Bound States of 2D Bosonic Oscillator in a Magnetic Field in Anti deSitter Space

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***Abstract* - We conduct an analytical investigation of the two-dimensional deformed bosonic oscillator equation, which describes the behavior of charged particles (both spin 0 and spin 1 particles) in the presence of a uniform magnetic field. This study takes into account a minimal uncertainty in momentum arising from the anti-de Sitter model. To solve the system, we employ the Nikiforov-Uvarov (NU) method. Our analysis yields exact energy eigenvalues and corresponding wave functions for scalar Duffin-Kemmer-Petiau (DKP) case. We observe that even for large principal quantum numbers, the deformed spectrum remains discrete. For the spin 1 DKP case, we elucidate the behavior of the DKP equation and determine the nonrelativistic energies. We demonstrate that the space deformation introduces a new spin-orbit interaction proportional to its parameter.**

Index Terms - DKP equation; anti-de Sitter space; Scalar case; Vector case; Magnetic field.

I. INTRODUCTION

Quantum field theory in curved space through modifications of the Heisenberg algebra, such as the extended uncertainty principle (EUP), has sparked significant interest among researchers attempting to include gravity into quantum mechanics. This extended concept incorporates gravity into quantum mechanics by accounting for its quantum fluctuations. This unification resulted in a minimal length scale of the Planck order [1]. The minimum length can be linked to a modification of the standard Heisenberg algebra by adding small corrections to the canonical commutation relations. This shifts the standard algebra. Mignemi's work shows that the Heisenberg relations are modified in the (anti-)de Sitter space by adding corrections proportional to the cosmological constant. The modifications were driven by DSR [2], string theory [3], noncommutative geometry [4], and black holes [5].

The (DKP) equation, a first-order relativistic equation for spin-0 and spin-1 particles, has recently received renewed attention due to its applicability to various nuclear and particle physics problems [6–8]. Research on relativistic quantum harmonic oscillators has expanded to include scalar and vector bosons, in addition to particles with arbitrary spins. In a previous experiment, we incorporated a new possibility into the DKP equation. We dubbed the system a DKP oscillator because, in the non-relativistic limit, the spin-0 representation of this DKP equation provides the ordinary harmonic oscillator, whereas the spin-1 representation yields a harmonic oscillator with a Thomas form spin-orbit coupling. This oscillator is a relativistic version of the quantum harmonic oscillator, applicable to both scalar and vector bosons.

In this study, we are interested in phenomenological models of quantum gravity. We analyze scalar and vector DKP equations in 2D spaces using the position space representation for deformed quantum mechanics with EUP. For this system, we consider an oscillator-like interaction with an external uniform magnetic field.

II. OSCILLATOR DKP IN A MAGNETIC FIELD

The massive scalar and vector particles m's free DKP equation can be expressed as follows [9-11]:

$$\left[c\beta\cdot p+mc^2\right]\Psi(r,t)=i\hbar\beta^0\partial_0\Psi(r,t) \tag{1}$$

where β and β^0 are the DKP matrices [12].

We present the nonminimal substitution by writing the 2D DKP oscillator in the same manner as the Dirac oscillator

$$p\rightarrow p-im\omega\eta^0 r \tag{2}$$

Where ω is the oscillator's frequency. η^0 is defined by the relation $\eta^0=2\left(\beta^0\right)^2-1$.

The equation of the DKP oscillator is obtained by using the relation (2) in Eq. (1), to which a vector potential is added $A=\frac{1}{2}B\times r$;

$$\left[c\beta\cdot\left(p-\frac{e}{c}A-im\omega\eta^0 r\right)+mc^2\right]\Psi=i\hbar\beta^0\left(\frac{\partial\Psi}{\partial t}\right) \tag{3}$$

We formulate the 2D deformed stationary DKP oscillator equation for the stationary solutions by using the definition of the AdS algebra (see ref [13]).

$$\left[c\beta\cdot\left(\sqrt{1-\lambda r^2}\,p-\frac{e}{c}B\times\frac{r}{\sqrt{1-\lambda r^2}}-im\omega\eta^0\frac{r}{\sqrt{1-\lambda r^2}}\right)+mc^2\right]\Psi=E\beta^0\Psi \tag{4}$$

A. Scalar particle case

For a scalar particle (spin 0), the wave function consists of a vector with five components:

$$\Psi(r)=\begin{pmatrix}\Phi\\ i\psi\end{pmatrix}\text{ with }\Phi\equiv\begin{pmatrix}\phi\\ \chi\end{pmatrix}\text{ and }\psi\equiv\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} \tag{5}$$

If we replace Eq. (5) with Eq. (4), we get the coupled system that follows.

$$
\begin{aligned}
&mc^2\phi = E\,\chi + icp^+\cdot\psi,\\
&mc^2\psi = icp^-\phi, \qquad\qquad (6)\\
&mc^2\chi = E\,\phi.
\end{aligned}
$$

where the expressions of p+ and p- are given below;

$$p^{\pm} = p' \pm im\omega\frac{r}{\sqrt{1-\lambda r^2}},\; p' = \sqrt{1-\lambda r^2}\,p - \frac{e}{c}B\times\frac{r}{\sqrt{1-\lambda r^2}} \quad (7)$$

The system can now be directly transformed to the same differential equation of the 2D deformed KG case because it is uncoupled in favor of ϕ.

$$\left[\left(1-\lambda r^2\right)p^2 + \frac{\eta r^2}{1-\lambda r^2} + i\hbar\lambda r\cdot p - \frac{eB}{c}L_z - \varepsilon\right]\phi = 0, \quad (8)$$

With

$$\eta = m^2\omega^2 + \frac{e^2B^2}{4c^2} - \lambda\hbar m\omega \text{ and } \varepsilon = \frac{\left(E^2 - m^2c^4\right)}{c^2} + 2m\omega\hbar. \quad (9)$$

In accordance with condition, and using the same logic as in the KG example, the precise solution of the scalar DKP is provided by

$$\phi_n(r,\varphi) = C\,2^{l/2}\exp(il\varphi)\left(1-\lambda r^2\right)^{\mu/2}\left(\lambda r^2\right)^{l/2}P_n^{(l,\mu-1/2)}\left(1-2\lambda r^2\right) \quad (10)$$

and the associated energy spectrum is provided with the following

$$E_{n,l} = \pm mc^2\left[1 - \frac{2\omega\hbar}{mc^2} + \frac{2\hbar}{mc^2}\left\{(2n+l+1)\sqrt{\left(\omega - \frac{\lambda\hbar}{2m}\right)^2 + \omega^2} + \frac{\lambda\hbar}{2m}\left(4n(n+l+1)+2l+1\right) - \omega l\right\}\right]^{1/2} \quad (11)$$

The final expressions of Ψ(r) are as follows, while the other components are easy to evaluate:

$$\Psi_{n,l}(r,\varphi) = C\,2^{l/2}\exp(il\varphi)\left(1-\lambda r^2\right)^{\mu/2}\left(\lambda r^2\right)^{l/2} \times\left[\begin{pmatrix}1\\ \frac{E}{mc^2}\\ M(r)\\ N(r)\\ 0\end{pmatrix}P_n^{(l,\mu-1/2)}\left(1-2\lambda r^2\right) - \begin{pmatrix}0\\0\\ \Lambda(r)\\0\\0\end{pmatrix}P_n^{(l+1,\mu+1/2)}\left(1-2\lambda r^2\right)\right] \quad (12)$$

C is a normalization constant.

We can plot the variation of the energy according to its expression for different values of the parameter λ in function of the quantum number n, as it is shown in the upcoming figure;

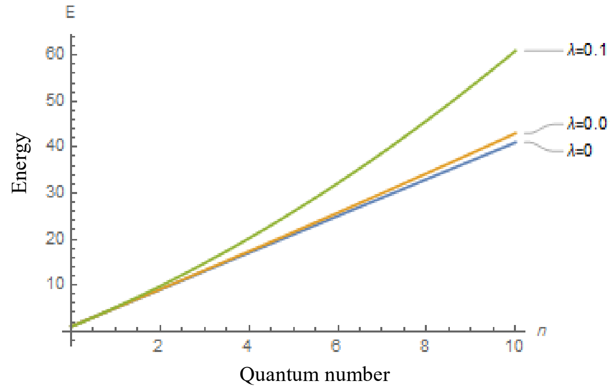

Fig. 1 Variation of the energy depending on the quantum number n for different values of the parameter of the deformation λ.

We can remark that by the increasing of the deformation parameter λ the variation is proportion to the rise of the quantum number n and for λ=0 refers to the ordinary case (i.e without deformation).

B. Vector particle case

The wave function of the spin 1 particle is a vector with ten components given as $\Psi(r)^T = \left(i\varphi, A(r), B(r), C(r)\right)$ with A_i, B_i and C_i $(i=1,2,3)$

The vectors A(r), B(r), and C(r) are represented by their components. Putting this form into Eq. (4), we get the following system:

$$
\begin{aligned}
&mc^2\varphi = -cp^-\cdot B,\\
&mc^2A = EB - cp^+\times C, \qquad\qquad (13)\\
&mc^2B = EA + cp^+\varphi,\\
&mc^2C = -cp^-\times A.
\end{aligned}
$$

To decouple this system, we remove φ, B, and C in terms of A and get

$$\left(E^2 - m^2c^4\right)A = c^2p^+\left(p^-\cdot A\right) - c^2p^+\times\left(p^-\times A\right) - \frac{1}{m^2}p^+\left[p^-\cdot\left[p^+\times\left(p^-\times A\right)\right]\right] \quad (14)$$

and we reword it to look like this:

$$\left(E^2 - m^2c^4\right)A = c^2\left[\left(p^+\cdot p^-\right)A - \left(p^+\times p^-\right)\times A\right] - \frac{1}{m^2}p^+\left[p^-\cdot\left[p^+\times\left(p^-\times A\right)\right]\right] \quad (15)$$

After the first two terms in Eq. (15) are directly calculated,

$$\varepsilon' A = \left[\left(1-\lambda r^2\right)p^2 + \left(m^2\omega^2 + \frac{e^2B^2}{4c^2} - \frac{eB}{c}\left(\lambda - \frac{2m\omega}{\hbar}\right)S_z - m\omega\hbar\lambda\right)\frac{r^2}{1-\lambda r^2} + i\hbar\lambda r\cdot p - \left(\frac{eB}{c} + 2\left(\lambda + \frac{2m\omega}{\hbar}\right)S_z\right)L_z\right]A - \frac{1}{(mc)^2}p^+\left[p^-\cdot\left[p^+\times\left(p^-\times A\right)\right]\right] \quad (16)$$

With

$$\varepsilon' = \frac{E^2 - m^2c^4}{c^2} + 2m\omega\hbar + \frac{2eB}{c}S_z \quad (17)$$

L_z and S_z represent the z-components of the orbital angular momentum and spinor, respectively.

Although there is no analytical way to solve equations like Eq. (16), we can nevertheless examine their nonrelativistic limit by treating the last term as unimportant due to its order m^{-3}. This approximation makes Eq. (16) identical to the KG and scalar DKP particles, respectively.

$$\varepsilon' A = \left[\left(1-\lambda r^2\right) p^2 + \left(m^2\omega^2 + \frac{e^2B^2}{4c^2} - \frac{eB}{c}\left(\lambda - \frac{2m\omega}{\hbar}\right) S_z - \lambda\hbar m\omega \right) \frac{r^2}{1-\lambda r^2} \right. \tag{18}$$
$$\left. + i\hbar\lambda r\cdot p - \left(\frac{eB}{c} + 2\left(\lambda + \frac{2m\omega}{\hbar}\right) S_z \right) L_z \right] A$$

When comparing this equation to Eqs. (8), we find four more terms. Two of these appear in the harmonic component (r^2): the pure spin term S_z and the interaction term 2m. The other two are spin-orbit types, with the standard 4m and an extra S_zL_z caused by space distortion. All of these words, as expected, are caused by the presence of the spin in this vectorial instance.

We utilize the same approach as in previous sections to obtain the energies.

$$E^{nr} = (2n+l+1)\hbar\sqrt{\left(\omega - \frac{\lambda\hbar}{2m}\right)^2 + \omega^2 - \frac{4}{\hbar}\omega\left(\omega - \frac{\lambda\hbar}{2m}\right) S_z} - \left(\hbar\omega - \omega S_z\right) \tag{19}$$
$$+ \frac{\lambda\hbar^2}{2m}\left(4n(n+l+1) + 2l + 1\right) - \left[\omega + \frac{2}{\hbar}\left(\omega + \frac{\lambda\hbar}{2m}\right) S_z \right] l.$$

The result demonstrates the contributions of all terms in Eq. (85) and especially those owing to the presence of both spin and deformation, as well as the additional spin-orbit term 2~SzLz, which can be read as attributable to the interaction between the two. The similar conclusion was found in the case of the Dirac oscillator with EUP [14-15].

III. Conclusion

Within the context of deformed quantum mechanics with anti-de Sitter commutation relations, we have examined the exact solutions of the 2D oscillator for DKP equations with an external uniform magnetic field in this research. The measurement of the momentum has a minimal uncertainty that is nonzero due to these AdS deformations. The analytical expressions of the bound state energies and the system's wave functions were found by the use of the Nikiforov Uvarov method. In the scalar situation, we analytically defined the system's eigenfunctions in terms of the Jacobi polynomials and the associated eigenenergies, with extra adjustments based on the deformation parameter. Based on our findings, the distorted spectrum is discrete even at high principal quantum number values, indicating that the EUP.

We restricted our investigation to the nonrelativistic limit of the vector DKP oscillator situation because it was nearly hard to discover an exact solution to the issue. In this instance, the spin's nonzero value caused correction terms to appear in addition to those resulting from the deformation. With the orbital moment (the typical spin-orbit term) and a new spin-orbit" term that represents the interaction of the spin with the orbital moment and the space deformation simultaneously, we were able to obtain the corrective terms that represent the effects of the spin and its interactions with the magnetic field. Thus, the EUP improves the spin-orbit term with a contribution proportionate to the new interaction.

References

[1] M. King, B. Zhu, and S. Tang, "Optimal path planning," *Mobile Robots*, vol. 8, no. 2, pp. 520-531, March 2001.

[2] H. Simpson, *Dumb Robots*, 3rd ed., Springfield: UOS Press, 2004, pp.6-9.

[3] M. King and B. Zhu, "Gaming strategies," in Path Planning to the West, vol. II, S. Tang and M. King, Eds. Xian: Jiaoda Press, 1998, pp. 158-176.

[4] B. Simpson, et al, "Title of paper goes here if known," unpublished.

[5] J.-G. Lu, "Title of paper with only the first word capitalized," *J. Name Stand. Abbrev.*, in press.

[6] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interface," *IEEE Translated J. Magn. Japan*, vol. 2, pp. 740-741, August 1987 [*Digest 9th Annual Conf. Magnetics Japan*, p. 301, 1982].

[7] M. Young, *The Technical Writer's Handbook*, Mill Valley, CA: University Science, 1989.