surface of the rotor, resulting in a uniform air gap and a symmetrical magnetic field distribution. Consequently, the reactance of the direct and perpendicular axes are close ($L_d = L_q$), and therefore the impedance torque is weak or negligible. Torque generation relies primarily on the interaction of the rotor's magnetic field with the stator's rotating field. This type is characterized by its simple design and ease of control, making it suitable for applications requiring good dynamic response with low control system complexity.

The second type, the internal magnet motor (IPMSM), has permanent magnets embedded within the rotor structure, resulting in an asymmetry of magnetic properties between the two axes ($L_d \neq L_q$). This asymmetry allows for the generation of an additional torque, known as the reluctance torque, alongside the magnet torque, thus improving torque density and increasing operating efficiency,

especially at high speeds. This design also enables operation in a field-weakening environment, making it suitable for applications requiring a wide speed range, such as electric drive systems.[3].

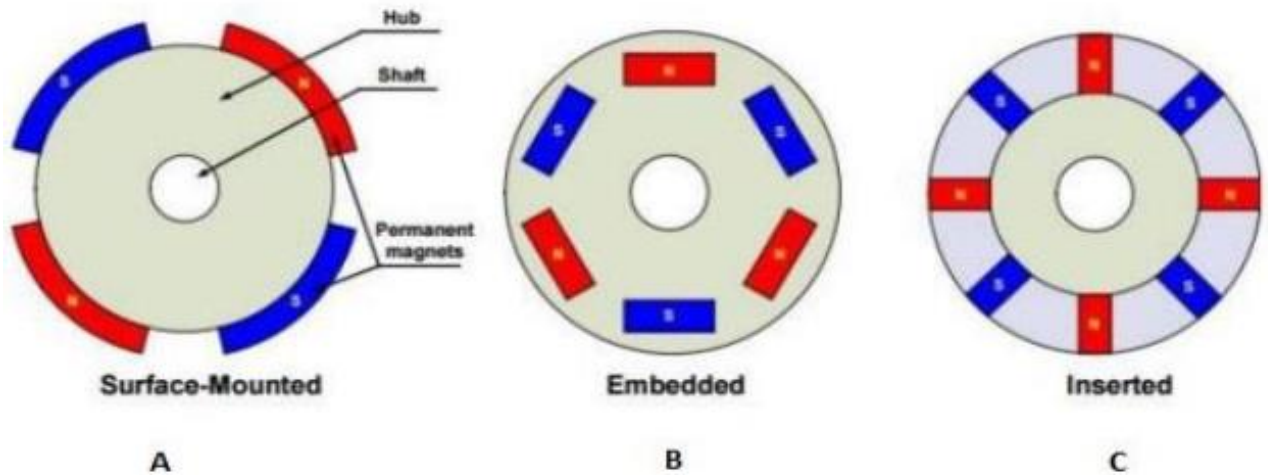


Figure (I. 1): Different structure of PMSM rotor

I.3 Multiphase Machines

I.3.1. Definition of multiphase machines

Multiphase electrical machines are defined as a class of electrical machines that operate on a power supply system consisting of more than three phases, with the stator windings distributed across a greater number of phases than in a conventional three-phase system. This arrangement aims to improve electromagnetic performance characteristics by generating a more uniform rotating magnetic field and reducing torque fluctuations.

These machines offer higher reliability, as they can continue operating even in the event of a phase loss. They also improve current distribution and reduce thermal stress on motor components. This architecture enables higher power density and better dynamic performance compared to conventional three-phase machines, making them suitable for advanced applications such as electric drive systems and demanding industrial systems.[4]. Among these machines is a five-phase machine, which will be studied in the next chapter and is shown in Figure (1.1).

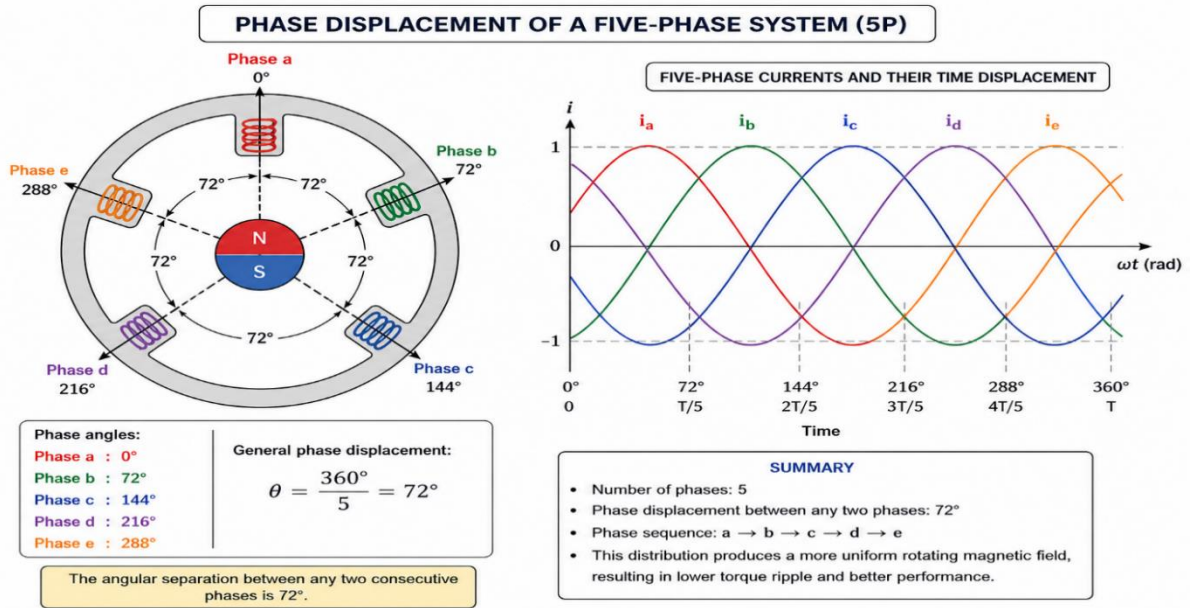


Figure (I.2): Multiphase Machines

I.3.2. Difference between three-phase and five-phase systems

Three-phase systems are the most commonly used in conventional electrical machinery. They consist of three windings spaced at an electrical angle of 120°, used to generate a uniform rotating magnetic field. This system is characterized by its simple structure, ease of control, and widespread use in industrial applications. However, it is more susceptible to torque fluctuations in some cases, and its ability to continue operating in the event of a failure is relatively limited.

In contrast, five-phase systems rely on the distribution of five windings in the stator with an electrical angle of 72° between each phase, allowing for the generation of a smoother and more stable magnetic field. This distribution contributes to reducing torque ripple and improving the dynamic performance of the motor, as well as increasing reliability, since the system can continue operating even if one or more phases are lost without a complete shutdown.

In general, five-phase systems offer improved performance in terms of efficiency, reduced vibrations, and increased failure tolerance compared to three-phase systems, but they are more complex in terms of control and electronic architecture [4].

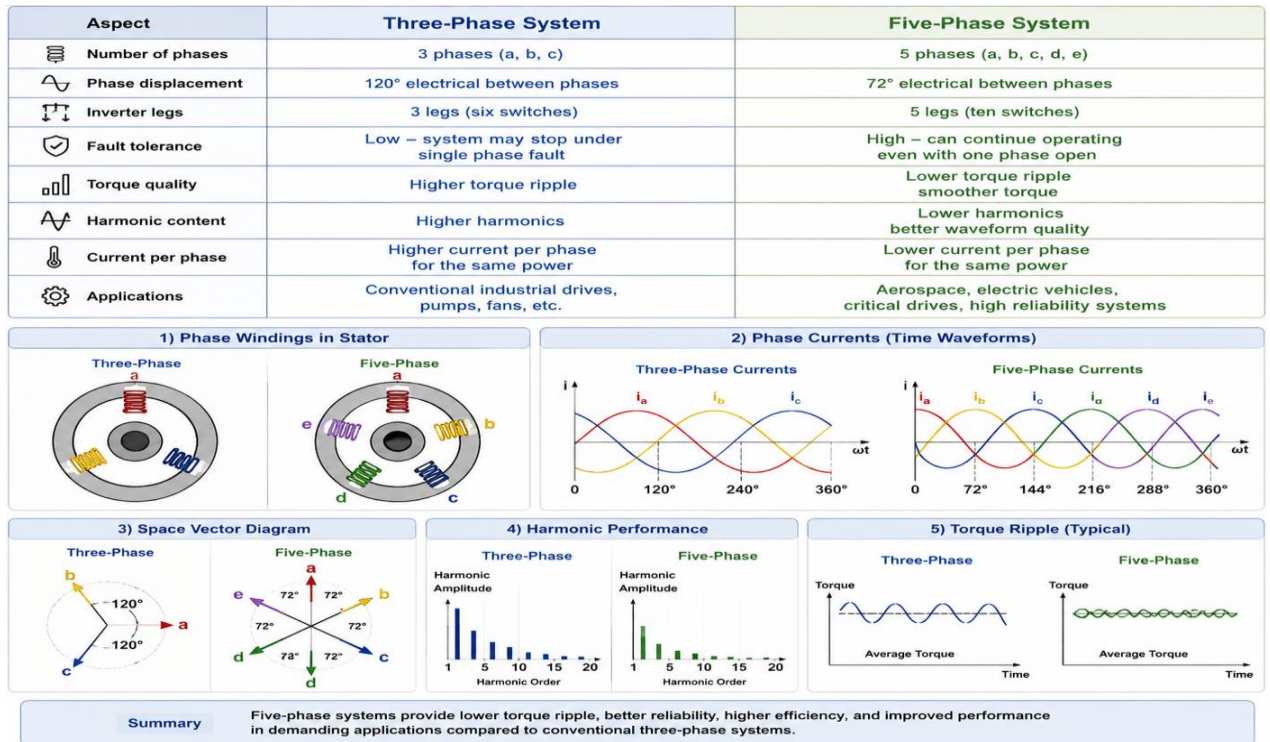


Figure (I.3): Overall Difference between three and five-phase systems

I.3.3. Advantages of five-phase PMSM

Five-phase synchronous permanent magnet motors offer a range of advantages over conventional three-phase motors, primarily due to the increased number of phases and improved current distribution within the machine.

Among the most significant advantages is reduced torque ripple. The uniform phase angular distribution generates a smoother and more stable magnetic field, resulting in improved rotational performance and reduced vibration and mechanical noise. These motors also boast high reliability in the event of failures, as they can continue operating even when one or more phases are lost, thus enhancing system reliability.

Furthermore, the five-phase architecture provides improved power density and increased operating efficiency, especially under dynamic and variable operating conditions. It also allows for greater flexibility in designing control strategies, enabling faster response times and better performance compared to three-phase systems [5].

I.4. Applications of Multiphase Machines

Five-phase machines (5-Phase PMSMs) are widely used in applications that require high efficiency, superior reliability, and continuous operation even under fault conditions. Their main applications include the following:

1. Electric Vehicles:

Five-phase motors provide smoother torque and lower torque ripple compared to conventional three-phase machines, making them highly suitable for modern electric vehicles.

2. Hybrid Electric Vehicles:

In hybrid electric vehicles, multiphase machines help improve fuel economy and reduce harmful emissions while maintaining high performance and operational reliability.

3. Aerospace Propulsion Systems:

Multiphase machines are employed in aerospace propulsion applications due to their high reliability, efficiency, and fault-tolerant capability. They contribute to lower fuel consumption and improved aircraft performance.

4. Ship Propulsion Systems:

In marine applications, multiphase machines are used in electric ship propulsion systems to provide better maneuverability, higher efficiency, and reduced fuel consumption.

5. High-Power Industrial Drives:

Multiphase machines are widely utilized in industrial drive systems because they ensure high efficiency, excellent torque performance, and reduced operating costs in heavy-duty industrial processes.

6. Wind Turbine Systems:

Multiphase machines are also applied in wind energy conversion systems, where they enable efficient and reliable electrical power generation under variable wind conditions [22].

The following Figure illustrates the areas of use the 5P-PMSM:

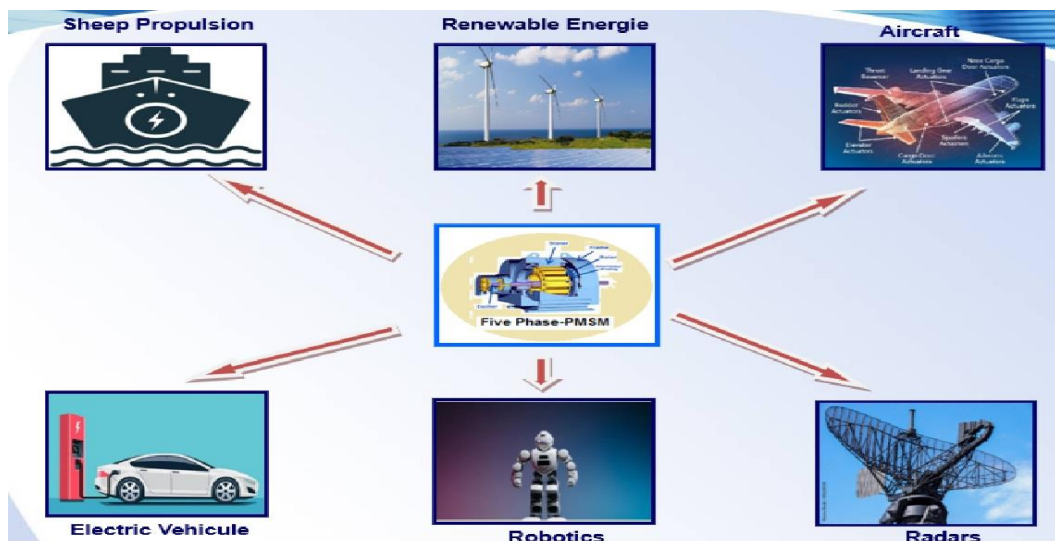


Figure (I. 4): Applications of Multiphase Machines

I.5. Sensorless Control strategy

I.5.1. Concept of sensorless control

Sensorless control in electrical machinery refers to control strategies that rely on estimating the fundamental mechanical variables of a motor, such as speed and position, without using direct mechanical sensors like position or speed sensors. Instead, it relies on available electrical measurements, such as currents and voltages at the motor terminals, and then processes these signals using mathematical models or estimation algorithms to extract the unmeasured mechanical information [6].

I.5.2. Importance of eliminating sensors

Eliminating mechanical sensors in electric motor control systems, particularly in AC and PMSM motors, is a significant step toward improving system performance and reducing complexity. This approach contributes to lowering the overall system cost by removing mechanical sensors such as position and speed sensors, as well as reducing the maintenance requirements associated with these components.

Furthermore, eliminating sensors enhances system reliability, as sensors are among the most vulnerable components to failure in harsh industrial environments such as high temperatures or severe vibrations. Additionally, this approach allows for the design of smaller and more flexible motors, which is crucial in applications requiring high power density and limited installation space.

In practice, this method relies on mathematical models and estimation algorithms to extract operating variables such as speed and position based on electrical measurements, providing an efficient solution that combines good performance with low cost [6].

I.6. Speed and Position Estimation Methods

Speed and position estimation methods in sensorless control systems rely on inferring the motor's mechanical variables from electrical measurements such as current and voltage. This is achieved using mathematical models or advanced estimation algorithms, eliminating the need for traditional mechanical sensors. These techniques are fundamental in modern synchronous motor control systems, such as PMSMs.

Among the common speed and position estimation methods are model-based approaches, such as the State Observer and Sliding Mode Observer. These methods construct a mathematical model of the motor and compare its response with real-world measurements to gradually identify and correct errors. The Kalman filter or extended Kalman filter is also used to improve estimation accuracy in areas with noise and measurement uncertainty.

Additionally, back-EMF methods utilize induced signals in the motor windings to estimate the rotor speed and position. These methods are particularly effective at medium and high speeds. Modern techniques based on high-frequency signal injection have also emerged, particularly useful at low speeds or in idle conditions where back-EMF signals are weak. Generally, the appropriate estimation method is selected based on the speed range, operating conditions, and accuracy requirements of the system [8].

I.6.1. Definition of MRAS (Model Reference Adaptive System)

MRAS (Model Reference Adaptive System) is an adaptive estimation technique used in sensorless motor control. It compares the outputs of a reference model and an adaptive model to generate an error signal used for estimating unknown parameters such as rotor speed and position. MRAS is widely applied in PMSM drives because it provides accurate estimation without requiring mechanical sensors. [7].

I.6.2. Justifications for choosing the MRAS (Model Reference Adaptive System) method

The Reference Model Adaptive System (MRAS) method is a popular choice in industrial applications for estimating speed and position in sensorless control systems, as it offers a balance between accuracy and simplicity. This approach compares the output of a reference model with a tunable model and then uses an adaptive algorithm to minimize error, allowing for efficient estimation of mechanical variables without the need for additional sensors.

A key rationale for choosing MRAS is its simpler mathematical structure compared to other methods, such as the Extended Kalman Filter (EKF). This reduces computational complexity and makes it suitable for real-time implementation. Furthermore, MRAS offers good adaptability to changes in motor parameters, such as resistance and inductance, which enhances performance stability under varying operating conditions.

In addition, MRAS provides acceptable accuracy across a wide speed range, particularly in medium- and high-speed applications, with relatively low computational costs, making it well-suited for embedded systems and industrial control units. [7].

I.7. Advanced technologies in electric motor control

Advanced control technologies for electric motors refer to a range of modern strategies aimed at improving the performance of electric drive systems in terms of accuracy, dynamic response, efficiency, and robustness under various operating conditions. These technologies were developed to overcome the limitations of traditional control methods such as PI and PID, particularly in nonlinear and complex systems like PMSM and 5P-PMSM motors.

Among the most prominent of these technologies is Field-Oriented Control (FOC), which relies on transforming electrical variables into a rotating reference frame. This allows for the independent decoupling of torque and magnetic field control, significantly improving system performance. Model-based control techniques, such as Model Predictive Control (MPC), are also employed. MPC predicts the system's future behavior to select the optimal control signal for optimal performance.

Furthermore, sensorless control technologies are a significant recent development. These technologies rely on estimating speed and position using mathematical models and estimation algorithms such as MRAS, SMO, and EKF, thereby reducing system costs and increasing reliability. Intelligent control techniques such as neural networks and fuzzy logic are also used to improve performance in nonlinear and complex systems.

Overall, these techniques aim to achieve more precise, flexible, and responsive control in modern propulsion systems, while improving efficiency and reducing reliance on sensors and mechanical components [9].

I.7.1. Backstepping Control

Backstepping control is an advanced nonlinear control technique widely used in complex dynamic systems such as AC motors and PMSM permanent magnet motors. This method relies on stepwise control design, where the system is decomposed into several interconnected subsystems, and a controller is designed sequentially for each stage until the final control signal is reached.

The concept of backstepping is based on the Lyapunov function to ensure system stability at each design step. Virtual controls are defined and used to gradually reduce error until overall system stability is achieved.

This technique is highly effective at handling nonlinearity and uncertainty in system models, making it suitable for applications requiring high accuracy and robust dynamic response. However, it requires precise system modeling and can be more complex than traditional control methods such as PI and PID [10].

I.7.2. Justification for Choosing Backstepping Control

The selection of the Backstepping control strategy for the five-phase Permanent Magnet Synchronous Motor (5P-PMSM) is mainly motivated by the nonlinear nature of the motor model and the high-performance requirements of modern electrical drive systems. Unlike conventional linear controllers, Backstepping is specifically designed to deal with nonlinear

and strongly coupled dynamic systems, making it particularly suitable for PMSM drives operating under varying load conditions and parameter uncertainties.

One of the main advantages of Backstepping control is its ability to guarantee system stability through Lyapunov theory. By constructing Lyapunov functions at each design step, the controller ensures the convergence of tracking errors toward zero while maintaining the overall stability of the system. This property is especially important in high-performance applications where precise speed and torque tracking are required.

In addition, Backstepping control provides better dynamic response and robustness compared to classical PI controllers. It effectively compensates for nonlinearities and coupling effects between the d- and q-axis variables, which improves transient performance and reduces sensitivity to disturbances and load variations. This makes the strategy highly appropriate for multiphase machines such as the 5P-PMSM, where the system dynamics are more complex than those of conventional three-phase motors.

Another important reason for choosing Backstepping is its compatibility with advanced control techniques such as Sensorless Control and Field-Oriented Control (FOC). The method can be easily integrated with observers and estimation algorithms to achieve accurate control without requiring mechanical sensors, thereby improving system reliability and reducing hardware cost.

Furthermore, the five-phase PMSM is often used in applications demanding high reliability and fault tolerance, such as electric vehicles, aerospace systems, and industrial drives. In these applications, Backstepping control offers excellent tracking accuracy, smooth torque response, and enhanced operational stability even under adverse operating conditions.

For these reasons, Backstepping control represents an effective and reliable solution for achieving high-performance control of five-phase permanent magnet synchronous motors.[19]

I.8. Conclusion:

In summary, Permanent Magnet Synchronous Motors, especially five-phase PMSMs, have emerged as highly effective solutions for advanced electric drive systems because of their high efficiency, excellent reliability, and strong fault-tolerant characteristics. The application of sensorless estimation techniques such as MRAS, combined with advanced nonlinear control approaches like Backstepping Control, significantly improves system stability, estimation accuracy, and dynamic response. Therefore, integrating multiphase PMSM technology with intelligent control strategies offers a powerful and reliable framework for developing high-performance, energy-efficient, and robust industrial drive applications.

Chapter II: Modeling of the Five-Phase PMSM

II.1. Introduction:

The modeling of the Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) is essential for understanding its electrical and mechanical behavior and for designing advanced control strategies for high-performance applications. The machine consists of five stator windings and a rotor equipped with permanent magnets that produce the excitation flux. Compared with conventional three-phase machines, the 5P-PMSM offers important advantages such as higher reliability, reduced torque ripple, improved fault tolerance, and better power distribution.

To establish the mathematical model, several simplifying assumptions are usually considered, such as neglecting magnetic saturation and iron losses while assuming a sinusoidal distribution of the stator magnetic field. The modeling process begins in the natural reference frame by deriving the voltage, flux linkage, and mechanical equations that describe the electromagnetic torque and rotor dynamics. In order to simplify analysis and control design, Clarke and Park transformations are applied to convert the five-phase variables into stationary and rotating reference frames. These transformations lead to simplified dynamic models in the (α, β) and $(d-q)$ frames, making the system easier to analyze, simulate, and control.

II.2. Structure and configuration of the Motor

The five-phase synchronous motor (5P-PMSM) consists of two main components: the stator and the rotor, along with auxiliary elements such as a dynamic frame, a mechanical shaft, and an external housing. This integrated system converts electrical energy into magnetic energy through the interaction between the magnetic field generated in the stator and the magnetic field produced by the magnets operating in the micro-rotor.

II.2.1 The Stator:

The stator is the stationary part of the motor. It consists of a laminated iron core containing slots in which copper windings are mounted. These windings are distributed across five phases (a, b, c, d, e), spaced at an electrical angle of 72° . When these windings are powered by a five-phase commutator, a rotating magnetic field is generated, which drives the rotor. The following figure (II.01) represents the types of 5P-PMSM stator.

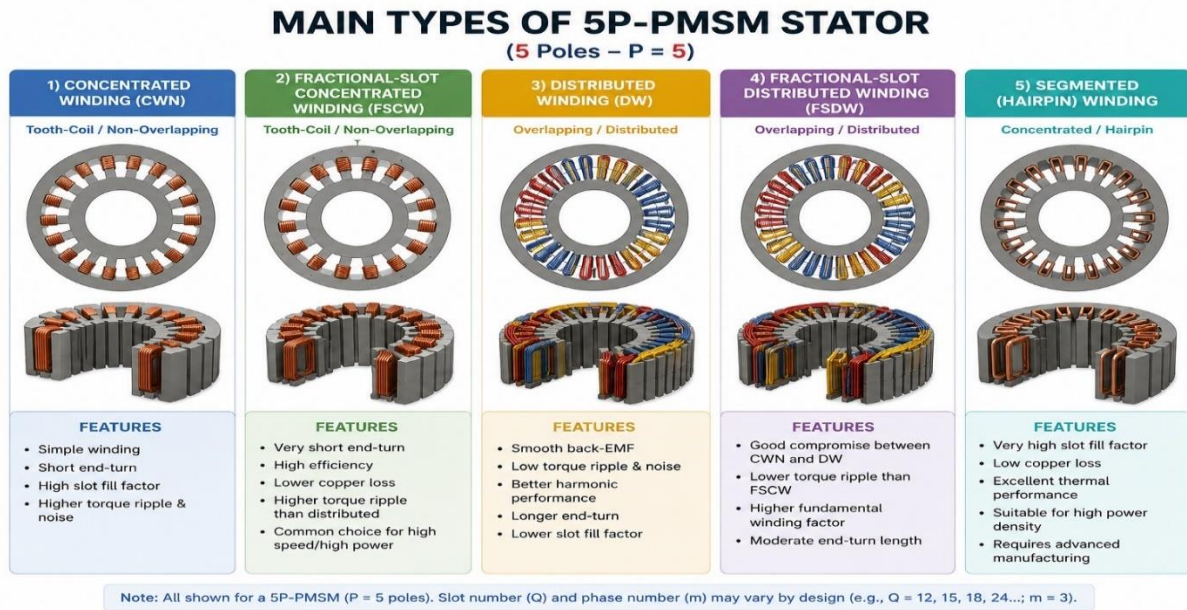


Figure (II.1): the types of 5P-PMSM stator.

II.2. 2. The Rotor:

The rotor is the moving part directly connected to the shaft. It consists of a magnetic core with permanent magnets mounted on or inside it. Two main constructions are commonly used:

SPMSM: Magnets mounted on the rotor surface.

IPMSM: Magnets embedded inside the rotor, providing greater durability and higher torque density [3] [16].

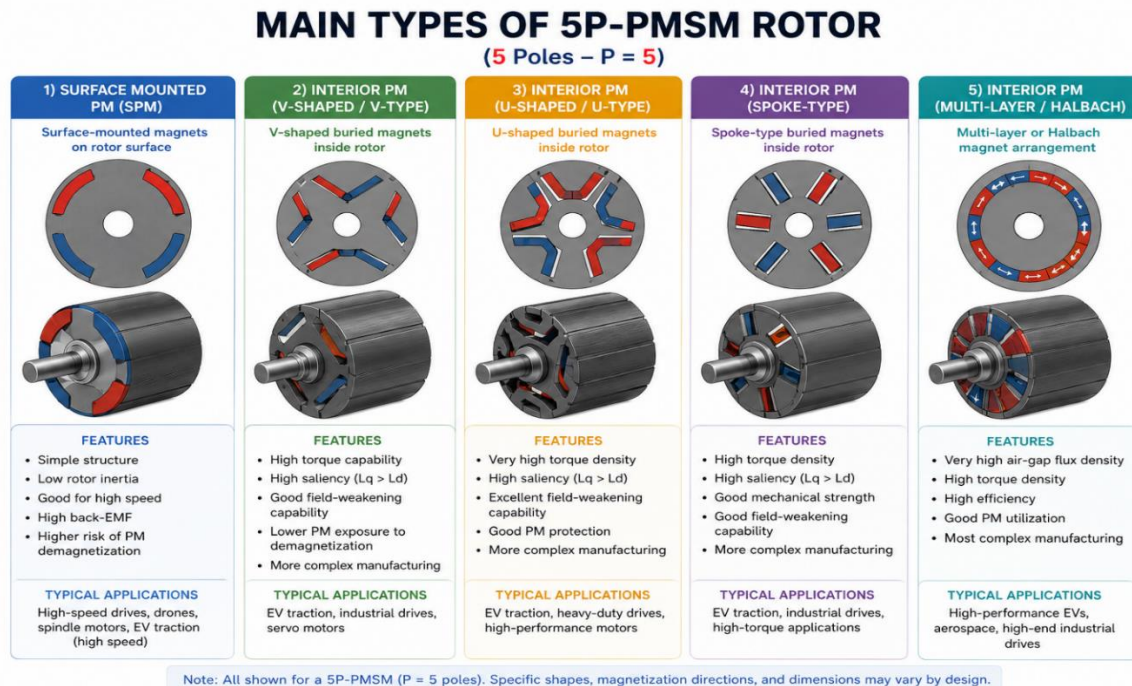


Figure (II. 2): the types of 5P-PMSM rotor.

II.3. Simplification Assumptions

In the mathematical modeling and control of the Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM), a set of simplifying assumptions is employed to reduce system complexity and facilitate the derivation of the dynamic equations. These assumptions allow for a more accurate representation of the motor by considering its fundamental electromagnetic and mechanical characteristics. The most important assumptions are as follows [3] [11]:

- The magnetic circuit is assumed to be linear, i.e., magnetic saturation is neglected.
- The stator terminals are assumed to be well-distributed and perfectly aligned.
- A decimeter is assumed between the stator and rotor.
- Railway losses, hysteresis losses, and cyclic current losses are neglected.
- Therefore, the magnets are assumed to have stable and insignificant effects on temperature variations.
- Inter faith harmonics are neglected.
- The stability of motor parameters such as resistance and inductance is assumed.
- The rotor is assumed to operate without mechanical vibrations or eccentricity.
- The electronic inverter alternative is ideal, neglecting switching losses and time lags.
- Sequential-to-segment communication is neglected due to the use of an isolated neutral point.

Based on these assumptions, the mathematical model becomes increasingly sophisticated, leading to many advanced control technologies such as field-directed control (FOC), backstepping, and sensorless control [12].

II.3. Modeling of 5P-PMSM

The modeling of a Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) represents an essential step for analyzing, understanding, and improving the performance of the motor drive system. It provides an accurate mathematical representation of the motor by describing the electrical, magnetic, and mechanical phenomena occurring within the machine. This modeling process involves the development of dynamic equations that characterize the behavior of the motor, including electromagnetic interactions, torque production, flux distribution, and control system response.

However, due to the inherent complexity of multiphase machines such as the 5P-PMSM, the resulting mathematical model may become highly complicated and computationally demanding. Therefore, several simplification approaches are commonly introduced to reduce the complexity of the system while preserving the essential dynamic characteristics of the motor. These simplifications facilitate analysis, controller design, and simulation implementation in a more efficient and practical manner [21].

II.3.1. Mathematical model of the 5P-PMSM in natural frame

Modeling a Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) in its natural reference frame consists of establishing the mathematical equations that describe the dynamic behavior of the machine based on the intrinsic characteristics of its five stator phases and internal components. In this framework, the electrical and mechanical dynamics of the motor are represented using space-vector formulations, which provide an effective description of the interactions between voltages, currents, flux linkages, and electromagnetic torque in the natural reference frame [28].

II.3.1.1. Voltages equations

As is well known, the rotor of a Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) contains permanent magnets that generate a constant magnetic flux. On the other hand, the stator is composed of five-phase winding coils responsible for producing the electromagnetic interaction within the machine. According to Faraday's law, the stator voltage equations describing the electrical behavior of the motor can be expressed as follows [27]:

$$[V_s] = [R_s][I_s] + \frac{d[\lambda_s]}{dt} \quad (\text{II.01})$$

The following definitions represent the voltages, the currents, and the flux linkages used in the previous equation:

$$[V_s] = \begin{pmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{pmatrix}; [I_s] = \begin{pmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \end{pmatrix} \text{ and } [\lambda_s] = \begin{pmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \\ \lambda_e \end{pmatrix} \quad (\text{II.02})$$

The notations (a,b,c,d,e) represent the indices of the five stator phases. Moreover, $[R_s]$ denotes the stator resistance matrix. Considering that the 5P-PMSM is assumed to be a perfectly symmetrical machine, the stator phase resistances are considered identical, and the stator resistance matrix is written by:

$$[R_s] = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 \\ 0 & 0 & 0 & R_s & 0 \\ 0 & 0 & 0 & 0 & R_s \end{bmatrix} \quad (\text{II.03})$$

II.3.1.2. Flux linkage equations

The total flux linkage in the air gap is produced by both the stator inductances and the magnetic flux generated by the permanent magnets mounted on the rotor, which passes through the stator windings [29]. Accordingly, the overall flux linkage can be expressed by the following relation:

$$[\lambda_s] = [\lambda_{ss}] + [\lambda_m] = [L_s][I_s] + [\lambda_m] \quad (\text{II.04})$$

Where:

$[\lambda_m]$: Matrix Vector of the flux created by the permanent magnets through the stator windings can be expressed it as:

$$[\lambda_m] = |\lambda_m| \begin{bmatrix} \cos(\theta) \\ \cos(\theta - 2\pi/5) \\ \cos(\theta - 4\pi/5) \\ \cos(\theta - 6\pi/5) \\ \cos(\theta - 8\pi/5) \end{bmatrix} \quad (\text{II.05})$$

And θ is the electrical angle of the rotor, $|\lambda_m|$ is the flux amplitude of permanent magnet [21].

$[L_s]$: is the total stator inductance matrix involving the proper and mutual matrix of its windings, then can be written it as:

$$[L_s] = [L_{s0}] + [L_{s1}] \quad (\text{II.06})$$

With:

$[L_{s0}]$: is the proper inductances matrix of stator windings, and:

$[L_{s1}]$: The mutual inductances matrix of stator windings.

II.3.1.3. Mechanical equations of 5P-PMSM

The mechanical model of a Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) represents the rotational dynamics of the machine and describes the relationship between the developed electromagnetic torque and the applied mechanical load. This model explains how the electromagnetic phenomena inside the motor are converted into mechanical motion. The dynamic behavior of the mechanical system is generally derived from Newton's second law of motion and can be expressed as follows:[21]

$$J \frac{d\Omega}{dt} = T_{em} - T_L - F\Omega \quad (\text{II.07})$$

Where:

J : is inertia moment of the motor and load combined.

F : is the friction coefficient

T_{em} : is the net torque generated by the electromagnetic interaction of the motor's phases.

T_L : is the external load torque acting on the motor.

Ω : is the rotor speed, as well can be write $\Omega P = w$, with w is the electrical speed, and P is pair of pol number.

II.3.2. Park and Clark transformation

Park's transformation, also referred to as the d-q transformation, is a mathematical method widely used to simplify the analysis and control of electrical machines such as the Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM). This transformation consists of converting the original five-phase reference frame into an equivalent fictitious two-axis orthogonal reference frame. As a result, the stator and rotor variables, including voltages, currents, and flux linkages, are expressed in a rotating reference frame independent of the electrical angle θ . This transformation eliminates the time-varying nature of the machine variables and considerably simplifies the differential equations governing the dynamic behavior of the 5P-PMSM [26]. The Park transformation matrix proposed for the 5P-PMSM can therefore be expressed as follows:

$$[G(\theta_{co})] = K \begin{bmatrix} \cos \theta_{co} & \cos(\theta_{co} - \frac{2\pi}{5}) & \cos(\theta_{co} - \frac{4\pi}{5}) & \cos(\theta_{co} + \frac{4\pi}{5}) & \cos(\theta_{co} + \frac{2\pi}{5}) \\ \sin \theta_{co} & \sin(\theta_{co} - \frac{2\pi}{5}) & \sin(\theta_{co} - \frac{4\pi}{5}) & \sin(\theta_{co} + \frac{4\pi}{5}) & \sin(\theta_{co} + \frac{2\pi}{5}) \\ \cos \theta_{co} & \cos(\theta_{co} + \frac{4\pi}{5}) & \cos(\theta_{co} - \frac{2\pi}{5}) & \cos(\theta_{co} + \frac{2\pi}{5}) & \cos(\theta_{co} - \frac{4\pi}{5}) \\ \sin \theta_{co} & \sin(\theta_{co} + \frac{4\pi}{5}) & \sin(\theta_{co} - \frac{2\pi}{5}) & \sin(\theta_{co} + \frac{2\pi}{5}) & \sin(\theta_{co} - \frac{4\pi}{5}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (\text{II.8})$$

Where θ_{co} represents the angle between the a-axis of the original reference frame and the d1-axis of the arbitrary reference frame. Moreover, K denotes a constant coefficient related to the power transformation from the natural reference frame to the transformed reference frame. Depending on the selected transformation approach, the coefficient K can take two different values as follows:

$$K = \frac{2}{5} : \text{For the transformation preserving of amplitude.}$$

$$K = \sqrt{\frac{2}{5}} : \text{For the transformation preserving the power}$$

Through Park's transformation, which depends on the angular position coordinate, the new reference frame can be defined in three main ways according to the chosen position and angular speed orientation, as follows:

II.3.3. Stator reference frame

It is also known as the stationary reference frame, and its axes are fixed with respect to the stator; in this system, the reference axes remain immobile relative to the stator, where:

$$\theta_{co} = 0 \Rightarrow w_{co} = 0$$

The voltage in this system varying as a function of time as sinusoidal magnitudes, which can be represented by the following method:

$$[X_{\alpha 1 \beta 1 \alpha 2 \beta 2}] = [T(\theta_{co} = 0)][X_{abcde}] \quad (\text{II.9})$$

The inverse Park transform is compatible to return to five-phase variables, it is defined by:

$$[X_{abcde}] = [T(\theta_{co} = 0)]^{-1} [X_{\alpha 1 \beta 1 \alpha 2 \beta 2}] \quad (\text{II.10})$$

II.3.4. Rotor reference frame

This reference frame is fixed to the rotor and rotates at the electrical angular speed, and its corresponding indices are given as $w_{co} = w, \theta_{co} = \theta$. It is particularly useful for analyzing the transient behavior of the machine and is therefore well suited for control purposes. The machine variables in this frame are obtained by:

$$\begin{cases} [X_{d1q1d2q2}] = [T(\theta_{co} = \theta)][X_{abcde}] \\ [X_{abcde}] = [T(\theta_{co} = \theta)]^{-1}[X_{d1q1d2q2}] \end{cases} \quad (\text{II.11})$$

II.3.7. Synchronous reference frame

This system is rotated with the speed of the electromagnetic field, created by the stator windings, where the coordinate speed of this reference is the synchronous speed

$$w_{co} = w_s \Rightarrow \theta_{co} = \theta_s$$

Figure (II.4) presents a schematic illustration of the different reference frames of the 5P-PMSM. These three reference frames are commonly used in the literature of machine control. Most researchers adopt either the stationary stator reference frame ($\alpha 1, \beta 1, \alpha 2, \beta 2$) or the rotating rotor reference frame ($d1, q1, d2, q2$). In the following, these frames are described under the assumption of a smooth-pole machine.

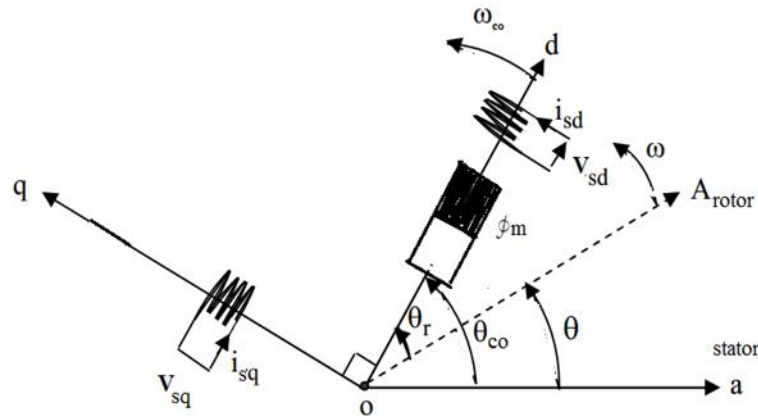


Figure (II.3): Schematic representation of Different reference frames of 5P-PMSM

II.4. Dynamic model of 5P-PMSM (in a stationary frame (α, β))

The selection of this reference frame is suitable for implementing observation techniques. In this case, the 5P-PMSM model expressed in the stationary frame ($\alpha 1, \beta 1, \alpha 2, \beta 2$) can be derived from the natural-frame mathematical model using Park's transformation matrix with ($\theta_{co} = 0$), or alternatively by transforming the dynamic variables of the model—such as

voltage, current, and flux—defined in the (d1, q1, d2, q2) frame through an appropriate transformation matrix, as follows:

$$\begin{bmatrix} X_{d1q1d2q2} \end{bmatrix} = \begin{bmatrix} T(\theta) \end{bmatrix}^{-1} \begin{bmatrix} X_{\alpha1\beta1\alpha2\beta2} \end{bmatrix} \quad (\text{II.12})$$

Where the matrix [T] expressed in following form:

$$T(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{II.13})$$

The final dynamic model of 5P-PMSM in ($\alpha1, \beta1, \alpha2, \beta2$) frame written as follow:

$$\left\{ \begin{array}{l} V_{\alpha1} = R_s I_{\alpha1} + L_{\alpha1} \frac{dI_{\alpha1}}{dt} - w\lambda_m \sin \theta \\ V_{\beta1} = R_s I_{\beta1} + L_{\beta1} \frac{dI_{\beta1}}{dt} + w\lambda_m \cos \theta \\ V_{\alpha2} = R_s I_{\alpha2} + L_{\alpha2} \frac{dI_{\alpha2}}{dt} \\ V_{\beta2} = R_s I_{\beta2} + L_{\beta2} \frac{dI_{\beta2}}{dt} \\ J \frac{d\Omega}{dt} = T_{em} - T_L - F\Omega \\ T_{em} = \frac{5}{2} P \lambda_m (I_{\beta1} \cos \theta - I_{\alpha1} \sin \theta) \end{array} \right. \quad (\text{II.14})$$

II.5. Dynamic model of 5P-PMSM in rotating frame (d-q)

The selection of this reference frame is more suitable for the application of machine control techniques. The representation of the 5P-PMSM in the (d1, q1, d2, q2) frame is obtained by multiplying the stator voltage equations (II.01) by Park's transformation matrix (II.8), rotating at the electrical speed $\omega_{co} = \omega$. This operation yields the model in the following form:

II.5.1. Electrical equation

The electrical equation presented in the following equation Where its fifth row is zero-sequence component can be removed.

$$\left\{ \begin{array}{l} V_{d1} = R_s I_{d1} + \frac{d\lambda_{d1}}{dt} - \omega \lambda_{q1} \\ V_{q1} = R_s I_{q1} + \frac{d\lambda_{q1}}{dt} + \omega \lambda_{d1} \\ V_{d2} = R_s I_{d2} + \frac{d\lambda_{d2}}{dt} \\ V_{q2} = R_s I_{q2} + \frac{d\lambda_{q2}}{dt} \\ V_0 = R_s I_0 + \frac{d\lambda_0}{dt} \end{array} \right. \quad (\text{II.15})$$

II.5.2. Flux linkage equation

For the representation of the stator flux linkage in the (d1, q1, d2, q2) reference frame, it should be noted that only the direct-axis component exists. Based on this consideration, the flux linkage equations of the 5P-PMSM can be written as follows:

$$\left\{ \begin{array}{l} \lambda_{d1} = L_d I_{d1} + \lambda_m \\ \lambda_{q1} = L_q I_{q1} \\ \lambda_{d2} = L_l I_{d2} \\ \lambda_{q2} = L_l I_{q2} \end{array} \right. \quad (\text{II.16})$$

With:

L_l : is the inductance of secondary frame (d2, q2).

II.5.3. Mechanical Equation

The mechanical equation of the 5P-PMSM, given in (II.7) and derived from Newton's law, remains unchanged in form. The only modification concerns the expression of the electromagnetic torque when expressed in the (d1, q1, d2, q2) reference frame. This torque is given by the following equation:

$$T_{em} = \frac{5}{2} P (\lambda_{d1} I_{q1} - \lambda_{q1} I_{d1}) \quad (\text{II.17})$$

By setting equation (II.16) in (II.17) can be write the mechanical equation as follow:

$$\left\{ \begin{array}{l} J \frac{d\Omega}{dt} = T_{em} - T_L - F\Omega \\ T_{em} = \frac{5}{2} P (\lambda_m I_{q1} + (L_d - L_q) I_{q1} I_{d1}) \end{array} \right. \quad (\text{II.18})$$

The final dynamic model of 5P-PMSM in (d1, q1, d2, q2) frame defined by the combination of the stator voltage and current equations, flux and mechanical equation, as follow:

$$\left\{ \begin{array}{l} V_{d1} = R_s I_{d1} + L_{d1} \frac{dI_{d1}}{dt} + \omega L_{q1} I_{q1} \\ V_{q1} = R_s I_{q1} + L_{q1} \frac{dI_{q1}}{dt} + \omega L_{d1} I_{d1} + \omega \lambda_m \\ V_{d2} = R_s I_{d2} + L_{d2} \frac{dI_{d2}}{dt} \\ V_{q2} = R_s I_{q2} + L_{q2} \frac{dI_{q2}}{dt} \\ J \frac{d\Omega}{dt} = T_{em} - T_L - F\Omega \\ T_{em} = \frac{5}{2} P (\lambda_m I_{q1} + (L_{d1} - L_{q1}) I_{q1} I_{d1}) \end{array} \right. \quad (\text{II.19})$$

II.6. The state space model of a (5P-PMSM)

Expressing the machine model in state-space form helps simplify the system by highlighting its controllable variables, particularly torque, position, and machine speed. Assuming a smooth-pole machine condition, where $L_{d1} = L_{q1}$, the model can be further simplified and written in the following form [13]:

$$\left\{ \begin{array}{l} \frac{dX}{dt} = [A]X + [B]U + [E]D \\ Y = [C]X \end{array} \right. \quad (\text{II.20})$$

Where: X represent the state vector of the system, Y is the output vector, U indicate the control vector, and D is the disturbance input of system, [A] is the system (or state) matrix, [B] is the input matrix, [C] represent the output matrix, and [E] represent the disturbance input matrix.

From the equation (II.19) can be rewritten it as follow:

$$\left\{ \begin{array}{l}
\frac{dI_{d1}}{dt} = \frac{1}{L_{d1}}V_{d1} - \frac{R_s}{L_{d1}}I_{d1} + P\Omega \frac{L_{q1}}{L_{d1}}I_{q1} \\
\frac{dI_{q1}}{dt} = \frac{1}{L_{q1}}V_{q1} - \frac{R_s}{L_{q1}}I_{q1} - P\Omega \frac{L_{d1}}{L_{q1}}I_{d1} - P\Omega \frac{\lambda_m}{L_{q1}} \\
\frac{dI_{d2}}{dt} = \frac{1}{L_{d2}}V_{d2} - \frac{R_s}{L_{d2}}I_{d2} \\
\frac{dI_{q2}}{dt} = \frac{1}{L_{q2}}V_{q2} - \frac{R_s}{L_{q2}}I_{q2} \\
\frac{d\Omega}{dt} = \frac{T_{em}}{J} - \frac{T_l}{J} - \frac{F}{J}\Omega \\
T_{em} = \frac{5}{2}P\lambda_m I_{q1}
\end{array} \right. \quad (II.21)$$

The general state-space representation is formulated by defining an input–output model that depends on the choice of the state vector. For the 5P-PMSM model in the rotor reference frame, the input vector consists of the machine voltages $V_{d1q1d2q2}$, while the output vector corresponds to the stator currents $I_{d1q1d2q2}$. Consequently, the state-space model of the 5P-PMSM can be expressed as in (II.20), as detailed below:

$$X = [I_{d1} \quad I_{q1} \quad I_{d2} \quad I_{q2} \quad \Omega]^T, \quad U = [V_{d1} \quad V_{q1} \quad V_{d2} \quad V_{q2}], \quad \text{and} \quad D = [0 \quad 0 \quad 0 \quad 0 \quad T_l].$$

$$[A] = \begin{bmatrix}
-\frac{R_s}{L_1} & \frac{P\Omega L_{q1}}{L_{di}} & 0 & 0 & 0 \\
\frac{-P\Omega L_{d1}}{L_1} & -\frac{R_s}{L_1} & 0 & 0 & \frac{P\lambda_m}{L_1} \\
0 & 0 & -\frac{R_s}{L_2} & 0 & 0 \\
0 & 0 & 0 & -\frac{R_s}{L_2} & 0 \\
0 & \frac{5P\lambda_m}{2} & 0 & 0 & -\frac{F}{J}
\end{bmatrix}, \quad [B] = \begin{bmatrix}
\frac{1}{L_p} & 0 & 0 & 0 \\
0 & \frac{1}{L_p} & 0 & 0 \\
0 & 0 & \frac{1}{L_s} & 0 \\
0 & 0 & 0 & \frac{1}{L_s} \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad [E] = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-\frac{F}{J}
\end{bmatrix}$$

The output Y is selected component to control it as a torque, the output matrix in this case:

$$[C] = \begin{bmatrix} 0 & \frac{5P\lambda_m}{2} & 0 & 0 \end{bmatrix}$$

II.7. Association inverter-5P-PMSM

Generally, an inverter is used to drive the 5P-PMSM by converting DC power into AC power with the appropriate phase shifting. This machine is characterized by fast dynamic response, low inertia, a high torque-to-inertia ratio, good low-speed performance, and precise phase control; therefore, it relies on the inverter to ensure efficient operation. Compared with conventional three-phase machines, the five-phase motor requires more advanced control strategies and more complex power electronic structures, which are typically implemented through the inverter. As a result, controlling a five-phase machine is more challenging [23].

II.7.1. Description of five phase inverter

To control the speed or position of a five-phase PMSM, a voltage inverter is required to supply the machine. An inverter is a static power converter that converts electrical energy from a DC source into AC electrical energy [30]. Its main structure consists of ten power switches, typically controllable semiconductor devices such as transistors or IGBTs, each equipped with antiparallel diodes and arranged in five legs. During operation, only one switch in each leg is allowed to conduct at a time in order to prevent a short circuit of the DC source. The schematic diagram of the VSI inverter feeding the five-phase PMSM is shown in Figure (II.5).

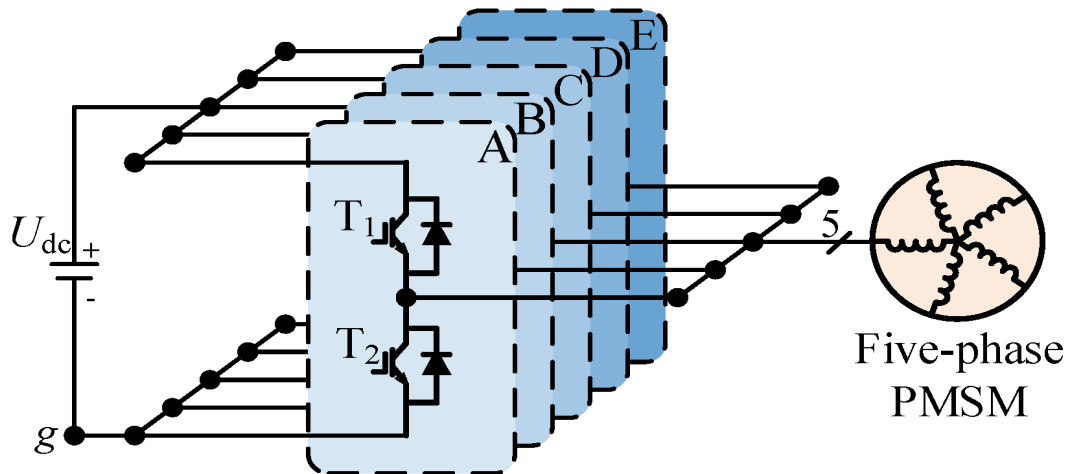


Figure (II.5): Five-phase two-level inverter

The continuity of current flow in the reverse direction is ensured by the antiparallel switching diodes. These switching devices are connected in parallel across a constant DC voltage source. By appropriately controlling the switching sequence (turning the switches on and off),

the inverter modulates the phase voltages and currents, thereby generating a five-phase AC supply with adjustable amplitude and frequency. Where:

V_{dc} : is the DC-link source of constant value, (T_1, T_2, \dots, T_{10}) are ten symmetrical controllable semiconductors. (D_1, D_2, \dots, D_{10}) are a ten symmetrical protective diode.

II.7.2. Five phase voltage source inverters

In an inverter–machine system, determining the output voltage of a five-phase Voltage Source Inverter (VSI) requires consideration of the phase relationships and the modulation techniques used to synthesize the desired voltage waveform. To simplify the inverter modeling process, several assumptions are made [23]:

- The 5P-PMSM is equilibrium coupled in star-coupled with isolated neutral.
- The chute of voltage across the switches is negligible;
- The switching of controllable semiconductors is instantaneous.
- The DC source is a perfect voltage continue and constant does not change value.

Also, to define a relationship between the phase voltage and the closing and opening states of the inverter switches as follow:

$S_i=0$ if when switch number 'i' is open, and $S_i=1$ if it is on close, with $i = (1, 2, 3, 4, 5)$.

The voltages between each leg of inverter and the neutral point (N) are defined by the following relationships:

$$\begin{cases} V_{AO} = V_{AN} + V_{NO} = S_1 V_{dc} \\ V_{BO} = V_{BN} + V_{NO} = S_2 V_{dc} \\ V_{CO} = V_{CN} + V_{NO} = S_3 V_{dc} \\ V_{DO} = V_{DN} + V_{NO} = S_4 V_{dc} \\ V_{EO} = V_{EN} + V_{NO} = S_5 V_{dc} \end{cases} \quad (\text{II.22})$$

Given that V_{dc} is the DC link voltage of the inverter, V_{NO} represents the voltage between the neutral point of the machine and the neutral point of the source, and V_{AN} denotes the phase voltage of the machine, equation (II.24) can be rewritten as follows:

$$\begin{cases} V_{AN} = V_{AO} - V_{NO} \\ V_{BN} = V_{BO} - V_{NO} \\ V_{CN} = V_{CO} - V_{NO} \\ V_{DN} = V_{DO} - V_{NO} \\ V_{EN} = V_{EO} - V_{NO} \end{cases} \quad (\text{II.23})$$

According to previous equation, either the equilibrium of machine, we can write the following:

$$V_{NO} = \frac{V_{AO} + V_{BO} + V_{CO} + V_{DO} + V_{EO}}{5} \quad (\text{II.24})$$

By setting (II.22) and (II.23) in (II.24) can be obtain the phase output voltage of inverter as follow:

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \\ V_{DN} \\ V_{EN} \end{bmatrix} = \frac{V_{dc}}{5} \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} \quad (\text{II.25})$$

II.8. The pulse width Modulation (PWM) technique

The PWM technique is considered an effective and widely adopted solution in many applications. It enables accurate control of the inverter output voltage by adjusting the pulse width while keeping the switching frequency constant. This method is extensively used in motor drives, voltage regulation systems, and uninterruptible power supplies due to its simple implementation and relatively low computational requirements, making it suitable when processing resources are limited.

The principle of PWM is based on generating switching instants through the comparison between a reference modulating signal (produced by the control system) and a high-frequency triangular carrier wave. To ensure that the power delivered to the motor and its load mainly follows the modulating signal, the PWM carrier frequency must be much higher than the fundamental frequency of the reference signal [24].

PWM is a technique used to control the power delivered to a load without changing the amplitude of the supply voltage. In inverter control, both single-pulse PWM and multi-pulse PWM methods can be employed. The required output voltage is achieved by varying both the number of pulses and their corresponding widths.

II.9. Conclusion:

In conclusion, the modeling of the Five-Phase Permanent Magnet Synchronous Motor constitutes a fundamental stage for analyzing the behavior of the machine and designing high-performance control systems. Through the establishment of electrical and mechanical equations, the application of Clarke and Park transformations, and the development of dynamic and state-space models, the 5P-PMSM can be accurately represented under different operating conditions. Moreover, the integration of the five-phase inverter and PWM techniques ensures efficient energy conversion and improved drive performance. Therefore, this theoretical modeling provides the necessary basis for simulation, analysis, and implementation of advanced control and estimation strategies for five-phase PMSM drive systems.

***Chapter III: Backstepping control
of the Five-Phase PMSM***

III.1.Introduction:

Backstepping control is a powerful nonlinear control technique widely used in Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) drives because of its ability to guarantee system stability, robustness, and accurate dynamic performance. This method is based on a recursive design procedure that decomposes the nonlinear system into interconnected subsystems while ensuring stability through Lyapunov theory. In 5P-PMSM applications, Backstepping control mainly involves stabilizing the speed tracking error, generating suitable reference currents, and regulating stator currents through appropriate reference voltages. Owing to its strong robustness against disturbances and parameter uncertainties, this control strategy provides fast dynamic response, reduced overshoot, and high tracking accuracy, making it highly suitable for advanced industrial applications such as electric vehicles, robotics, and aerospace systems.

III.2. Principal of backstepping control

Backstepping control is an effective nonlinear control technique used to ensure the stability of complex dynamic systems. It works by dividing the nonlinear system into interconnected subsystems and recursively designing controllers using virtual control variables until the final control law is obtained. The method relies on Lyapunov stability theory, where suitable Lyapunov functions are constructed to guarantee error convergence and asymptotic stability. Although selecting an appropriate Lyapunov function can be challenging, Backstepping remains highly attractive due to its robustness, flexibility, and ability to handle nonlinearities, parameter uncertainties, and external disturbances. Consequently, it is widely applied in advanced electrical drive systems, especially PMSM and sensorless control applications.[21]

III.3.Stability by Lyapunov theory

Lyapunov stability theory is one of the most important mathematical tools used in the analysis and design of nonlinear control systems. It provides a systematic method for determining whether a dynamic system remains stable without requiring the explicit solution of its differential equations. This theory is widely employed in advanced control strategies such as Backstepping control for Permanent Magnet Synchronous Motors (PMSMs) and other nonlinear electromechanical systems.

The basic principle of Lyapunov theory is based on defining a scalar energy-like function called the **Lyapunov function**, generally denoted by $V(x)$. This function must be positive definite, meaning that:

$$\begin{cases} \dot{x} = f(x, u) \\ f(x_0) = 0 \end{cases} \quad (\text{III.01})$$

The time derivative of the Lyapunov function is then analyzed to determine the stability of the system. If the derivative satisfies:

$$\dot{V}(x) < 0 \quad (\text{III.02})$$

the system is considered asymptotically stable because the system energy decreases over time and the state variables converge toward the equilibrium point.

In Backstepping control design, Lyapunov theory is applied recursively. At each step, a Lyapunov function is constructed for a subsystem, and the control law is designed to ensure that the derivative of this function remains negative. By repeating this procedure for all subsystems, the stability of the complete nonlinear system can be guaranteed.

III.4. Backstepping control theory

Consider the following second-order dynamic system described by the differential equation below:

$$\begin{cases} \dot{x}_1 = f(x_1) + x_2 \\ \dot{x}_2 = u \end{cases} \quad (\text{III.03})$$

It is assumed that the system output is required to track the reference signal in order to illustrate the recursive design procedure of the Backstepping control strategy and analyze the stability of the system. Subsequently, a suitable Lyapunov function $V(x)$ is selected, and the control design process is carried out through two main steps:

III.4.1. First step

We construct the first Lyapunov numerical function for a system with the previous equation as follows:

$$V_1 = \frac{1}{2} e_1^2 \quad (\text{II.04})$$

Where e_1 represents the tracking error described in the following form:

$$e_1 = x_1^* - x_1 \quad (\text{III.05})$$

With x_1^* is the desired reference output.

For guarantee the stability of our system and ensure the negativity of Lyapunov function must be make derivation of V_1 which written on the next equation as follow:

$$e_1^* = e_1^* e_1^* \quad (\text{III.0.6})$$

The derivative of error equation is:

$$\dot{e}_1 = \dot{x}_1^* - \dot{x}_1 = \dot{x}_1^* - f(x_1) - \dot{x}_2 \quad (\text{III.07})$$

Setting the equation gives us the following:

$$V_1 = e_1^* (x_1^* - x_1) = e_1^* (x_1^* - f(x_1) - x_2) \quad (\text{III.08})$$

To ensure the negation of the Lyapunov function derivative and to realize the stability condition, introducing a positive constant k_1 such that:

$$\dot{V}_1 = \dot{e}_1^* (x_1^* + k_1 e_1 - k_1 e_1 - f(x_1) - x_2) = -k_1 e_1^2 + (x_1^* + k_1 e_1 - f(x_1) - x_2) \quad (\text{III.9})$$

For guarantee $v_1^* < 0$ must be the virtual command determined for assure the condition $(x_1^{**} + k_1 e_1 - f(x_1) - x_2) = 0$. So that realize $v_1^* = -k_1 e_1^2 < 0$; consequently, the virtual control written on the following expression as follow:

$$x_{c2}^* = x_1^* + k_1 e_1 - f(x_1) \quad (\text{III.10})$$

III.4.2. Second step

In the second step, a new error variable e_2 is introduced based on the previously defined virtual control, which is considered as a new reference signal, and can be expressed as follows:

$$e_2 = x_2^* - x_2 = k_1 e_1 + x_2^* - f(x_1) - x_2 = e_1^* + k_1 e_1 \quad (\text{III.11})$$

The derivative of this error calculated from the previous equation equal:

$$\dot{e}_2 = \dot{x}_2^{**} - \dot{x}_2 = k_1 \dot{e}_1 + \dot{x}_2^{**} - f(x_1) - \dot{x}_2 = k_1 \dot{e}_1 + \dot{x}_2^{**} - f(x_1) - u \quad (\text{III.12})$$

The Lyapunov function V_2 can be written in to take account the global error as follow:

$$v_2 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \quad (\text{III.13})$$

The derivative of the second Lyapunov function V_2 is:

$$\dot{v}_2 = \dot{e}_1 e_1 + \dot{e}_2 e_2 = -k_1 e_1^2 + e_2 (e_1 + k_1 e_2 - k_1^2 e_1 + x_2^{**} - f(x_1) - u) \quad (\text{III.14})$$

To ensure the negative of the derivative of the Lyapunov function we defined a positive k_2 that realizes the following condition:

$$v_2^* = -k_1 e_1^2 - k_2 e_2^2 < 0 \quad (\text{III.15})$$

the control input U can be expressed as follows:

$$\begin{aligned} u &= e_1 + k_1 e_2 - k_1^2 e_1 + x_1^{**} - f(x_1) + k_2 e_2 \\ &= (k_1 + k_2) e_2 - (1 - k_1^2) e_1 + x_1^{**} - f(x_1) \end{aligned} \quad (\text{III.16})$$

The positive constants k_1 and k_2 play an essential role in determining the dynamic response and stability of the system. Therefore, selecting suitable control gains in Backstepping control requires both theoretical understanding and practical experience. This selection depends on the characteristics of the controlled system and often involves several tuning trials to achieve an appropriate balance between stability, rapid response, and overall control performance under different operating conditions.

III.5. The Backstepping control of 5P-PMSM

The objective of the proposed Backstepping control strategy is to ensure that the motor electrical speed accurately tracks the desired reference speed. The main principle of Backstepping control consists of transforming the complex nonlinear system into an equivalent structure composed of interconnected cascade subsystems, which simplifies the controller design and stability analysis. Furthermore, the use of the Lyapunov approach ensures the stability of the overall system. The nonlinear model of the 5P-PMSM drive, including the effect of the third harmonic discussed in the previous chapter, can therefore be reformulated in the rotating reference frame (d1-q1-d2-q2) as follows:

$$\begin{cases} \frac{di_{d1}}{dt} = F_1 + \frac{V_{d1}}{L_s} \\ \frac{di_{q1}}{dt} = F_2 + \frac{V_{q1}}{L_s} \\ \frac{di_{d2}}{dt} = F_3 + \frac{V_{d2}}{L_{ls}} \\ \frac{di_{q2}}{dt} = F_4 + \frac{V_{q2}}{L_{ls}} \end{cases} \quad (\text{III.17})$$

And the mechanical equation expressed in next form as:

$$\begin{cases} T_{em} = K \varphi_f P i_{q1} \\ \frac{d\Omega}{dt} = F_5 \end{cases} \quad (\text{III.18})$$

Where $K = \sqrt{\frac{5}{2}}$ also:

$$\begin{cases} F_1 = -\frac{R_s}{L_s} i_{d1} + \omega i_{q1} \\ F_2 = -\frac{R_s}{L_s} i_{q1} - \omega i_{d1} - \frac{\lambda_m}{L_s} \omega \\ F_3 = -\frac{R_s}{L_{ls}} i_{d2} + 3\omega i_{q2} \\ F_4 = -\frac{R_s}{L_{ls}} i_{q2} - 3\omega i_{d2} \\ F_5 = \frac{T_{em}}{J} - \frac{T_L}{J} - \frac{F\Omega}{J} \end{cases} \quad (\text{III.19})$$

III.5.1. Calculation of the reference currents

At this stage, the speed controller is designed to ensure accurate tracking of the reference speed trajectory. Therefore, the speed tracking error and its time derivative can be expressed as follows:

$$\begin{cases} e_1 = \omega^* - \omega \\ \dot{e}_1 = \dot{\omega}^* - \dot{\omega} \end{cases} \quad (\text{III.20})$$

The derivative of speed error can be established as:

$$\dot{e}_1 = \dot{\omega}^* - F_5 \quad (\text{III.21})$$

The first Lyapunov function related with speed error is established to check the tracking performances as:

$$v_1 = \frac{1}{2} e_1^2 \quad (\text{III.22})$$

the derivative is computed as follows:

$$\dot{V}_1 = e_1 (\dot{\omega}^* - F_5) \quad (\text{III.23})$$

Thus, can be rewritten as:

$$\dot{V}_1 = e_1 (\dot{\omega}^* - F_5) = -H_1 e_1^2 \quad (\text{III.24})$$

Where, $H_1 > 0$ and the derivative of speed error gives:

$$\dot{e}_1 = \dot{\omega}^* - \dot{\omega} = -H_1 e_1 \quad (\text{III.25})$$

The i_{q1} component contributes to developed torque, while the i_{d1} , i_{d2} , i_{q2} components contribute to the power losses. Then the references currents can be described as:

$$\begin{cases} i_{d1}^* = 0 \\ i_{q1}^* = \left(\dot{\omega}^* + \frac{T_L}{J} + \frac{B\omega}{J} + H_1 e_1 \right) / \left(\frac{K\lambda_m P}{J} \right) \\ i_{d2}^* = 0 \\ i_{q2}^* = 0 \end{cases} \quad (\text{III.26})$$

III.5.2. Calculation of the reference voltages

Obtaining the reference voltages based on the preceding phase is the aim of this phase., where the current errors are derived as follows:

$$\begin{cases} e_2 = i_{d1}^* - i_{d1} \\ e_3 = i_{q1}^* - i_{q1} \\ e_4 = i_{d2}^* - i_{d2} \\ e_5 = i_{q2}^* - i_{q2} \end{cases} \quad (\text{III.27})$$

Hence, their derivative is expressed as follows:

$$\begin{cases} \dot{e}_2 = \dot{i}_{d1}^* - \dot{i}_{d1} \\ \dot{e}_3 = \dot{i}_{q1}^* - \dot{i}_{q1} \\ \dot{e}_4 = \dot{i}_{d2}^* - \dot{i}_{d2} \\ \dot{e}_5 = \dot{i}_{q2}^* - \dot{i}_{q2} \end{cases} \quad (\text{III.28})$$

Using, the current errors are given as follows:

$$\begin{cases} e_2 = -i_{d1} \\ e_3 = \left(\dot{\omega}^* + \frac{T_L}{J} + \frac{B\omega}{J} + H_1 e_1 \right) / \left(\frac{K\lambda_m P}{J} \right) - i_{q1} \\ e_4 = -i_{d2} \\ e_5 = -i_{q2} \end{cases} \quad (\text{III.29})$$

can be obtained as:

$$\dot{e}_1 = \frac{K\lambda_m P}{J} z_3 - H_1 e_1 \quad (\text{III.30})$$

One obtins :

$$\left\{ \begin{array}{l} \dot{e}_2 = \dot{i}_{d1}^* - F_1 - \frac{V_{d1}}{L_s} \\ \dot{e}_3 = \dot{i}_{q1}^* - F_2 - \frac{V_{q1}}{L_s} \\ \dot{e}_4 = \dot{i}_{d2}^* - F_3 - \frac{V_{d2}}{L_{ls}} \\ \dot{e}_5 = \dot{i}_{q2}^* - F_4 - \frac{V_{q2}}{L_{ls}} \end{array} \right. \quad (\text{III.31})$$

Therefore, to prove the overall stability of the studied control algorithm, a new Lyapunov function is defined as:

$$V_2 = \frac{e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2}{2} \quad (\text{III.32})$$

The derivative becomes:

$$\begin{aligned} \dot{V}_2 = & -H_1 e_1^2 - H_2 e_2^2 - H_3 e_3^2 - H_4 e_4^2 - H_5 e_5^2 + e_2 \left(H_2 e_2 + \dot{i}_{d1}^* - F_1 - \frac{V_{d1}}{L_s} \right) \\ & + e_3 \left(H_3 e_3 + K\phi_f n_p e_2 + \dot{i}_{q1}^* - F_2 - \frac{V_{q1}}{L_s} \right) \\ & + e_4 \left(H_4 e_4 + \dot{i}_{d2}^* - F_3 - \frac{V_{d2}}{L_s} \right) + e_5 \left(H_4 e_5 + \dot{i}_{q2}^* - F_4 - \frac{V_{q2}}{L_s} \right) \end{aligned} \quad (\text{III.33})$$

Consequently, if the variables within parentheses in the same expression are equal to zero, the derivative of the overall Lyapunov is negative.

$$\left\{ \begin{array}{l} H_2 e_2 + \dot{i}_{d1}^* - F_1 - \frac{V_{d1}}{L_s} = 0 \\ H_3 e_3 + K\phi_f n_p e_2 + \dot{i}_{q1}^* - F_2 - \frac{V_{q1}}{L_s} = 0 \\ H_4 e_4 + \dot{i}_{d2}^* - F_3 - \frac{V_{d2}}{L_{ls}} = 0 \\ H_4 e_5 + \dot{i}_{q2}^* - F_4 - \frac{V_{q2}}{L_{ls}} = 0 \end{array} \right. \quad (\text{III.34})$$

Finally, the reference voltages are obtained as:

$$\begin{cases} V_{d1}^* = L_s (H_2 e_2 + \dot{i}_{d1}^* - F_1) \\ V_{q1}^* = L_s (H_3 e_3 + k \varphi_m P z_1 + \dot{i}_{q1}^* - F_2) \\ V_{d2}^* = L_{ts} (H_4 e_4 + \dot{i}_{d2}^* - F_3) \\ V_{q2}^* = L_{ts} (H_4 e_5 + \dot{i}_{q2}^* - F_4) \end{cases} \quad (\text{III.35})$$

Where: H1, H2, H3, and H4 are that have been employed in our suggested control are positive and have been selected to meet the enhanced requirements of the suggested backstepping control, including a quicker dynamic of the rotor speed and stator currents, with minimum overshoot. Whereby their precise value selection will enhance the closed loop's dynamic and hence ensure the regulated system's stability. In this chapter, the values of these parameters that have been selected from multiple simulation sessions where fitting the optimal dynamic demand of the rotor speed and stator current served as the primary selection criterion.

III.5. Conclusion:

In conclusion, Backstepping control represents an effective and robust nonlinear control strategy for the Five-Phase Permanent Magnet Synchronous Motor due to its capability to guarantee system stability and high dynamic performance. By relying on the Lyapunov stability theory and the recursive design procedure, this method ensures accurate tracking of speed and current references while maintaining robustness against disturbances and parameter variations. Moreover, the calculation of suitable reference currents and voltages allows the 5P-PMSM drive to achieve fast response, reduced tracking errors, and improved operational efficiency. Therefore, Backstepping control constitutes a promising solution for advanced multiphase motor drive applications requiring high precision, reliability, and stability.

***Chapter IV: MRAS-Based Sensorless
Control of 5P-PMSM with load
variation***

IV.1. Introduction:

Model Reference Adaptive System (MRAS)-based sensorless control is widely used in Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) drives because of its simplicity, robustness, and accurate estimation performance. This technique estimates rotor speed and position without mechanical sensors, reducing system cost and improving reliability. The MRAS method operates by comparing a reference model with an adaptive model and minimizing the estimation error through an adaptive mechanism. When combined with Backstepping control, the system achieves stable operation, accurate speed tracking, fast dynamic response, and strong robustness against disturbances and load variations. Simulation results in MATLAB/Simulink confirm the effectiveness of the proposed approach for high-performance five-phase PMSM applications.

IV.2. Principle of Sensorless Control using MRAS

The Model Reference Adaptive System (MRAS) is one of the most widely used technologies for sensorless control of permanent magnet synchronous motors (PMSMs). MRAS-based control aims to estimate rotor speed and position without the need for mechanical sensors such as encoders or angular resolvers. This approach improves system reliability, reduces equipment costs, and increases system durability in harsh industrial environments [7].

The MRAS operating principle is based on comparing two mathematical models of the engine:

IV.2. a. The Reference Model

The reference model represents the actual behavior of the system and does not depend on the parameter to be estimated. It uses measurable signals such as stator voltages and currents to generate a reference output.

The reference model of MRAS described from state model of the 5P-PMSM can be written as[21]:

$$[\dot{X}] = [A][X] + [B][V] + [C] \quad (IV.01)$$

$$[Y] = [I][X]$$

With:

The MRAS observer input is $[V] = [Vd1, Vq1, Vd2, Vq2]^T$, the output variable of system is $[X] = [Id1, Iq1, Id2, Iq2]^T$, and $[I]$ the unitary matrix. Then the state-space matrices of

model defined as:

$$[A] = \begin{bmatrix} -\frac{R_s}{L_{d1}} & \frac{L_{q1}\omega}{L_{d1}} & 0 & 0 \\ -\frac{L_{d1}\omega}{L_{q1}} & -\frac{R_s}{L_{q1}} & 0 & 0 \\ 0 & 0 & -\frac{R_s}{L_{d2}} & 0 \\ 0 & 0 & 0 & -\frac{R_s}{L_{q1}} \end{bmatrix}$$

and

$$[B] = \begin{bmatrix} \frac{1}{L_{d1}} & 0 & 0 & 0 \\ 0 & \frac{1}{L_{q1}} & 0 & 0 \\ 0 & 0 & \frac{1}{L_{d2}} & 0 \\ 0 & 0 & 0 & \frac{1}{L_{q2}} \end{bmatrix}, [C] = \begin{bmatrix} 0 \\ \frac{\varphi_m \omega}{L_{q1}} \\ 0 \\ 0 \end{bmatrix}$$

IV.2.b. Adaptive (Adjustable) Model

The adaptive model has a structure similar to the reference model, but its output depends on the estimated variable, such as estimated rotor speed $\hat{\omega}$ or estimated rotor position $\hat{\theta}$.

2. Adaptive Model: Depends on the estimated speed or position of the rotor.

The outputs of these two models are continuously compared to generate an estimation error signal, and this signal is then processed by an adaptive mechanism in order to correct the estimated speed and reduce the difference between the two models [21].

$$\begin{cases} \dot{X} = [A^*][X] + [B][V] + [C] \\ Y = [I][X] \end{cases} \quad (IV.02)$$

Where $[X] = [I_d^*, I_q^*, I_x^*, I_y^*]^T$ is the estimated state vector, so the state-space matrices of adjustable

model written in the form:

$$[\hat{A}] = \begin{bmatrix} -\frac{R_s}{L_{d1}} & \frac{L_{q1}\hat{\omega}}{L_{d1}} & 0 & 0 \\ -\frac{L_{q1}\hat{\omega}}{L_{q1}} & -\frac{R_s}{L_{q1}} & 0 & 0 \\ 0 & 0 & -\frac{R_s}{L_{d2}} & 0 \\ 0 & 0 & 0 & -\frac{R_s}{L_{q2}} \end{bmatrix} \text{ and } [\hat{C}] = \begin{bmatrix} 0 \\ -\frac{\lambda_m \hat{\omega}}{L_{q1}} \\ 0 \\ 0 \end{bmatrix}$$

The error in estimation is expressed by the following relationship:

$$e = y_{ref} - y_{adp} \quad (IV.03)$$

where:

- **yref**: Output of the reference model
- **yadp** : Output of the adaptive model

The adaptation mechanism is typically designed based on Lyapanov's stability theory to ensure that the estimation error approaches zero and that the observer is stabilized [16].

The estimated speed is continuously updated according to an adaptive law of the form:

$$\hat{\omega} = K_{pe} + K_i \int edt \quad (IV.04)$$

$$\hat{W} = \left(K_p + \frac{K_i}{p} \right) \left(\frac{L_{q1}}{L_{d1}} \in_{d1} I_{q1} - \frac{L_{d1}}{L_{q1}} \in_{q1} I_{d1} - \frac{\phi_m}{L_{q1}} \in_{q1} \right) \quad (IV.05)$$

Where:

$\hat{\omega}$: Rated speed of the rotor

Kp: Proportionality coefficient

Ki: Integration coefficient

In permanent magnet synchronous motor drive systems, an MRAS monitor is typically integrated with field-directed control (FOC) or backstepping control strategies. Estimated speed and position are used instead of signals from sensors, enabling fully sensorless operation [17] [18].

One of the major advantages of MRAS-based sensorless control is its simplicity and low computational complexity compared with other observer techniques. In addition, it provides [20]:

- Fast speed estimation
- Good Dynamic response
- High robustness against parameter variations
- Reduced system cost
- Improved reliability

IV.3. Integration with the Backstepping Controller of 5P-PMSM

The integration of a Model Reference Adaptation System (MRAS) with a Backstepping controller in a five-phase permanent magnet synchronous motor drive system (5P-PMSM) represents an advanced sensorless control strategy aimed at achieving high dynamic performance, robust stability, and high-speed tracking accuracy under load and parameter variations.

In this architecture, the MRAS controller is responsible for estimating the rotor speed and position without the use of mechanical sensors, while the Backstepping controller ensures the stability and regulation of motor currents and speed based on Lyapunov's stability theory. The estimated speed and position are continuously transmitted to the control algorithm to compensate for the signals from conventional sensors.

IV.4.1. Simulink Results of normal operating conditions

In this test, a backstepping control system and MRAS monitor, designed to test a five-phase PMSM motor without introducing any system errors, were used. The applied speed was set at 100 radians per second. However, Figures (IV,1) show that the estimated speed is as close as possible to the actual speed with minimal error, as the error shown in Figures (IV, 2) is very close to zero, ranging from 0 radians per second to 0.04 radians per second at steady state. On the other hand, the calculated load torque shown in Figure (IV, 3) accurately tracks the imposed reference load torque, indicating that the backstepping control of five-phase PMSM motor typically offers better efficiency performance, as well as effective control. Therefore, based on these figures, MRAS provides a suitable estimation method for normal operation.

The figure illustrates the speed response of the 5P-PMSM motor, where the red curve represents the reference speed w_r and the blue curve represents the actual speed w . We observe that the actual speed follows the reference speed very quickly with a short response time, demonstrating the effectiveness of the control system. A slight overshoot is also present at the beginning of the response, shown within the enlarged circle, where the speed exceeds the reference value by approximately 100.2 rad/s before quickly stabilizing around the target value of 100 rad/s. Furthermore, the fluctuations after stabilization are very small and limited, reflecting the system's stability and the accuracy of the reference speed tracking control.

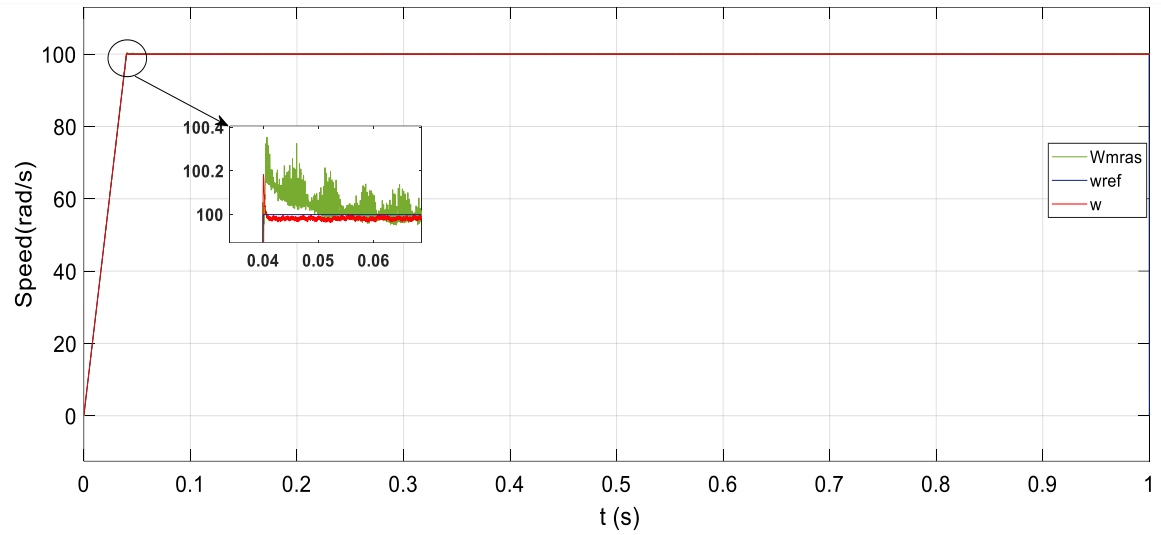


Figure (IV. 1): Speed(rad/s) with Wmras

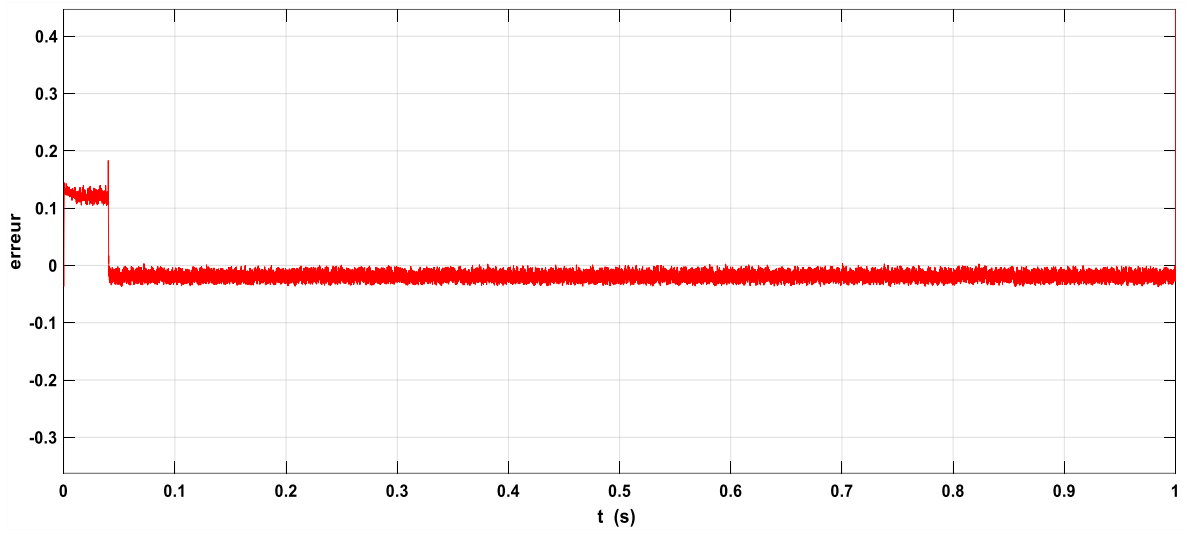


Figure (IV.2): Erreur (W-Wr)

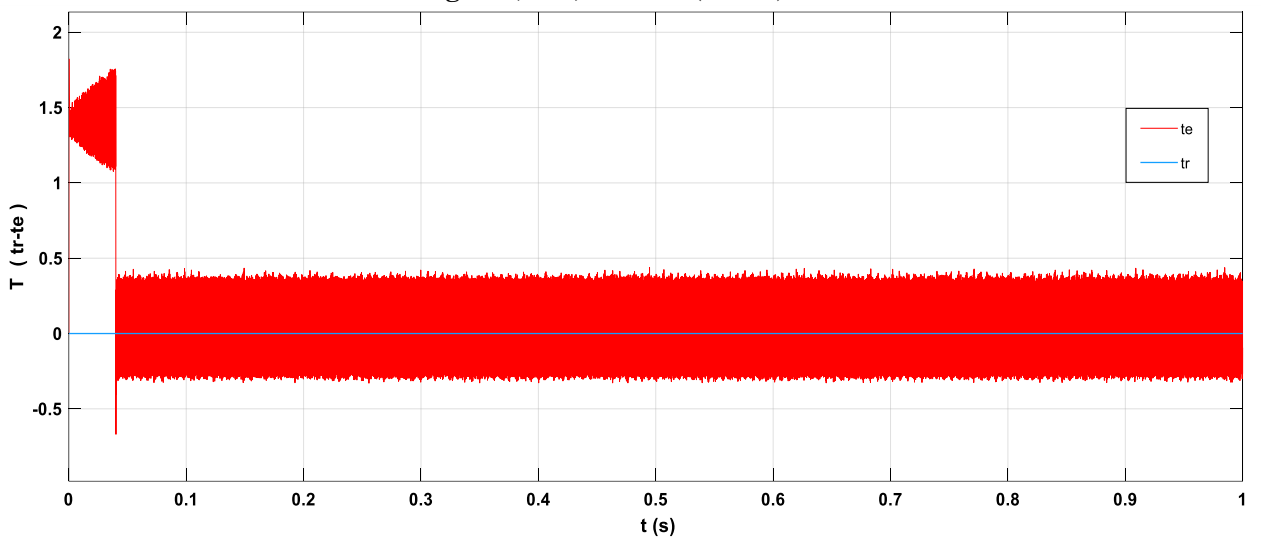


Figure (IV. 3): Torque (te,tr)

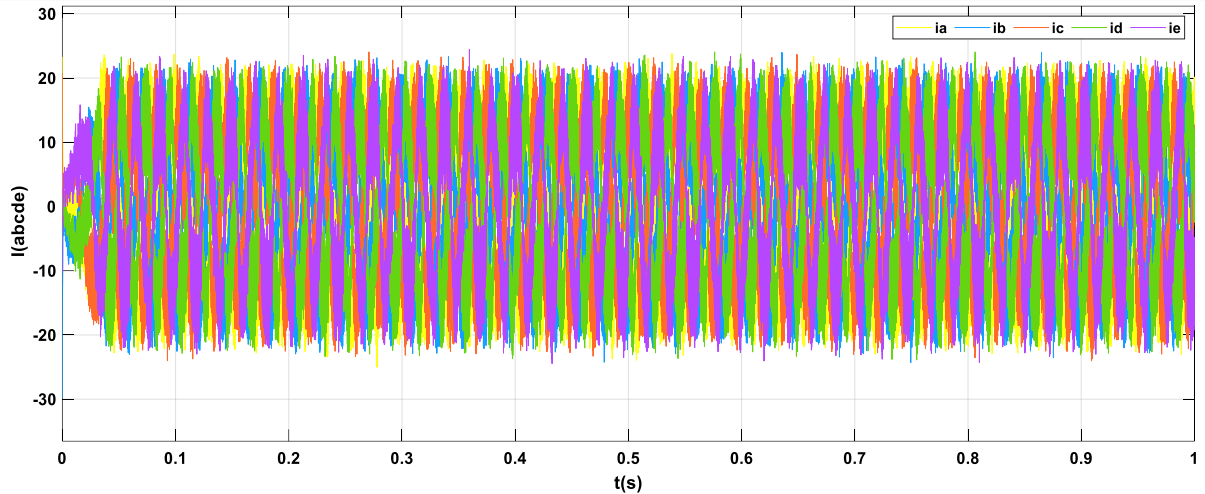


Figure (IV. 4): Motor currents in the presence of a load (ia,ib,ic,id,ie)

IV.4.2.Simulink simulation results for load change of 5P-PMSM

During a load change test of a five-phase permanent magnet synchronous motor (5P-PMSM), the load was intentionally changed to simulate a turbulent state. The objective of this test was to evaluate the motor's performance under these conditions. The same conditions used in the first test were employed: a rotational speed of 100 radians/second at time $t = 0$ seconds. The load change point, shown in Figure (IV, 5), was applied at time $t = 0.3$ seconds. The speed response curve shown in Figure (IV,5) tracks the rated rotational speed. Additionally, the oscillation of the electromagnetic torque response is also shown in Figure (IV,7). These results demonstrate the 5P-PMSM's. They also show effective control even when using an MRAS monitor, which is affected by changes in external parameters such as load torque.

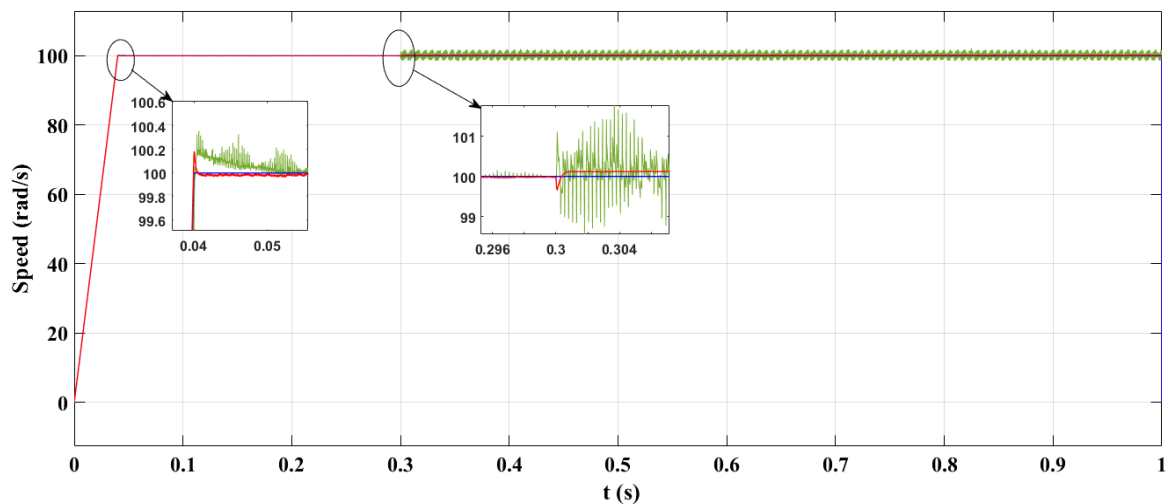
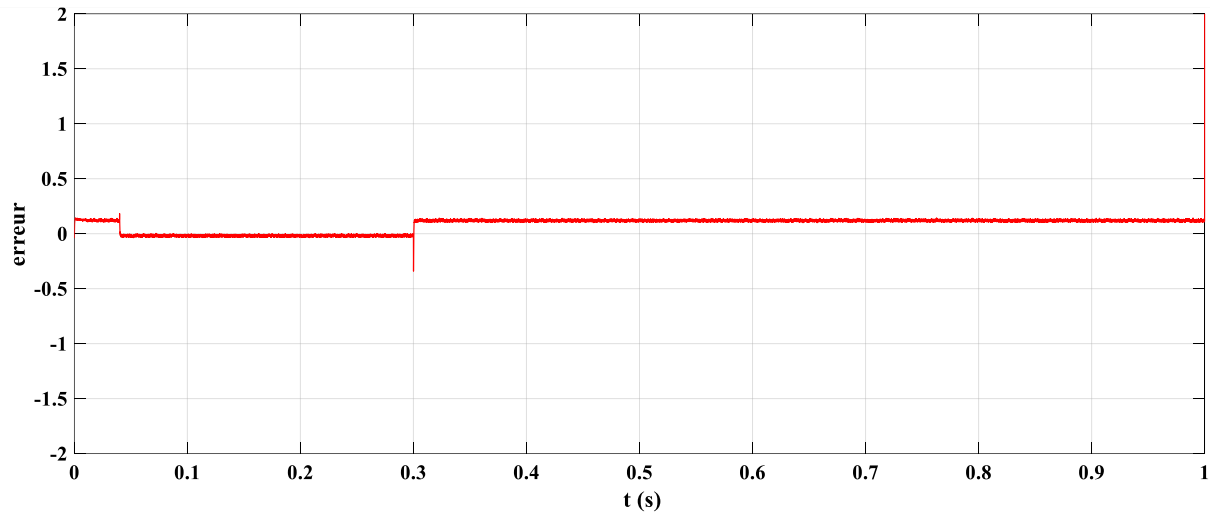
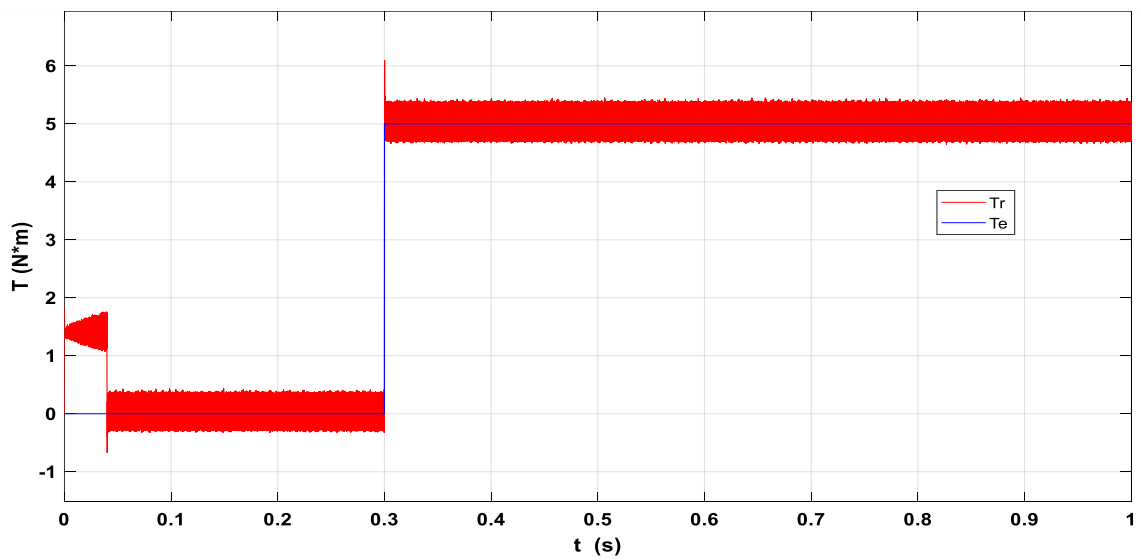
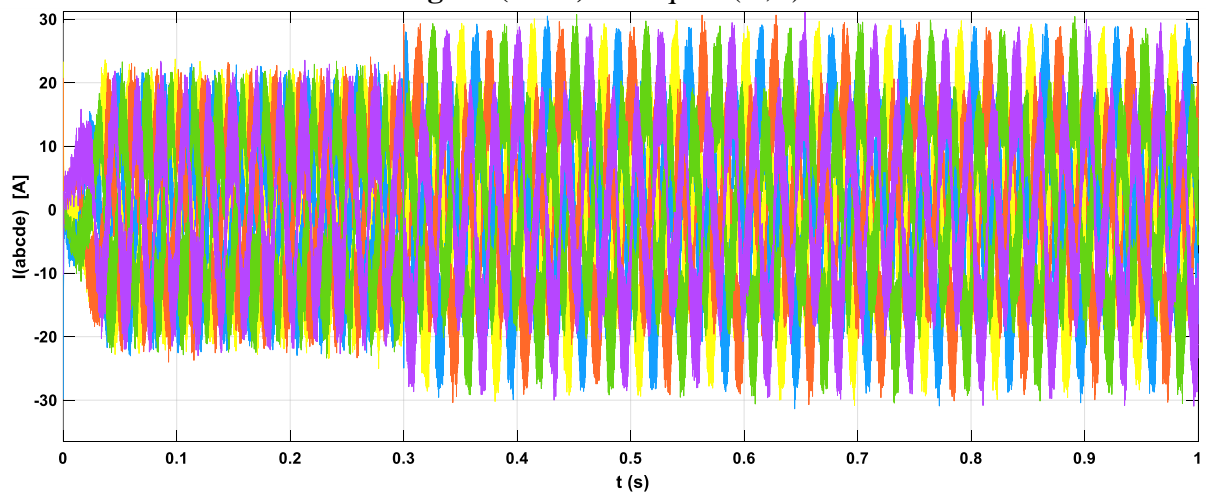


Figure (IV. 5): Speed(rad/s) with Wmras

Figure (IV. 6): Erreur ($W-W_r$)Figure (IV. 7): Torque (t_e, t_r)Figure (IV. 8): Motor currents in the presence of a load (i_a, i_b, i_c, i_d, i_e)

The figure illustrates the speed response of a 5P-PMSM motor to a load change at time $t = 0.3$ s. The red curve represents the reference speed W_r , while the green curve (W_{mras}) and the other curve represent the actual motor speed. Upon startup, the speed rapidly increases to the reference value of 100 radians/second with very slight overshoot, as shown in the first magnification, and then stabilizes around the target value with a short response time. When a load change is applied at $t = 0.3$ s, the second magnification shows slight disturbance and minor speed fluctuations. However, the control system was able to quickly eliminate the effect of the load and return the speed to its reference value without permanent error. This demonstrates the high robustness of the controller and its ability to withstand load disturbances, maintain system stability, and accurately track the reference speed.

General Conclusion



Conclusion General

In conclusion, the study of the Five-Phase Permanent Magnet Synchronous Motor (5P-PMSM) combined with advanced control and estimation techniques represents a significant contribution to the development of modern high-performance electric drive systems. This work presented the theoretical background of PMSMs and multiphase machines, highlighting the advantages of five-phase systems in terms of reliability, reduced torque ripple, and improved dynamic performance. The mathematical modeling of the 5P-PMSM in different reference frames provided a solid analytical basis for understanding the electrical and mechanical behavior of the machine and for developing suitable control strategies. Furthermore, the application of the nonlinear Backstepping Control technique demonstrated its effectiveness in ensuring system stability, accurate speed tracking, and robustness against disturbances through the Lyapunov stability approach. In addition, the implementation of the MRAS-based sensorless control strategy enabled accurate estimation of rotor speed and position without mechanical sensors, thereby reducing system complexity and improving reliability. The integration of MRAS estimation with Backstepping control proved to be an efficient and robust solution for controlling the 5P-PMSM under both normal operating conditions and load variations. Finally, the simulation results obtained through MATLAB/Simulink confirmed the high performance, stability, and robustness of the proposed control strategy, making it a promising approach for advanced industrial applications such as electric vehicles, aerospace systems, robotics, and high-performance.

References and bibliography

Références et bibliographiques

- [1] **.N. Manias,**” Power Electronics and Motor Drive Systems” – Stefanos, Academic Press, 2017.
- [3]. **Paul C. Krause**’ Analysis of Electric Machinery and Drive Systems” , Wiley-IEEE Press(2013).
- [4]. **Emil Levi** “Multiphase Electric Drives: Modeling and Control “, Wiley-IEEE Press, 2015.
- [5] **.C. B. Gupta** “ Electric Machines and Drives: Principles, Control, Modeling, and Simulation” ‘Academic Press, 2019.
- [6]. **R. Krishnan** ” Electric Motor Drives: Modeling, Analysis, and Control” – ‘IEEE Press / Wiley, 2010.
- [7]. **Seung-Ki Sul** ” Control of Electric Machine Drive Systems” – ‘Wiley-IEEE Press, 2011.
- [8]. **Va P. Popov** “Sensorless Control of Electric Drives”, Springer, 2019.
- [9]. **R. Krishnan** “Advanced Electric Drives: Analysis, Control and Modeling Using Simulation Tools” Prentice Hall / CRC Press, 2001.
- [10]. **Hassan K. Khalil** “Nonlinear Systems “ ‘Prentice Hall, 2002.
- [11]. **R. Krishnan,** “*Permanent Magnet Synchronous and Brushless DC Motor Drives.* CRC Press”. (2010).
- [12]. **E. Levi** “Multiphase Electric Machines for Variable-Speed Applications,” *IEEE Transactions on Industrial Electronics*, (2008) .
- [13]. **Ned Mohan,**” Power Electronics: Converters, Applications, and Design “ ‘ Wiley, 2003.
- [14]. **Bimal K. Bose,** “ Modern Power Electronics and AC Drives” ‘Prentice Hall, 2002.
- [15] **J.-J. E. Slotine** and **W. Li,** Applied Nonlinear Control. Englewood Cliffs, NJ: Prentice Hall, 1991.
- [16] **Katsuhiko Ogata** "Modern Control Engineering " (Prentice Hall / Pearson) 2010.
- [16] **Sundarapandian Vaidyanathan** and **Ahmad Taher Azar** "Adaptive Backstepping Control for PMSM Drives" 2021.

- [17] G. Wang, R. Ni and D. Xu, "*Position Sensorless Control Techniques for Permanent Magnet Synchronous Motor Drives* ", Singapore: Springer, 2020.
- [18] X. Zhang, "Sensorless Control of Permanent Magnet Synchronous Motor Based on Model" Hangzhou, China, 2017
- [19] J. Li and Y. Li, "Intelligent Backstepping Control for AC Motor Drive Systems". Singapore: Springer, 2021.
- [20] M. H. Boughazala, "Performance Analysis of the Five Phase Permanent-Magnet Synchronous Motor Open Fault Phase," in *Proc. 1st National Conference on Renewable Energies and Advanced Electrical Engineering (NC-REAAE'25)*, M'Sila, Algeria, May 6–7, 2025.
- [21] M. H. Boughazala " **Contribution to the Control of Five Phase Permanent Magnet Synchronous Motor**" **Cycle Doctoral /2024**
- [22] A. Iqbal, S. Moinuddin, and M. R. Khan, "Space Vector Model of A Five-Phase Voltage Source Inverter," pp. 488–493, 2006.
- [23] F. Acosta-Cambranis, J. Zaragoza, L. Romeral, and N. Berbel, "Comparative analysis of svm techniques for a five-phase vsi based on sic devices," *Energies*, vol. 13, no. 24, 2020, doi: 10.3390/en13246581.
- [24] M. A. Patel, A. R. Patel, and D. R. Vyas, "Use of PWM Techniques for Power Quality Improvement," no. May, pp. 2–6, 2009.
- [25] K. Tounsi, D. Abdelkader, and S. Barkat, "Vector Control of Five-Phase Permanent Magnet Synchronous Motor Drive," no. October 2016, 2015, doi: 10.1109/INTEE.2015.7416853.
- [26] M. JANASZEK, "EXTENDED CLARKE TRANSFORMATION FOR n-PHASE SYSTEMS," *Proc. Electrotech. Inst.*, vol. 63, no. 0, pp. 5–26, 2016
- [27] M. A. Mossa, H. Echeikh, and A. Ma'arif, "Dynamic Performance Analysis of a Five-Phase PMSM Drive Using Model Reference Adaptive System and Enhanced Sliding Mode Observer," *J. Robot. Control*, vol. 3, no. 3, pp. 289–308, 2022,
- [28] Huang. W; Huang, Y.; Xu, D. Model-Free Predictive Current Control of Five-Phase PMSM Drives. *Electronics* **2023**, 12, 4848. [https:// doi.org/10.3390/electronics12234848](https://doi.org/10.3390/electronics12234848)
- [29] H. De Jong, "Modelling and Simulation of Five Phase Permanent Magnet Synchronous Motor," vol. I, no. 2, pp. 111–118, 2003.
- [30] A. Mushtaq, "What are the types of Pulse Width Modulation Techniques," *May 25, 2022, 2022*.

Annexe

ANNEXES

A)

Table .1. parameters of 5P-PMSM drive

P	L_s	L_{ls}	R_s	φ_m	J
2	2.1 mH	0.13 mH	0.18 Ω	0.163 T	0.0011Kg.m2

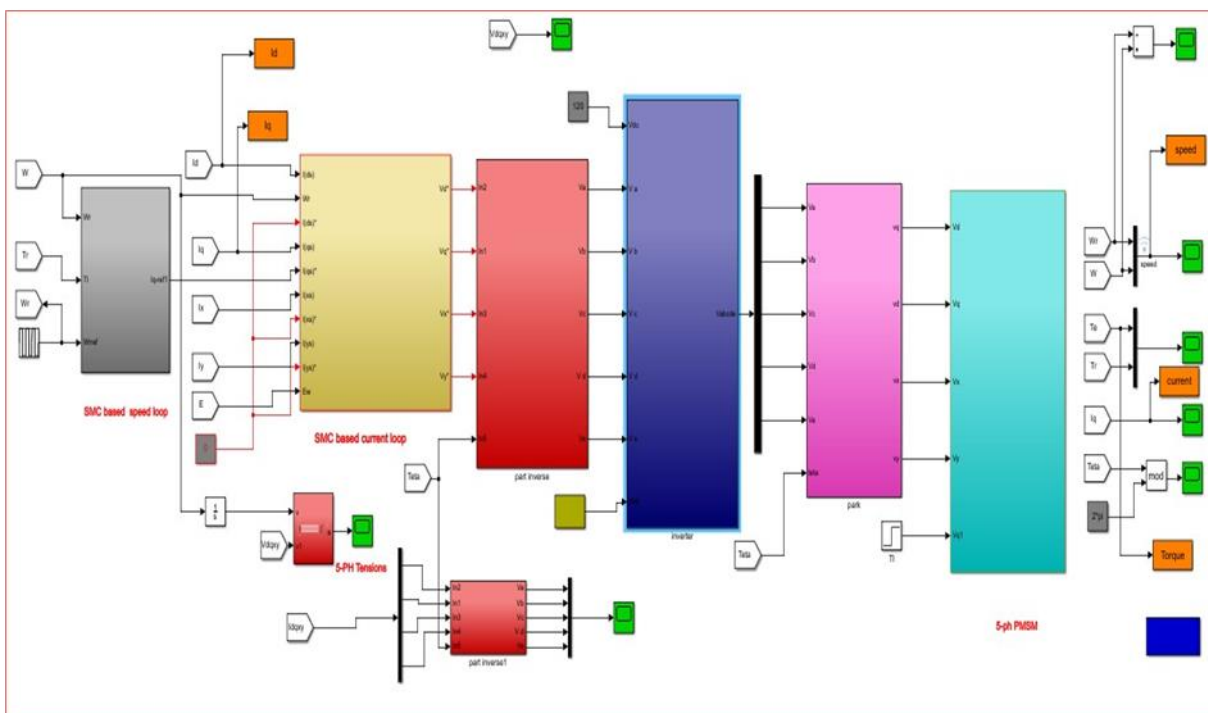
B)

Table 2. Gains of backstepping controller

$c1$	$c2$	$c3$	$c4$
10000	5000	2500	1000

C)

MATLAB-Simulink of proposed Backstepping Control



Abbreviations list

- PMSMs : permanent magnet synchronous Motors
- 5-phase PMSM, 5P- PMSM: five Phase Permanent Magnet Synchronous Motor
- MRAS: model reference adaptive system.
- SMO: Sliding Mode Observer
- AI: Artificial Intelligence
- EMF: Electromotive force
- MMF: Magnetomotive force
- HF: High Frequency
- DSP: Digital signal processor
- DC machine: Direct Current machine
- AC machine: Alternative Current machine
- VSI: Voltage Source Inverter
- CSI: Current Source Inverter
- PWM: Pulse-Width Modulation
- IGBT: Insulated gate bipolar transistors
- MOSFET: metal-oxide-semiconductor field-effect transistor
- DCMI: Diode Clamped Inverter.
- FCMI: Flying Capacitor Inverter
- PI: proportional integral control
- PID: proportional integral Derivative control.
- DTC: direct torque control
- FOC: field-oriented control
- RFOC: rotor-flux oriented control
- SMC: sliding mode control

-
- MPC: model predictive control
 - I_q : q-axis current
 - I_d : d- current
 - $X(t)$: state variable or state vector
 - $S(x)$: Sliding control Surface
 - ρ : is a positive constant
 - r : degree relative of sliding surface
 - $e(t)$: state error relative
 - A : is the state matrix of the observer
 - B : is a control input matrix
 - C : is output matrix
 - V_s : is input vector of Luenberger observer
 - G : gain matrix of Luenberger observer
 - \dot{X} : the derivative of state variable
 - \hat{X} : the estimated value of the state variable
 - Y : the output system
 - L_d : direct inductance
 - L_q : Quadratic inductance
 - L_l : leakage inductance
 - PWM: Pulse width modulation
 - $(\alpha-\beta)$ ($\alpha_1, \beta_1, \alpha_2, \beta_2$): stationary frame or the stator reference frame
 - $(d-q)$ (d_1, q_1, d_2, q_2): rotational frame or rotor reference frame
 - $(U V)$: Synchronous reference frame
 - (a, b, c, d, e) : natural frame for 5P-PMSM representation
 - GA: Genetic Algorithms

- [Vs]: stator voltages
- [Is]: stator currents
- [Φ_s]: stator Flux
- [φ_m]: Matrix Vector of the flux created by the permanent magnets
- [Rs]: stator resistances
- [L_{s0}]: is the proper inductances matrix of stator windings,
- [L_{s1}]: The mutual inductances matrix of stator windings.
-
- Lm: is the main stator inductance
- Ls: is linkage inductance.
- P: is pair of pol number.
- Θ : rotor angular position
- Ω : mechanical speed
- w: is the electrical speed
- Tem: electro-magnetic torque
- TL: eternal load torque
- J: is inertia moment of the motor and load combined.
- F: is the friction coefficient
- Ω : is the mechanical energy delivered by rotor
- [I_s]^T is the transposed matrix of stator current
- θ_{co} is the angle between the a-axis of referential frame and the d1-axis (coordinate angle)
- wco: is the speed coordinate
- ws: is the synchronous speed.

- K : is a constant factor depend of the power transformation from original frame to new reference frame
- $[T]$: Park matrix transformation
- $[I_{\alpha 1}, I_{\beta 1}, I_{\alpha 2}, I_{\beta 2}]$: Stator currents in stationary frame
- $[V_{\alpha 1}, V_{\beta 1}, V_{\alpha 2}, V_{\beta 2}]$: Stator voltages in stationary frame
- $[\varphi_{\alpha 1}, \varphi_{\beta 1}, \varphi_{\alpha 2}, \varphi_{\beta 2}]$: Stator voltages in stationary frame
- $[I_{q1}, I_{d1}, I_{q2}, I_{d2}]$: Stator currents in rotational frame
- $[V_{q1}, V_{d1}, V_{q2}, V_{d2}]$: Stator voltages in rotational frame
- $[\varphi_{q1}, \varphi_{d1}, \varphi_{q2}, \varphi_{d2}]$: Stator voltages in rotational frame
- L_{d1}, L_{q1} : are the direct and quadratic inductances of principal frame
- L_{d3}, L_{q3} : are the direct and quadratic inductances of secondar frame
- $[A]$: is the system (or state) matrix
- $[B]$: is the input matrix
- $[C]$: represent the output matrix,
- $[E]$: represent the disturbance input matrix
- D : is the disturbance input of system
- V_{dc} : is the DC-link source
- $f(x, u)$: system function
- $V_1(x)$: first Lyapunov function
- $\dot{V}_1(x)$: derivative of first Lyapunov function
- $f(x_1, x_2)$: system of the second order
- e_1 : the tracking error
- \dot{e}_1 : Derivative of the tracking error
- \dot{X} : is the derivative of desired output.
- x_1^* is the desired reference output

- $V_2(x)$: second Lyapunov function
- $\dot{V}_2(x)$: derivative of second Lyapunov function
- k_1, k_2 : positives constants of Lyapunov functions
- X_{c2} : virtual command of control
- u : the global control
- \ddot{x}^* : second derivative of variable.
- K_1^2 : square of gain.
- T_{em} : electromagnet torque
- T_L : Load charge torque.
- J : motor inertia
- B : viscous friction
- Φ_m : Permanent Electromagnet
- w^* : reference speed
- I_{d1ref}, V_{d1ref} : reference current and voltage in rotational frame
- H_1, H_2, H_3, H_4 : are the backstepping control gains
- e_1, e_2, e_3, e_4 : are the backstepping control error
- L_s : Principal frame inductance
- L_{Ls} : secondaire frame inductance
- n : random numbers
- $\Delta\omega$: the relative errors of speed
- Δi : errors of current
- $\text{sgn}(s)$: signum function

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ