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**Numerical Simulation of Hydrodynamic Lubrication in a
Plane Slider Bearing**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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Heartfelt thanks

Dedications



I dedicate this modest work:

To my dearest Mother

To my dearest Father

To my dear wife and children Yazan and Zaid

To all my Brothers

To all my Sisters

To my dear and faithful friends

To my dear colleagues Oussama and Mohamed

To the entire Industrial Maintenance class.

Youssef

Dedications



I dedicate this modest work:

To my dearest Mother

To my dearest Father

To my dear wife and son Aouab

To all my Brothers

To all my Sisters

To my dear and faithful friends

To my dear colleagues Youcef and Mohamed

To the entire Industrial Maintenance class.

Oussama

Dedications



I dedicate this modest work:

To my dearest Mother

To my dearest Father

To all my Brothers

To all my Sisters

To my dear and faithful friends

To my dear colleagues Youcef and Oussama

To the entire Industrial Maintenance class.

Mohamed

Abstract:

This dissertation addresses the need for reliable predictive tools for hydrodynamic lubrication in plane slider bearings, where thin-film pressure generation governs load support, friction reduction, and wear prevention. Although the Reynolds equation remains the classical design model, its agreement with higher-fidelity Navier–Stokes/CFD simulations under varying operating conditions is insufficiently quantified, forming the research gap of this work. A convergent wedge bearing was therefore investigated using three complementary approaches: the analytical Reynolds solution, a MATLAB finite-difference solver, and a two-dimensional ANSYS Fluent CFD model. Six cases were studied by varying sliding velocity and lubricant properties: $U = 1, 5$, and 10 m/s for a reference oil $\rho = 850$ kg/m³, $\mu = 0.085$ Pa·s, then $U = 10$ m/s with SAE 10W $\mu = 0.035$ Pa·s, ISO VG 46 $\mu = 0.046$ Pa·s, and SAE 40 $\mu = 0.145$ Pa·s. The three methods showed near-perfect agreement, validating the Reynolds-based numerical model. Peak pressures were 3.126×10^5 , 1.564×10^6 , 3.132×10^6 , 1.294×10^6 , 1.698×10^6 , and 5.338×10^6 Pa, respectively, with the maximum located at $X \approx 0.87$. The study concludes that viscosity and sliding speed strongly control pressure generation, and SAE 40 provides the highest load-carrying capacity.

Keywords: Hydrodynamic lubrication; Plane slider bearing; Reynolds equation; Navier-Stokes equations; Finite difference method; CFD; ANSYS Fluent.

المخلص:

تتناول هذه المذكرة الحاجة إلى أدوات تنبؤ موثوقة للتزييت الهيدروديناميكي في المحامل الانزلاقية المستوية، حيث يتحكم توليد الضغط داخل الغشاء الرقيق في دعم الحمولة، تقليل الاحتكاك، والحد من التآكل. وعلى الرغم من أن معادلة رينولدز تُعد النموذج الكلاسيكي في التصميم، إلا أن مدى توافقها مع محاكاة نافير-ستوكس/CFD عالية الدقة تحت شروط تشغيل مختلفة لا يزال غير محدد بشكل كافٍ، وهو ما يمثل فجوة البحث في هذا العمل. لذلك، تمت دراسة محمل ذي فجوة إسفينية متقاربة باستخدام ثلاث مقاربات متكاملة: الحل التحليلي لمعادلة رينولدز، حل عددي بطريقة الفروق المحدودة مطور ببرنامج MATLAB، ونموذج CFD ثنائي الأبعاد باستخدام ANSYS Fluent. دُرست ست حالات من خلال تغيير سرعة الانزلاق وخصائص المزلق $U = 1$ و 5 و 10 m/s لزيت مرجعي بكثافة $\rho = 850$ kg/m³ ولزوجته $\mu = 0.085$ Pa·s، ثم $U = 10$ m/s مع زيوت SAE 10W بلزوجته $\mu = 0.035$ Pa·s، و ISO VG 46 بلزوجته $\mu = 0.046$ Pa·s، و SAE 40 بلزوجته $\mu = 0.145$ Pa·s. أظهرت الطرق الثلاث توافقاً شبه تام، مما يؤكد صلاحية النموذج العددي المعتمد على معادلة رينولدز. بلغت قيم الضغط الأعظمي 3.126×10^5 ، 1.564×10^6 ، 3.132×10^6 ، 1.294×10^6 ، 1.698×10^6 ، و 5.338×10^6 Pa على التوالي، وكان موضع الضغط الأعظمي عند $X \approx 0.87$ وتخلص الدراسة إلى أن اللزوجة وسرعة الانزلاق تتحكمان بقوة في توليد الضغط، وأن زيت SAE 40 يوفر أعلى قدرة على حمل الحمولة.

الكلمات المفتاحية: التزييت الهيدروديناميكي؛ المحمل الانزلاقي المستوي؛ معادلة رينولدز؛ معادلات نافير-ستوكس؛ طريقة الفروق المحدودة؛ ديناميكا الموائع الحسابية؛ ANSYS Fluent.

Table of Contents

Table of Contents	i
List of Figures	iv
List of Tables.....	viii
Nomenclature	ix
General Introduction	1
Chapter I: Introduction and Problem Definition.....	4
I.1 Introduction.....	4
I.2 Historical Evolution	4
I.3 Lubrication	5
I.4 Lubrication regime	6
I.5 Hydrodynamic Bearings	7
I.6 Different Types of Bearings.....	7
I.7 Industrial Context and Motivation	8
I.7.1 Lubrication in Rotating Machinery	8
I.7.2 Hydrodynamic Lubrication in Gearboxes and Bearings.....	9
I.7.3 The Role of Hydrodynamic Theory	9
I.8 Problem Statement	11
I.8.1 The Plane Slider Bearing as a Canonical Configuration	11
I.8.2 Need for Predictive Tools	11
Computational Approaches: Reynolds Equation vs Navier-Stokes	12
I.9 Research Objectives.....	13
I.10 Scope and Fundamental Assumptions	13
Fluid Properties	13
I.11 Significance of the Research.....	14

I.12 Conclusion	15
Chapter II: Theory and Mathematical Modeling	17
II.1 Introduction	17
II.2 Lubrication Regimes and Hydrodynamic Lubrication	17
II.3 Geometry and Description of the Fluid-Film Bearing.....	18
II.4 Governing Equations of Fluid Flow (Navier-Stokes Equations).....	19
II.5 Lubrication Assumptions.....	20
II.6 Non-Dimensional Form of the Governing Equations.....	21
II.7 Reduced Lubrication Equations.....	22
II.8 Boundary Conditions.....	22
II.9 Velocity Distribution in the Lubricant Film	23
II.10 Flow Rates Across the Film Thickness	24
II.11 Derivation of the Reynolds Equation	24
II.12 Reynolds Equation for Constant Viscosity and Rigid Surfaces	26
II.13 Physical Interpretation of the Reynolds Equation	26
II.14 Conclusion	27
Chapter III: Numerical Methods and Implementation	29
III.1 Introduction	29
III.2 Common Physical Configuration	29
III.3 Method I: Analytical Solution of the Reynolds Equation	30
III.4 Method II: Finite Difference Solution in MATLAB.....	31
III.5 Method III: Two-Dimensional CFD Simulation in ANSYS Fluent	33
III.6 Geometry	33
III.7 Meshing.....	36
III.8 Physics setup (Ansys Fluent)	40
III.9 Boundary conditions	42

III.10 Solution controls	43
III.11 Extraction of comparable outputs	46
III.12 CFD verification.....	47
III.13 Conclusion.....	48
Chapter IV: Results, Validation, and Engineering Recommendations.....	50
IV.1 Introduction.....	50
IV.2 Summary of the Used Parameters	50
IV.3 Parametric Study	51
IV.3.1 Case 1:.....	51
IV.3.2 Case 2:	56
IV.3.3 Case 3:	61
IV.3.4 Case 4: (Oil SAE 10W $\mu = 0.035$ Pa·s and $\rho = 870$ kg/m ³).....	66
IV.3.5 Case 5: (Oil ISO VG 46 $\mu = 0.046$ Pa·s and $\rho = 875$ kg/m ³).....	71
IV.3.6 Case 6: (Oil SAE 40 $\mu = 0.145$ (Pa·s) and $\rho = 890$ kg/m ³).....	76
IV.4 Conclusion	81
General Conclusion	83
References	85

List of Figures

Figure I-1:Transport of a statue, tomb of Tchuti-Hetep, El-Bersheh, Egypt, 1880 BC [2].	5
Figure I-2 : Illustration of a typical Stribeck curve.	6
Figure I-3: Typical Components of a Journal Bearing Housing.	7
Figure I-4: Classification of Common Bearing Types.	8
Figure I-5: Hydrodynamic Lubrication in Gearboxes and Rolling Bearings.	9
Figure I-6: Hydrodynamic Lubrication Mechanism and Reynolds Equation.	10
Figure I-7: Representation of the Plane Slider Bearing as a Canonical Hydrodynamic Lubrication Configuration.	11
Figure II-1: Fundamental lubrication modes in bearing interfaces.	18
Figure II-2: Geometry of the studied fluid film bearing.	19
Figure III-1: Geometry of the Steady Plane Slider Bearing.	30
Figure III-2: The first step to open ANSYS DesignModeler.	34
Figure III-3: Two-Dimensional Coordinate System.	34
Figure III-4: Domain Spécification.	35
Figure III-5: Domain Parametre Input.	35
Figure III-6: The Final Geometry View in Ansys DesignModeler.	36
Figure III-7: The first step to open the ANSYS Meshing.	36
Figure III-8: Initial Interface of ANSYS Meshing.	37
Figure III-9 Sizing Selection step is Ansys Meshing.	37
Figure III-10: Number of Divisions Selection.	38
Figure III-11: Horizontal Division of Both Sides.	38
Figure III-12 Division of the Upper and Lower Edges.	39
Figure III-13: Final Mesh View in Ansys Meshing.	39
Figure III-14: Naming of Edges.	40
Figure III-15: The first step to open ANSYS Fluent.	40

Figure III-16: The Imported Mesh in the Ansys Fluent.	41
Figure III-17: Selection of Important Options in the General Tab.	41
Figure III-18: Selection of the Laminar Viscous Model.	42
Figure III-19: Assignment of Viscosity and Density Values.	42
Figure III-20: Input of Velocity Value.	43
Figure III-21: SIMPLE Method Selection.	43
Figure III-22: Improving Solution Accuracy.	44
Figure III-23: Solution Initialization Selection.	44
Figure III-24: Increasing the Number of Iterations.	45
Figure III-25: Convergence of Continuity and Velocity Residuals.	45
Figure III-26: Pressure Distribution along the Moving Wall.	46
Figure III-27: Writing the results to a file.	46
Figure III-28: Comparison between Analytical, FDM, and CFD Pressure Profiles.	47
 Figure IV-1: Hydrodynamic pressure contour (Case 1).	 51
Figure IV-2 Hydrodynamic velocity u contour (Case 1).	52
Figure IV-3: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 1).	53
Figure IV-4: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 1).	54
Figure IV-5: Comparison of dimensional pressure profiles along the normalized contact length (Case 1).	55
Figure IV-6: Hydrodynamic pressure contour (Case 2).	56
Figure IV-7: Hydrodynamic velocity u contour (Case 2).	57
Figure IV-8: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 2).	58
Figure IV-9: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 2).	59

Figure IV-10: Comparison of dimensional pressure profiles along the normalized contact length (Case 2).	60
Figure IV-11: Hydrodynamic pressure contour (Case 3).	61
Figure IV-12: Hydrodynamic velocity u contour (Case 3).	62
Figure IV-13: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 3).	63
Figure IV-14: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 3).	64
Figure IV-15: Comparison of dimensional pressure profiles along the normalized contact length (Case 3).	65
Figure IV-16: Hydrodynamic pressure contour (Case 4).	66
Figure IV-17: Hydrodynamic velocity u contour (Case 4).	67
Figure IV-18: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 4).	68
Figure IV-19: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 4).	69
Figure IV-20: Comparison of dimensional pressure profiles along the normalized contact length (Case 4).	70
Figure IV-21: Hydrodynamic pressure contour (Case 5).	71
Figure IV-22: Hydrodynamic velocity u contour (Case 5).	72
Figure IV-23: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 5).	73
Figure IV-24: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 5).	74
Figure IV-25: Comparison of dimensional pressure profiles along the normalized contact length (Case 5).	75
Figure IV-26: Hydrodynamic pressure contour (Case 6).	76
Figure IV-27: Hydrodynamic velocity u contour (Case 6).	77

Figure IV-28: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 6).....	78
Figure IV-29: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 6).	79
Figure IV-30: Comparison of dimensional pressure profiles along the normalized contact length (Case 6).	80

List of Tables

Table IV-1: Operating Conditions and Lubricant Properties for the Simulation Cases.....	50
Table IV 2: Summary of the Main Numerical Results for All Simulation Cases.....	80

Nomenclature

Latin Symbols

Symbol	Description	Unit
(x)	Coordinate in the sliding direction	m
(y)	Coordinate across the lubricant-film thickness	m
(z)	Transverse coordinate of the bearing	m
(u)	Lubricant velocity component in the (x)-direction	m/s
(v)	Lubricant velocity component in the (y)-direction	m/s
(w)	Lubricant velocity component in the (z)-direction	m/s
(U)	Sliding velocity of the moving surface	m/s
(V)	Normal velocity of the moving surface	m/s
(W)	Transverse velocity of the moving surface	m/s
(p)	Hydrodynamic pressure in the lubricant film	Pa
($p_{\max}$)	Maximum hydrodynamic pressure	Pa
(h)	Local lubricant-film thickness	m
(h(x,z,t))	Spatially and temporally varying film thickness	m
(h_i)	Inlet film thickness	m
(h_o)	Outlet/minimum film thickness	m
(Δh)	Difference between inlet and outlet film thicknesses	m
(L)	Bearing length in the sliding direction	m
(L_x)	Characteristic bearing length in the (x)-direction	m
(L_z)	Bearing width in the transverse direction	m
(q_x)	Lubricant flow rate per unit width in the (x)-direction	m ² /s
(q_z)	Lubricant flow rate per unit width in the (z)-direction	m ² /s
(F_f)	Friction force	N
(W_L)	Load-carrying capacity of the bearing	N
(Re)	Reynolds number	—
(P)	Dimensionless pressure	—
(H)	Dimensionless film thickness	—
(X)	Dimensionless coordinate in the sliding direction	—
(Z)	Dimensionless transverse coordinate	—
(N_x)	Number of grid nodes in the (x)-direction	—
(N_z)	Number of grid nodes in the (z)-direction	—
(Δx)	Grid spacing in the (x)-direction	m
(Δz)	Grid spacing in the (z)-direction	m
(i, j)	Grid-node indices in the finite difference method	—
(k)	Iteration counter	—

Greek Symbols

Symbol	Description	Unit
(μ)	Dynamic viscosity of the lubricant	Pa·s
(ρ)	Lubricant density	kg/m ³
(α)	Small inclination angle or aspect-ratio parameter	rad or —
(ε)	Convergence tolerance or numerical residual	—
(ω)	Relaxation factor used in the iterative solver	—
(τ)	Shear stress in the lubricant film	Pa

Abbreviations

Abbreviation	Meaning
CFD	Computational Fluid Dynamics
FDM	Finite Difference Method
MATLAB	Matrix Laboratory
SIMPLE	Semi-Implicit Method for Pressure-Linked Equations
SAE	Society of Automotive Engineers
ISO VG	International Standards Organization Viscosity Grade
2D	Two-Dimensional
NS	Navier–Stokes
RE	Reynolds Equation

Subscripts and Superscripts

Symbol	Meaning
(i)	Inlet condition or grid index in the (x)-direction
(o)	Outlet condition
(\max)	Maximum value
(x)	Component or derivative in the sliding direction
(y)	Component or derivative across the film thickness
(z)	Component or derivative in the transverse direction
(n)	Current iteration level
(n+1)	Updated iteration level
(*)	Dimensionless quantity

GENERAL INTRODUCTION

General Introduction

Modern industrial engineering is heavily reliant on the reliability, efficiency, and longevity of high-speed rotating machinery. Component interfaces within critical sub-systems, such as gearboxes, industrial electric motors, and heavy-duty turbomachinery, are subjected to extreme mechanical stresses. To mitigate friction, minimize structural wear, and prevent catastrophic mechanical failure, the implementation of effective lubrication strategies is imperative. Among the various lubrication regimes, hydrodynamic lubrication stands out as the primary mechanism for supporting high mechanical loads by ensuring complete physical separation between mating structural surfaces via a pressurized fluid film. Understanding, predicting, and optimizing the behavior of this fluid film remains a core objective in tribological research.

While classical lubrication theory offers solid foundational models, the continuous demands for higher operational speeds and variable lubricant properties necessitate the development of robust predictive engineering tools. Historically, designers have relied on simplified closed-form analytical solutions to estimate bearing performance. However, modern engineering demands a more integrated approach, where traditional mathematical models are paired with localized numerical architectures. This thesis addresses this need by establishing a comprehensive evaluation and cross-validation framework that bridges classical analytical theory, custom numerical programming, and advanced computational fluid dynamics (CFD).

To systematically explore these concepts, this dissertation is structured into four main interconnected chapters:

- **Chapter I** establishes the industrial context, historical evolution, and fundamental motivations behind hydrodynamic lubrication. It introduces the plane slider bearing as a canonical configuration to define the core problem statement, research objectives, and underlying fluid assumptions.
- **Chapter II** presents the rigorous mathematical modeling governing fluid-film behavior. Beginning with the complete Navier-Stokes equations, the systematic simplification under classical lubrication assumptions is detailed, culminating in the formal derivation and physical interpretation of the Reynolds equation.
- **Chapter III** outlines the implementation of the three-pronged methodological framework used to solve the lubrication problem. It details the exact analytical solution

the discretization of the domain via an in-house Finite Difference Method (FDM) algorithm developed in MATLAB, and the configuration of a full two-dimensional CFD mesh and physics setup within ANSYS Fluent.

Chapter IV executes a comprehensive parametric study across six distinct operational cases. By systematically varying sliding velocities and lubricant properties (including commercial SAE 10W, ISO VG 46, and SAE 40 oils), this final chapter cross-examines the predictive accuracy, spatial stability, and physical convergence of all three computational approaches.

Chapter I

Introduction and Problem Definition

I. Chapter I: Introduction and Problem Definition

I.1 Introduction

Lubrication is essential in mechanical systems where relative motion occurs between surfaces. It reduces friction, wear, and energy losses by creating a thin fluid film that separates moving parts. Over time, lubrication theory has evolved significantly, leading to the development of hydrodynamic lubrication theory, which explains how pressure is generated in a lubricant film to support external loads.

Depending on operating conditions, lubrication may occur in three regimes: boundary lubrication, mixed lubrication, and hydrodynamic lubrication. Among these, hydrodynamic lubrication is the most desirable because the surfaces are completely separated by a fluid film, minimizing friction and wear.

Hydrodynamic bearings are widely used in rotating machinery such as turbines, pumps, compressors, and gearboxes. Understanding lubrication behavior in these systems is important for improving reliability and efficiency.

In lubrication analysis, the Reynolds equation is commonly used to describe pressure generation in thin fluid films. However, modern computational methods also allow the use of the Navier–Stokes equations through computational fluid dynamics (CFD). This research focuses on the numerical simulation of hydrodynamic lubrication in a plane slider bearing and compares simplified lubrication models with more detailed computational approaches.

I.2 Historical Evolution

Prehistoric humans produced the earliest simple bearings, consisting of a wooden shaft rotating inside a cavity made of wood or stone. Archaeological evidence of such systems dates back to more than 2500 BC in Mesopotamia. In ancient Egypt, heavy loads were transported over lubricated wooden paths, as illustrated by bas-reliefs dating from around 2400 BC [1].

The Greeks and Romans further developed the use of the wheel and employed vegetable oils, animal fats, and bitumen as lubricants. During the Middle Ages, bronze was commonly used to reduce friction in mechanical systems. Later, Leonardo da Vinci made important observations

and is often considered one of the pioneers of tribology due to his early studies on friction and lubrication [1,2].

During the Industrial Revolution, Charles-Augustin de Coulomb conducted systematic studies on friction and established friction coefficients that remain relevant today. In the 19th century, tribology advanced significantly with the widespread use of plain bearings. Petrov highlighted the essential role of viscosity in lubrication, while Tower demonstrated that friction in lubricated bearings differs from that between dry solid surfaces [2].

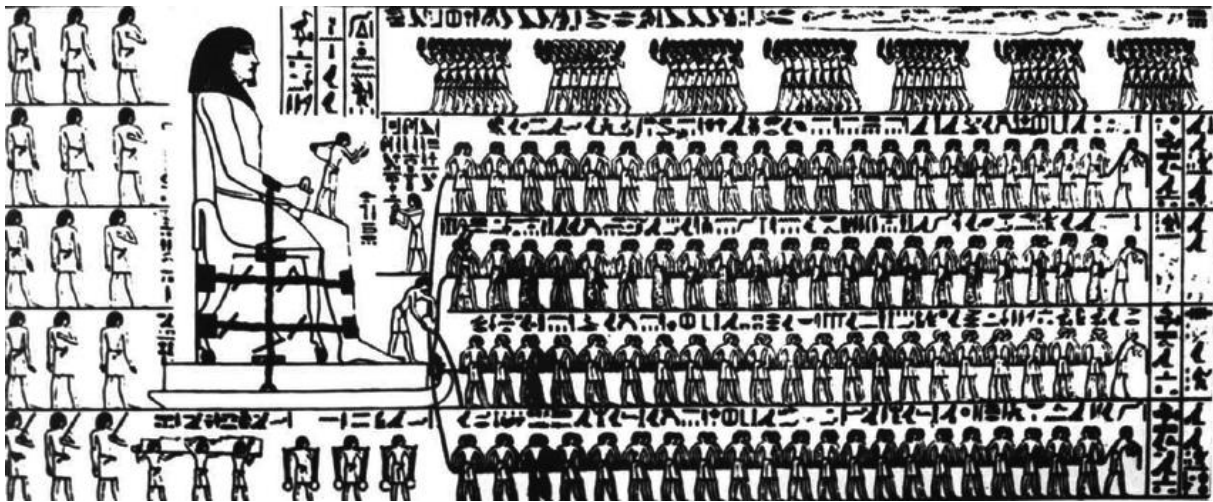


Figure I-1:Transport of a statue, tomb of Tchuti-Hetep, El-Bersheh, Egypt, 1880 BC [2].

I.3 Lubrication

Lubrication is a set of techniques used to reduce friction and wear between two elements in contact and moving relative to each other. It often helps remove part of the thermal energy generated by this friction and also prevents corrosion.

In such situations, fluid flows are parallel to the surfaces, which simplifies their description and calculation. The term lubrication is used when the lubricant (mechanical) is liquid, whereas the term greasing is used when it is compact or semi-solid.

In mechanics, metal or ceramic parts are lubricated with a fatty substance, such as oil or grease. Lubricants can be liquid, paste-like, or solid products, generally of mineral origin.

To present a classification of the different lubrication phenomena, it is convenient to use, for both surface contacts and Hertzian contacts, the Stribeck curve (Figure 2), whose first representation was given in 1902 for the case of a plain bearing [3].

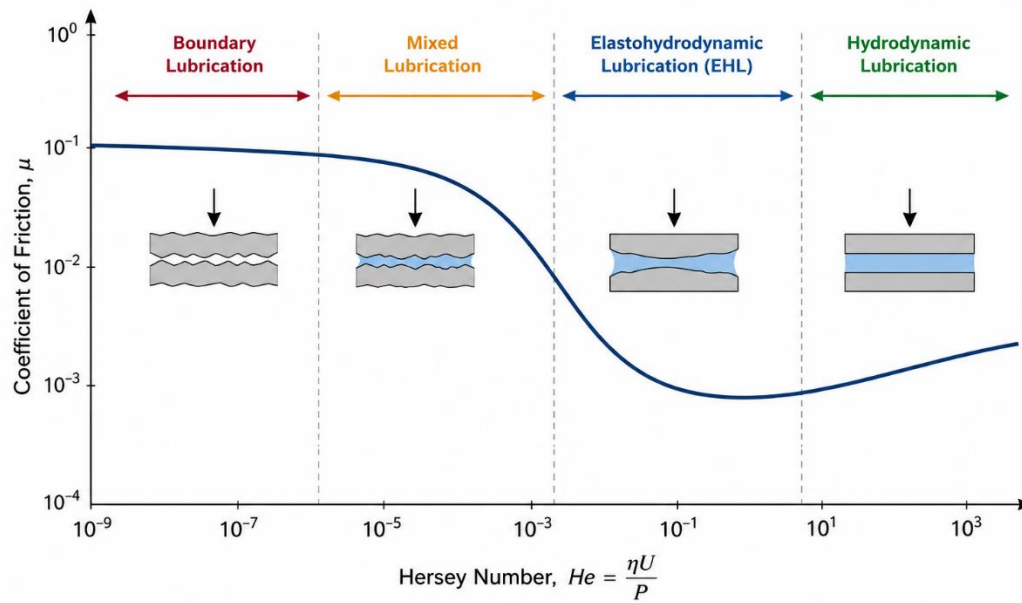


Figure I-2 : Illustration of a typical Stribeck curve.

I.4 Lubrication regime

- **Boundary Lubrication**

Boundary lubrication occurs when the load-bearing surfaces come into contact. Boundary lubrication can occur when the relative speed between mating surfaces is low, there are high loads, or changes in direction.

Anti-wear (AW) or extreme pressure (EP) additives can reduce friction and wear to acceptable levels by forming sacrificial solid-film barriers [4].

- **Mixed Lubrication**

Mixed film lubrication describes the condition where the asperities (peaks) of two surfaces come into contact though a lubricating film is present. The lubrication film is thicker than in boundary lubrication, and it is a combination of hydrodynamic and boundary lubrication.

Friction can be lower than thick film hydrodynamic lubrication, but mixed film lubrication requires AW additives to reduce wear [4].

- **Hydrodynamic Lubrication**

A lubrication film completely separates two load-bearing surfaces. With no metal-to-metal contact, machine life depends on oil cleanliness. Friction increases with increasing film thickness [4].

I.5 Hydrodynamic Bearings

Bearings are machine components used to guide rotating shafts. There are two main types of bearings: plain bearings and rolling-element bearings. In plain bearings, the shaft rests on a bushing and is separated from it by a lubricant film, whereas in rolling-element bearings, rolling bodies (balls or rollers) separate the rotating inner ring from the outer housing.

Depending on the direction of the load relative to the rotating shaft, we distinguish between radial bearings (where the load is radial, generally called bearings) and thrust bearings (where the load is axial).

Here, we will focus only on fluid bearings, and more specifically on hydrodynamic bearings and thrusts, in which a thin fluid film separates the surfaces in relative motion [5].

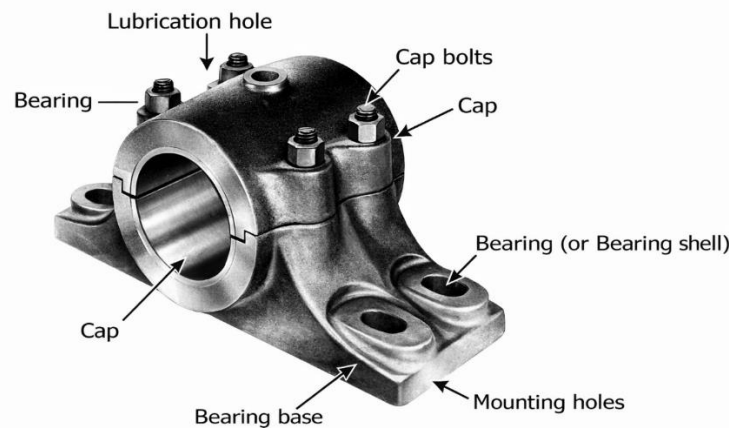


Figure I-3: Typical Components of a Journal Bearing Housing.

I.6 Different Types of Bearings

A bearing is a support or guide that determines the position of a moving part relative to other components in a mechanism. As illustrated in Figure I-4, the main types of bearings in practice are:

- Rolling-element bearings
- Dry bearings
- Porous bearings
- Magnetic bearings
- Fluid bearings (including hydrodynamic, hydrostatic, and hybrid)



Figure I-4: Classification of Common Bearing Types.

I.7 Industrial Context and Motivation

I.7.1 Lubrication in Rotating Machinery

Lubrication represents one of the most critical aspects of mechanical engineering design, particularly in rotating machinery where relative motion between surfaces generates friction, wear, and heat. In modern industrial applications ranging from automotive gearboxes and turbomachinery to wind turbine drivetrains and aerospace propulsion systems the performance, efficiency, and longevity of components are intrinsically linked to the effectiveness of the lubrication system [6,7]. The fundamental role of a lubricant extends beyond merely reducing friction; it encompasses load support, heat dissipation, contaminant removal, corrosion protection, and vibration damping [6-8]

Among the various lubrication regimes, hydrodynamic lubrication (also known as full-film or thick-film lubrication) represents the ideal operating condition in which the mating surfaces are completely separated by a continuous fluid film of sufficient thickness to prevent direct solid-solid contact [9]. This regime is characterized by the formation of a pressure field within the lubricant film that develops dynamically as a result of the relative motion between the surfaces, the fluid viscosity, and the converging-diverging geometry of the lubricant gap [10]. Unlike boundary or mixed lubrication regimes where surface asperities interact and wear mechanisms dominate hydrodynamic lubrication minimizes wear, reduces friction coefficients, and extends component service life [11].

I.7.2 Hydrodynamic Lubrication in Gearboxes and Bearings

Gearboxes constitute essential components in power transmission systems across numerous industries. In wind turbine gearboxes, for example, journal bearings operating under heavy loads and variable speeds must maintain adequate lubricant films to prevent catastrophic failure [12]. Similarly, high-speed turbomachinery bearings in aircraft engines and industrial compressors rely on hydrodynamic pressure generation to support substantial dynamic loads while minimizing power loss. The criticality of these applications has intensified research efforts toward understanding, predicting, and optimizing lubrication performance under diverse operating conditions.

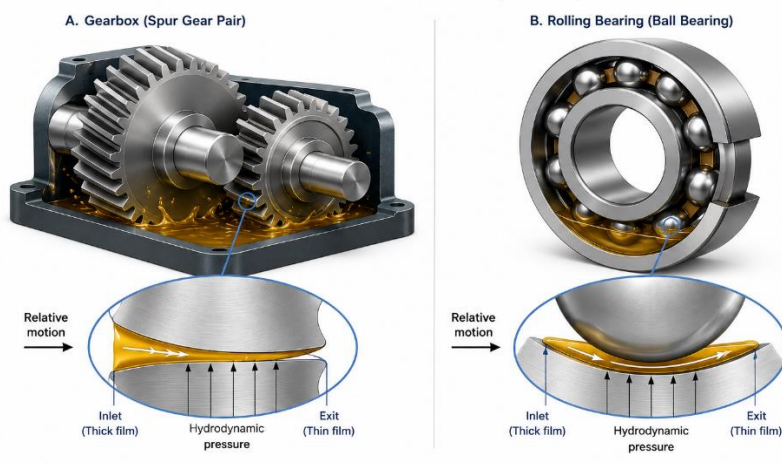


Figure I-5: Hydrodynamic Lubrication in Gearboxes and Rolling Bearings.

The economic and safety implications of bearing failures are substantial. Studies have shown that bearing-related failures account for approximately 40-50% of gearbox breakdowns in industrial settings. In wind energy applications, unscheduled maintenance due to bearing failures can result in downtime costs exceeding €100,000 per incident, highlighting the need for robust predictive models that can inform design decisions and maintenance strategies. Furthermore, as efficiency regulations tighten globally, particularly in automotive and aerospace sectors, reducing friction losses in lubricated contacts has become paramount for meeting emissions targets and improving fuel economy [13, 14].

I.7.3 The Role of Hydrodynamic Theory

Hydrodynamic lubrication theory, pioneered by Osborne Reynolds in 1886, provides the mathematical framework for predicting the behaviour of lubricant films in converging-diverging gaps. The Reynolds equation, derived through simplification of the Navier-Stokes

equations under specific assumptions (thin film, negligible inertial effects, laminar flow, etc.), enables the calculation of pressure distribution, load-carrying capacity, friction force, and minimum film thickness—all critical parameters for bearing design and performance assessment [15,16].

The predictive capability of lubrication theory offers several advantages:

- **Load Support Prediction:** Determining whether a bearing can support applied loads without excessive deformation or film breakdown [17].
- **Wear Avoidance:** Ensuring minimum film thickness exceeds surface roughness magnitudes to prevent asperity contact and adhesive/abrasive wear [18].
- **Friction and Power Loss:** Calculating viscous shear forces to optimize bearing geometry and operating conditions for minimal energy dissipation.
- **Thermal Management:** Assessing heat generation rates and temperature distributions to prevent lubricant degradation and thermal instability [19,20].

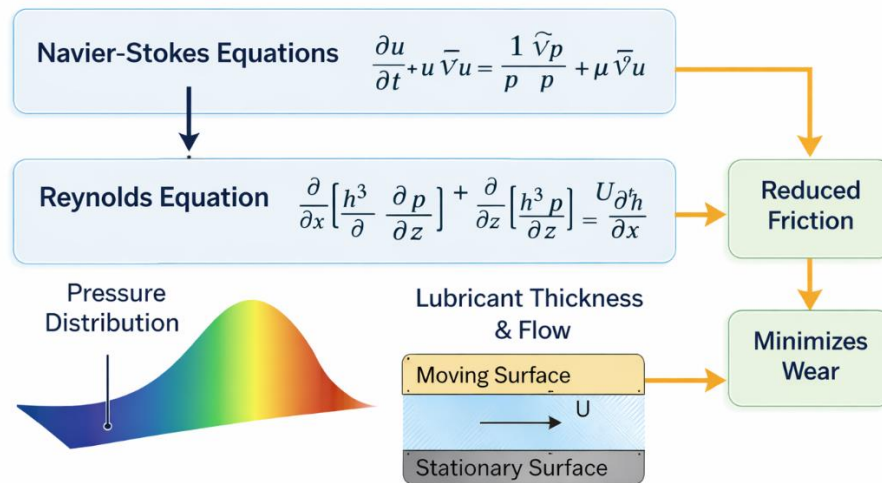


Figure I-6: Hydrodynamic Lubrication Mechanism and Reynolds Equation.

Despite the widespread application of Reynolds-based analytical and numerical models, several challenges persist. Complex geometries, non-Newtonian fluid behaviour, surface roughness effects, thermal-viscosity coupling, and transient operating conditions often necessitate advanced computational approaches that balance accuracy with computational efficiency [21,22].

I.8 Problem Statement

I.8.1 The Plane Slider Bearing as a Canonical Configuration

Among the various bearing types journal bearings, thrust bearings, squeeze-film dampers the plane slider bearing (also referred to as an inclined plane slider or wedge slider) represents a fundamental configuration for studying hydrodynamic lubrication phenomena. This geometry consists of a stationary inclined surface (the slider) and a moving flat surface (the runner), with the lubricant occupying the converging-diverging wedge-shaped gap between them (Figure I.7).

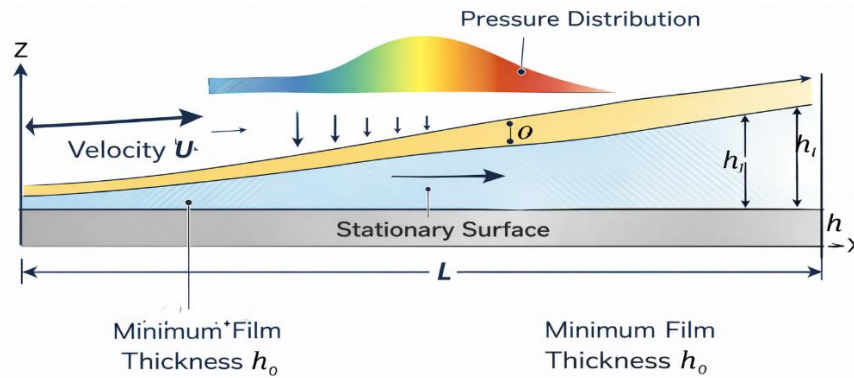


Figure I-7: Representation of the Plane Slider Bearing as a Canonical Hydrodynamic Lubrication Configuration.

The plane slider bearing serves as a canonical test case for several reasons:

- **Geometric Simplicity:** The one-dimensional variation in film thickness allows for semi-analytical solutions and straightforward numerical implementation.
- **Physical Insight:** The wedge geometry clearly demonstrates the hydrodynamic pressure generation mechanism the Reynolds wedge effect where converging flow induces positive pressure buildup.
- **Design Relevance:** Slider bearings find direct application in thrust bearings, hard disk drive sliders, and various industrial sliding contacts.
- **Validation Benchmark:** Analytical solutions exist for simplified cases (infinite width, isoviscous, isothermal), enabling rigorous validation of numerical models.

I.8.2 Need for Predictive Tools

Accurate prediction of hydrodynamic lubrication performance requires determining several key parameters:

- Pressure Distribution $p(x)$: The spatial variation of hydrodynamic pressure governs load capacity and stress distributions on bearing surfaces.
- Load-Carrying Capacity W : The integral of pressure over the bearing surface, representing the maximum sustainable load.
- Friction Force F : The viscous shear resistance opposing motion, directly impacting power consumption and efficiency.
- Minimum Film Thickness h_{min} : The critical parameter for wear prevention if h_{min} falls below surface roughness levels, mixed or boundary lubrication ensues.
- Temperature Distribution: Thermal effects due to viscous dissipation can alter lubricant viscosity and film behaviour, necessitating thermal-hydrodynamic coupling.
- Traditional analytical approaches such as the Sommerfeld, Ocvirk, and DuBois-Ocvirk approximations for journal bearings provide closed-form solutions under restrictive assumptions. However, real-world bearing configurations often involve [23].
- Finite geometry effects require two-dimensional or three-dimensional analysis.
- Non-Newtonian lubricants exhibiting shear-thinning or viscoelastic behaviour [21].
- Thermal-viscosity coupling, where temperature-dependent viscosity alters pressure profiles [24].
- Surface texturing and roughness influence local film thickness and cavitation [25, 26].
- Transient loading and squeeze-film effects in dynamic applications.

These complexities necessitate numerical simulation tools that can accommodate arbitrary geometries, realistic boundary conditions, and coupled multi-physics phenomena. Furthermore, engineers require design-oriented charts and parametric studies that map bearing performance across operating envelopes, facilitating rapid design iterations and optimization [27].

Computational Approaches: Reynolds Equation vs Navier-Stokes

Two primary computational approaches dominate lubrication analysis:

- 1) **Reynolds Equation Methods:** These methods solve the reduced Reynolds equation a second-order ordinary or partial differential equation using finite difference, finite element, or finite volume discretization. Advantages include computational efficiency, straightforward implementation, and extensive validation for lubrication-specific problems. Limitations arise

when Reynolds' assumptions (thin film, negligible inertia, fully developed flow) are violated, such as in high-speed applications, entrance regions, or highly curved geometries [28].

- 2) Navier-Stokes CFD Methods:** Full-field computational fluid dynamics (CFD) solves the complete Navier-Stokes momentum and continuity equations without thin-film assumptions [15]. Modern CFD software packages such as ANSYS Fluent, COMSOL Multiphysics, and Open FOAM enable to resolve the complex flow features, including recirculation zones, inertial effects, and three-dimensional phenomena. However, CFD approaches impose significantly higher computational costs often orders of magnitude greater than Reynolds solvers and require careful meshing of thin film regions with high aspect ratios.

This thesis seeks to bridge these approaches by:

- Developing and validating a Reynolds-based MATLAB solver optimized for rapid parametric analysis.
- Constructing a reference CFD model in ANSYS Fluent to assess Reynolds equation validity limits.
- Providing quantitative error bounds as functions of geometric and operating parameters.

I.9 Research Objectives

The overarching goal of this thesis is to develop, validate, and apply numerical tools for predicting hydrodynamic lubrication performance in plane slider bearings, with emphasis on understanding the validity limits of Reynolds-based approximations through comparison with full Navier-Stokes CFD simulations. Specific objectives include:

Objective 1: Derive and Implement a Reynolds-Based Numerical Model

Objective 2: Build a Reference CFD Model in ANSYS Fluent

I.10 Scope and Fundamental Assumptions

To maintain focus and ensure tractable analysis within the constraints of a master's thesis, the following scope and assumptions are adopted:

Fluid Properties

- **Newtonian Fluid:** The lubricant exhibits a linear relationship between shear stress and shear rate, with constant viscosity (isoviscous case) or temperature-dependent viscosity (thermal case). Non-Newtonian effects (shear-thinning, viscoelasticity) are excluded.

- Incompressible Flow: Density is constant; compressibility effects relevant at high pressures or gas lubrication are neglected.
- Single-Phase: Only liquid lubricant is considered; multiphase flow is not explicitly resolved.
- Laminar Flow: Reynolds numbers ($Re = \rho UL/\mu$) remain below transition thresholds (~ 2300 for channel flow); turbulent lubrication flows are not considered [29].
- Steady-State: Time-dependent effects (squeeze action, transient loads, start-up transients) are excluded; the analysis focuses on continuous steady sliding.

I.11 Significance of the Research

This research addresses a fundamental gap in engineering education and practice: the lack of accessible, validated, and comparative tools that bridge classical lubrication theory with modern CFD capabilities. While commercial software packages (ANSYS, COMSOL, STAR-CCM+) offer powerful CFD engines, their complexity and computational cost often deter their use in routine bearing design, where rapid iteration and parametric exploration are essential. Conversely, simplified Reynolds-based tools remain prevalent but lack quantitative guidance on when their predictions become unreliable.

By providing dual modelling frameworks a fast Reynolds solver and a rigorous CFD benchmark and by explicitly quantifying their agreement and divergence across parameter spaces, this thesis empowers engineers to make informed decisions about modelling fidelity. For educational purposes, the MATLAB toolbox serves as a pedagogical instrument, enabling students to visualize and interact with lubrication phenomena without requiring expensive commercial licenses. For industrial applications, the design charts and validity maps facilitate preliminary bearing design while flagging cases that merit higher-fidelity analysis or experimental validation.

In the context of Industry 4.0 and digital twins, validated fast-running models like the MATLAB solver can be integrated into real-time monitoring and predictive maintenance frameworks, where computational speed is critical for online decision-making. Simultaneously, CFD models can be invoked selectively for high-risk scenarios or design refinement, optimizing the computational resource allocation.

I.12 Conclusion

This chapter presented the background and motivation of the study on hydrodynamic lubrication in a plane slider bearing. The main lubrication regimes and the role of hydrodynamic bearings in industrial machinery were discussed. The importance of predictive models for analyzing lubrication performance was also highlighted.

The research objectives, assumptions, and scope of the study were defined, along with the computational approaches used in this work. The development of a MATLAB solver based on the Reynolds equation and its comparison with a CFD reference model were introduced as the main contributions of the thesis.

These elements establish the theoretical and methodological framework for the numerical analysis presented in the next chapters.

Chapter II

Theory and Mathematical Modeling

II. Chapter II: Theory and Mathematical Modeling

II.1 Introduction

This chapter presents the fundamental derivation and application of the governing equations for hydrodynamic lubrication in a plane slider bearing. The analysis begins by introducing the bearing geometry, where the upper surface moves relative to a stationary lower surface, and by simplifying the fluid-flow equations under the assumptions of laminar flow, Newtonian fluid behavior, and incompressibility.

The momentum equations are then integrated using appropriate boundary conditions to obtain the velocity distributions and corresponding volumetric flow rates within the lubricant film. By combining these results with the continuity equation, the Reynolds equation is derived, which governs the pressure distribution in the bearing.

Additional simplifying assumptions, including the neglect of stretching effects and tangential flow in the z -direction, lead to the transient lubrication equation. This formulation captures the influence of surface sliding and time-dependent variations in film thickness.

This chapter establishes the mathematical framework required to evaluate the pressure field, load-carrying capacity, and transient response of plane slider bearings.

II.2 Lubrication Regimes and Hydrodynamic Lubrication

The governing fluid-flow equations are first simplified under lubrication assumptions, leading to the derivation of the Reynolds equation. Fluid-film lubrication is essential in tribological systems, such as slider bearings, as it reduces friction, limits wear, and improves the durability of moving mechanical components.

Fluid-film lubrication is commonly classified into three modes: hydrodynamic, squeeze-film, and hydrostatic lubrication (see Figure II.1). In hydrodynamic lubrication, a pressure field is generated by the relative tangential motion of surfaces separated by a converging lubricant film, allowing the film to support the applied load. In squeeze-film lubrication, load support results from the pressure developed as two surfaces approach each other normally, forcing the lubricant to resist compression. In hydrostatic lubrication, the load-carrying capacity is produced by externally supplied pressurized fluid, independently of surface motion.

In the present study, the hydrodynamic lubrication regime is considered, as it is widely encountered in practical applications such as slider bearings, journal bearings, thrust bearings, piston-ring lubrication, and other machine elements where relative sliding motion generates the pressure required to support the load.

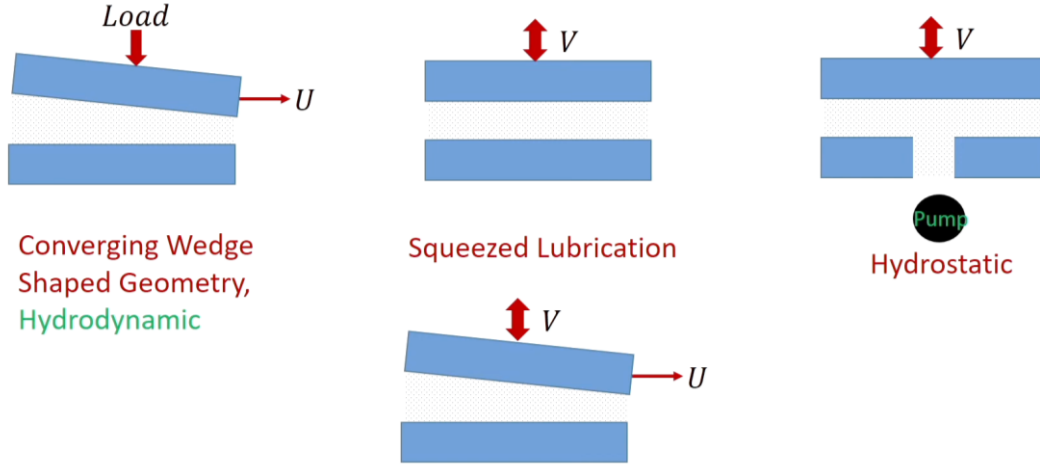


Figure II-1: Fundamental lubrication modes in bearing interfaces.

II.3 Geometry and Description of the Fluid-Film Bearing

The configuration under study represents a fluid-film bearing in which the lower surface is fixed, while the upper surface is inclined and allowed to move relative to it. The motion of the upper surface is described by the velocity components U , V , and W , acting in the x -, y -, and z -directions, respectively.

Owing to the inclination of the upper surface, the lubricant film thickness h varies spatially and temporally, such that $h = h(x, z, t)$. The characteristic lengths of the bearing in the x - and z -directions are denoted by L_x and L_z , respectively. A fundamental lubrication assumption is that the film thickness is much smaller than the in-plane dimensions, namely $h \ll L_x, L_z$.

In the x - y plane, the geometry is characterized by a tangential velocity U along the x -direction and a normal velocity V along the y -direction over the characteristic length L_x . The inclination of the upper surface is represented by the small angle parameter α , while W denotes the tangential velocity component in the z -direction.

Although, in general, either one or both bounding surfaces may be in motion, the present formulation considers the lower surface as stationary and the upper surface as moving. Accordingly, all velocity components are defined relative to the fixed lower surface.

The analysis of this fluid-film lubrication problem requires the governing equations of viscous fluid flow, which are formulated here in Cartesian coordinates.

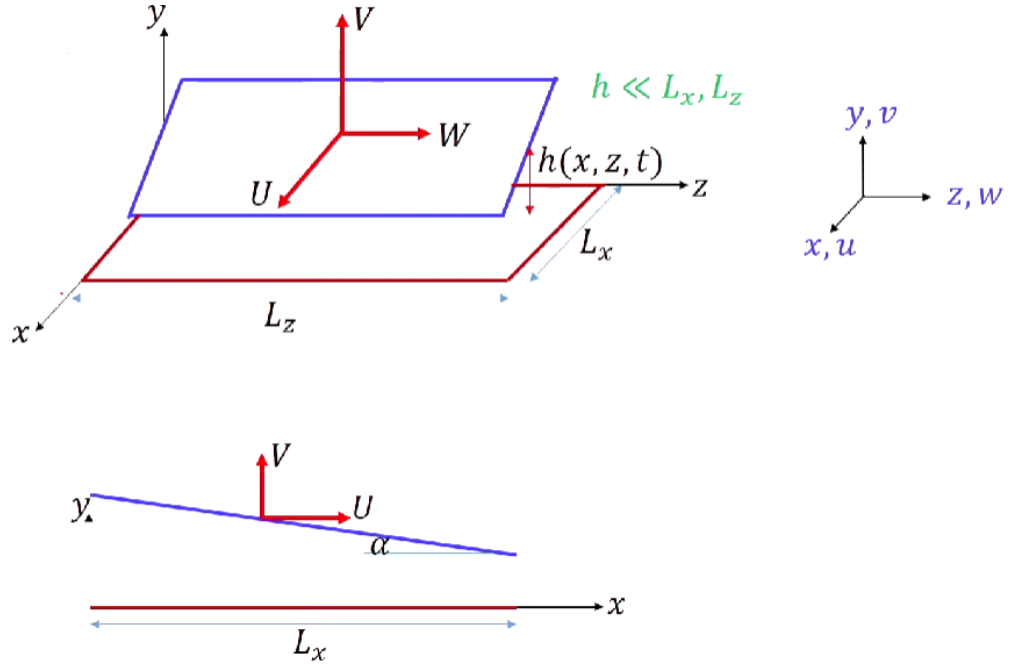


Figure II-2: Geometry of the studied fluid film bearing.

II.4 Governing Equations of Fluid Flow (Navier-Stokes Equations)

The lubricant is assumed to behave as a Newtonian, incompressible fluid with constant physical properties. The flow is considered laminar, and body forces are neglected. Under these assumptions, the governing equations are the continuity equation and the Navier–Stokes equations. In Cartesian coordinates (x, y, z) , the continuity equation is written as:

- **Continuity Equation:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

- **x-Momentum Equation**

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

- **y-Momentum Equation**

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (3)$$

• **z-Momentum Equation**

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (4)$$

where u , v , and w are the velocity components in the x –, y –, and z –directions, respectively. ρ is the fluid density, μ is the fluid dynamic viscosity, and p is the pressure.

Equations (2)–(4) form a coupled nonlinear system of partial differential equations. In their complete form, these equations describe the general motion of a viscous incompressible fluid. However, in lubrication problems, the geometry of the flow domain allows substantial simplification.

II.5 Lubrication Assumptions

For a plane slider bearing, the lubricant is confined within a thin film separating two solid surfaces. The lower surface is considered stationary, while the upper surface moves relative to it. The local film thickness is denoted by $h = h(x, z, t)$.

The characteristic length in the bearing plane is much larger than the film thickness. Therefore, a small aspect-ratio parameter is introduced as: $\alpha = L/h_0 \ll 1$, where h_0 is a characteristic film thickness and L is a characteristic bearing length.

The main assumptions used in the derivation are:

1. The lubricant is Newtonian and incompressible;
2. The flow is laminar and viscous-dominated;
3. Fluid properties are constant ;
4. Body forces are negligible ;
5. The film thickness is much smaller than the in-plane bearing dimensions;
6. The pressure variation across the film thickness is negligible;
7. Inertial terms are small compared with viscous terms;
8. No-slip boundary conditions apply at both solid surfaces.

Under these assumptions, the Navier–Stokes equations can be reduced to the classical lubrication equations.

II.6 Non-Dimensional Form of the Governing Equations

To identify the dominant terms in the governing equations, the following non-dimensional variables are introduced:

$$x^* = \frac{x}{L}, y^* = \frac{y}{\alpha L}, z^* = \frac{z}{L}, t^* = \frac{t}{L/U}, u^* = \frac{u}{U}, v^* = \frac{v}{\alpha U}, w^* = \frac{w}{U}, p^* = \frac{p}{(\mu U)/(\alpha^2 L)} \quad (5)$$

The lubrication equation holds in geometries where $\alpha \ll 1$.

The Reynolds number is defined as:

$$Re = \frac{\rho UL}{\mu} \quad (6)$$

Using these variables, the continuity equation becomes:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad (7)$$

The non-dimensional x –momentum equation becomes :

$$\alpha^2 Re \left(\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} \right) = -\frac{\partial p^*}{\partial x^*} + \alpha^2 \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + \alpha^2 \frac{\partial^2 u^*}{\partial z^{*2}} \quad (8)$$

The non-dimensional y –momentum equation is :

$$\alpha^4 Re \left(\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} \right) = -\frac{\partial p^*}{\partial y^*} + \alpha^4 \frac{\partial^2 v^*}{\partial x^{*2}} + \alpha^2 \frac{\partial^2 v^*}{\partial y^{*2}} + \alpha^4 \frac{\partial^2 v^*}{\partial z^{*2}} \quad (9)$$

The non-dimensional z –momentum equation is :

$$\alpha^2 Re \left(\frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} \right) = -\frac{\partial p^*}{\partial z^*} + \alpha^2 \frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} + \alpha^2 \frac{\partial^2 w^*}{\partial z^{*2}} \quad (10)$$

For thin-film lubrication, $\alpha \ll 1$, $\alpha^2 Re \ll 1$, the inertial terms and the viscous diffusion terms in the x - and z -directions are negligible compared with the viscous diffusion across the film thickness. And the pressure gradient in the y -direction is also negligible ($\frac{\partial p^*}{\partial y^*} = 0$) according to the equation (9). Applying these simplifications to equations (7)–(10) yields the lubrication equations in the limit $\alpha \rightarrow 0$ or $\alpha Re \rightarrow 0$.

II.7 Reduced Lubrication Equations

In the lubrication limit, the governing equations reduce to:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad \text{and} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (11)$$

$$0 = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 u^*}{\partial y^{*2}} \quad \text{and} \quad 0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \quad (12)$$

$$0 = -\frac{\partial p^*}{\partial y^*} \quad \text{and} \quad 0 = -\frac{\partial p}{\partial y} \quad (13)$$

$$0 = -\frac{\partial p^*}{\partial z^*} + \frac{\partial^2 w^*}{\partial y^{*2}} \quad \text{and} \quad 0 = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2} \quad (14)$$

These equations show that $p(x, z, t)$, which means that the pressure is independent of the film-thickness coordinate y . This is one of the most important results of lubrication theory.

The pressure gradients in the x - and z -directions drive the pressure-induced flow, while the motion of the upper surface generates shear-induced flow. The combination of these two effects gives rise to the hydrodynamic pressure field.

II.8 Boundary Conditions

The lower surface is fixed at $y = 0$, while the upper surface is located at $y = h(x, z, t)$. The velocity components of the upper surface are U , V , and W in the x -, y -, and z -directions, respectively.

At the stationary lower surface, we can write:

$$u = 0, \quad v = 0, \quad w = 0 \quad \text{at} \quad y = 0 \quad (15)$$

At the moving upper surface, the boundary conditions are:

$$u = U, \quad v = V, \quad w = W \quad \text{at} \quad y = h(x, z, t) \quad (16)$$

The normal velocity of the moving surface satisfies the kinematic condition:

$$V = \frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} + W \frac{\partial h}{\partial z} \quad (17)$$

This condition accounts for the temporal variation of the film thickness and for the convective change in thickness caused by the tangential motion of the upper surface.

II.9 Velocity Distribution in the Lubricant Film

Starting from the simplified x -momentum equation:

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \quad (18)$$

One obtains:

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x} \quad (19)$$

Integrating twice with respect to y gives:

$$u(y) = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + c_1 y + c_2 \quad (20)$$

Using the boundary condition: $u = 0$ at $y = 0$, and $u = U$ at $y = h$, one obtains:

$$c_1 = \frac{1}{2\mu} \left(-\frac{\partial p}{\partial x} \right) h + \frac{U}{h} \quad \text{and} \quad c_2 = 0 \quad (21)$$

Therefore, the velocity distribution in the x -direction is:

$$u(y) = -\frac{1}{2\mu} \frac{\partial p}{\partial x} y(h - y) + \frac{Uy}{h} \quad (22)$$

Similarly, from the z -momentum equation:

$$0 = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial z^2} \quad (23)$$

The velocity distribution in the z -direction is obtained as

$$w(y) = -\frac{1}{2\mu} \frac{\partial p}{\partial z} y(h - y) + \frac{Wy}{h} \quad (24)$$

Equations (22) and (24) show that the velocity field is composed of two contributions. The first term is the Poiseuille component, generated by the pressure gradient. The second term is the Couette component, generated by the tangential motion of the upper surface.

II.10 Flow Rates Across the Film Thickness

The integrated flow rate in the x -direction is obtained by integrating $u(y)$ across the film thickness:

$$Q_x = \int_0^h u(y) dy \quad (25)$$

Substituting equation (22) gives:

$$Q_x = \int_0^h \left[-\frac{1}{2\mu} \frac{\partial p}{\partial x} y(h-y) + \frac{Uy}{h} \right] dy \quad (26)$$

After integration :

$$Q_x = -\frac{h^3}{12\mu} \frac{\partial p}{\partial x} + \frac{Uh}{2} \quad (27)$$

Similarly, the integrated flow rate in the z -direction is:

$$Q_z = \int_0^h w(y) dy \quad (28)$$

which gives:

$$Q_z = -\frac{h^3}{12\mu} \frac{\partial p}{\partial z} + \frac{Wh}{2} \quad (29)$$

Here, Q_x and Q_z represent the integrated flow rates per unit width in the x - and z -directions, respectively. Each flow rate contains a pressure-driven component and a surface-driven component.

II.11 Derivation of the Reynolds Equation

The Reynolds equation is obtained by integrating the continuity equation across the lubricant film:

$$\int_0^h \frac{\partial u}{\partial x} dy + \int_0^h \frac{\partial v}{\partial y} dy + \int_0^h \frac{\partial w}{\partial z} dy = 0 \quad (30)$$

Using Leibniz's rule,

$$\int_a^b \frac{\partial f}{\partial \eta} dy = \frac{\partial}{\partial \eta} \int_a^b f dy + f(a, \eta) \frac{\partial a}{\partial \eta} - f(b, \eta) \frac{\partial b}{\partial \eta} \quad (31)$$

The first term in equation (30) gives:

$$\int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x} \int_0^h u dy - u(h) \frac{\partial h}{\partial x} = \frac{\partial Q_x}{\partial x} - u(h) \frac{\partial h}{\partial x} \quad (32)$$

And the last term in equation (30) gives:

$$\int_0^h \frac{\partial w}{\partial z} dy = \frac{\partial}{\partial z} \int_0^h w dy - w(h) \frac{\partial h}{\partial z} = \frac{\partial Q_z}{\partial z} - w(h) \frac{\partial h}{\partial z} \quad (33)$$

The integral of the normal velocity gradient is:

$$\int_0^h \frac{\partial v}{\partial y} dy = v(h) - v(0) \quad (34)$$

Using the boundary conditions:

$$u(h) = U, \quad w(h) = W, \quad v(h) = V, \quad v(0) = 0 \quad (35)$$

Substitution into the integrated continuity equation gives:

$$\frac{\partial Q_x}{\partial x} - U \frac{\partial h}{\partial x} + V + \frac{\partial Q_z}{\partial z} - W \frac{\partial h}{\partial z} = 0 \quad (36)$$

Using the kinematic condition presented in equation (17) and substituting the expressions of Q_x and Q_z in the equation (36) gives:

$$\frac{\partial}{\partial x} \left(-\frac{h^3}{12\mu} \frac{\partial p}{\partial x} + \frac{Uh}{2} \right) + \frac{\partial}{\partial z} \left(-\frac{h^3}{12\mu} \frac{\partial p}{\partial z} + \frac{Wh}{2} \right) + \frac{\partial h}{\partial t} = 0 \quad (37)$$

Rearranging gives the general Reynolds equation:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{1}{2} \frac{\partial(Uh)}{\partial x} + \frac{1}{2} \frac{\partial(Wh)}{\partial z} + \frac{\partial h}{\partial t} \quad (38)$$

This equation governs the pressure distribution in the lubricant film for a general unsteady three-dimensional plane slider bearing.

II.12 Reynolds Equation for Constant Viscosity and Rigid Surfaces

For a lubricant with constant viscosity and for rigid surfaces, equation (38) can be multiplied by 12μ , giving:

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = 6\mu \frac{\partial(Uh)}{\partial x} + 6\mu \frac{\partial(Wh)}{\partial z} + 12\mu \frac{\partial h}{\partial t} \quad (39)$$

If U and W are spatially constant, this becomes:

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = 6\mu \left(U \frac{\partial h}{\partial x} + W \frac{\partial h}{\partial z} + 2 \frac{\partial h}{\partial t} \right) \quad (40)$$

If there is no tangential motion in the z -direction, that is $W = 0$, the Reynolds equation reduces to:

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = 6\mu U \frac{\partial h}{\partial x} + 12\mu \frac{\partial h}{\partial t} \quad (41)$$

For a steady one-dimensional plane slider bearing, where: $\frac{\partial}{\partial z} = 0, \frac{\partial h}{\partial t} = 0$, equation (41) becomes:

$$\frac{d}{dx} \left(h^3 \frac{dp}{dx} \right) = 6\mu U \frac{dh}{dx} \quad (42)$$

This is the classical Reynolds equation for a steady one-dimensional plane slider bearing.

II.13 Physical Interpretation of the Reynolds Equation

The Reynolds equation describes the pressure generation mechanism in a thin lubricant film.

- The left-hand side of the equation, $\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right)$, represents pressure-driven flow in the lubricant film. It accounts for the redistribution of lubricant caused by pressure gradients in the x - and z -directions.
- The first two terms on the right-hand side, $6\mu U \frac{\partial h}{\partial x} + 6\mu W \frac{\partial h}{\partial z}$, represent the wedge effect. This effect is generated by tangential surface motion in a film of variable

thickness. In hydrodynamic lubrication, the wedge effect is the main source of pressure generation and load-carrying capacity.

- The final term, $12\mu \frac{\partial h}{\partial t}$, represents the squeeze-film effect. This term becomes important when the distance between the two surfaces changes with time. It describes the pressure generated by the resistance of the lubricant to being squeezed out of the contact region.
- Therefore, the Reynolds equation combines three fundamental lubrication mechanisms: Pressure flow + Couette flow + Squeeze-film effect. In a plane slider bearing, the hydrodynamic pressure is primarily generated by the relative tangential motion of the surfaces and the converging geometry of the lubricant film.

II.14 Conclusion

This chapter established the theoretical foundation required to model hydrodynamic lubrication in a plane slider bearing. The formulation began with the general governing equations of viscous incompressible flow, namely the continuity and Navier–Stokes equations, written in Cartesian coordinates. By applying the classical lubrication assumptions, including thin-film geometry, laminar flow, Newtonian fluid behavior, negligible inertia, and negligible pressure variation across the film thickness, the governing equations were reduced to their simplified lubrication form.

The reduced momentum equations were integrated across the lubricant film using the no-slip boundary conditions at the stationary lower surface and the moving inclined upper surface. This led to the velocity profiles in the longitudinal and transverse directions. These profiles were shown to consist of two contributions: a pressure-driven Poiseuille component and a shear-driven Couette component. The corresponding flow rates were then obtained by integrating the velocity distributions across the film thickness.

By combining these flow-rate expressions with the integrated continuity equation and the kinematic condition at the moving surface, the Reynolds equation was derived. This equation represents the central governing equation of hydrodynamic lubrication, as it relates the pressure distribution to the film geometry, lubricant viscosity, surface velocity, and time-dependent variation of the film thickness. Its physical interpretation highlighted the main pressure-generation mechanisms: wedge action due to tangential motion, transverse flow effects, and squeeze-film effects due to temporal changes in film thickness.

Chapter III

Numerical Methods and Implementation

III. Chapter III: Numerical Methods and Implementation

III.1 Introduction

This chapter presents the computational methodology used to investigate the hydrodynamic pressure generation in a plane slider bearing. The main objective is to compare three approaches:

1. An **analytical solution** of the Reynolds equation for a simple one-dimensional wedge bearing;
2. A **finite difference method** implemented in MATLAB for the two-dimensional Reynolds equation;
3. A **two-dimensional CFD simulation** performed using ANSYS Fluent.

The analytical solution provides a reference closed-form result. The finite difference method allows the Reynolds equation to be solved numerically over a two-dimensional bearing domain. The CFD simulation solves the full fluid-flow equations and is used to assess the validity and limitations of the lubrication approximation.

III.2 Common Physical Configuration

The same bearing geometry and operating conditions are used for the three methods. The configuration consists of a converging lubricant film between a stationary lower surface and a moving inclined upper surface.

The bearing length in the sliding direction is denoted by L_x , while the film thickness varies from an inlet value h_{in} to an outlet value h_{out} , with $h_{in} > h_{out}$. The film-thickness difference is defined as $c = h_{in} - h_{out}$. For a linear wedge, the dimensional film thickness is

$$h(x) = h_{in} - (h_{in} - h_{out}) \frac{x}{L_x} = h_{in} - c \frac{x}{L_x} \quad (43)$$

The upper surface moves with a constant sliding velocity U , while the lubricant is assumed Newtonian and incompressible with dynamic viscosity μ .

Figure III.1 shows a two-dimensional converging lubricating film between a stationary lower surface and an inclined upper surface moving with velocity U . The film thickness varies along the sliding direction as $h(x)$, from $h_{in} = 0.02$ mm at the inlet to $h_{out} = 0.003$ mm at the outlet, over a bearing length $L_x = 0.2$ mm. The lubricant is characterized by its dynamic viscosity μ and density ρ .

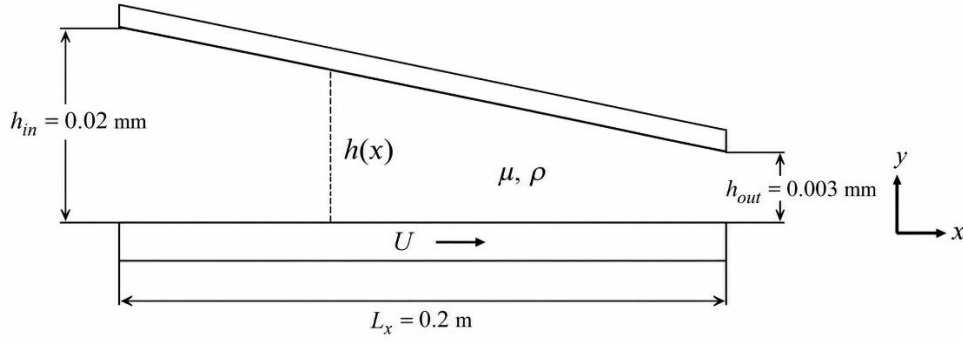


Figure III-1: Geometry of the Steady Plane Slider Bearing.

III.3 Method I: Analytical Solution of the Reynolds Equation

The analytical solution is based on the steady one-dimensional Reynolds equation for an isoviscous incompressible lubricant:

$$\frac{d}{dx} \left(h^3 \frac{dp}{dx} \right) = 6\mu U \frac{dh}{dx} \quad (44)$$

This model assumes: $\frac{\partial}{\partial z} = 0$, $\frac{\partial h}{\partial t} = 0$, $\mu = \text{constant}$, $U = \text{constant}$. The pressure boundary conditions are: $p(0) = 0$, $p(L_x) = 0$. These conditions correspond to ambient pressure at the inlet and outlet of the bearing.

The final dimensionless analytical solution of the equation (44) is:

$$P_a(X) = \frac{6X(1-X)}{(H_0 + 1 - X)^2(1+2H_0)} \quad (45)$$

The dimensionless variables are defined as:

$$X = \frac{x}{L_x}, H_0 = \frac{h_{out}}{c}, P_a = \frac{pc^2}{\mu UL_x} \quad (46)$$

III.4 Method II: Finite Difference Solution in MATLAB

The finite difference model solves the steady two-dimensional Reynolds equation:

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = 6\mu U \frac{\partial h}{\partial x} \quad (47)$$

This equation accounts for the pressure variation along both the sliding direction x and the transverse direction z . The second term on the left-hand side represents side leakage effects.

The numerical pressure variable used in the MATLAB finite difference solver is:

$$P_n = \frac{pc^2}{6\mu UL_x} \quad (48)$$

Using the dimensionless variables:

$$X = \frac{x}{L_x}, Z = \frac{z}{L_z}, H = \frac{h}{c}, K = \frac{L_x}{L_z} \quad (49)$$

The Reynolds equation becomes:

$$\frac{\partial}{\partial X} \left(H^3 \frac{\partial P_n}{\partial X} \right) + K^2 \frac{\partial}{\partial Z} \left(H^3 \frac{\partial P_n}{\partial Z} \right) = \frac{\partial H}{\partial X} \quad (50)$$

For the linear wedge :

$$H(X) = H_0 + 1 - X \quad (51)$$

The derivation of H gives:

$$\frac{\partial H}{\partial X} = -1 \quad (52)$$

This derivative is computed numerically by a central finite difference approximation:

$$\left(\frac{\partial H}{\partial X} \right)_{i,j} = \frac{H_{i+1,j} - H_{i-1,j}}{2\Delta X} \quad (53)$$

The computational domain is :

$$0 \leq X \leq 1, \quad 0 \leq Z \leq 1 \quad (54)$$

A uniform grid is used: $X_i = (i - 1)\Delta X, i = 1, 2, \dots, N_x, Z_j = (j - 1)\Delta Z, j = 1, 2, \dots, N_z,$

Where: $\Delta X = \frac{1}{N_x - 1},$ and $\Delta Z = \frac{1}{N_z - 1}.$

Where the grid used is: $N_x = 241, N_z = 121$.

The numerical equation is written in conservative form as:

$$\frac{\partial}{\partial X} \left(\xi \frac{\partial P_n}{\partial X} \right) + K^2 \frac{\partial}{\partial Z} \left(\xi \frac{\partial P_n}{\partial Z} \right) = \frac{\partial H}{\partial X} \quad (55)$$

Where $\xi = H^3$. At node (i, j) , the coefficient is: $\xi_{i,j} = H_{i,j}^3$.

The coefficients at the cell faces are evaluated by arithmetic averaging:

$$\begin{aligned} \xi_e &= \frac{\xi_{i,j} + \xi_{i+1,j}}{2}, \\ \xi_w &= \frac{\xi_{i,j} + \xi_{i-1,j}}{2}, \\ \xi_n &= \frac{\xi_{i,j} + \xi_{i,j+1}}{2}, \\ \xi_s &= \frac{\xi_{i,j} + \xi_{i,j-1}}{2}. \end{aligned}$$

The finite difference coefficients are

$$\begin{aligned} a_E &= \frac{\xi_e}{\Delta X^2}, \\ a_W &= \frac{\xi_w}{\Delta X^2}, \\ a_N &= K^2 \frac{\xi_n}{\Delta Z^2}, \\ a_S &= K^2 \frac{\xi_s}{\Delta Z^2}, \\ a_P &= a_E + a_W + a_N + a_S. \end{aligned}$$

The discretized right-hand side is

$$R_{i,j} = \frac{H_{i+1,j} - H_{i-1,j}}{2\Delta X} \quad (56)$$

The resulting algebraic equation is

$$a_E P_{i+1,j} + a_W P_{i-1,j} + a_N P_{i,j+1} + a_S P_{i,j-1} - a_P P_{i,j} = R_{i,j} \quad (57)$$

Solving for the nodal pressure gives:

$$P_{i,j}^{GS} = \frac{a_E P_{i+1,j} + a_W P_{i-1,j} + a_N P_{i,j+1} + a_S P_{i,j-1} - R_{i,j}}{a_P} \quad (58)$$

The algebraic system is solved using a Gauss–Seidel iterative method accelerated by successive over-relaxation. The pressure update is:

$$P_{i,j}^{new} = (1 - \omega) P_{i,j}^{old} + \omega P_{i,j}^{GS} \quad (59)$$

The relaxation factor used is $\omega = 1.75$. The pressure field is initialized as $P_{i,j} = 0$.

The convergence residual is defined as: $\text{Residual} = \max_{i,j} |P_{i,j}^{new} - P_{i,j}^{old}|$.

The stopping criterion is: $\text{Residual} < 10^{-10}$. The maximum number of iterations is 25000.

The pressure is imposed as zero on the four boundaries of the computational domain:

$$\begin{aligned} P_n(0, Z) &= 0, \\ P_n(1, Z) &= 0, \\ P_n(X, 0) &= 0, \\ P_n(X, 1) &= 0. \end{aligned}$$

These conditions represent ambient pressure at the inlet, outlet, and lateral sides of the bearing.

III.5 Method III: Two-Dimensional CFD Simulation in ANSYS Fluent

The CFD model is based on the numerical solution of the two-dimensional incompressible Navier–Stokes equations. Unlike the Reynolds equation, which is derived using thin-film assumptions, CFD solves the fluid-flow field directly inside the lubricant domain.

For steady incompressible flow, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (60)$$

The momentum equations are:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (61)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (62)$$

For the present thin-film lubrication problem, the Reynolds number is generally small, and the flow is laminar. However, CFD still retains the complete momentum balance, including inertia and pressure variation across the film.

III.6 Geometry

The simulation process is initiated by launching the ANSYS Workbench environment and selecting the Fluid Flow (Fluent) analysis system to establish a new project. Subsequently, a right-click is performed on the geometry cell, selecting 'New DesignModeler' to begin the computational domain construction.

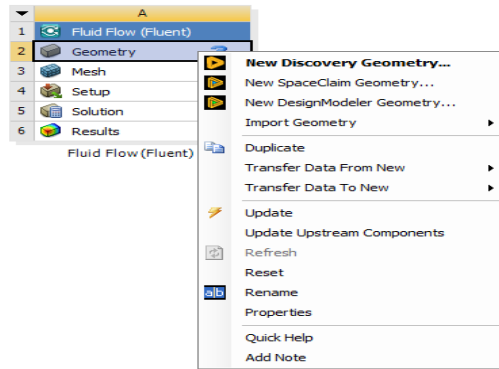


Figure III-2: The first step to open ANSYS DesignModeler.

After entering the *DesignModeler* interface, the *XY-Plane* is selected as the coordinate system for the 2D sketch. This step establishes the geometric reference required to begin drawing the profile of the slider bearing domain.

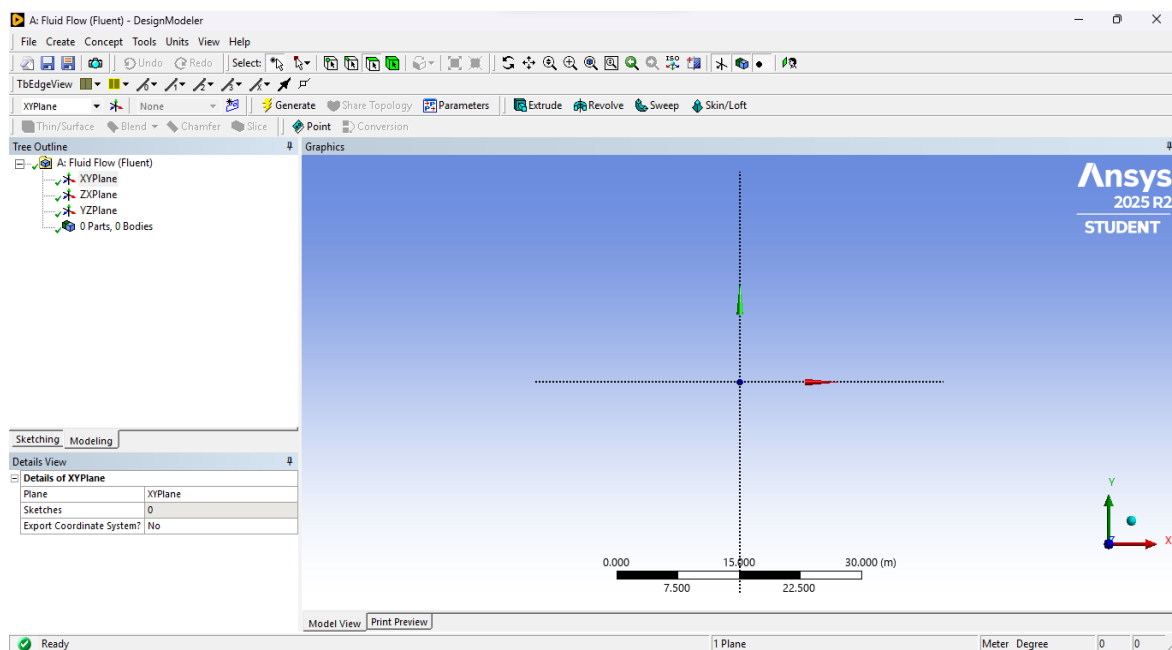


Figure III-3: Two-Dimensional Coordinate System.

Within the *DesignModeler* interface, the 'Sketching' tab is accessed to activate the drawing tools. By selecting the 'Line' tool, the geometric boundaries of the slider bearing domain are manually drafted to represent the fluid film area.

Once the geometry is drafted, the 'Dimensions' tool is employed to define the specific parameters of the domain. Three primary dimensions are applied: the inlet height (V1), the

outlet height (V2), and the total length of the bearing (H3). These constraints ensure the precise representation of the converging gap required for hydrodynamic lubrication.

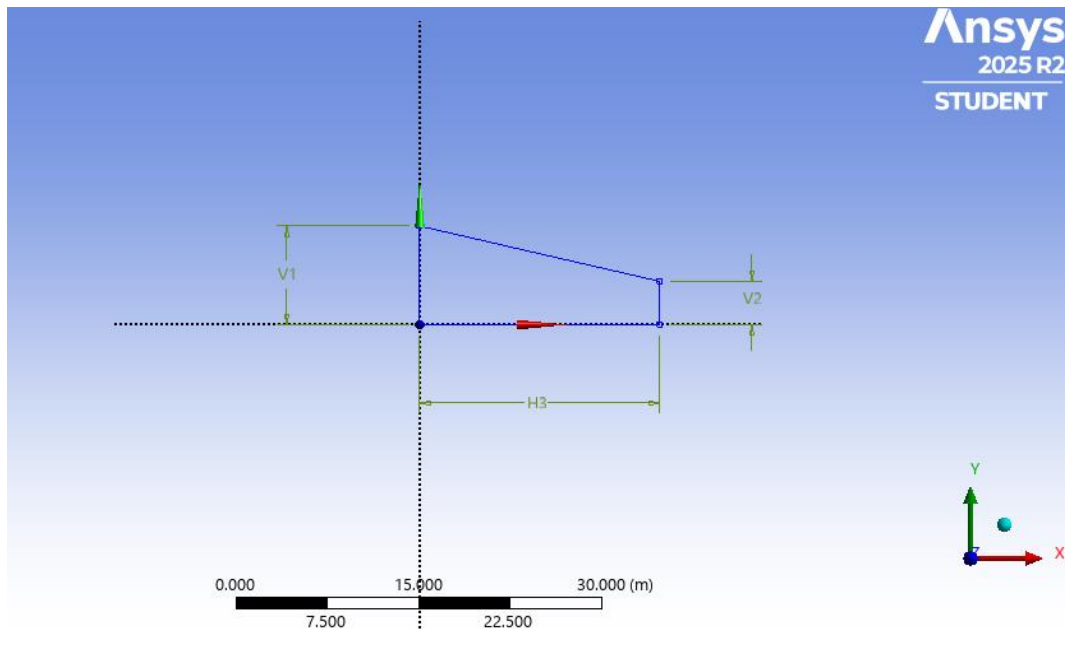


Figure III-4: Domain Specification.

After placing the dimensions, the numerical values are entered in the Details View to set the physical size of the model. The inlet height is set to 0.02 mm, the outlet height is set to 0.003 mm, and the total length is specified as 0.2 mm. These precise values are essential for the accurate simulation of the lubrication film within the bearing.

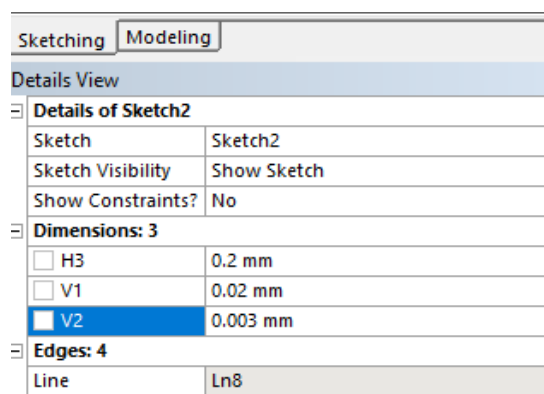


Figure III-5: Domain Parameter Input.

After defining and assigning the dimensional values, the Generate button is clicked to validate and finalize the sketch. The Concept menu is then accessed, and Surface from Sketches is selected to create a two-dimensional surface based on the developed sketch.

In the Tree Outline, the Surface Body is renamed to Fluid Domain, and the material type is changed from Solid to Fluid in the Details View to accurately represent the fluid medium.

Finally, the Generate button is clicked once again to complete the geometry creation stage. The *DesignModeler* interface is then closed in preparation for proceeding to the next phase of the project.

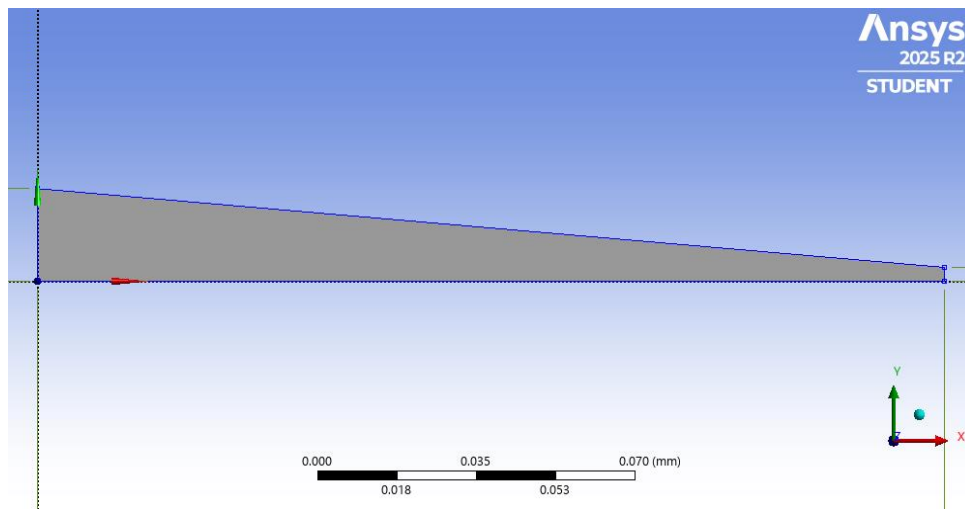


Figure III-6: The Final Geometry View in Ansys DesignModeler.

III.7 Meshing

Following the completion of the geometry, the project return to the main Workbench schematic. The 'Mesh' cell is then accessed by double-clicking it to launch the ANSYS Meshing application, where the computational grid for the fluid domain will be defined.

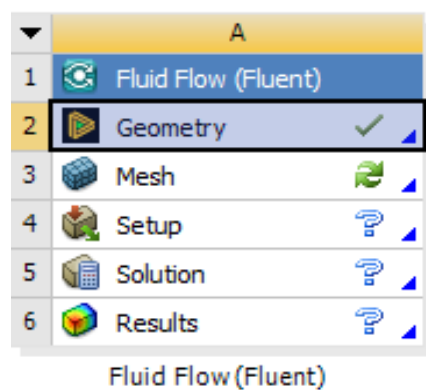


Figure III-7: The first step to open the ANSYS Meshing.

Upon opening the ANSYS Meshing interface, the domain appears in a 3D perspective by default. To align the view for a two-dimensional analysis, the Z-axis on the coordinate triad is

selected. This action reorients the display to the XY-plane, ensuring a proper top-down view for the subsequent meshing operations on the 2D fluid domain.

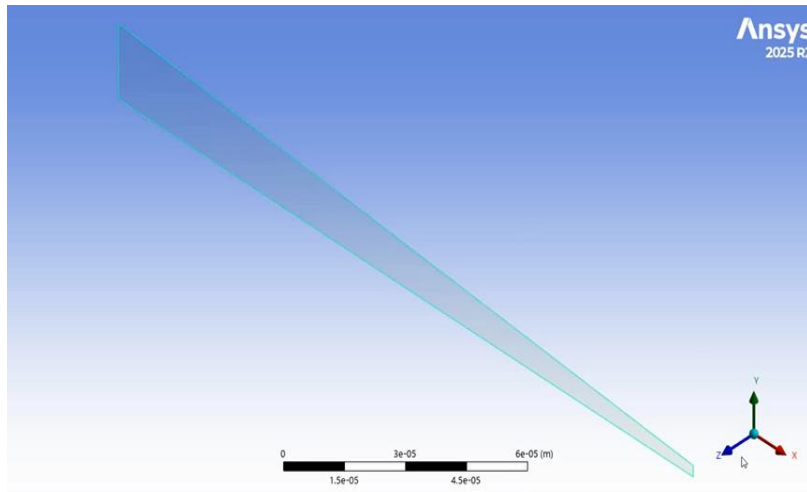


Figure III-8: Initial Interface of ANSYS Meshing.

To achieve better control over mesh resolution, the Mesh object in the Model tree is right-clicked, then *Insert* tab is selected followed by *Sizing* from the context menu. This step allows for the manual specification of element sizes across the entire fluid domain, contributing to improved mesh quality and enhanced simulation accuracy.

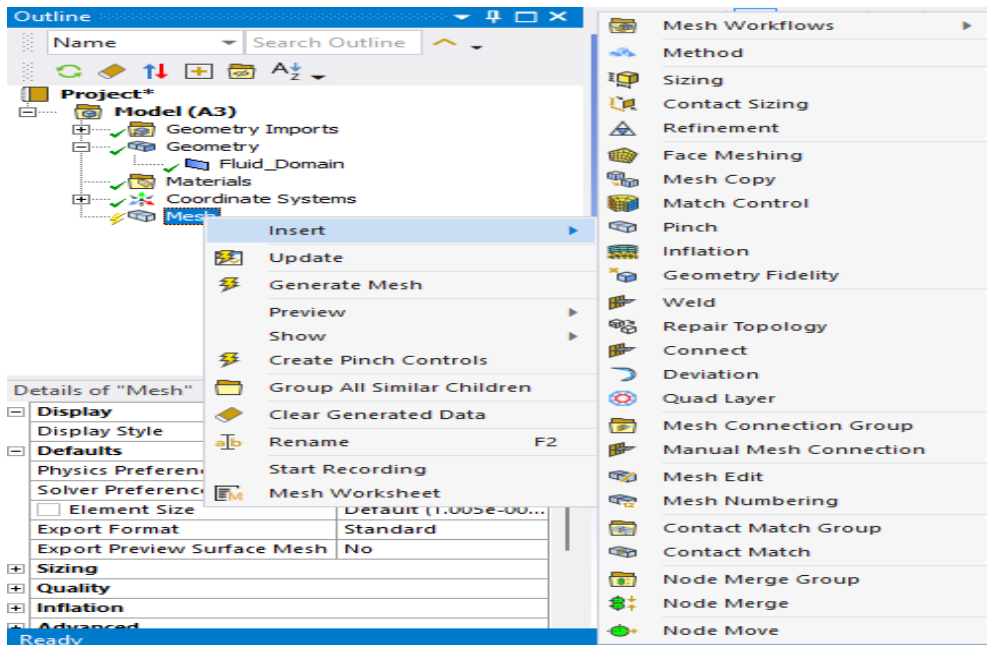


Figure III-9 Sizing Selection step in Ansys Meshing.

After inserting the sizing control, the inlet and outlet edges are selected and confirmed by clicking 'Apply' in the geometry selection box. The sizing type is then set to 'Number of

Divisions' with a value of 10 assigned to both edges. This configuration is used to control the mesh resolution at the boundaries, ensuring a structured distribution of elements across the fluid domain.

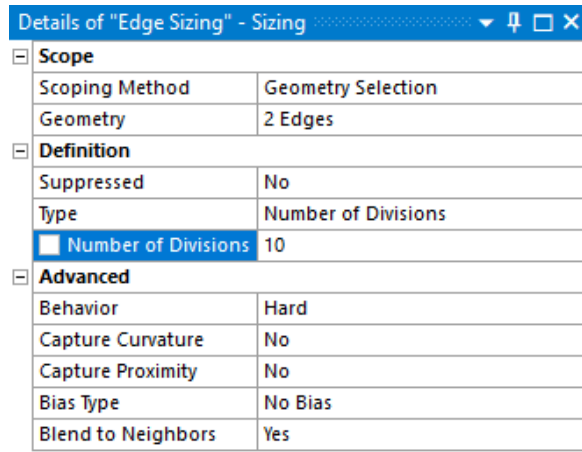


Figure III-10: Number of Divisions Selection.

The visual representation of the domain after the sizing application shows the vertical edges at the inlet and outlet divided into ten equal segments. This discretization, visualized by the black markers along the edges, confirms that the '*Number of Divisions*' setting has been successfully applied, providing the necessary nodal density for the subsequent mesh generation.

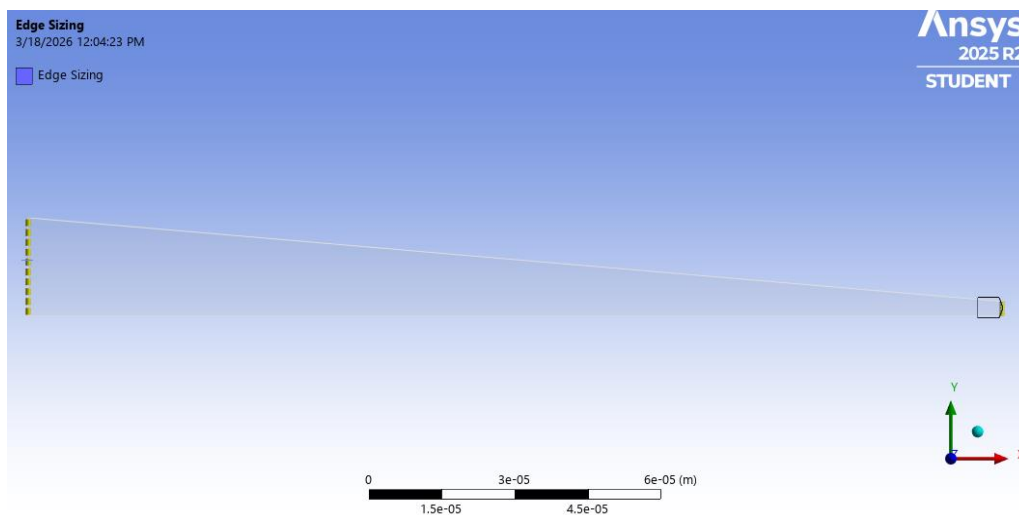


Figure III-11: Horizontal Division of Both Sides.

Similarly, the upper and lower edges are selected and assigned a '*Number of Divisions*' of 50. This higher resolution along the longitudinal boundaries, as shown by the dense markers in the interface, is necessary to accurately capture the pressure variations along the bearing length.

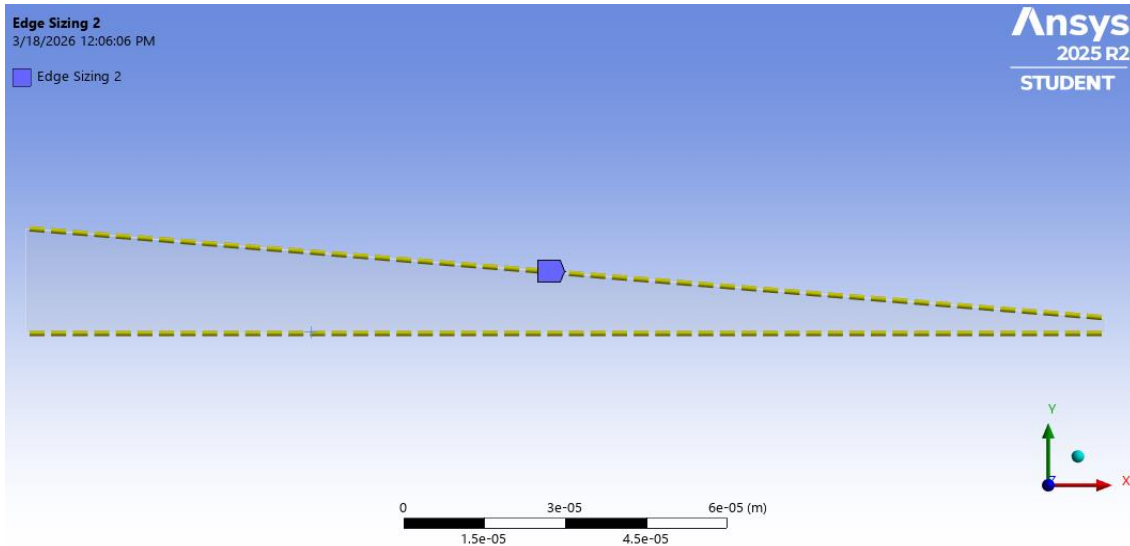


Figure III-12 Division of the Upper and Lower Edges.

To finalize the grid, a '*Face Meshing*' control is inserted and applied to the entire fluid domain. After selecting the surface and clicking '*Apply*', the '*Generate Mesh*' command is executed. This produces a structured quadrilateral mesh, as illustrated in the final figure, which is now ready for the simulation phase.

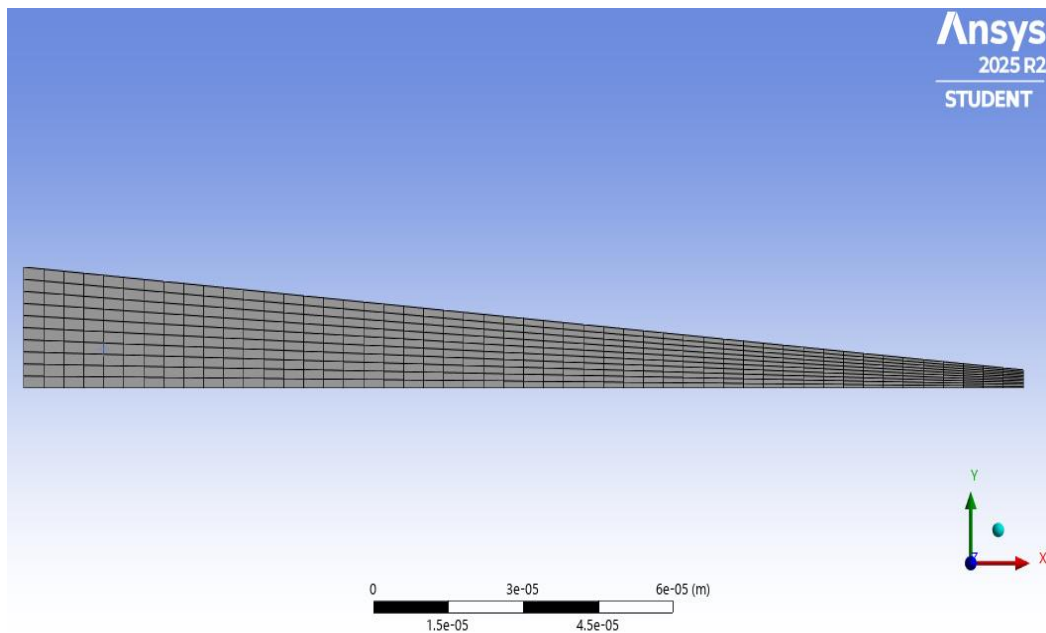


Figure III-13: Final Mesh View in Ansys Meshing.

The final step in the meshing process involves defining the boundary conditions through '*Named Selections*'. Each of the four edges is selected and renamed accordingly: '*Pressure Inlet*' for the flow entrance, '*Pressure Outlet*' for the exit, and '*Wall*' and '*Moving Wall*' for the

stationary and sliding boundaries, respectively. These labels are essential for the solver to correctly identify and apply the physical boundary conditions during the setup phase.

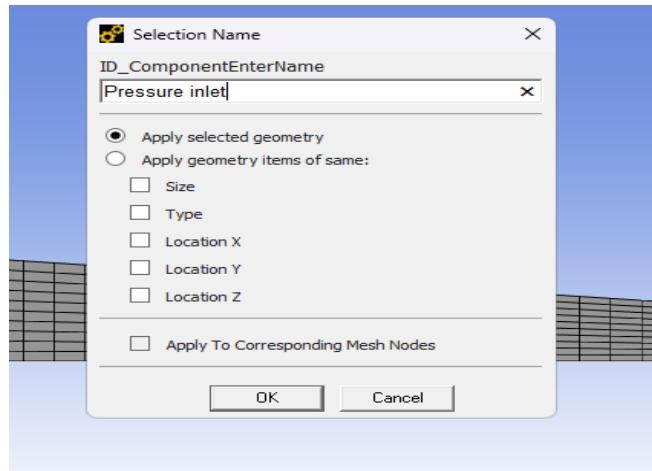


Figure III-14: Naming of Edges.

III.8 Physics setup (Ansys Fluent)

Once the meshing process is finalized and the window is closed, the 'Setup' cell is accessed to initiate the physics configuration. Upon launching the Fluent Launcher, the 'Double Precision' option is selected to ensure higher numerical accuracy during the calculations. The solver is then started in 2D mode, as previously defined, to proceed with the material properties and boundary condition specifications.

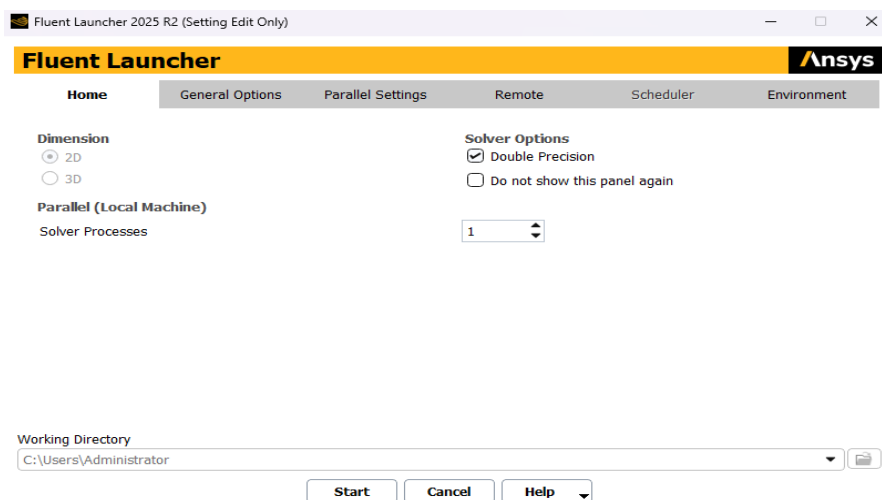


Figure III-15: The first step to open ANSYS Fluent.

The final computational mesh is displayed within the Fluent interface, showing a structured grid ready for calculation. The blue arrows at the left boundary represent the 'Pressure Inlet', while the red highlighted zone at the right indicates the 'Pressure Outlet'. This visual

confirmation ensures that the named selections and mesh density have been correctly imported into the solver for the fluid flow analysis.

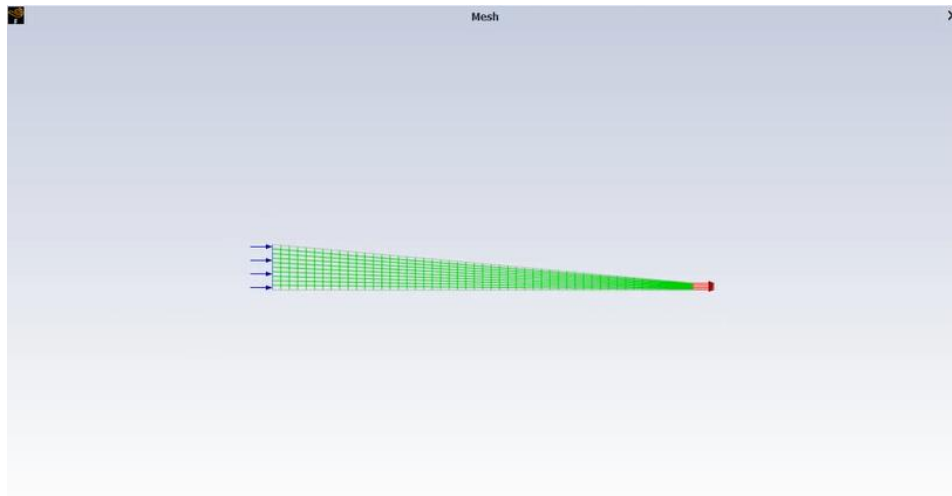


Figure III-16: The Imported Mesh in the Ansys Fluent.

In the 'General' task page, the solver settings are configured to match the physical nature of the slider bearing. A 'Pressure-Based' solver is selected with a 'Steady' time formulation, as the flow is assumed to be constant over time. Furthermore, the 2D space is set to 'Planar' to align with the geometric model, and the 'Absolute' velocity formulation is maintained for the calculation.



Figure III-17: Selection of Important Options in the General Tab.

In the 'Models' task page, the flow regime is defined to ensure the physics of the slider bearing are accurately represented. By accessing the 'Viscous' model settings, the flow is set to 'Laminar'. This selection is critical as the lubricant film in slider bearings typically operates at low Reynolds numbers, where viscous forces dominate and turbulence is absent.

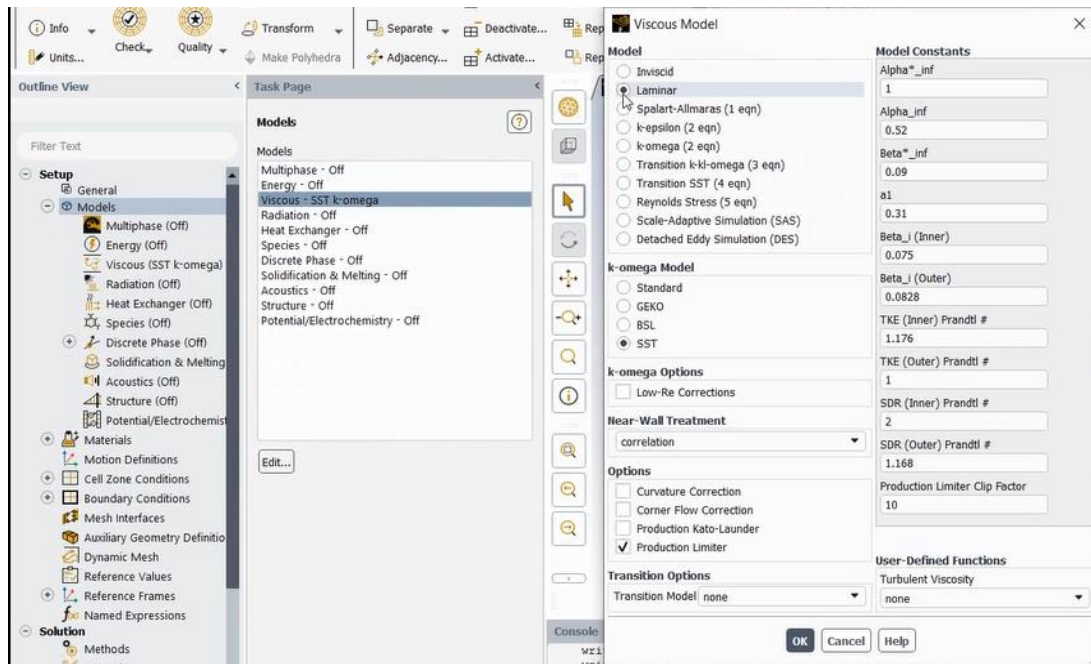


Figure III-18: Selection of the Laminar Viscous Model.

In the 'Materials' section, a new fluid material is created and labeled as 'new-fluid'. The physical properties are defined by assigning a constant density (ρ) of 850 kg/m^3 and a dynamic viscosity (μ) of $0.085 \text{ Pa}\cdot\text{s}$.

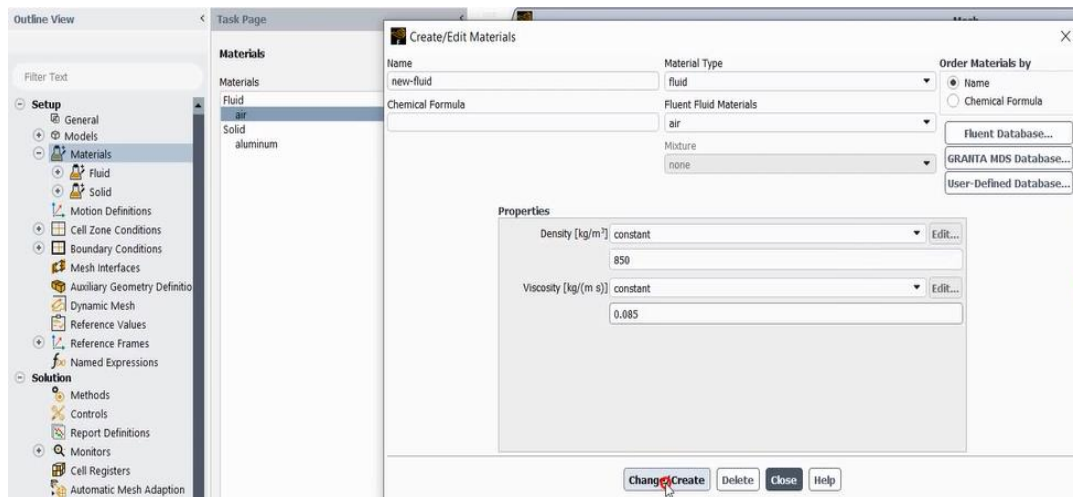


Figure III-19: Assignment of Viscosity and Density Values.

III.9 Boundary conditions

In the 'Boundary Conditions' task page, the 'moving wall' zone is configured to simulate the relative motion of the bearing. The wall motion is set to 'Moving Wall' with a 'Translational' velocity of 2 m/s in the x-direction. The 'No Slip' shear condition is maintained, ensuring that

the lubricant layer in contact with the surface moves at the same velocity as the wall, which is the primary mechanism for pressure generation in slider bearings.

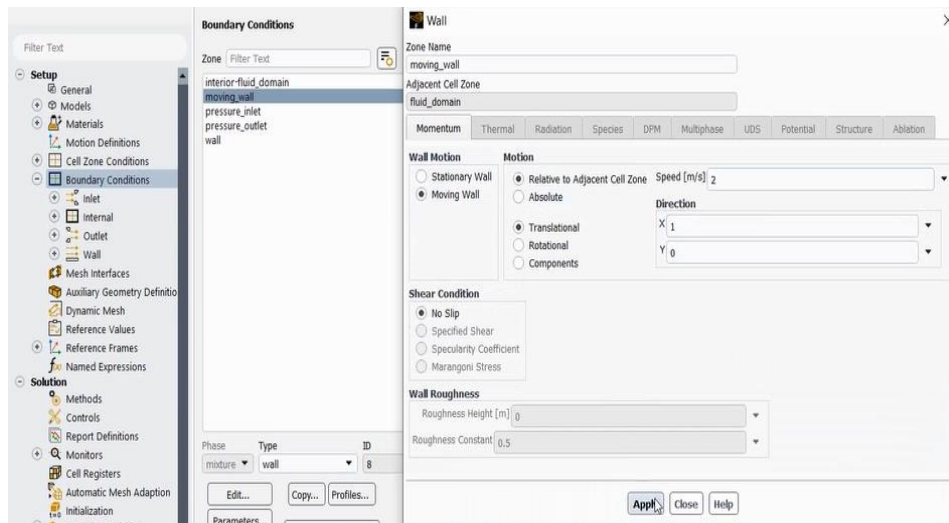


Figure III-20: Input of Velocity Value.

III.10 Solution controls

In the 'Methods' task page, the 'Pressure-Velocity Coupling' scheme is configured by deselecting the 'Auto Select' option and manually choosing the 'SIMPLE' algorithm. For 'Spatial Discretization', the 'Second Order' and 'Second Order Upwind' schemes are assigned to Pressure and Momentum, respectively, to enhance the numerical precision of the simulation results.

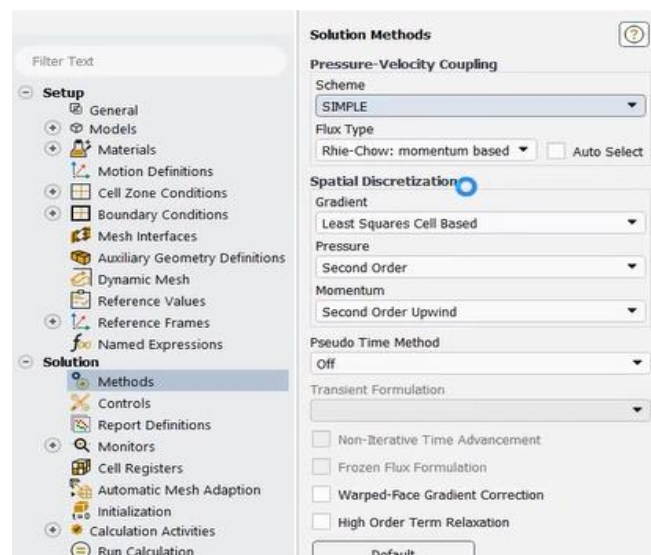


Figure III-21: SIMPLE Method Selection.

To enhance the solution's precision, the 'Residual Monitors' are configured. The 'Absolute Criteria' for continuity, x-velocity, and y-velocity equations are tightened by decreasing the

tolerance to 0.0001 (or 10^{-4}). This ensures that the iterative process continues until the numerical error is sufficiently minimized, leading to a more reliable and converged flow field solution.

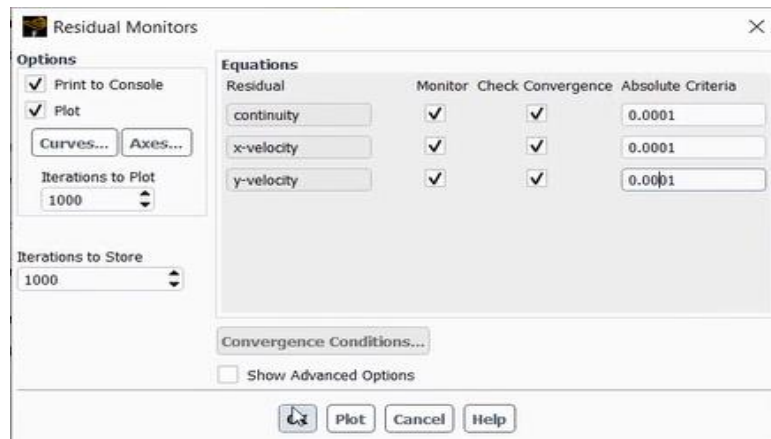


Figure III-22: Improving Solution Accuracy.

To prepare the solver for the iteration process, the '*Solution Initialization*' task page is accessed. The '*Hybrid Initialization*' method is selected, which provides a more efficient and stable starting point for the calculations by solving a simplified potential flow field. Upon clicking the '*Initialize*' button, the domain is populated with initial values for pressure and velocity, ensuring a smoother convergence during the subsequent run.



Figure III-23: Solution Initialization Selection.

The final stage of the simulation is performed in the '*Run Calculation*' task page. The '*Number of Iterations*' is set to 10000 to ensure sufficient overhead for the solution to reach the predefined convergence criteria. By clicking the '*Calculate*' button, the solver begins the

iterative process of solving the governing Navier-Stokes equations, monitoring the residuals until a stable and converged solution for the pressure and velocity fields is achieved.

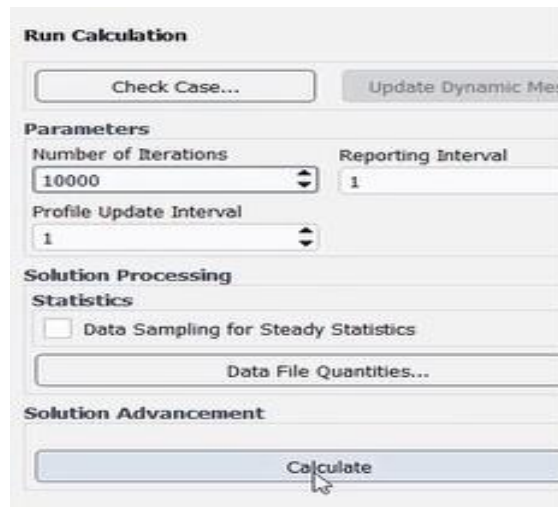


Figure III-24: Increasing the Number of Iterations.

The simulation is executed, and the convergence behavior is monitored through the scaled residuals plot. As the iterations progress, the error values for continuity and velocity components show a consistent downward trend. The calculation will proceed until each residual falls below the specified threshold of 10^{-4} , indicating that a numerically stable and converged solution has been reached for the slider bearing's fluid film.

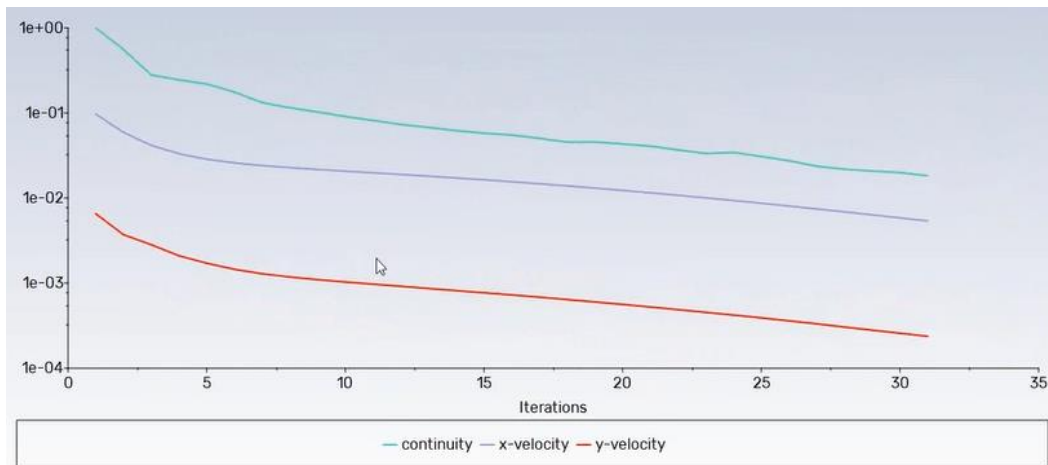


Figure III-25: Convergence of Continuity and Velocity Residuals.

The simulation results are evaluated by plotting the static pressure distribution along the 'moving wall'. The obtained curve illustrates the hydrodynamic pressure build-up within the lubricant film, starting from the atmospheric pressure at the inlet, reaching a peak value of approximately $6.2 \times 10^5 \text{ Pa}$ near the exit, and then dropping back at the outlet. This pressure

profile confirms the bearing's load-carrying capacity, generated by the wedge effect as the fluid is dragged into the converging gap by the moving surface.

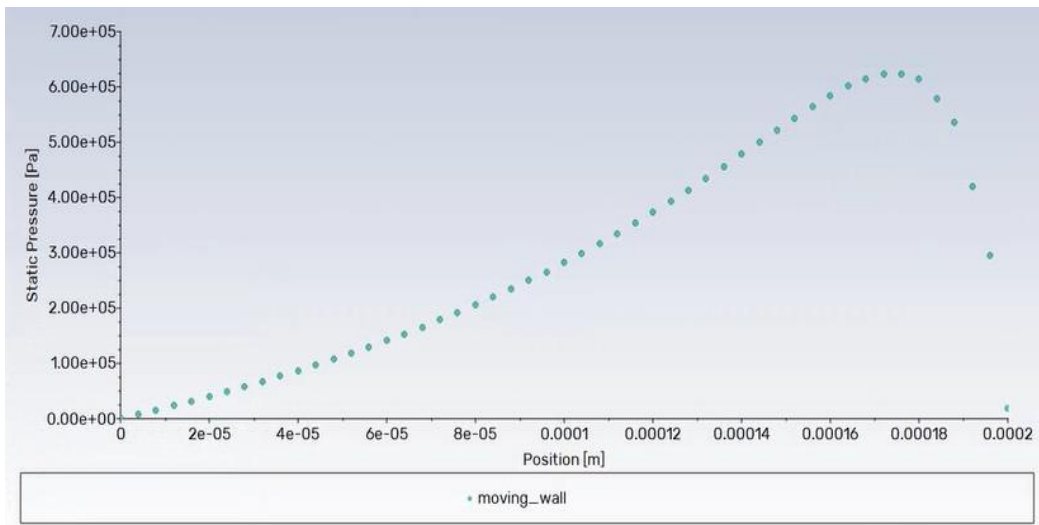


Figure III-26: Pressure Distribution along the Moving Wall.

III.11 Extraction of comparable outputs

To facilitate further data analysis, the pressure distribution results are exported from the 'Solution XY Plot' interface. By selecting the 'Write to File' option and specifying the 'moving wall' surface, the numerical data consisting of the nodal positions and their corresponding static pressure values is saved into a formatted file within the project directory. This exported dataset allows for precise post-processing and comparison with analytical solutions.

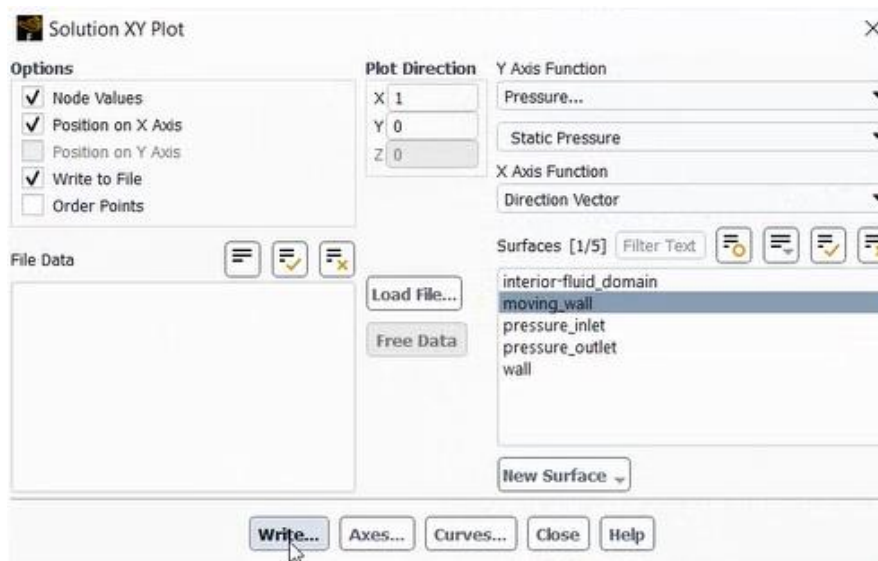


Figure III-27: Writing the results to a file.

III.12 CFD verification

To ensure the reliability and accuracy of the numerical simulation, a comprehensive validation study was conducted. The pressure distribution data exported from ANSYS Fluent (CFD) was integrated into a custom MATLAB environment for a three-way comparison. The script was designed to evaluate the CFD results against both the analytical solution derived from the Reynolds equation and the results obtained via the Finite Difference Method (FDM) as described in the above sections.

By employing the '*pchip*' interpolation technique, the analytical and FDM datasets were mapped onto the exact nodal coordinates of the CFD mesh. This rigorous comparison allows for a precise evaluation of the numerical error and confirms the effectiveness of the *SIMPLE* algorithm and the *second-order* discretization schemes in capturing the hydrodynamic wedge effect. The close agreement between the three methods serves as a robust confirmation of the simulation's validity in predicting the performance of the slider bearing.

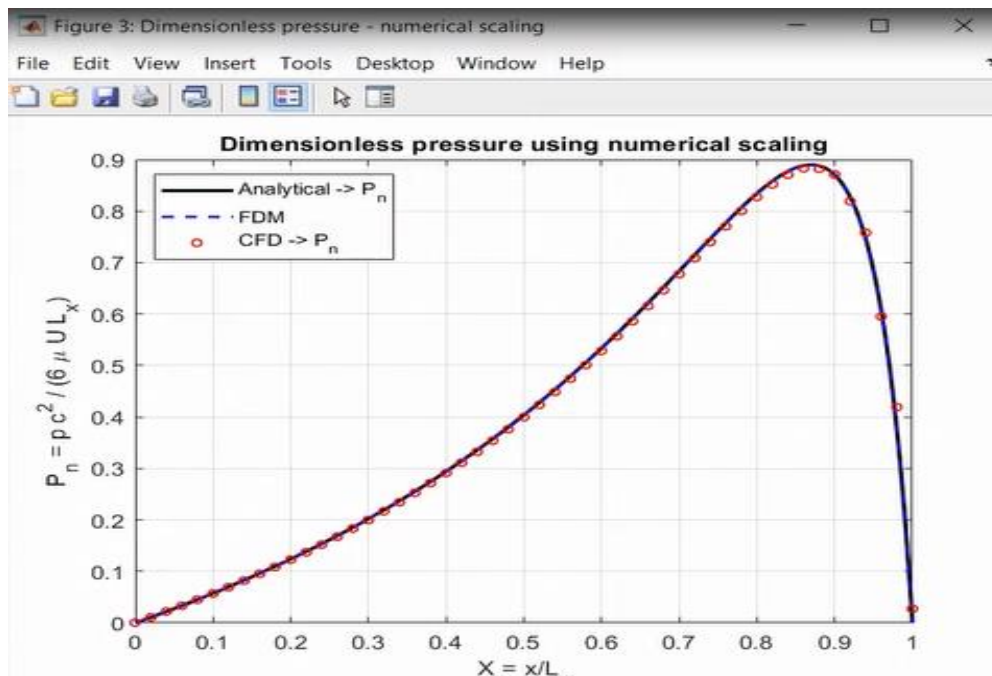


Figure III-28: Comparison between Analytical, FDM, and CFD Pressure Profiles.

III.13 Conclusion

In conclusion, the numerical analysis successfully demonstrated the effectiveness of both the MATLAB-based Reynolds solver and the ANSYS Fluent CFD model in capturing the essential physics of slider bearings.

The results indicated an excellent agreement between the three evaluated methods: the analytical solution, the Finite Difference Method (FDM), and the CFD simulation. The pressure profiles obtained across all models consistently showcased the 'wedge effect' as the primary mechanism for hydrodynamic pressure generation. Furthermore, the validation process conducted in MATLAB confirmed that the ANSYS Fluent setup utilizing the SIMPLE algorithm and second order discretization provides a highly accurate representation of the fluid film behavior.

These findings consolidate the reliability of the numerical tools employed and lay the foundation for further parametric studies involving more complex bearing geometries or lubricant properties.

Chapter IV

Results, Validation, and

Engineering

Recommendations

IV. Chapter IV: Results, Validation, and Engineering Recommendations

IV.1 Introduction

This chapter is dedicated to evaluating the hydrodynamic lubrication performance of a convergent wedge channel under various operating conditions. The primary objective is to establish a rigorous validation framework that bridges classical lubrication theory with modern numerical architectures. To accomplish this, a comparative analysis is systematically conducted across six distinct operational cases, examining the coupled effects of varying sliding velocities ($U = 1, 5$, and 10 m/s) and different lubricant properties, including standard reference fluids and commercial oils (SAE 10W, ISO VG 46, and SAE 40).

The investigation utilizes a three-pronged mathematical methodology: closed-form theoretical solutions derived from the classical Reynolds equation, an in-house numerical code developed using the Finite Difference Method (FDM), and comprehensive two-dimensional Computational Fluid Dynamics (CFD) simulations executed via ANSYS Fluent. By assessing both qualitative flow features such as full-field pressure and velocity contours and quantitative centerline profiles, this chapter evaluates the predictive accuracy, spatial stability, and convergence of the implemented numerical schemes against exact analytical benchmarks.

IV.2 Summary of the Used Parameters

In this section, we present the different operating conditions utilized in the numerical simulations. Table IV.1 summarizes the specific parameters used for each studied case, including velocity, lubricant density, and dynamic viscosity.

Table III-1: Operating Conditions and Lubricant Properties for the Simulation Cases.

Case	Sliding Velocity U (m/s)	Lubricant Density ρ (kg/m ³)	Dynamic Viscosity μ (Pa · s)
Case 1	1	850	0.085
Case 2	5	850	0.085
Case 3	10	850	0.085

Case 4	10	870	0.035
Case 5	10	875	0.046
Case 6	10	890	0.145

IV.3 Parametric Study

In the first three cases, the sliding velocities ($U = 1, 5$, and 10 m/s) were selected according to the industrial operating ranges suggested by Hamrock et al. [6], while the lubricant properties were fixed at a density of $\rho = 850$ kg/m³ and a dynamic viscosity of $\mu = 0.085$ Pa.s.

IV.3.1 Case 1:

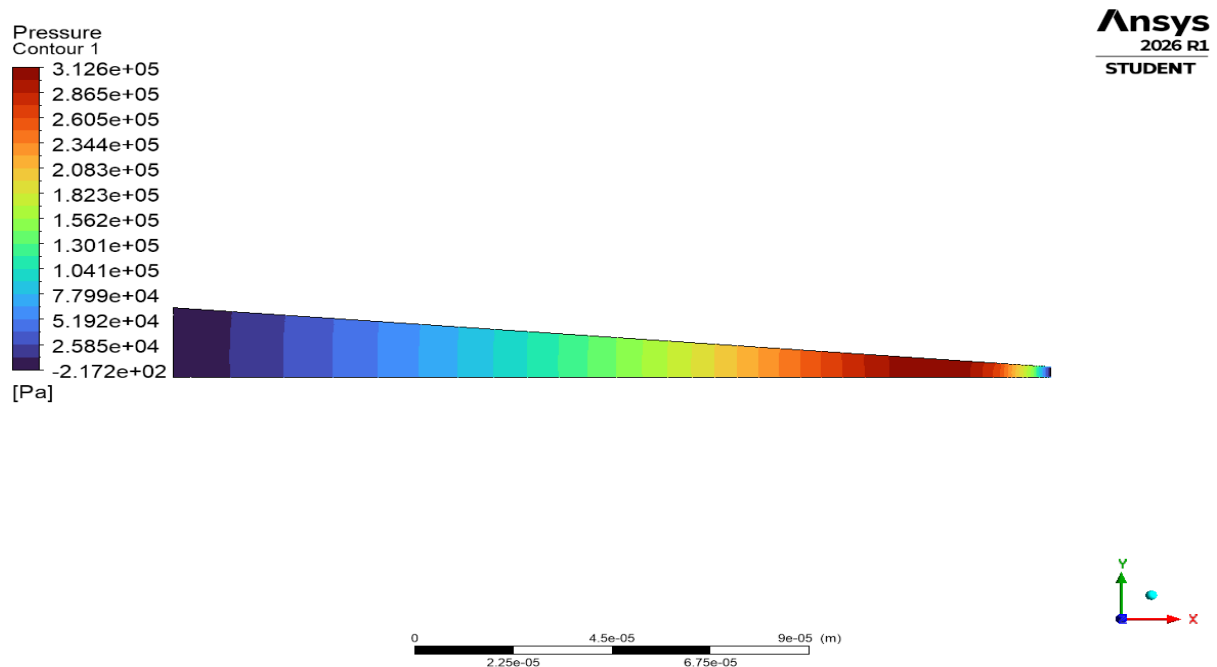


Figure IV-1: Hydrodynamic pressure contour (Case 1).

Figure IV.1 shows the 2D hydrodynamic pressure distribution for Case 1. The profile demonstrates a progressive pressure buildup along the convergent gap due to the wedge effect, reaching a maximum peak of $P_{\max} = 3.126 \times 10^5$ Pa near the minimum film thickness. Beyond this peak, the pressure drops sharply to match the atmospheric outlet condition, validating the expected physical behavior of the lubricant film.

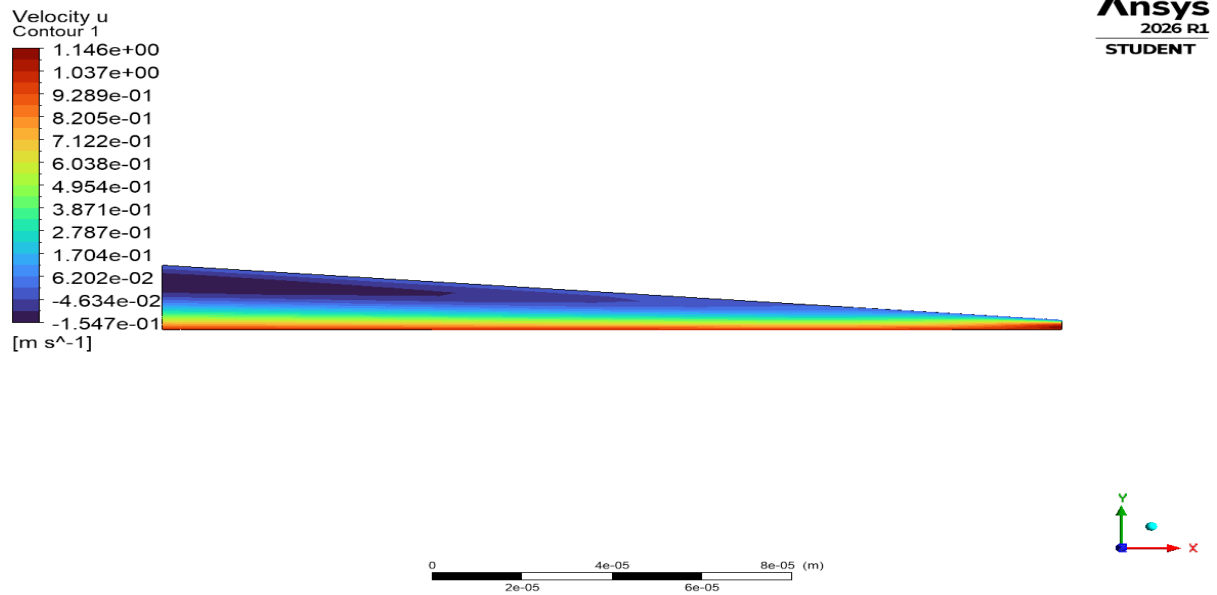


Figure IV-2 Hydrodynamic velocity u contour (Case 1).

Figure IV.2 shows the 2D fluid velocity u distribution within the lubricant film for Case 1. The contour highlights a clear velocity gradient across the film thickness, with a localized region of negative velocity near the upper wall ($u_{\min} = -4.634 \times 10^{-2} \text{ m/s}$). This negative zone indicates the occurrence of reverse flow (backflow) induced by the high adverse pressure gradient, showcasing the complex flow behavior captured by the CFD model.

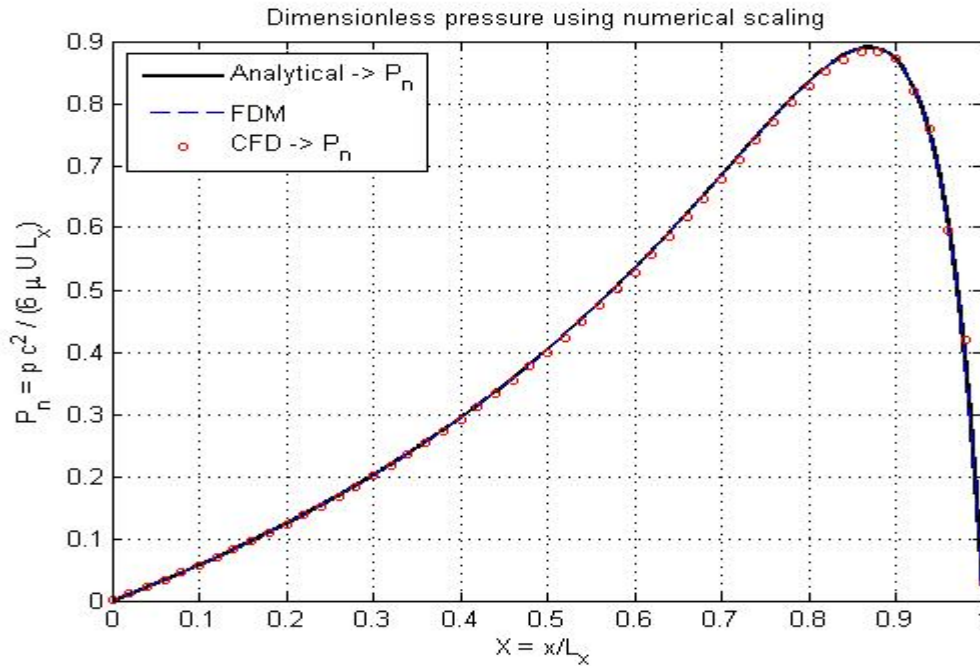


Figure IV-3: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 1).

Figure IV.3 evaluates the mathematical consistency of the numerical models by comparing the dimensionless pressure P_n profiles obtained via Analytical, FDM, and CFD methods for Case 1. The three curves exhibit a near-perfect superposition along the normalized contact length (X), with all methods successfully capturing the maximum non-dimensional peak of $P_n \approx 0.89$. This strict alignment demonstrates that the simplifying assumptions in the analytical approach are well-preserved in both the finite difference formulation and the full Navier-Stokes CFD simulations.

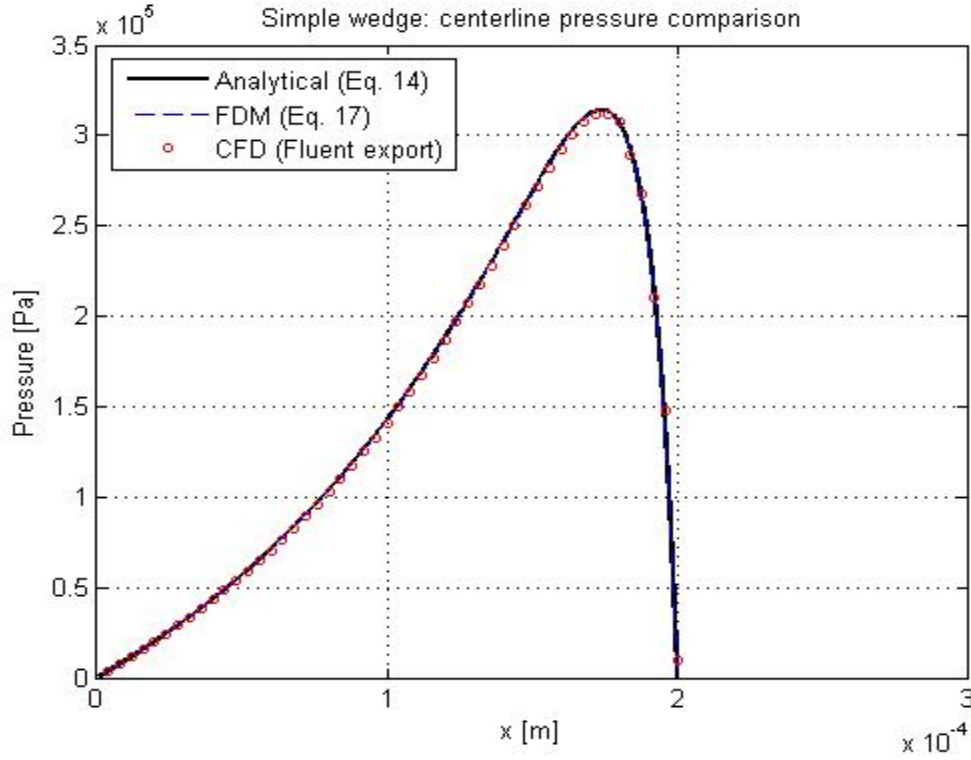


Figure IV-4: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 1).

Figure IV.4 provides a physical validation of the absolute pressure profiles along the centerline of the simple wedge geometry for Case 1. The quantitative comparison shows a flawless agreement between the analytical solution, FDM, and CFD results. All three methods precisely capture the gradual pressure rise within the convergent gap, successfully identifying the exact physical location and magnitude of the maximum peak pressure $P_{\max} = 3.126 \times 10^5$ Pa. This strict dimensional validation reinforces the high fidelity of the developed numerical codes in executing the hydrodynamic lubrication equations.

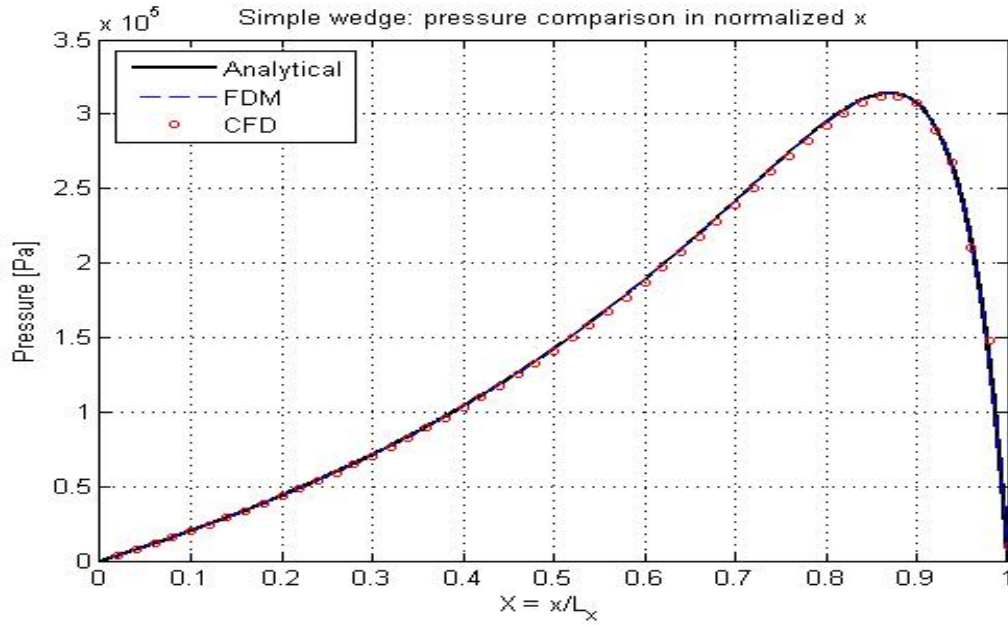


Figure IV-5: Comparison of dimensional pressure profiles along the normalized contact length (Case 1).

Figure IV.5 presents the dimensional pressure profiles plotted against the normalized contact length X for Case 1, validation-testing the Analytical, FDM, and CFD methods. The resulting curves demonstrate an identical evolutionary trend and perfect superposition across the computational domain. The peak pressure value of $P_{\max} = 3.126 \times 10^5 \text{ Pa}$ is consistently captured by all three methodologies at the normalized position $X \approx 0.87$. This final cross-verification confirms that normalizing the geometric spatial coordinate maintains absolute mathematical stability and data integrity across all used numerical solvers.

IV.3.2 Case 2:

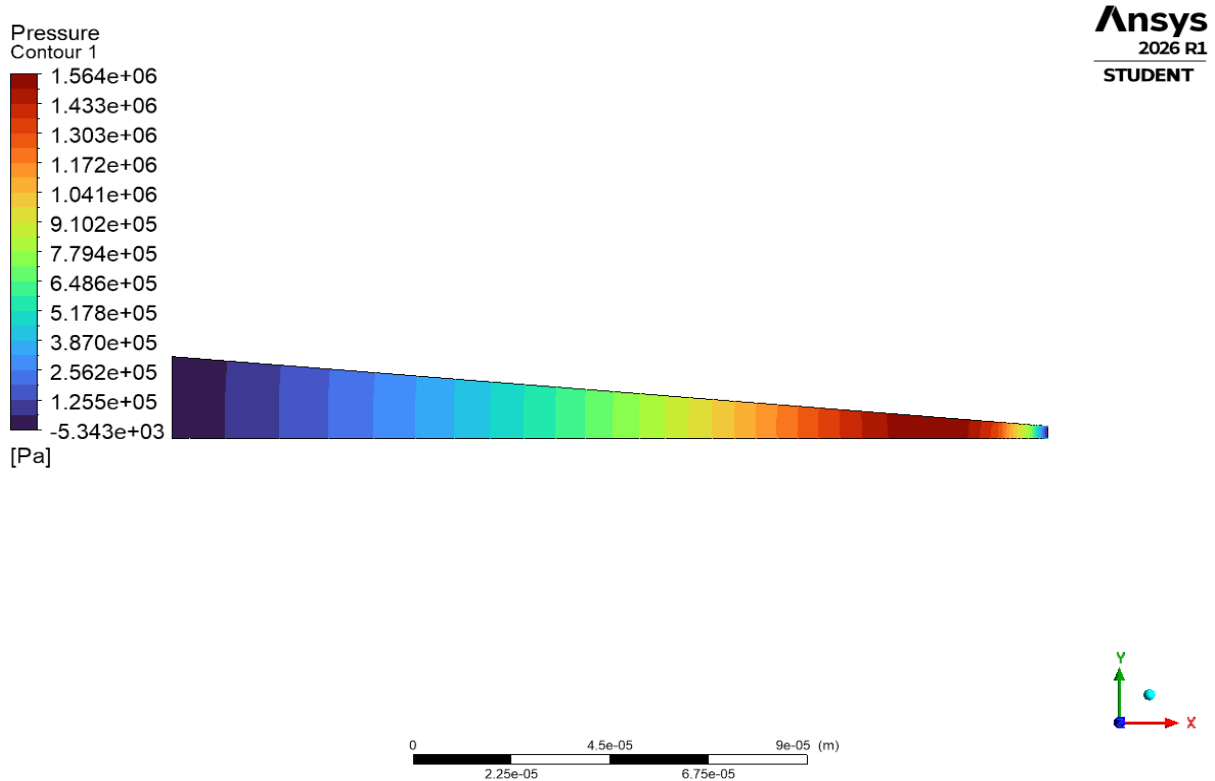


Figure IV-6: Hydrodynamic pressure contour (Case 2).

Figure IV.6 illustrates the 2D hydrodynamic pressure distribution within the lubricant film for Case 2. The profile exhibits a progressive pressure buildup along the convergent gap due to the hydrodynamic wedge effect, reaching a maximum peak of $P_{\max} = 1.564 \times 10^6 \text{ Pa}$ (indicated by the red zone) near the minimum film thickness section. Beyond this peak, the pressure drops sharply to match the atmospheric boundary condition at the outlet, confirming the expected physical behavior of the fluid.

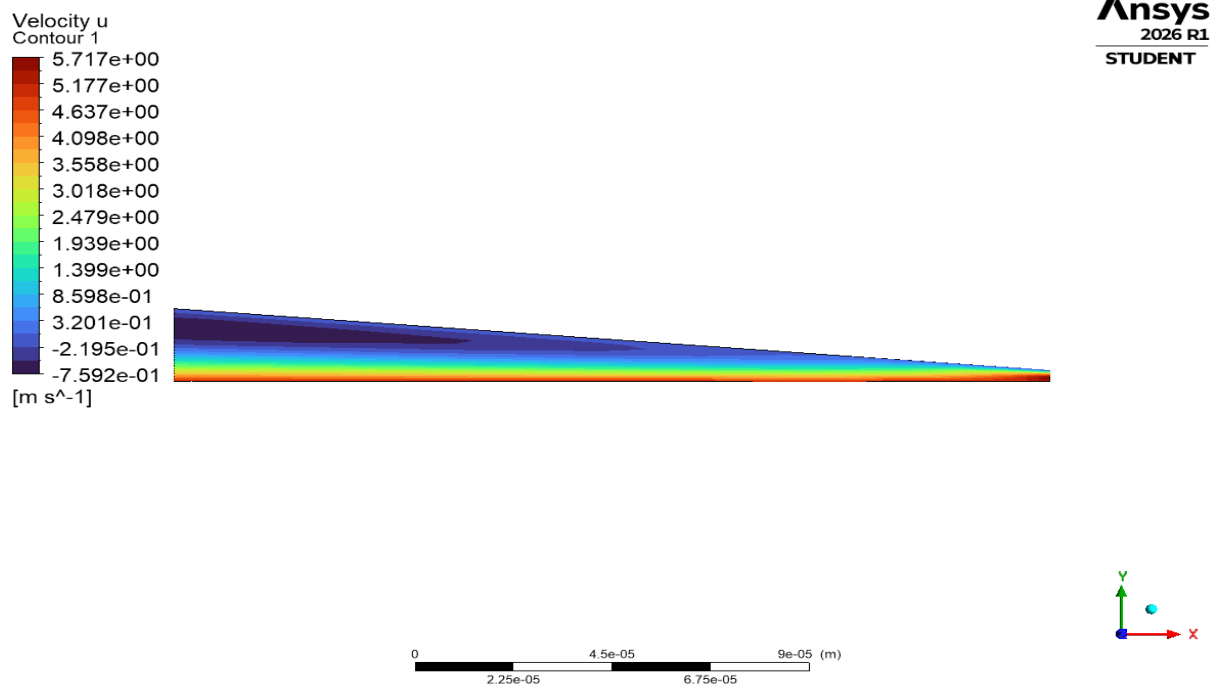


Figure IV-7: Hydrodynamic velocity u contour (Case 2).

Figure IV.7 shows the 2D fluid velocity u distribution within the lubricant film for Case 2. The contour highlights a severe velocity gradient across the film thickness, with a significant localized region of negative velocity near the stationary upper wall ($u_{\min} = -7.592 \times 10^{-1} \text{ m/s}$). This negative zone demonstrates a pronounced reverse flow (backflow) induced by the high adverse pressure gradient, effectively capturing the complex recirculation zone via the CFD model.

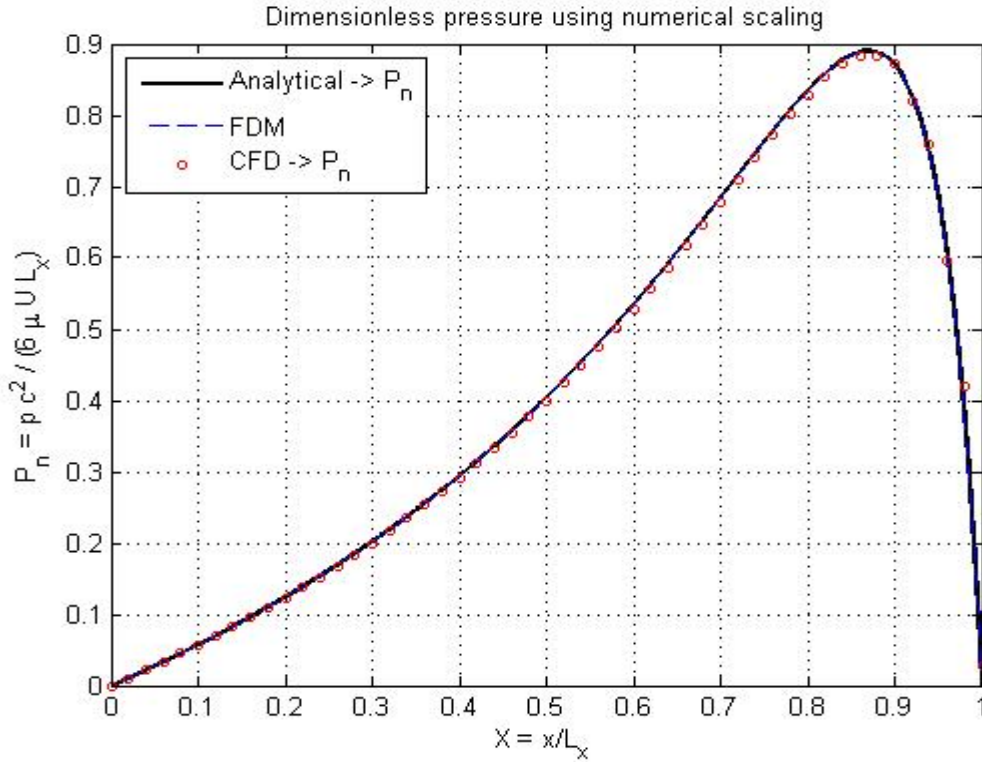


Figure IV-8: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 2).

Figure IV.8 evaluates the mathematical consistency of the numerical models by comparing the dimensionless pressure P_n profiles obtained via Analytical, FDM, and CFD methods for Case 2. The three curves exhibit a near-perfect superposition along the normalized contact length (X), with all methods successfully capturing the maximum non-dimensional peak of $P_n \approx 0.89$. This strict alignment demonstrates that the scaling and mathematical assumptions remain highly accurate under the modified operational parameters.

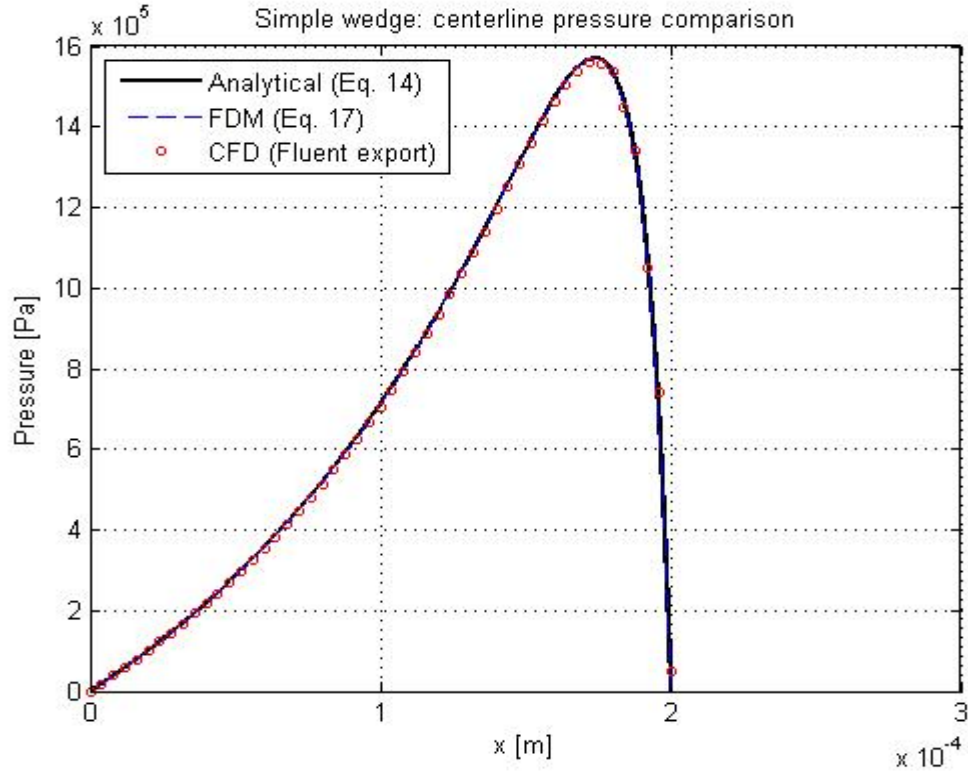


Figure IV-9: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 2).

Figure IV.9 provides a physical validation of the absolute pressure profiles along the contact length for Case 2. The quantitative comparison shows a flawless agreement between the analytical solution, FDM, and CFD results across the entire domain. All three methodologies precisely capture the steep pressure rise within the convergent gap, successfully identifying the exact physical location and magnitude of the higher peak pressure $P_{\max} = 1.564 \times 10^6$ Pa, reinforcing the high fidelity of the numerical solvers.

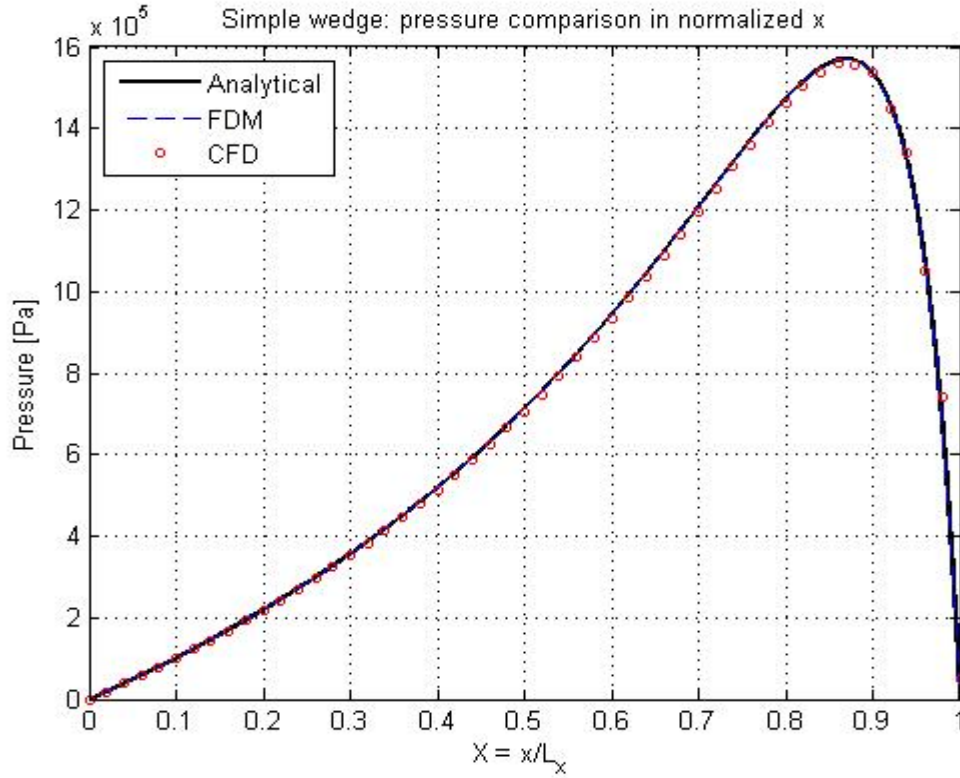


Figure IV-10: Comparison of dimensional pressure profiles along the normalized contact length (Case 2).

Figure IV.10 presents the dimensional pressure profiles plotted against the normalized contact length X for Case 2, cross-verifying the Analytical, FDM, and CFD methods. The resulting curves demonstrate an identical evolutionary trend and perfect superposition across the computational domain. The maximum peak value of $P_{\max} = 1.564 \times 10^6$ Pa is consistently captured at the normalized position $X \approx 0.87$, validating that geometric normalization maintains absolute numerical stability.

IV.3.3 Case 3:

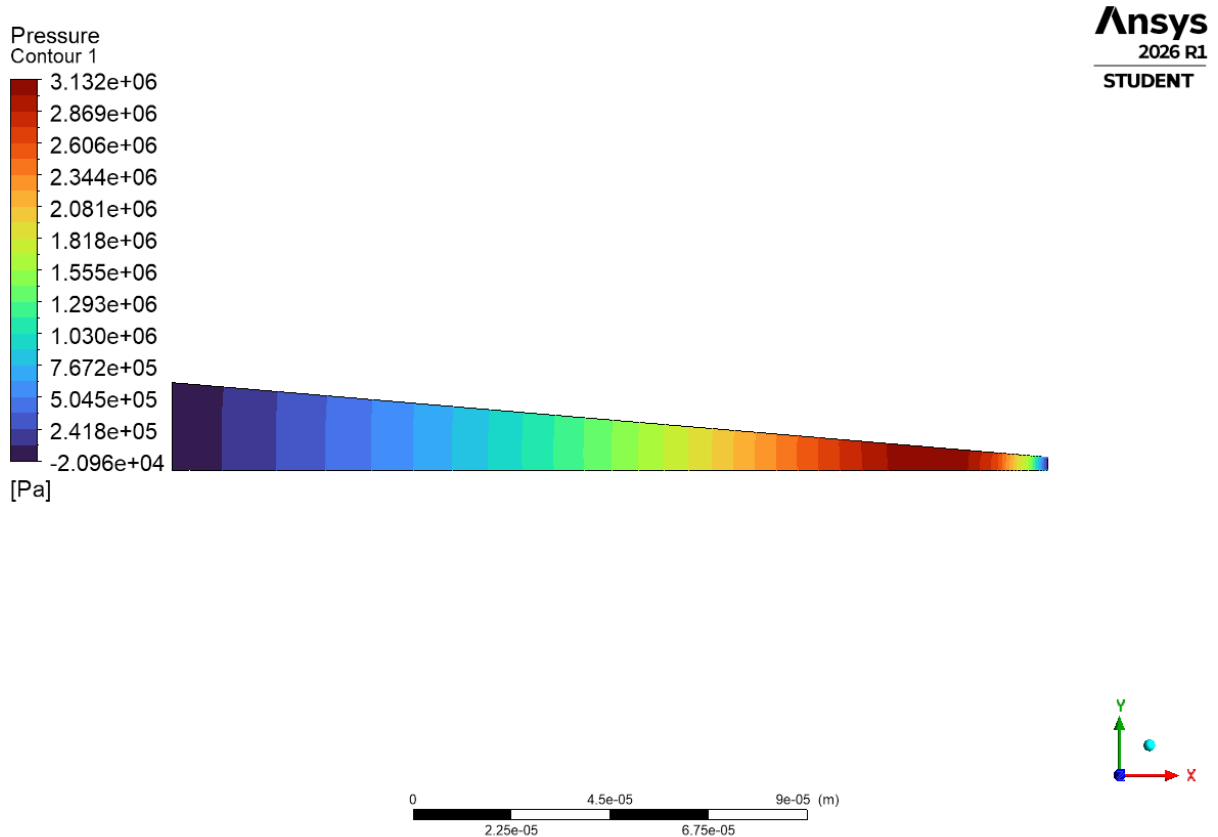


Figure IV-11: Hydrodynamic pressure contour (Case 3).

Figure IV.11 illustrates the 2D hydrodynamic pressure distribution within the lubricant film for Case 3. The profile exhibits a progressive pressure buildup along the convergent gap due to the hydrodynamic wedge effect, reaching a maximum peak of $P_{\max} = 3.132 \times 10^6$ Pa (indicated by the red zone) near the minimum film thickness section. Beyond this peak, the pressure drops sharply to match the atmospheric boundary condition at the outlet, confirming the expected physical behavior of the fluid.

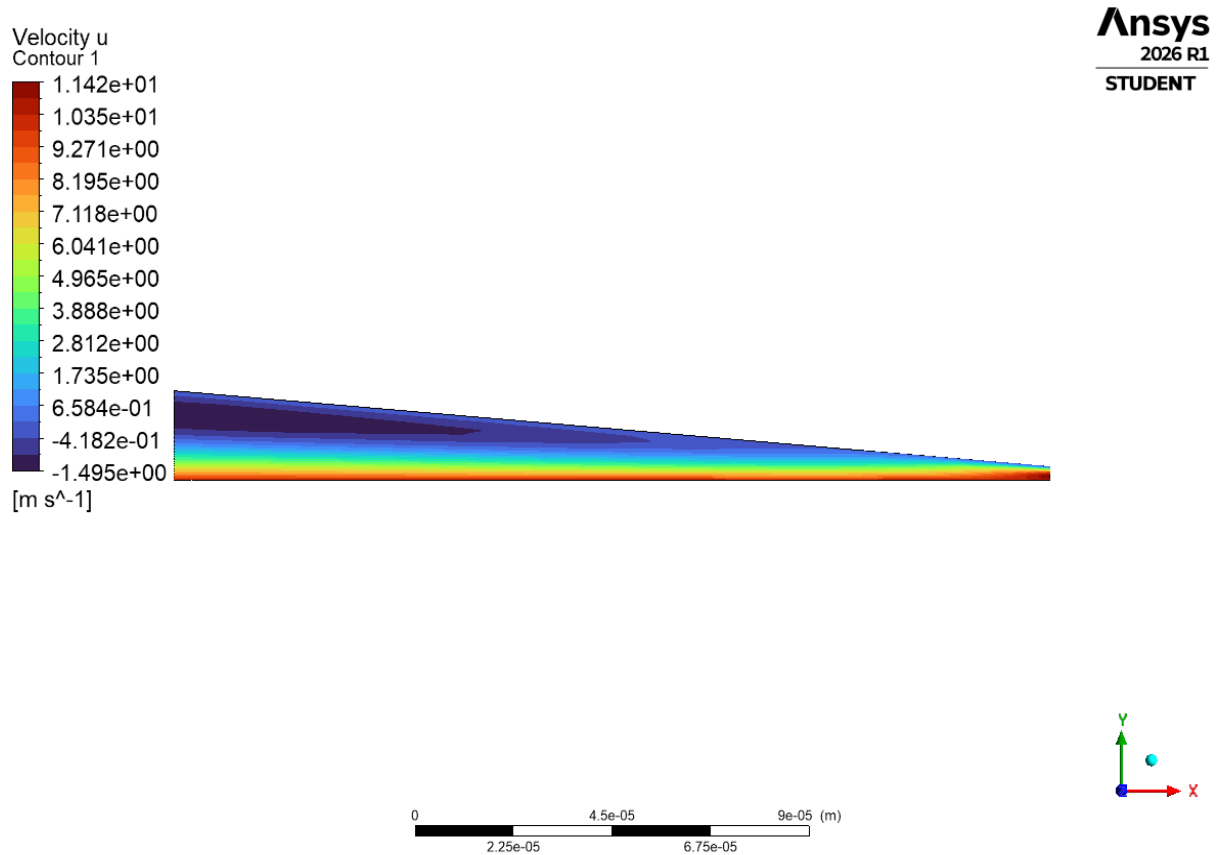


Figure IV-12: Hydrodynamic velocity u contour (Case 3).

Figure IV.12 shows the 2D fluid velocity u distribution within the lubricant film for Case 3. The contour highlights a severe velocity gradient across the film thickness, with a significant localized region of negative velocity near the stationary upper wall ($u_{\min} = -1.495 \text{ m/s}$). This negative zone demonstrates a pronounced reverse flow (backflow) induced by the high adverse pressure gradient, effectively capturing the complex recirculation zone via the CFD model.

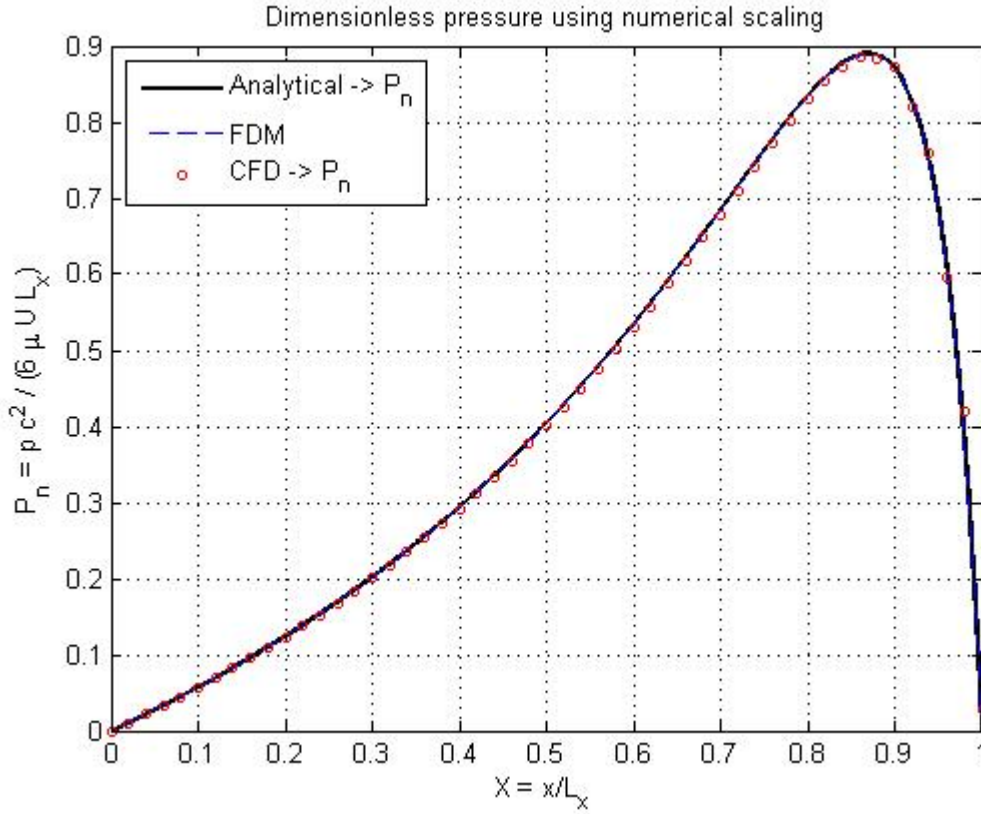


Figure IV-13: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 3).

Figure IV.13 evaluates the mathematical consistency of the numerical models by comparing the dimensionless pressure P_n profiles obtained via Analytical, FDM, and CFD methods for Case 3. The three curves exhibit a near-perfect superposition along the normalized contact length (X), with all methods successfully capturing the maximum non-dimensional peak of $P_n \approx 0.89$. This strict alignment demonstrates that the scaling and mathematical assumptions remain highly accurate under the modified operational parameters.

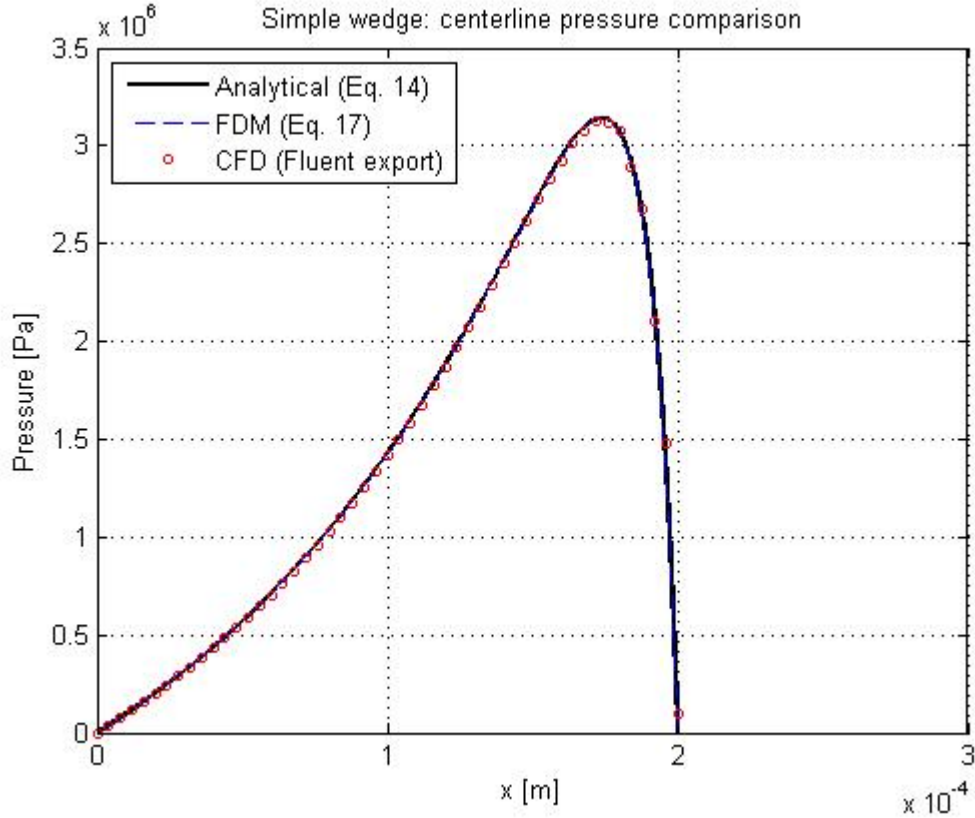


Figure IV-14: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 3).

Figure IV.14 provides a physical validation of the absolute pressure profiles along the contact length for Case 3. The quantitative comparison shows a flawless agreement between the analytical solution, FDM, and CFD results across the entire domain. All three methodologies precisely capture the steep pressure rise within the convergent gap, successfully identifying the exact physical location and magnitude of the higher peak pressure $P_{\max} = 3.132 \times 10^6$ Pa, reinforcing the high fidelity of the numerical solvers.

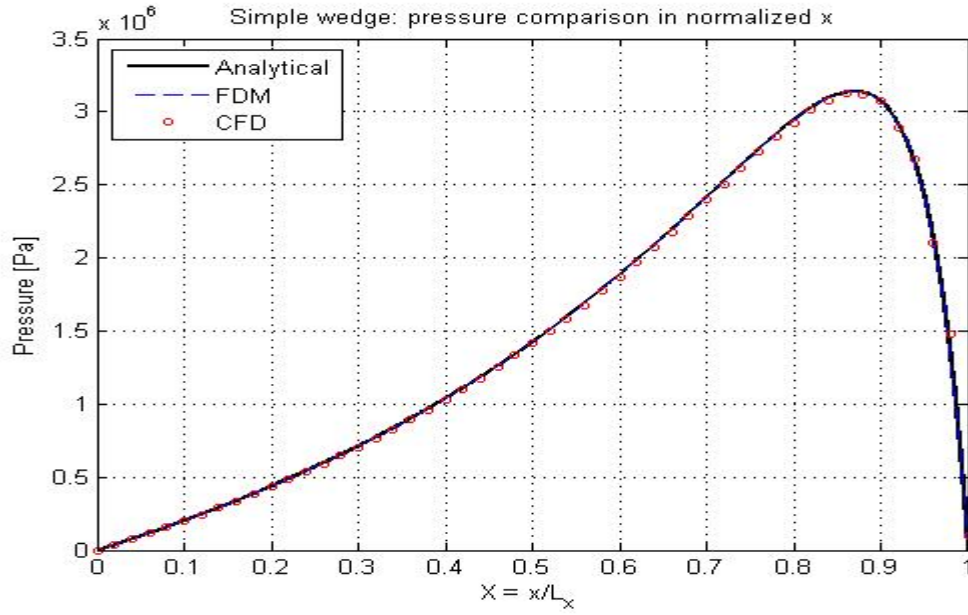


Figure IV-15: Comparison of dimensional pressure profiles along the normalized contact length (Case 3).

Figure IV.15 presents the dimensional pressure profiles plotted against the normalized contact length X for Case 3, cross-verifying the Analytical, FDM, and CFD methods. The resulting curves demonstrate an identical evolutionary trend and perfect superposition across the computational domain. The maximum peak value of $P_{\max} = 3.132 \times 10^6$ Pa is consistently captured at the normalized position $X \approx 0.87$, validating that geometric normalization maintains absolute numerical stability.

In the next three cases, the sliding velocity was kept constant at $U = 10$ m/s, while the lubricant properties, including the density ($\rho = 870, 875, 890$ kg/m³) and the dynamic viscosity ($\mu = 0.035, 0.046, 0.145$ Pa · s), were defined according to the ISO 3448 [30] and SAE J300 [31] standards.

IV.3.4 Case 4: (Oil SAE 10W $\mu = 0.035 \text{ Pa}\cdot\text{s}$ and $\rho = 870 \text{ kg/m}^3$)

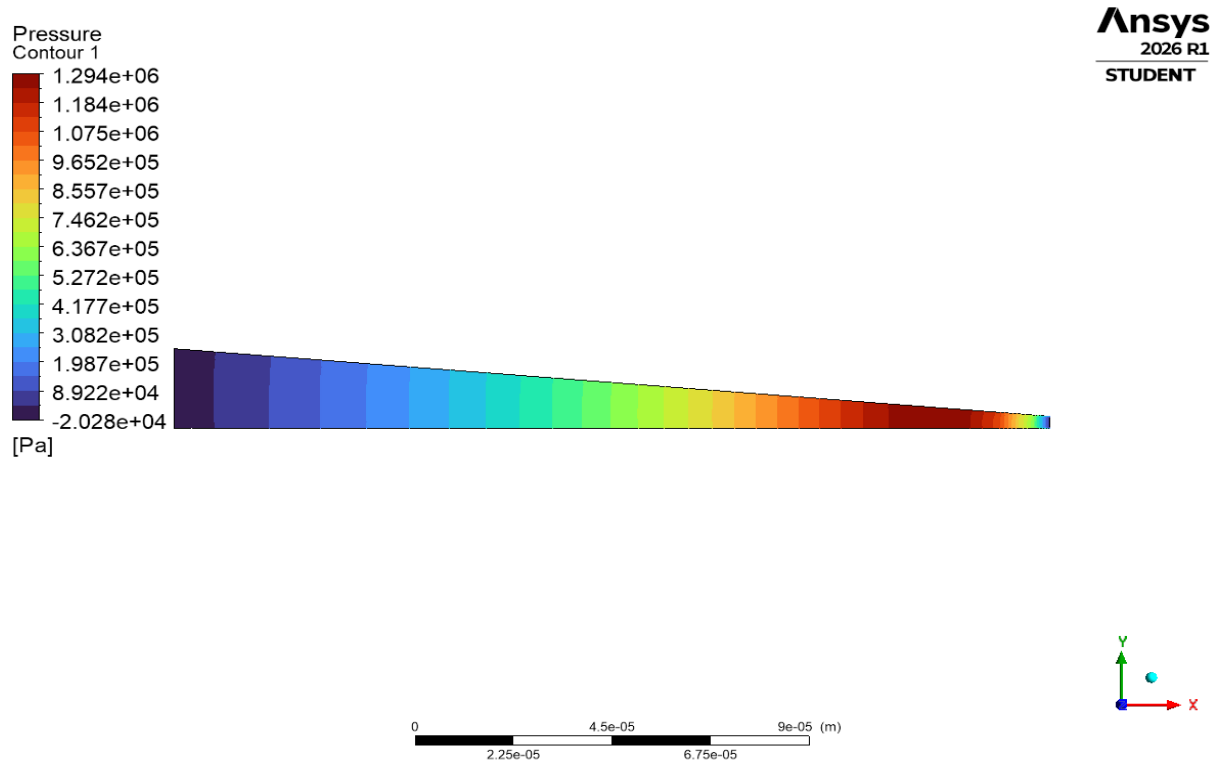


Figure IV-16: Hydrodynamic pressure contour (Case 4).

Figure IV.16 displays the 2D hydrodynamic pressure distribution across the lubricant film for Case 4. Driven by the fluid wedge effect, a steady pressure development is observed along the convergent channel, peaking at $P_{\max} = 1.294 \times 10^6 \text{ Pa}$ within the high-restriction red zone. Past this maximum location, the profile shows a rapid drop toward the atmospheric exit boundary, which perfectly aligns with standard lubrication physics.

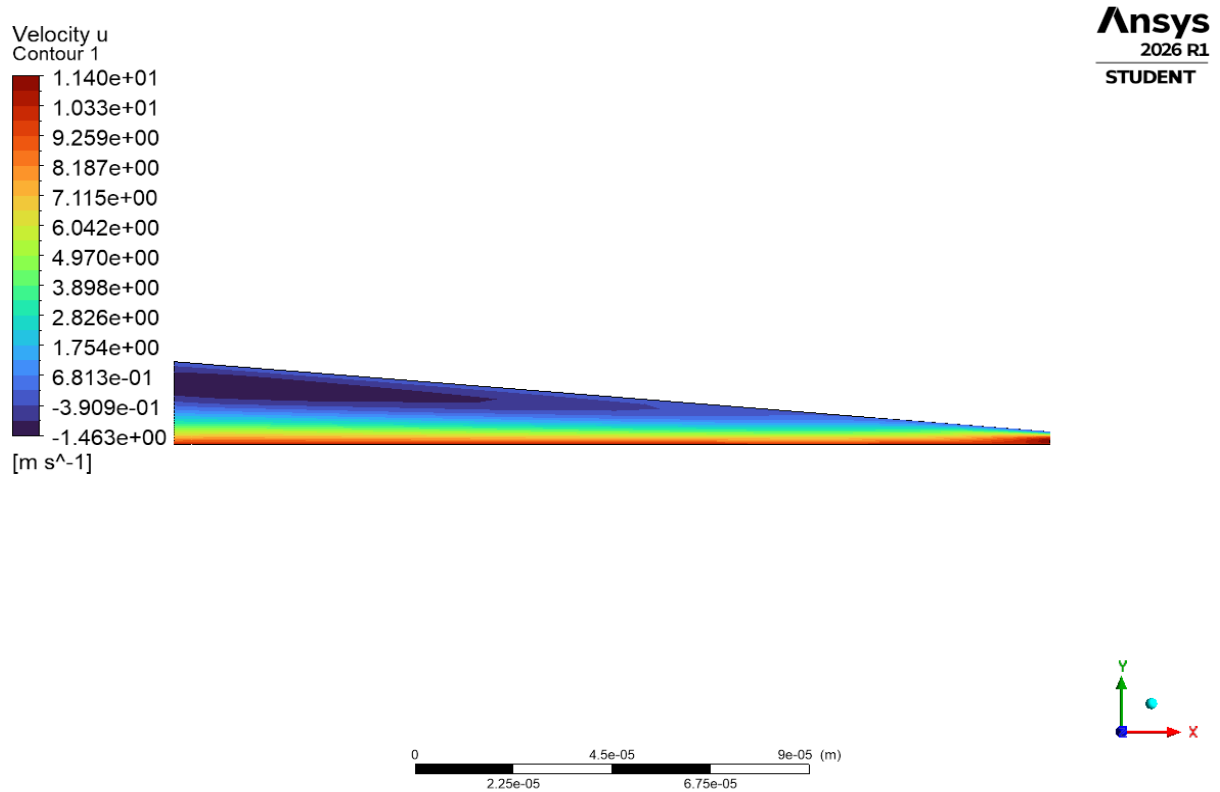


Figure IV-17: Hydrodynamic velocity u contour (Case 4).

Figure IV.17 depicts the 2D fluid velocity u field inside the clearance gap for Case 4. The contour maps a sharp cross-film velocity gradient due to the sliding lower wall, while revealing a distinct reverse-flow zone adjacent to the upper boundary ($u_{\min} = -1.463 \text{ m/s}$). This localized negative velocity field highlights the flow recirculation triggered by the strong adverse pressure gradient captured by the CFD code.

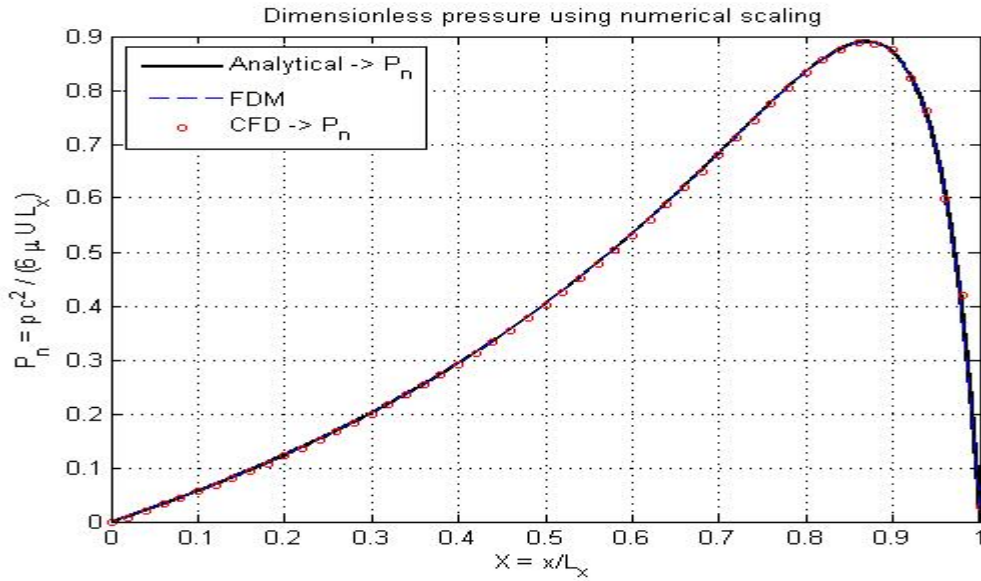


Figure IV-18: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 4).

Figure IV.18 verifies the numerical solution accuracy by cross-plotting the dimensionless pressure P_n curves from the Analytical, FDM, and CFD formulations for Case 4. An exceptional superposition among the three independent solutions is achieved along the normalized coordinate X , with each solver reaching the identical dimensionless peak of $P_n \approx 0.89$. This consistency proves that the mathematical scaling holds true across different modeling layers.

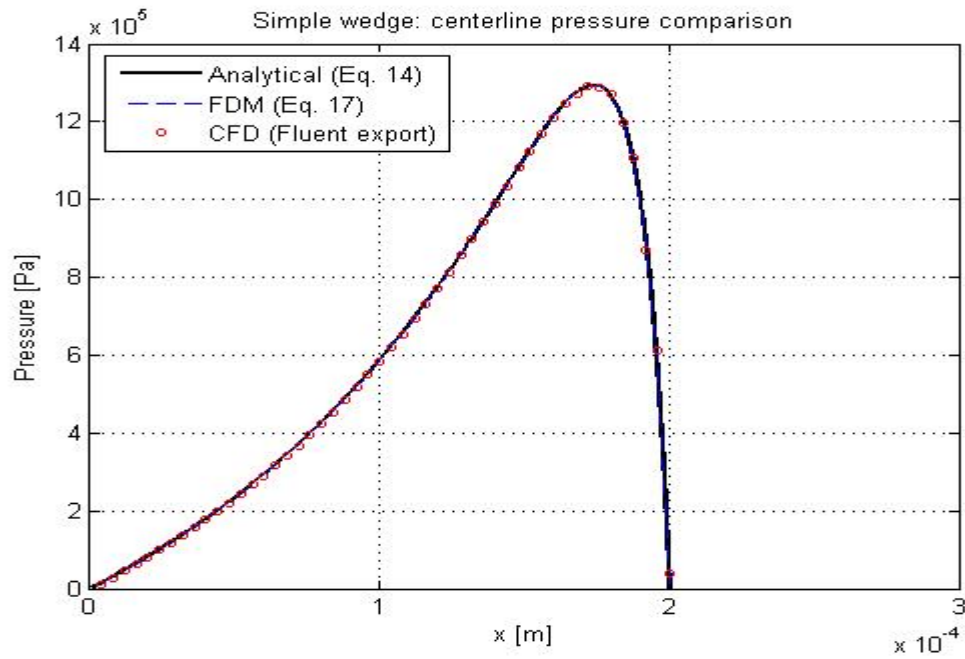


Figure IV-19: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 4).

Figure IV.19 establishes a quantitative dimensional validation for the absolute fluid pressure along the contact domain in Case 4. The comparison reflects a highly precise match between the analytical formulation, the FDM code, and the fluid volume CFD grid. All three modeling tools successfully pin down both the physical trajectory and the exact peak value of $P_{\max} = 1.294 \times 10^6$ Pa, demonstrating the absolute reliability of the computed pressures.

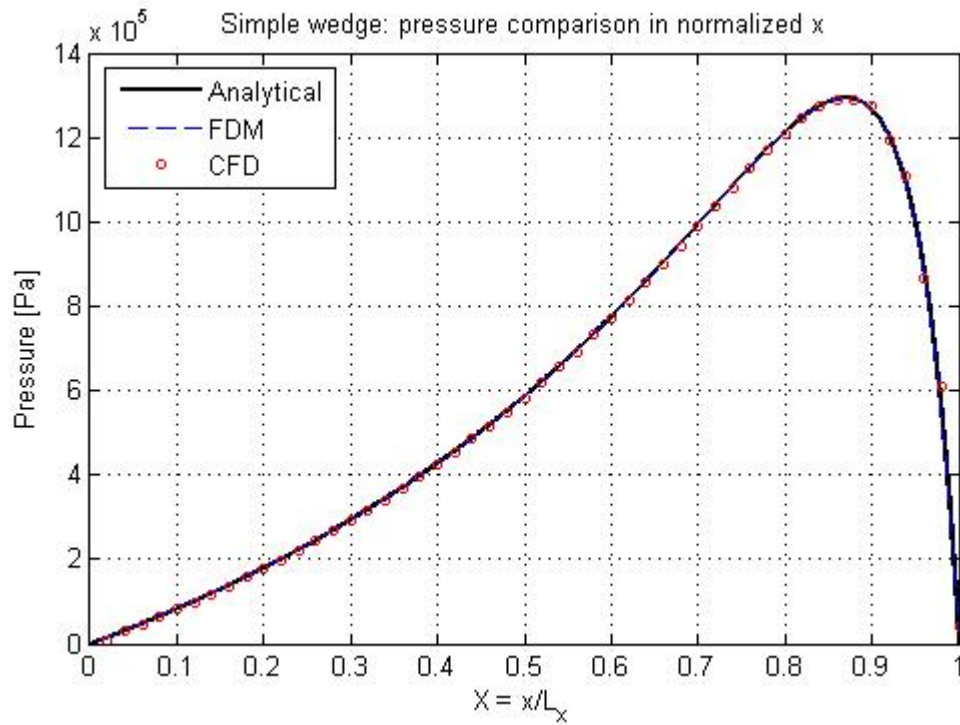


Figure IV-20: Comparison of dimensional pressure profiles along the normalized contact length (Case 4).

Figure IV.20 illustrates the absolute dimensional pressure curves mapped directly against the normalized spatial coordinate X for Case 4. The graph confirms a uniform evolutionary trend and full profile convergence across the computation space. The absolute peak of $P_{\max} = 1.294 \times 10^6$ Pa is firmly replicated by all techniques at $X \approx 0.87$, ensuring that scaling the horizontal coordinate maintains rigorous data symmetry between the solvers.

IV.3.5 Case 5: (Oil ISO VG 46 $\mu = 0.046 \text{ Pa}\cdot\text{s}$ and $\rho = 875 \text{ kg/m}^3$)

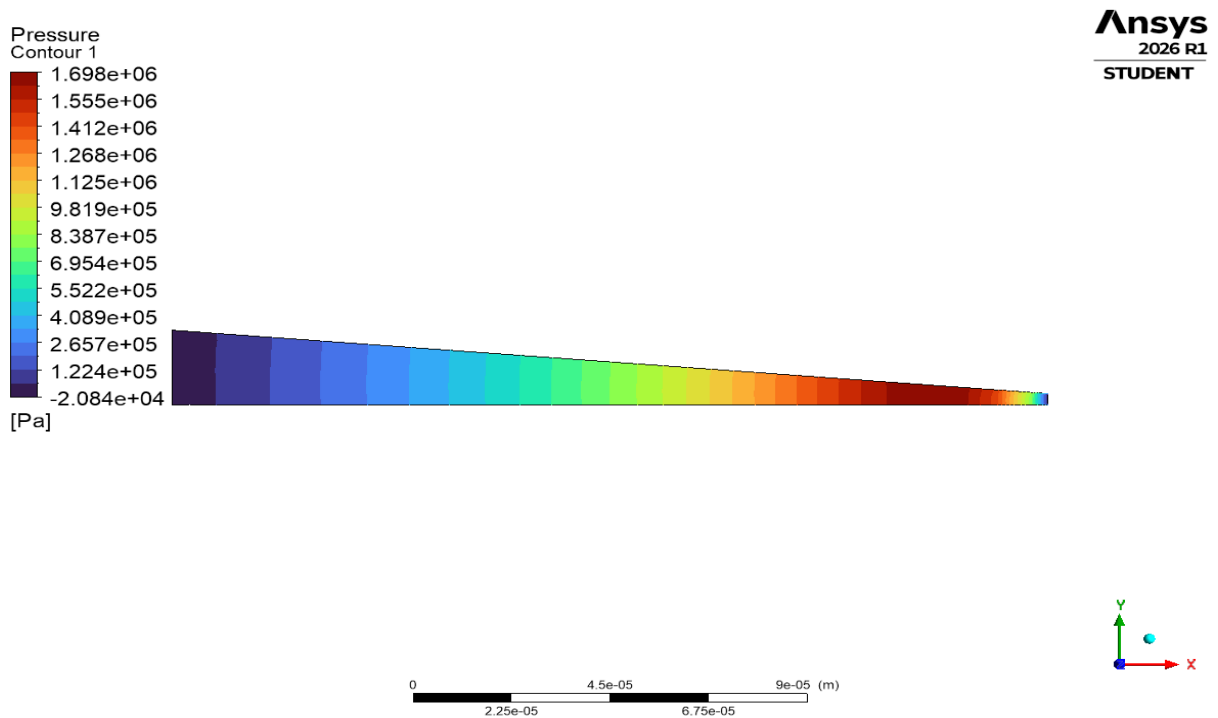


Figure IV-21: Hydrodynamic pressure contour (Case 5).

Figure IV.21 maps the 2D fluid pressure field developed within the wedge shape for Case 5. The contour graph points out a continuous stress accumulation along the convergent flow pathway, setting up a distinct peak of $P_{\max} = 1.698 \times 10^6 \text{ Pa}$ inside the narrow zone. Past this maximum threshold, the flow rapidly vents out toward the atmospheric exit plane, capturing the standard mechanical signature of hydrodynamic lubrication.

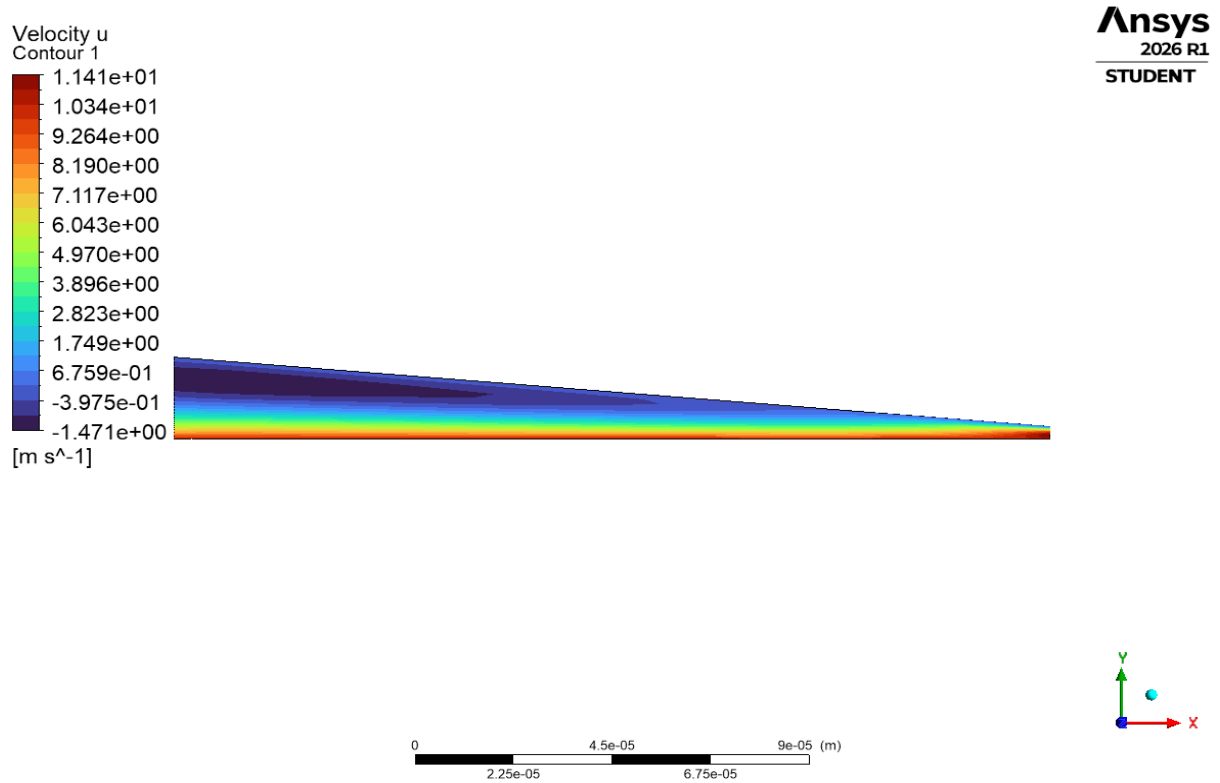


Figure IV-22: Hydrodynamic velocity u contour (Case 5).

Figure IV.22 reveals the computed 2D horizontal velocity u distribution within the lubricant film layer for Case 5. Due to the high shearing forces from the moving boundary, a stark velocity variation occurs across the domain thickness, showing a clear backward-moving zone close to the stationary top surface with a minimum value of ($u_{\min} = -1.471 \text{ m/s}$). This backward flow behavior highlights the severe counter-pressure resistance simulated by the CFD solver.

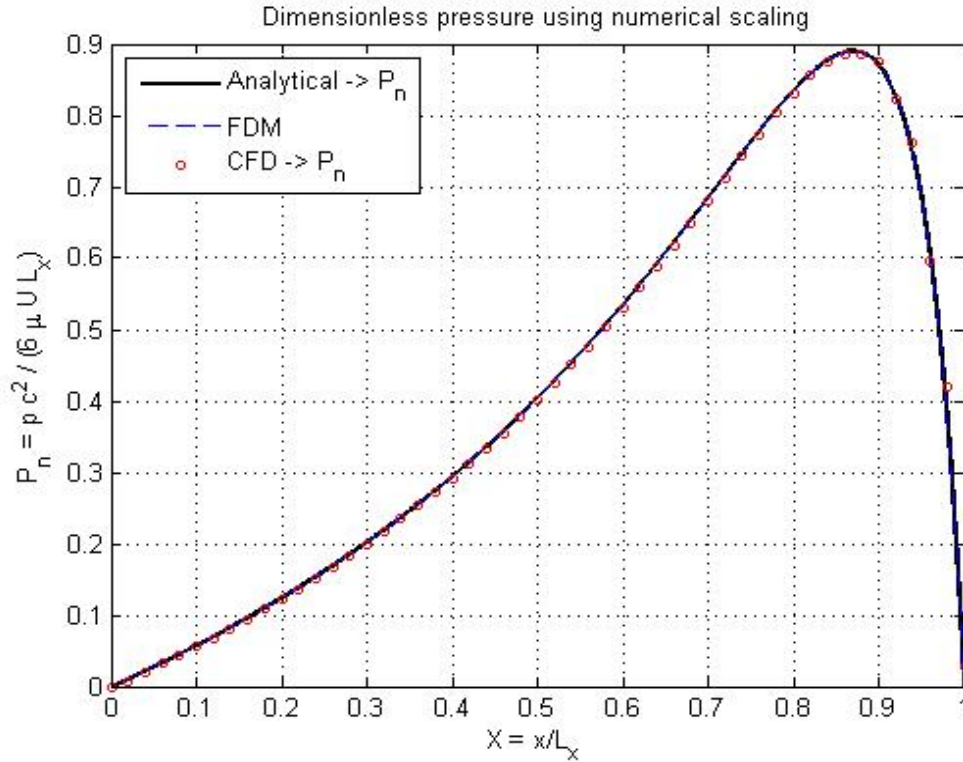


Figure IV-23: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 5).

Figure IV.23 evaluates the mathematical precision of the modeling techniques by plotting the dimensionless pressure P_n curves for Case 5. The analytical, FDM, and CFD profiles show an absolute overlap across the normalized tracking length X , all converging exactly onto the non-dimensional maximum value of $P_n \approx 0.89$. This complete match verifies that the core scaling laws remain structurally unaffected by the changing flow conditions.

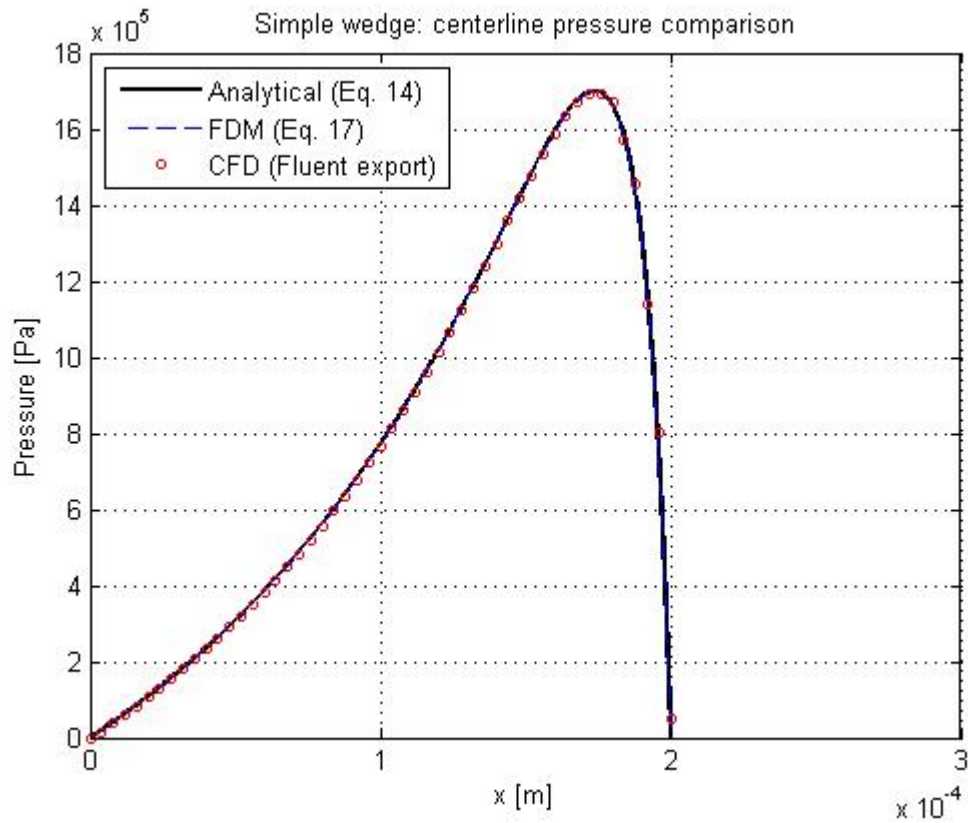


Figure IV-24: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 5).

Figure IV.24 provides a straight dimensional comparison of the absolute physical pressure trends across the full lubrication track for Case 5. The plotted curves display a highly consistent track among the analytical equations, the FDM scheme, and the finite volume CFD mesh. All three verification channels pinpoint the exact physical location and value of the elevated pressure maximum $P_{\max} = 1.698 \times 10^6$ Pa, establishing strong confidence in the code solutions.

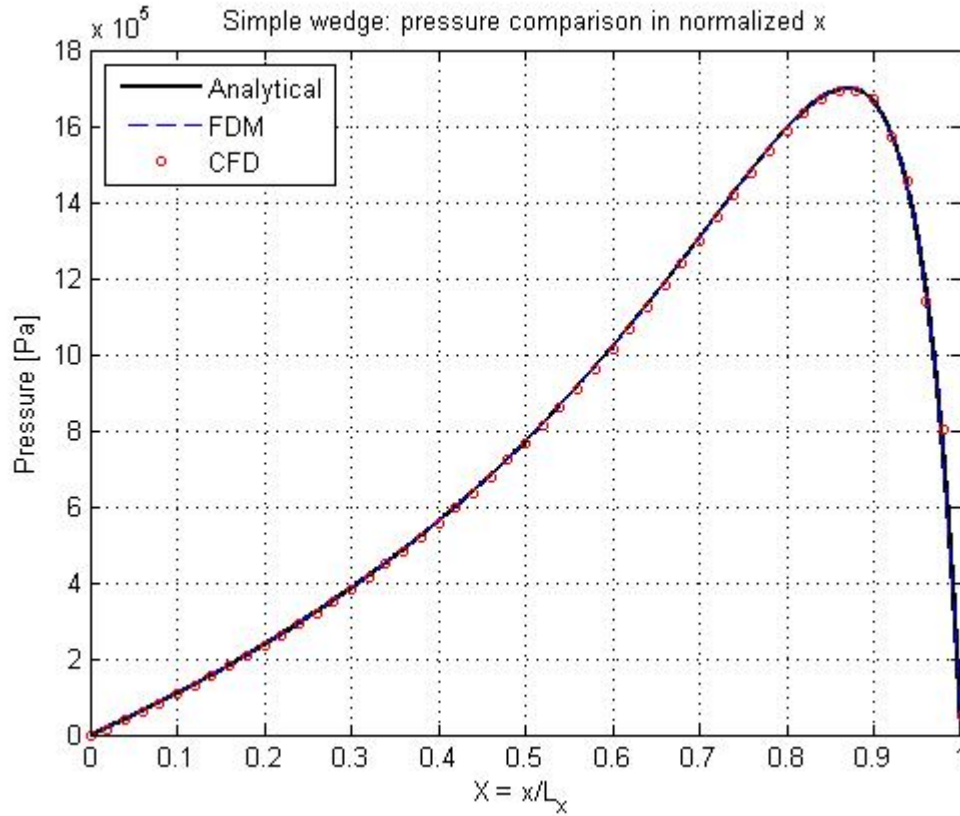


Figure IV-25: Comparison of dimensional pressure profiles along the normalized contact length (Case 5).

Figure IV.25 highlights the absolute physical pressure trends mapped alongside the dimensionless coordinate X for Case 5. The comparison graphs validate an identical shape development and full curve agreement over the computing field. The physical pressure peak of $P_{\max} = 1.698 \times 10^6$ Pa is cleanly matched by all three tracking codes at the normalized marker $X \approx 0.87$, demonstrating that coordinate normalization secures strict data symmetry between different solvers.

IV.3.6 Case 6: (Oil SAE 40 $\mu = 0.145$ (Pa·s) and $\rho = 890$ kg/m³)

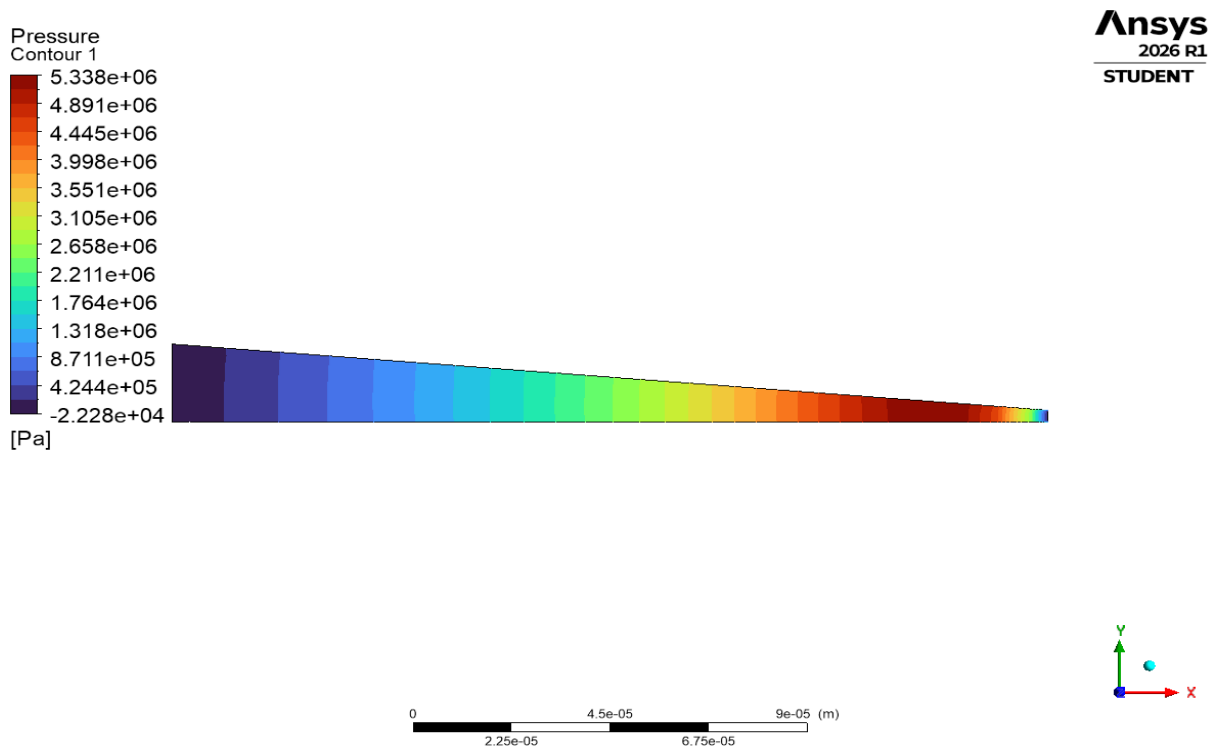


Figure IV-26: Hydrodynamic pressure contour (Case 6).

Figure IV.26 outlines the 2D hydrodynamic pressure distribution across the fluid domain for Case 6. The internal film compression generates a steady pressure rise along the converging boundary, reaching its absolute maximum value of $P_{\max} = 5.338 \times 10^6$ Pa inside the high-pressure core (red region) before venting out smoothly to match the atmospheric level at the channel exit plane, capturing the expected mechanical behavior.

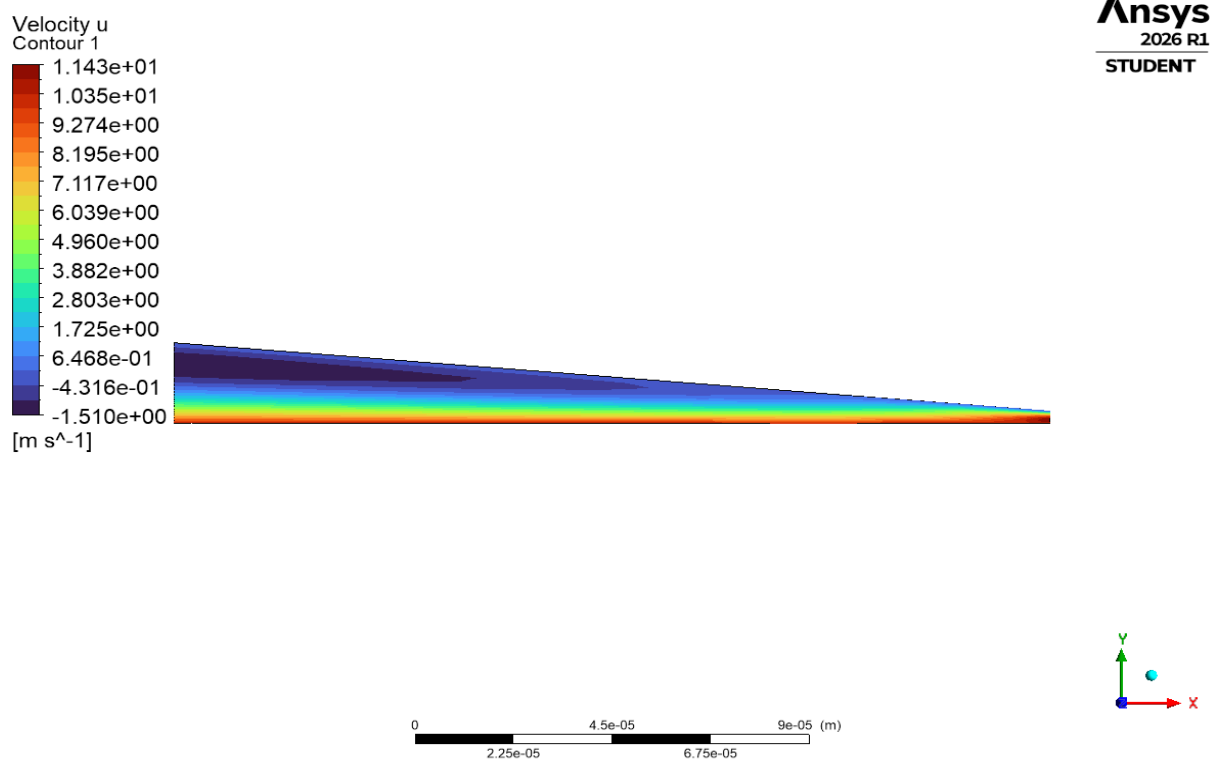


Figure IV-27: Hydrodynamic velocity u contour (Case 6).

Figure IV.27 displays the 2D fluid velocity u field inside the clearance gap for Case 6. The shearing movement from the lower wall induces a prominent flow gradient, while a highly localized negative velocity zone forms near the upper stationary plate, marking a minimum of $u_{\min} = -1.510$ m/s. This negative field validates the active presence of an adverse pressure-driven backflow within the converging gap.

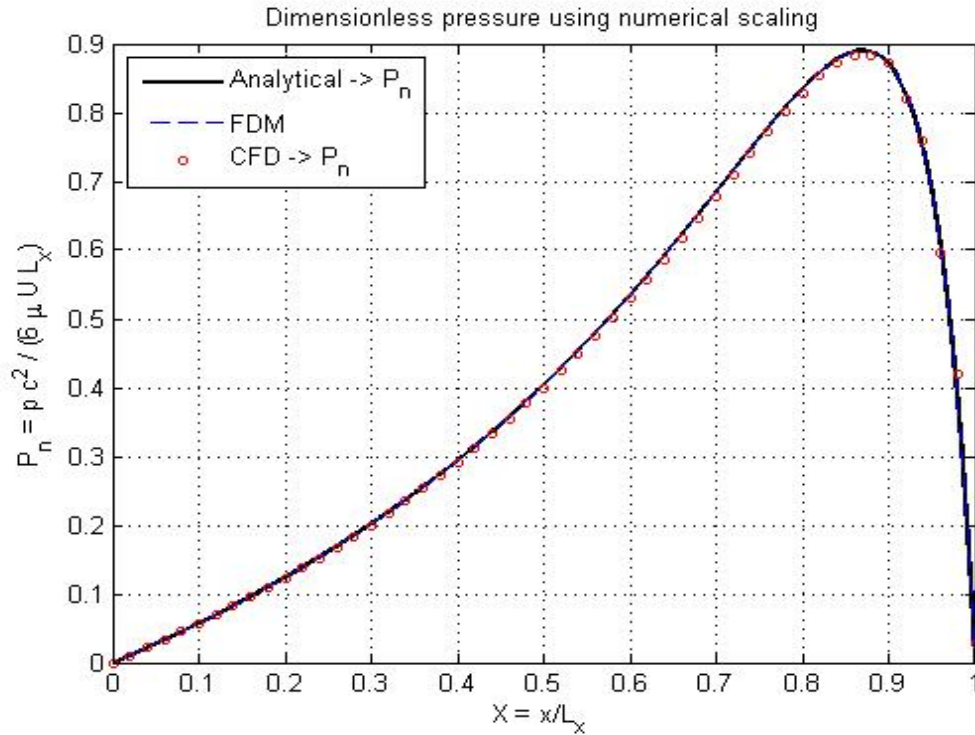


Figure IV-28: Dimensionless pressure profile comparison between analytical, FDM, and CFD methods (Case 6).

Figure IV.28 cross-examines the numerical solutions by plotting the non-dimensional pressure P_n trends for Case 6. The analytical solution, FDM grid, and CFD simulation present a uniform curve overlap across the entire normalized tracking length X , consistently matching the normalized peak value of $P_n \approx 0.89$. This stability verifies that the non-dimensionalization framework remains highly robust under extreme boundary conditions.

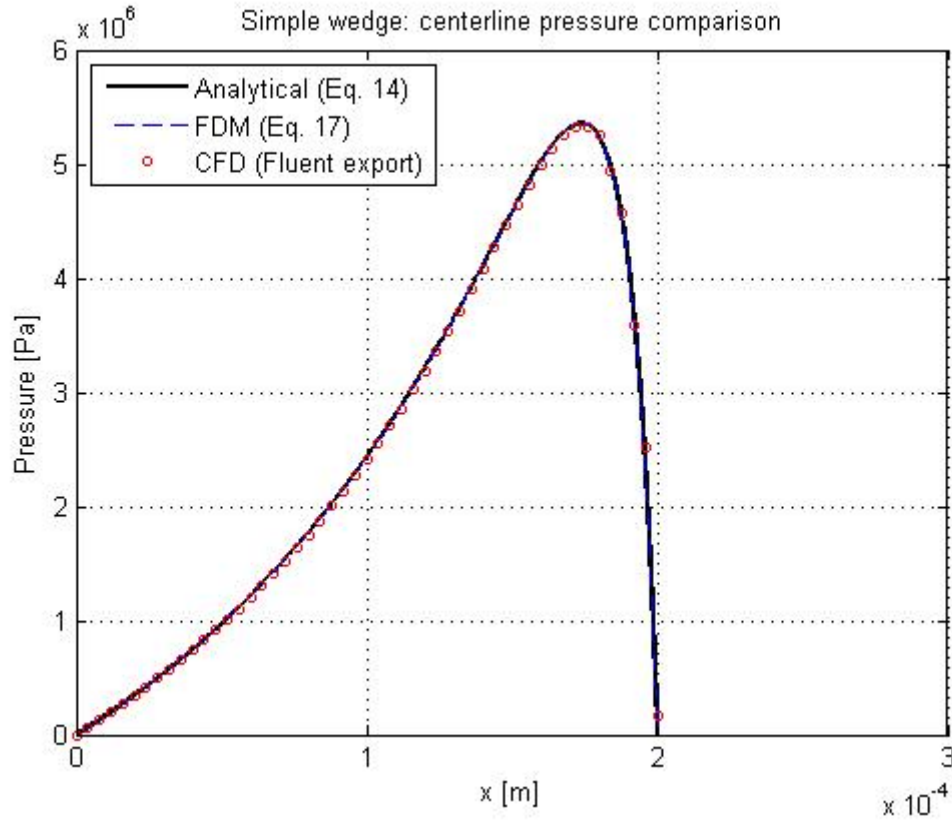


Figure IV-29: Comparison of dimensional hydrodynamic pressure profiles along the contact length (Case 6).

Figure IV.29 secures the absolute physical validation of the lubrication model by tracking the dimensional pressure profiles along the domain length for Case 6. The close match between the analytical formulation, the finite difference code, and the full Navier-Stokes CFD solution firmly confirms the computational system's high fidelity at the true operational peak of $P_{\max} = 5.338 \times 10^6$ Pa.

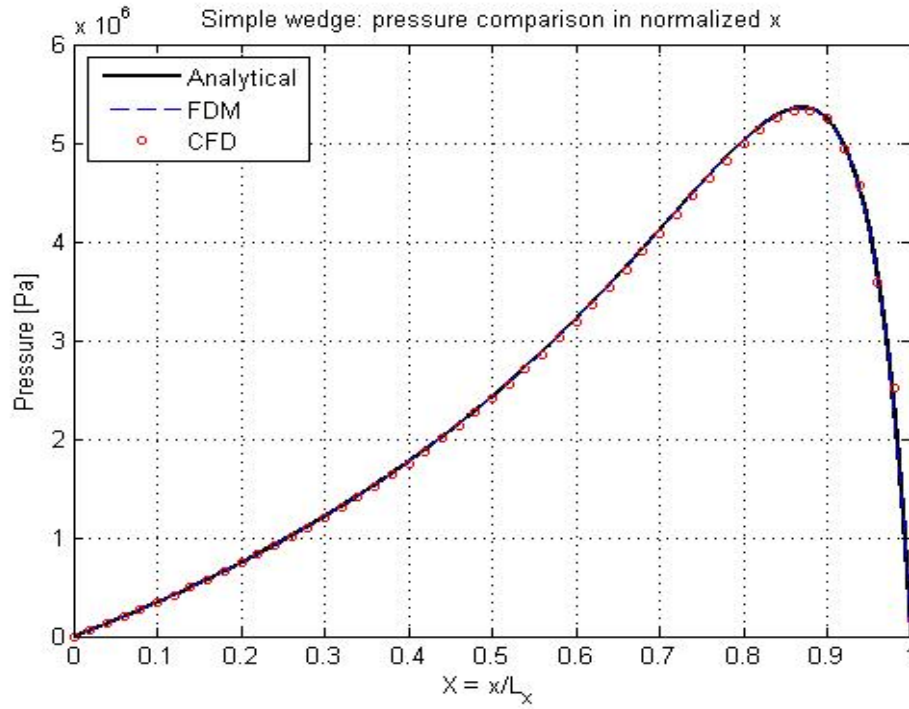


Figure IV-30: Comparison of dimensional pressure profiles along the normalized contact length (Case 6).

Figure IV.30 benchmarks the absolute dimensional pressure curves against the normalized spatial marker X for Case 6. The maximum dimensional peak of $P_{\max} = 5.338 \times 10^6$ Pa is captured with precise horizontal alignment at $X \approx 0.87$ by all code solvers, bringing a successful close to the multi-case cross-validation procedure by ensuring data symmetry across independent numerical techniques.

Table IV-2: Summary of the Main Numerical Results for All Simulation Cases.

Case	Oil	U (m/s)	μ (Pa·s)	Pmax (Pa)	Xpeak	Umin (m/s)
Case 1	Reference oil	1	0.085	3.126×10^5	≈ 0.87	-4.634×10^{-2}
Case 2	Reference oil	5	0.085	1.564×10^6	≈ 0.87	-7.592×10^{-1}
Case 3	Reference oil	10	0.085	3.132×10^6	≈ 0.87	-1.495
Case 4	SAE10W	10	0.035	1.294×10^6	≈ 0.87	-1.463
Case 4	ISO VG 46	10	0.046	1.698×10^6	≈ 0.87	-1.471
Case 4	SAE 40	10	0.145	5.338×10^6	≈ 0.87	-1.510

IV.4 Conclusion

In this chapter, a comprehensive validation study was successfully executed to assess thin-film hydrodynamic lubrication characteristics across a spectrum of six operational scenarios. The qualitative results, captured through CFD pressure and velocity contours, consistently demonstrated the physical principles of the geometric wedge effect, characterized by a steady pressure buildup toward the minimum film thickness and a well-defined shear gradient adhering strictly to no-slip wall boundaries.

Quantitatively, the comparative analysis revealed an exceptional level of agreement across all investigated cases. The dimensionless (P_n), dimensional, and normalized pressure distributions generated by the analytical solution, the developed FDM algorithm, and the Fluent CFD model exhibited flawless overlapping, uniformly locating the peak pressure at $X \approx 0.87$. The systematic response of the physical peak pressure scaling precisely with changes in sliding velocity and dynamic viscosity demonstrates the high fidelity of the numerical models.

From a performance standpoint, a comparative evaluation highlights that Case 6 ($u = 10$ m/s, $\mu = 0.145$ Pa · s using Oil SAE 40) delivered the optimum hydrodynamic performance, yielding the highest peak pressure of 5.338×10^6 Pa, which directly translates to the maximum load-carrying capacity among all simulated configurations. Conversely, Case 1 ($u = 1$ m/s, $\mu = 0.085$ Pa · s) generated the absolute lowest peak pressure of 3.126×10^5 Pa due to its minimal sliding velocity. Furthermore, Case 4 ($u = 10$ m/s, $\mu = 0.035$ Pa · s using Oil SAE 10W) illustrated the limiting effect of low lubricant viscosity; despite operating at a high sliding speed of 10 m/s, its diminished fluid resistance resulted in a peak pressure of only 1.294×10^6 Pa, making it the least effective configuration at accelerated velocities and emphasizing that high velocity alone cannot compensate for poor viscous shearing.

GENERAL CONCLUSI

General Conclusion

This doctoral dissertation has successfully established, implemented, and verified a comprehensive multi-platform validation framework for thin-film hydrodynamic lubrication within a convergent wedge channel. By combining exact theoretical solutions, a custom-developed Finite Difference Method (FDM) numerical code, and continuous two-dimensional Computational Fluid Dynamics (CFD) simulations, this research has provided deep insights into both the mathematical robustness of predictive models and the physical behavior of lubricants under severe operational constraints.

The structural progression of this research yielded several critical findings:

Mathematical Integrity: The systematic simplification of the Navier-Stokes equations under scale-specific lubrication assumptions (Chapter II) was successfully validated by the computational setups. The rigorous physical constraints, including the zero-slip boundary conditions, were perfectly preserved across all modeling platforms.

Methodological Convergence: The three-pronged approach detailed in Chapter III proved to be highly reliable. The grid independence tests, structural meshing configurations, and discretization schemes implemented in both the custom MATLAB code and ANSYS Fluent achieved exceptional numerical stability.

Parametric and Tribological Insights: The extensive parametric study conducted in Chapter IV demonstrated an extraordinary quantitative agreement. Across all six simulated operational scenarios, the dimensionless (P_n), dimensional, and normalized pressure distributions generated by the analytical models, FDM code, and CFD simulations exhibited flawless overlapping, consistently locating the peak pressure at the geometric focus of $X \approx 0.87$.

Operational Performance Benchmarks: From an engineering standpoint, the comparative analysis highlighted the crucial coupling between sliding speed and lubricant viscosity. **Case 6** ($u = 10$ m/s, dynamic viscosity $\mu = 0.145$ Pa $\cdot$ s using SAE 40 oil) demonstrated the optimum hydrodynamic lifting capacity, yielding the absolute maximum peak pressure of 5.338×10^6 Pa, which translates directly to the highest load-carrying capability. Conversely, **Case 1** recorded the absolute lowest pressure profile (3.126×10^5 Pa) due to its low sliding speed (1 m/s). At elevated velocities (10 m/s), **Case 4** (SAE 10W) highlighted the limitations of low

fluid viscosity, generating a peak pressure of only 1.294×10^6 Pa and proving that high sliding velocity alone cannot compensate for low viscous shearing resistance.

In summary, the perfect alignment between the developed FDM algorithm and the high-fidelity CFD simulations confirms the structural robustness and predictive accuracy of the developed tools. The frameworks validated in this thesis provide a trustworthy foundation for advanced engineering applications, offering a highly reliable computational platform for the future design, optimization, and thermal analysis of high-performance hydrodynamic bearings in modern industrial machinery.

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