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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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In the name of **ALLAH** most merciful

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ملخص: يسعى الإنسان دائما لمواكبة التطور التكنولوجي الذي يشهده العالم من خلال ابتكاره لتقنيات جديدة تسهل عليه التواصل عبر شبكات الإتصال خصوصا اللاسلكية منها، لذلك يسعى الباحثون لتطوير تقنيات خاصة بهدف استيعاب مستخدمين أكثر بتدفق عالي ودقة أفضل لأجل تكوين حزم تدعم الإرسال متعدد البث في الاتصالات اللاسلكية، كما تدعم تشكيل حزم كلاسيكية أحادية التدفق ترسل الإشارة نفسها من كل الهوائيات، وذلك بفضل تقنية الترميز المسبق الذي يمكنه ارسال تدفقات متعددة للبيانات وبأوزان مستقلة مناسبة لمن يعتمد على التقنيات الحديثة واهمها MIMO Massive متعدد المداخل ومتعدد المخارج. تعتمد هذه التقنية على الاستفادة من مسارات متعددة لتحسين سعة النظام، وهي مناسبة بشكل خاص للتكنولوجيا المليمترية.

يهدف هذا المشروع لدراسة محاكاة التحسينات التي ادخلت على التشفير من خلال استخدام تقنية MIMO المكثفة.

كلمات مفتاحية: MIMO الضخمة ، الترميز المسبق ، التشفير.

Abstract: Human beings always strive to keep up with the technological development that the world is witnessing by inventing new technologies that make it easier for them to communicate through communication networks, especially wireless ones. So, researchers seek to develop special technologies with the aim of accommodating more users with higher flow and better accuracy in order to form packages that support multi-broadcast transmission in wireless communications. Meanwhile, the developed technologies would support the formation of classic mono-flow packages that send the same signal from all antennas thanks to precoding technology that can send multiple data flows and appropriate independent weights. For those who rely on modern technologies, the most important of which is MIMO Massive multiple inputs and multiple outputs. This technology relies on multiple pathways to improve system capacity and is particularly suitable for Millimeter Wave Technology.

This project aims to study simulations of improvements to encryption using massive MIMO technology.

Keywords: MIMO massive, precoding, encryption.

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LISTS OF ACRONYMS AND ABBREVIATIONS

AMC Adaptive Modulation Coding

AP Access Point

BER Bit Error Rate

BPSK Binary Phase-Shift Keying

BS Base Station

CB Conjugate Beamforming

CDMA Code Division Multiple Access

Co-MIMO Cooperative MIMO

CoMP Coordinated Multipoint

CSI Channel State Information

CSIR Channel State Information Receiver

CSIT Channel-State information at the transmitter

DMT Diversity-Multiplexing Tradeoff

DPC Dirty Paper Coding

E_b/N_0 The Normalized Signal To Noise Ratio

- EGT** Equal Gain Transmission
- ISI** Intersymbol Interference
- LTE** Long Term Evolution
- LTE-A** Long Term Evolution-Advanced
- Mass-BS** MIMI Massive Base Station
- MIMO** Multi Input Multi Output
- MIMO BC** MIMO Broadcast
- MIMO MAC** MIMO Multiple-Access-Channel
- MIMO-OFDM** Orthogonal Frequency-Division Multiplexing
- MIMO-STBC** Space Time Block Coding
- MISO** Multi Input Single Output
- MMSE** Minimum Mean-Squared Error
- MMSE-LE** The Minimum Mean-Square Error Linear Equalizer
- MR** Maximum Ratio
- MS** Mobile Station
- MU-MIMO** Multi-User MIMO
- OFDM** Orthogonal Frequency-Division Multiplexing
- QAM** Quadrature Amplitude Modulation
- RF** Radio Frequency
- RS** Real Station
- RZF** Regularized Zero Forcing
- SDMA** Space-Division Multiple Access
- SIMO** Single Input Multi Output
- SISO** Single Input Single Output

SNR Signal-To-Noise Ratio

SNR Signal-to-Noise Ratio

SU-MIMO Single-User MIMO

SVD Singular Value Decomposition

TDD Time-Division Duplex

UL Up Link

UMTS Universal Mobile Telecommunication system

UT User Terminal

WLAN Wireless Local Area Network

ZF Zero Forcing

ZFE Zero Forcing Equalizer

GENERAL INTRODUCTIONS

There has been significant growth in demand for faster internet connectivity and rapid access to multimedia services in recent years. High capacity, great data rates, high spectrum efficiency, and high energy efficiency are the most important attributes. As a result, in the framework of multiple-input multiple-output (MIMO) schemes, communication systems have used new technologies. Traditional multi-antenna transmission schemes have recently undergone significant development in order to achieve high spectral efficiency and high link reliability.

The first chapter presents a short introduction about "system MIMO"; we dealt with the system in general and is based on its basis, applications, and its mathematical concepts.

After demonstrating the significance of MIMO and its applications, as well as how to use its technologies, we discussed in the second chapter the precoding technique MIMO after explaining the following after a brief introduction.

The third chapter proposes a graphical analysis. These interpretations present simulations of the effect of different parameters on the spectral efficiency of the MIMO system, and we end our work.

CHAPTER 1

MULTIPLE INPUT MULTIPLE OUTPUT - MIMO SYSTEM

1.1 Introductions

The quality of any wireless link is characterized by speed (or spectrum), range (coverage), and reliability (or security). The Multiple Input – Multiple Output (MIMO), a multiple antenna system, or a multidimensional wireless communication system, improves all these parameters. The MIMO technology exploits the use of multiple signals. Different antennas into the wireless medium transmit each signal. It enables many signal paths and all these signals are received through multiple antennas.[1]

Greater spectral efficiency, higher data rate, wider coverage, increased number of users, and enhanced reliability can be achieved by MIMO technology without using an additional frequency spectrum. Wi-Fi, Long Term Evolution (LTE), and much other radio, wireless and RF technologies are using the new MIMO wireless technology. Even now, there are many MIMO wireless routers in the market, and as this RF, technology is becoming more widespread, more MIMO routers and other items of wireless MIMO equipment will be seen.[1]

This chapter begins with the trade-off of high-speed wireless link design and the need for a MIMO system. The MIMO system is explained with the concept of diversity and then the types of diversity are discussed. Then we study the importance of MIMO and its applications and how to apply its technology.



Figure 1.1: Multiple-input and multiple-output (MIMO)[2]

1.2 Principles of MIMO

As explained above, the main source of disturbances, which a signal undergoes during its propagation, is the channel. Indeed, because multipath propagation phenomena, the signal undergoes fading, frequency or even time shifts. Unlike conventional systems, diversity systems take advantage of these types of propagation to improve system performance. To implement these improvements, MIMO systems exploit the techniques of:

Space Diversity: Also known as Antenna Diversity.

Frequency Diversity: This technique requires sending the same signal across different frequencies. However, attention must be paid to the bandwidth coherent and to the frequency range due to the multi-paths and the distances to cross by transmission.

Temporal diversity: When the sending of the same signal is separated by the coherence time of the channel, it is possible to take advantage of temporal diversity. Everything also depends on the moving speed of the mobile and the carrier frequency.[3]

1.3 Multiple types of antennas MIMO

There are four basic antenna configuration models, which include.

- SISO -single input, single output

- Single -Input Multi Output- SIMO
- MISO -Multiple Input Multiple Output
- MIMO -Multiple Input Multiple Output

1.3.1 MIMO-SISO

SISO (single input, single output) refers to a wireless communications system in which one antenna is used at the source (transmitter) and one antenna is used at the destination (receiver).



Figure 1.2: SISO- single input, single output [4]

The advantage of a SISO system is its simplicity. SISO does not require any treatment in terms of the various forms of diversity that can be used. However, the SISO channel performance is limited. Interference and fading will have more impact on the system than Shannon's law limits a MIMO system using some form of diversity, and the channel capacity - the rate depends on channel bandwidth and signal-to-noise ratio

$$C = B \log_2(1 + \text{SNR}) [\text{bit/s}] \quad (1.1)$$

Where **C** is the channel capacity, **B** is the channel bandwidth and **SNR** is the ratio signal to noise.[4]

1.3.2 MIMO-SIMO

SIMO Single Input and Multiple Output is a special case when the transmitter has one antenna.

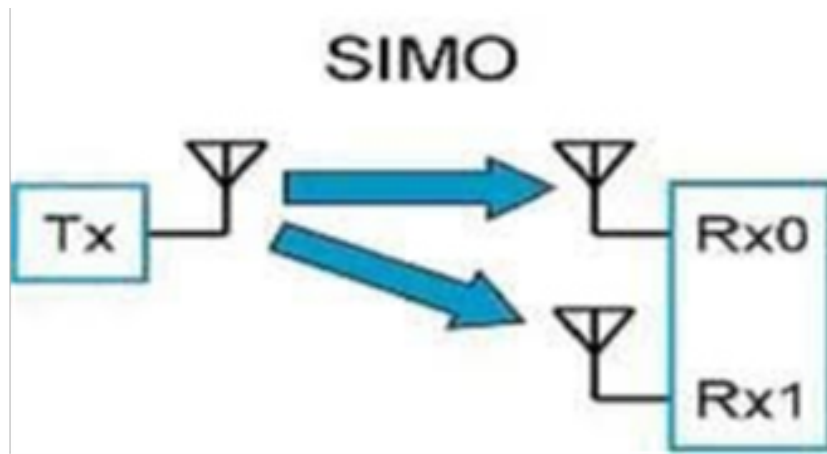


Figure 1.3: SIMO- Single Input and Multiple Output[4]

SIMO has the advantage of being relatively easy to implement, although processing is required in the receiver. Using SIMO can be quite acceptable in many applications, but when the receiver is located on a mobile device such as a mobile phone handset, the processing levels may be limited by size, cost and battery charge.[4]

1.3.3 MIMO-MISO

MISO is also called transmit diversity. In this case, the same data is transmitted redundantly from both transmitting antennas. The receiver is then able to receive the optimal signal that it can then use to extract the required data.

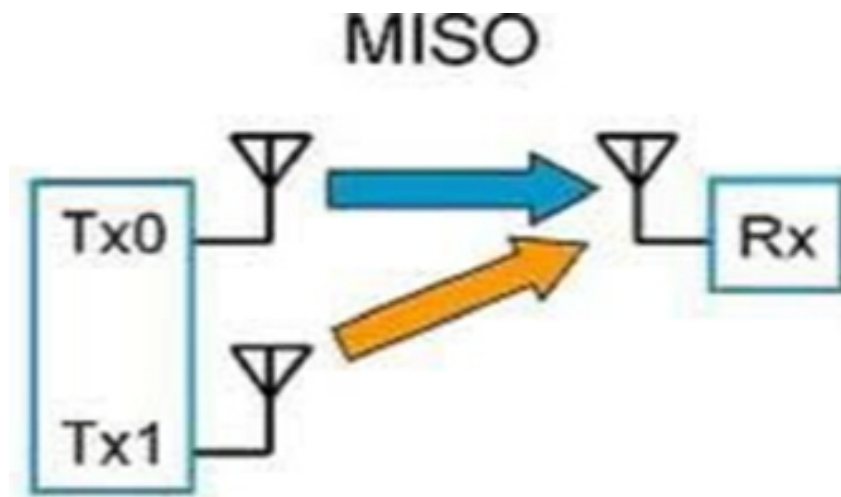


Figure 1.4: MISO - Multiple Input Single Output[4]

The advantage of using MISO is that the multiple antennas and the processing of redundancy are moved from the receiver to the transmitter. In cases such as cell phone user equipment, this can be an advantage significant in terms of space for antennas and reduction in processing level

required by the receiver for redundancy coding. This has a positive impact on the size, cost and battery life because the lower level of processing requires the battery consumption.[4]

1.3.4 MIMO-MIMO

MIMO is a radio antenna technology that uses multiple antennas at the transmitter and receiver level to allow a variety of signal paths transport the data, choosing separate paths for each antenna to allow the use of multiple signal paths.

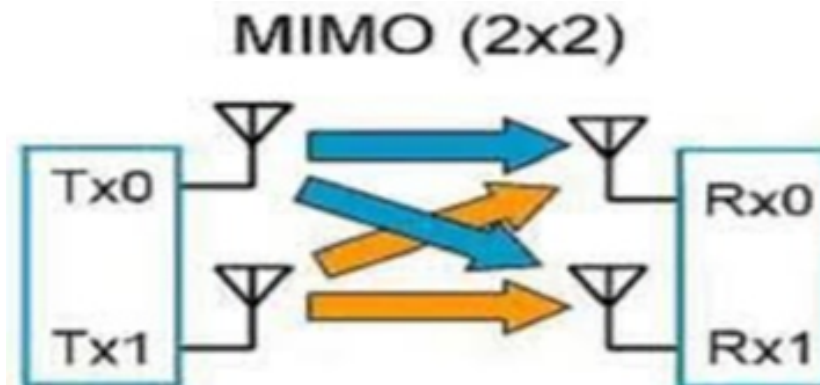


Figure 1.5: MIMO- Multiple Input Multiple Output[4]

MIMO communication channels provide an interesting solution to the problem of multipath propagation by requiring multiple signal paths indeed; MIMO systems use a combination of several antennas and several signal paths to gain knowledge about the channel of communication. Using the spatial dimension of communication link MIMO systems can achieve significantly higher data rates than those traditional SISO (Single Input, Single Output) channels.

In a 2*2 system MIMO, signals propagate along multiple paths from the transmitter to the receiver antennas.

With this channel knowledge, the receiver can retrieve the streams Independent of both transmitter antennas. Generates a 2x2 MIMO system two spatial streams to effectively double the maximum bitrate of this, which can be achieved through a traditional SISO 1 x 1 communication channel the maximum channel capacity of a MIMO system can be estimated by Streams N spatial function. Basic approximation of MIMO channel amplitude is a function of spatial fluxes, bandwidth, and signal-to-noise ratio (SNR). It is shown in the following equation:

$$C = NB \log_2(1 + SNR) [\text{bit /s}] \quad (1.2)$$

Where **C** is channel capacity, **N** is spatial stream number, **B** is bandwidth of the channel and

SNR is the signal to noise ratio. Given the MIMO channel capacity equation, it is possible to study the relationship between the number of spatial streams and the throughput of different implementations of SISO and MIMO configurations.[4]

1.4 Multiple user types MIMO

1.4.1 Multi-user MIMO (MU-MIMO)

Is a set of multiple-input and multiple-output (MIMO) technologies for multipath wireless communication, in which multiple users or terminals, each radioing over one or more antennas, communicating with one another. In contrast, single-user MIMO (SU-MIMO) involves a single multi-antenna-equipped user or terminal communicating with exactly one other similarly equipped node. Analogous to how OFDMA adds multiple-access capability to OFDM in the cellular-communications realm, MU-MIMO adds multiple-user capability to MIMO in the wireless realm. SDMA, massive MIMO, coordinated multipoint (CoMP), and ad hoc MIMO are all related to MU-MIMO; Each of those technologies often leverage spatial degrees of freedom to separate users.[5]

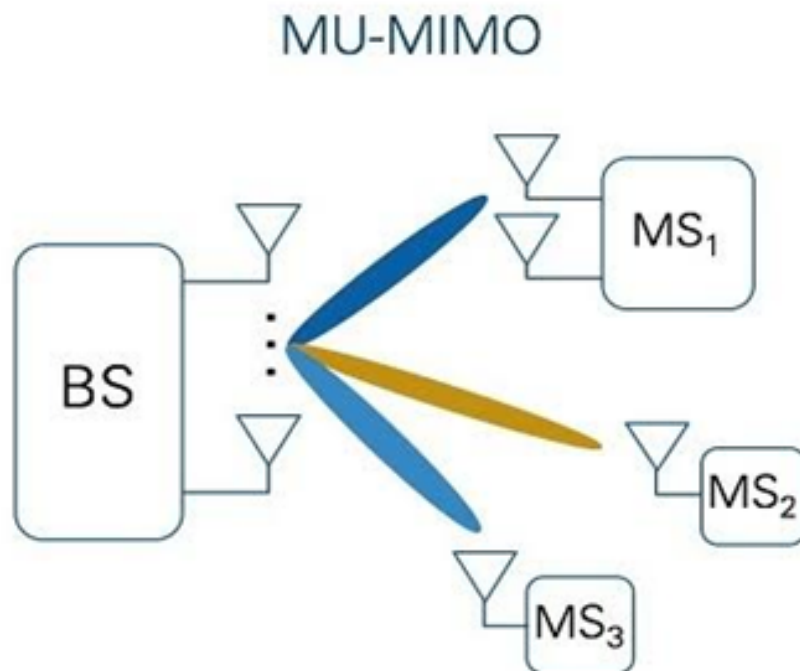


Figure 1.6: Multi-user MIMO (MU-MIMO)[4]

1.4.1.1 Technology

MU-MIMO leverages multiple users as spatially distributed transmission resources, at the

cost of somewhat more expensive signal processing. In comparison, conventional single-user MIMO (SU-MIMO) involves solely local-device multiple-antenna dimensions. MU-MIMO algorithms enhance MIMO systems where connections among users count greater than one. MU-MIMO may be generalized into two categories: MIMO broadcast channels (MIMO BC) and MIMO multiple-access channels (MIMO MAC) for downlink and uplink situations, respectively. Again in comparison, SU-MIMO may be represented as a point-to-point, pairwise MIMO.[6]

1.4.1.2 MIMO BC

Represents the downlink MIMO state where a single transmitter transmits to multiple receivers within the wireless network. Examples of advanced transmission processing for MIMO BC are interference-aware precoding and user scheduling for the SDMA-based downlink. For advanced dispatch processing, it must be defined in the transmitter (CSIT). That is, knowledge of CSIT allows improving productivity, and methods of obtaining CSIT have become of great importance. MIMO BC systems have a distinct advantage over point-to-point SU-MIMO systems, especially when the number of antennas in the transmitter or access point (AP) is greater than the number of antennas in each receiver (user) AP.

AP: Access point It is the transmitter and the user of the receiver.[8]

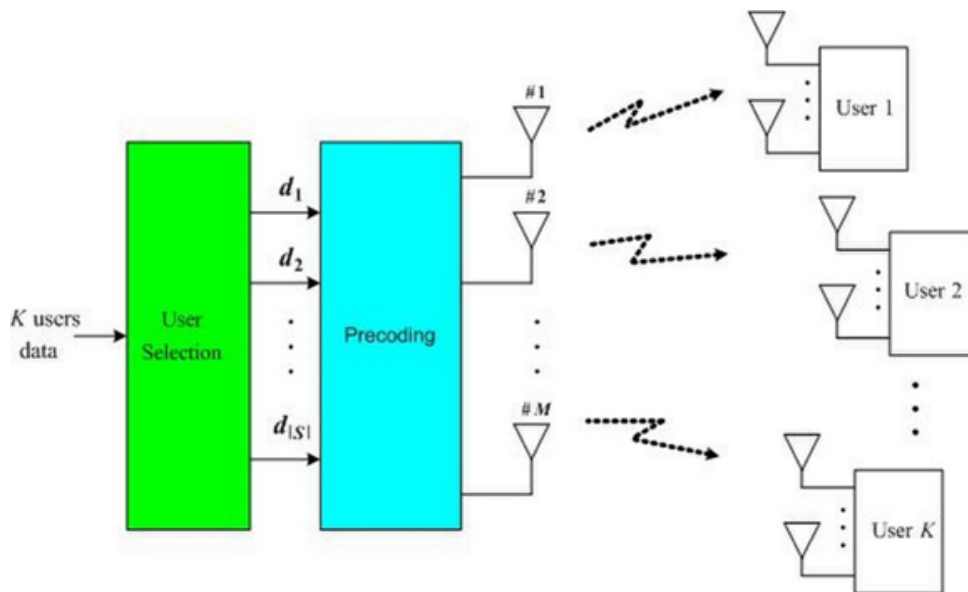


Figure 1.7: Multiuser MIMO System: MIMO BC case [9]

1.4.1.3 MIMO MAC

MIMO MAC represents the state of a multicaster MIMO uplink to a single receiver's wireless network. Examples of advanced MIMO MAC receive processing are co-interference

cancellation and SDMA-based uplink user scheduling. For advanced receive processing, the receiver must know the channel state information in the receiver (CSIR). Knowing CSIR in general is easier than knowing CSIT. However, knowing CSIR costs a lot of uplink resources to move assigned pilots from each user to the AP. MIMO MAC systems are superior to point-to-point MIMO systems especially when the number of receiving antennas at an access point is greater than the number of transmitting antennas at each user.[6]

1.4.1.4 Cross-layer MIMO

Cross-layer MIMO enhances the performance of MIMO links by solving certain cross-layer problems that may occur when MIMO configurations are employed in a system. Cross-layer techniques can be used to enhance the performance of SISO links as well. Examples of cross-layer techniques are Joint Source-Channel Coding, Adaptive Modulation and Coding (AMC, or "Link Adaptation"), Hybrid ARQ (HARQ), and user scheduling.[6]

1.4.2 Cooperative MIMO (CO-MIMO)

Cooperative multiple-input multiple-output (cooperative MIMO, CO-MIMO) is a technology that can effectively exploit the spatial domain of mobile fading channels to bring significant performance improvements to wireless communication systems. It is also called network MIMO, distributed MIMO, virtual MIMO, and virtual antenna arrays[7].

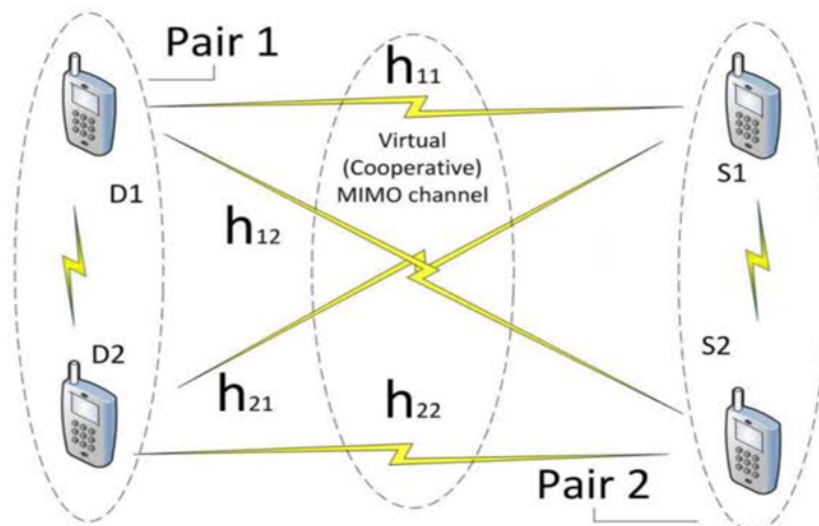


Figure 1.8: Cooperative MIMO [7]

1.4.2.1 Types CO-MIMO

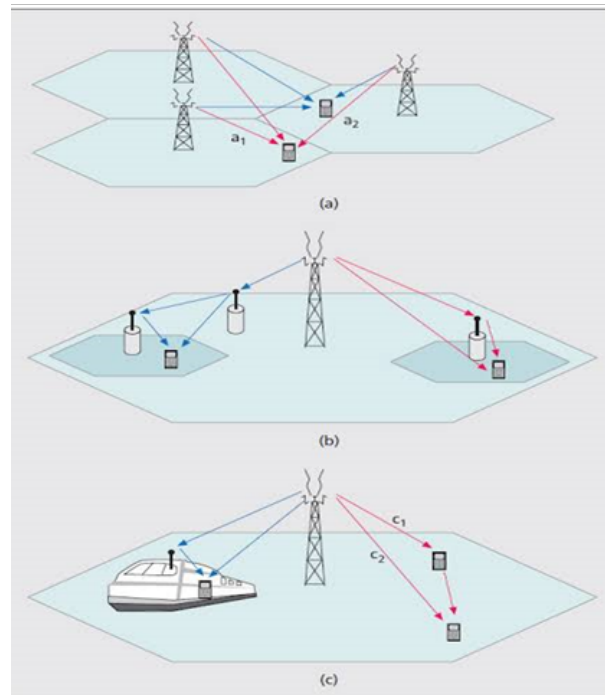


Figure 1.9: Three types of cooperative MIMO schemes in cellular systems:

- a) CoMP
- b) Fixed relay
- c) mobile relay.[10]

1.4.2.1.1 CoMP

coordinated multipoint (CoMP), data and channel state information (CSI) is shared among cellular base stations (BS) to coordinate their transmissions in the downlink and jointly process the received signals in the uplink. The system architecture is illustrated in **Fig 1.9(a)**. CoMP techniques can effectively turn otherwise harmful inter-cell interference into useful signals, significant power gain, channel rank advantage, and/or diversity gains to be exploited. CoMP requires a high-speed backhaul network for enabling the exchange of information (e.g., data, control information, and CSI) between the BS.[10]

1.4.2.1.2 Fixed relays

(Illustrated in **Fig 1.9(b)**) are low-cost and fixed radio infrastructures without wired backhaul connections. They store data received from the BS and forward to the mobile stations (MS), and vice versa. Fixed relay stations (RS) typically have smaller transmission powers and coverage areas than a BS. They can be strategically and cost effectively in cellular networks to extend coverage, reduce total transmission power, enhance the capacity of a specific region with high traffic demands, and/or improve signal.

1.4.2.1.3 Mobile relays

Mobile relays differ from fixed relays in the sense that the RSs are mobile and are not shared as the infrastructure of a network. Mobile relays are therefore more flexible in accommodating varying traffic patterns and adapting to different propagation environments.[11]

1.4.3 Overall diversity MIMO

A form of spatial diversity scheme that uses multiple transmitting or receiving stations to communicate coherently with single or multiple users that can be distributed in the coverage area, and at the same time and frequency resource.[12, 13]

Transmitters are very different in contrast to traditional micro-diversity MIMO schemes such as single-user MIMO. In a multiuser aggregate diversity MIMO scenario, users may also be far apart. Therefore, each link formed in a virtual MIMO link has a characteristic mean linkage SNR.

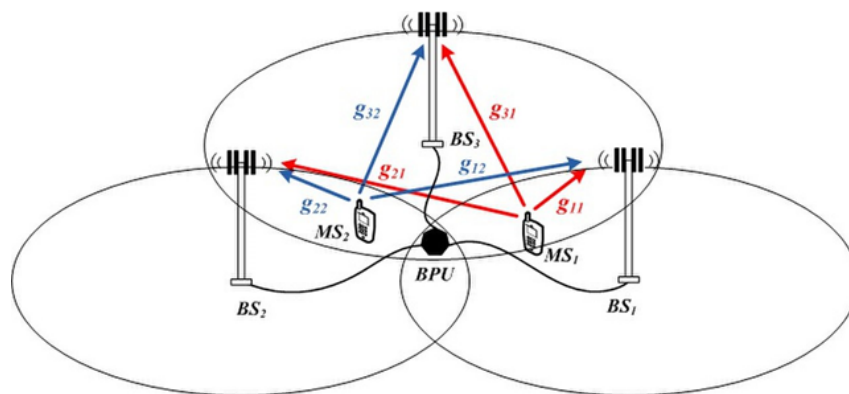


Figure 1.10: Overall diversity MIMO.[12, 13]

1.5 Applications of system MIMO:

Third generation (3G) (CDMA and UMTS) allows the implementation of transmission diversity schemes in space-time, in combination with the formation of transmission packets at base stations. 4G (4G) LTE and LTE Advanced define highly advanced antenna interfaces that rely heavily on MIMO technologies. LTE primarily focuses on single-link MIMO that relies on Spatial Multiplexing and space-time coding while LTE Advanced extends the design to multi-user MIMO. In wireless local area networks (WLAN), IEEE 802.11n (Wi-Fi).[14]

1.5.1 MIMO spatial diversity

The same message is transmitted simultaneously on different antennas during transmission. The signals received on each of the reception antennas are then rephrased and summed in a coherent manner. A simplified version uses the signal from only one of the antennas, the one that receives the best signal at a given time (polarized antennas). This makes it possible to increase the signal-to-noise ratio (thanks to the diversity gain) of the transmission. For this technique to be effective, the MIMO sub-channels must be decor related (independent) from each other.[15]

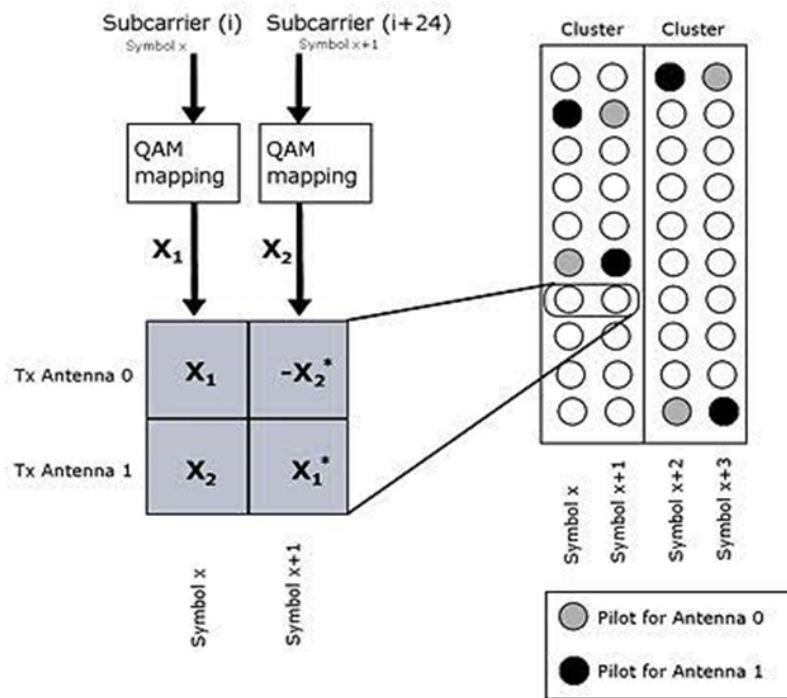


Figure 1.11: MIMO 2 antennas, spatial diversity [15]

1.5.2 MIMO spatial multiplexing

Each message is split into sub-messages. The different sub-messages are transmitted simultaneously on each of the transmit antennas. The signals received on the reception antennas are reassembled to reconstitute the original message. As with MIMO diversity, the propagation sub channels must be decor related. MIMO multiplexing makes it possible to increase the transmission rates (thanks to the multiplexing gain).[15]

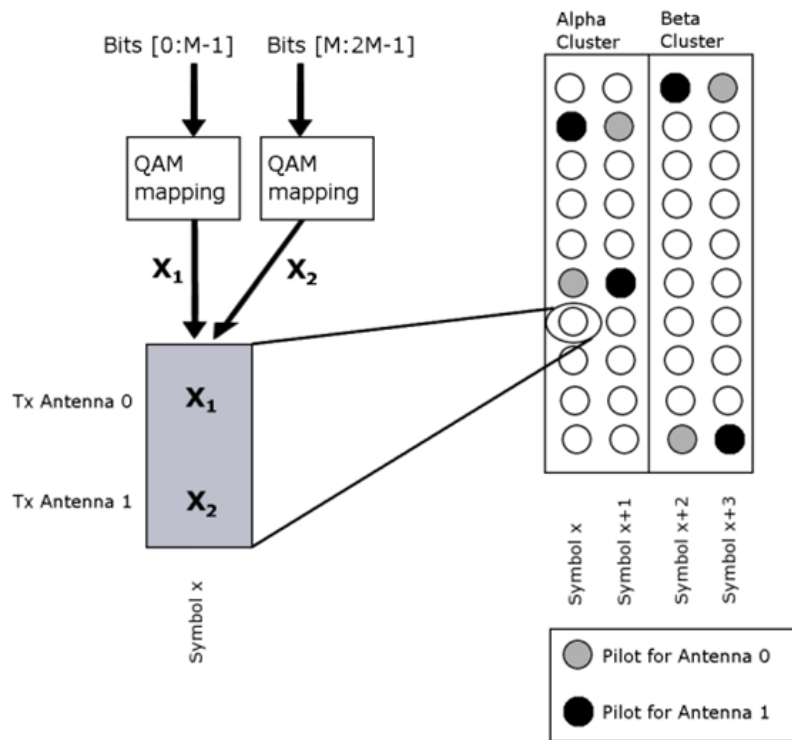


Figure 1.12: Spatial multiplexing: Two data streams transmitted at the same time, on the same frequency.[15]

1.5.3 Spatial multiplexing Technique

Spatial multiplexing techniques make receivers very complex, and thus they are usually combined with orthogonal frequency division multiplexing (OFDM) or orthogonal frequency division multiple access (OFDMA) modulation, where problems arising from a multipath channel are efficiently dealt with. The IEEE 802.16e standard includes MIMO-OFDMA technology. The IEEE 802.11n standard, released in October 2009, recommends the use of MIMO-OFDM.[16]

1.6 Massive MIMO

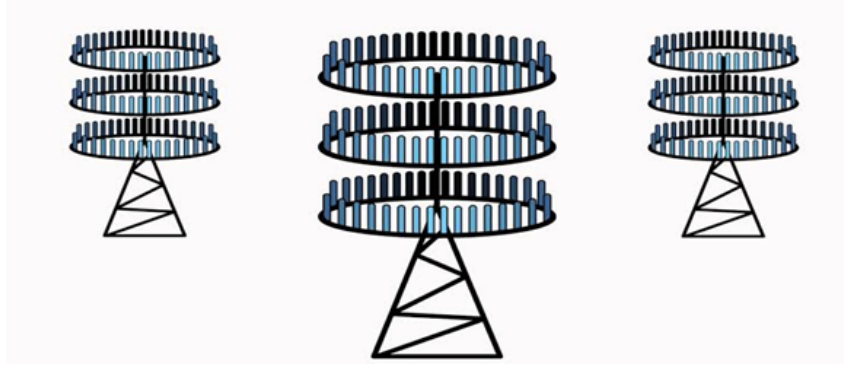


Figure 1.13: An illustration of massive MIMO.[17]

MIMO can be summarized according to one principle: MIMO spatial multiplexing consists of to simultaneously use the same radio frequencies to transmit different signals. This means that several transmitting antennas of a base station can transmit different signals and that several reception antennas of a device can receive and divide them simultaneously. Standard MIMO networks typically use two or four antennas to transmit the data and the same number to receive it. Massive MIMO, on the other hand, is a MIMO system with a particularly high number of antennas. Massive MIMO increases the number of transmit antennas (tens or more than 100 elements) on a base station (Figure 13). Massive MIMO offers two major innovations.[17]

1.6.1 Beamforming

Is a traffic signaling system for cellular base stations that identifies the most efficient data transmission path for a user particular and reduces interference for nearby users. At the stations of massive MIMO based, signal-processing algorithms draw the best transmission route over the air to each user. They can then send individual data packets in many different directions returning them from buildings and other objects in a perfectly coordinated. In summary, think of massive MIMO technology as a 3D structure massive increasing horizontal and vertical coverage capabilities.[17]

1.6.2 MU-MIMO

Further increases the total capacity per base station by allowing the communication with multiple devices using the same resources, thus creating a practically unified peripheral side. The simultaneous use of the antennas of multiple devices allows for the creation of large-scale

MIMO virtual channels. The combination of these two innovations makes it possible to increase the transmission speed wireless by increasing the number of base station antennas without consuming more bandwidth or increase modulation values.[18]

1.6.3 Features of Massive MIMO

Massive MIMO is a form of MU-MIMO structure in which the variety of BS antennas and the number of UTs are enormous. In Massive MIMO, thousands of BS antennas simultaneously serve tens or even hundreds of users with the same frequency resources. Some highlights of Massive MIMO is:

1.6.3.1 Scalability

The base station learns the channels via uplink learning, with TDD operation. The time required for channel estimation is independent of the number BS antennas. Therefore, the wide range of BS antennas can be as wide as desired without extending the channel estimation overhead. In addition, signal processing at each UT is essential and does not depend on the existence of other UTs; the Signal processing by demultiplexing is performed at the UTs. The addition or loss of some service UTs no longer affects other UT activities.

1.6.4 Advantages of Massive MIMO Systems

The main advantages of Massive MIMO systems can be summarized as follows:[19]

- High spectral efficiency gain
- High energy efficiency gain
- Simple processing
- Increased data rate
- Increased basic link signal-to-noise ratio
- Channel strengthening

1.7 Math Description for MIMO

In MIMO systems, the transmitter transmits multiple streams by means of multiple transmitter antennas. The transmission streams pass through a channel matrix consisting of all the

paths N_r between the N_t transmitting antennas of the transmitter and N_r receiving antennas in the receiver. Then, the receiver obtains the received vector signal by multiple receiving antennas and decodes the received vector signal into the original

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \ddots & \ddots & \vdots \\ h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_r} \end{bmatrix}$$

Flat Dispersion Narrow Band MIMO System Similar to:

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1.3)$$

where \mathbf{Y} and \mathbf{x} are the transmit and receive vectors respectively and \mathbf{H} and \mathbf{n} are the channel matrix and the noise vector, respectively.[20]

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ \vdots & \vdots & \cdots & \vdots \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}$$

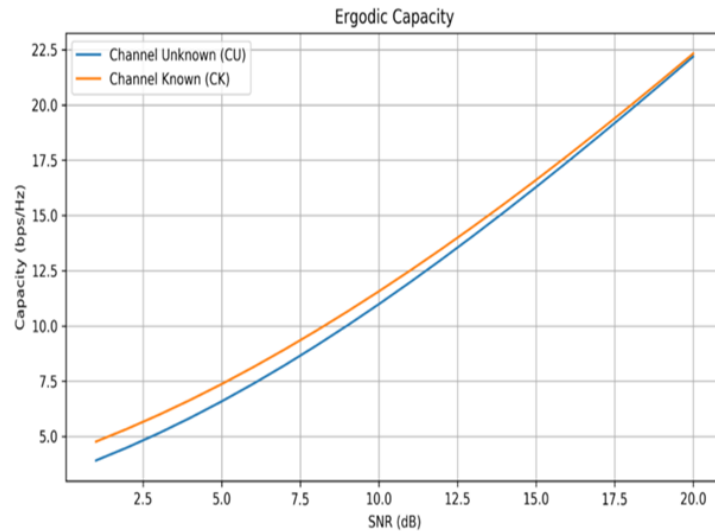


Figure 1.14: Closed loop ergodic (channel is known, CSI is ideal) and open loop amplitudes (channel is unknown, no CSI) the number of transmitting and receiving antennap.[21]

Referring to information theory, the channel amplitude is ergodic for MIMO systems where both the transmitter and receiver have an optimal instantaneous moment of channel state information

is:[22]

$$\begin{aligned} C_{\text{perfect}} - \text{CSI} &= E \left[\max \log_2 \det(\mathbf{I} + \rho \mathbf{H} \mathbf{Q} \mathbf{H}^H) \right] \\ &= E \left[\log_2 \det(\mathbf{I} + \rho \mathbf{D} \mathbf{S} \mathbf{D}) \right] \end{aligned} \quad (1.4)$$

Where \mathbf{H} Denotes the substitution of Hermit and ρ It is the ratio between the transmit power and noise power (ie, SNR) the optimum variance of the signal $\mathbf{Q}=\mathbf{V}\mathbf{S}\mathbf{V}$ It is achieved by decomposing the singular value of the channel matrix $\mathbf{H}=\mathbf{U}\mathbf{D}\mathbf{V}$ and a perfect diagonal assignment matrix $\mathbf{S} = \text{diag}(S_1, \dots, S_{\min(N_t, N_r)}, 0, \dots, 0)$. The optimal power allocation is achieved through waterfilling,[23] that is

$$s_i = \left(\mu - \frac{1}{\rho d_i^2} \right)^+, \quad \text{for } i = 1, \dots, \min(N_t, N_r) \quad (1.5)$$

where $d_1, \dots, d_{\min(N_t, N_r)}$ are the diagonal elements of \mathbf{D} , $(\cdot)^+$ is zero if its argument is negative, and μ is selected such that $S_1 + \dots + S_{\min(N_t, N_r)} = N_t$. If the transmitter has only statistical channel state information, then the ergodic channel capacity will decrease as the signal covariance \mathbf{Q} can only be optimized in terms of the average mutual information as:[22]

$$C_{\text{statistical-CSI}} = \max E \left[\log_2 \det(\mathbf{I} + \rho \mathbf{H} \mathbf{Q} \mathbf{H}^H) \right] \quad (1.6)$$

The spatial correlation of the channel has a strong impact on the ergodic channel capacity with statistical information. If the transmitter has no channel state information it can select the signal covariance \mathbf{Q} to maximize channel capacity under worst-case statistics, which means $\mathbf{Q} = \frac{1}{N_t} \mathbf{I}$ and accordingly

$$C_{\text{no-CSI}} = E \left[\log_2 \det \left(\mathbf{I} + \frac{\rho}{N_t} \mathbf{H} \mathbf{H}^H \right) \right] \quad (1.7)$$

Depending on the statistical properties of the channel, the ergodic capacity is no greater than $\min(N_t, N_r)$ times larger than that of a SISO system.

1.8 Conclusion

The use of multiple antennas at both the transmitter and receiver aims to improve performance or to increase the symbol rate of systems, but it usually requires higher implementation complexity. The antenna spacing must be larger than the coherence distance to ensure independent fading across different antennas. Alternatively, uncorrelated signals in different antennas can be assured through the use of orthogonal polarization. Multiple Input Multiple Output (MIMO) architectures can be used for :

- Combined diversity of transmitting and receiving
- For the parallel transmission of data
- For spatial multiplexing.

When used for spatial multiplexing, MIMO technology promises high bit rates in a narrow bandwidth.

Therefore, it is of high significance to the spectrum users. In this case, the MIMO system considers the transmission of different signals from each transmit element so that the receiving antenna array receives a superposition of all transmitted signals. All new ideas about how to improve performance, capacity, and/or spectrum efficiency while keeping computational cost at an acceptable level have been described. MIMO Systems and Applications can contribute to the concept of Green Radio Communications while supporting a reduction in energy consumption

CHAPTER 2

PRECODING TECHNIQUE MIMO

2.1 Introduction

Precoding is a preprocessing technique that exploits channel-state information at the transmitter (CSIT) to match the transmission to the instantaneous channel conditions. Linear and non-linear precoding designs are available in the literature. Linear precoding in particular provides a simple and efficient method to utilize CSIT. Linear precoding has been shown to be optimal in certain situations involving partial CSIT; however, in many instances the main motivation of linear precoders is to simplify the MIMO receiver.[24]

Precoding is a generalization of beam forming to support multi-stream (or multi-layer) transmission in multi-antenna wireless communications. In conventional single-stream beamforming the same signal is emitted from each of the transmit antennas with appropriate weighting (phase and gain) such that the signal power is maximized at the receiver output. When the receiver has multiple antennas, single-stream beam forming cannot simultaneously maximize the signal level at all of the receive antennas. Precoding improves error probability and transmission rate and thus System capacity by more efficient sharing of the channel with different users. It also allows suppression of intrusions and in this case reduces complexity receiver.[25]

In this chapter, we will study the precoding of MIMO systems, their linear and non-linear types, and their beam forming.

2.2 Linear Precoding

Linear precoding is a relatively simple method of MIMO signaling that can also be optimal in certain special cases

2.2.1 Diversity of MIMO Linear Precoding

This paper is dedicated to high-SNR analysis of MIMO linear precoding. The Diversity-Multiplexing Tradeoff (DMT) of a number of linear precoders is analyzed. Furthermore, since the diversity at finite rate (also known as the fixed-rate regime, corresponding to multiplexing gain of zero) does not always follow from the DMT, linear precoders are also analyzed for their diversity at fixed rates. In several cases, the diversity at multiplexing gain of zero is found not to be unique, but rather to depend on spectral efficiency. The analysis includes the zero-forcing (ZF), regularized ZF, matched filtering and Wiener filtering precoders. We calculate the DMT of ZF precoding under two common.[38]

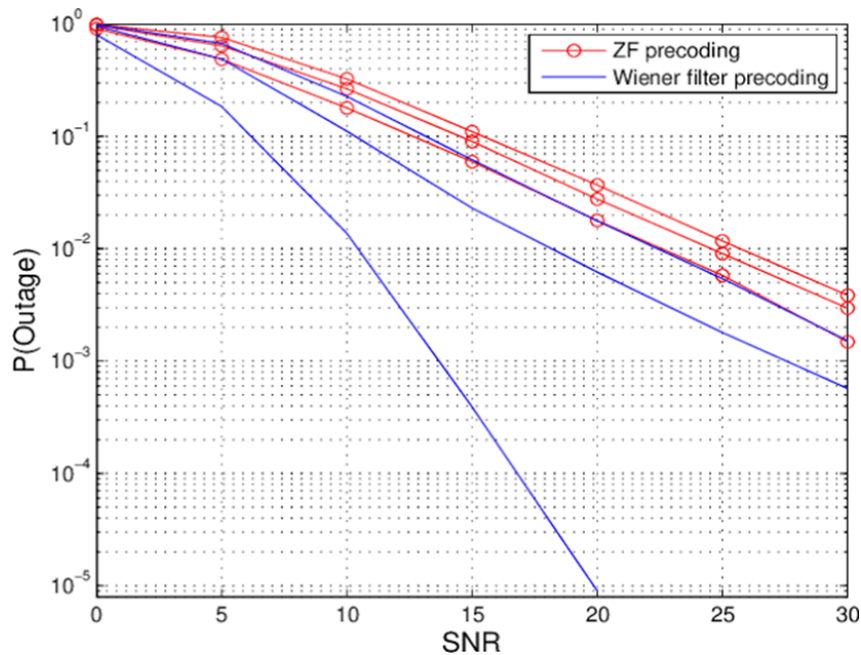


Figure 2.1: Outage probability of the ZF and Wiener filtering precoded MIMO 2×2 system for rates (left to right): $R = 1.9, 2.5$ and 3 b/s/Hz.[38]

Fig 2.1 shows the outage probabilities of the ZF and Wiener-filter precoded 2×2 MIMO systems. The diversity in the case of the ZF case is the same as the one predicted by the DMT. In the case of Wiener precoding, the diversity is the same as the one predicted by the DMT for high rate (R) values and it departs from the DMT for low rate values.[38]

2.2.2 Model system

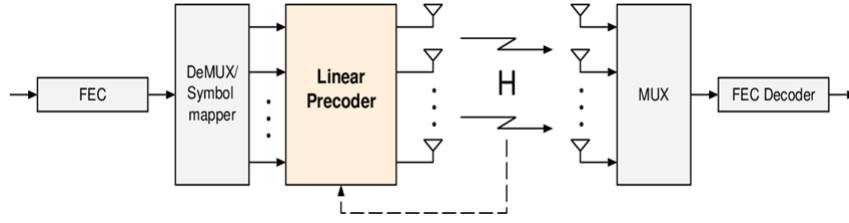


Figure 2.2: MIMO with linear precoder.[38]

MIMO system with linear precoding is depicted in Fig 2 This system uses the linear precoder to manage the interference between the streams in a MIMO system to avoid a lattice decoder in the receiver we consider a flat fading channel:

$$H \in \mathbb{C}^{N \times M} \quad (2.1)$$

Where \mathbf{M} and \mathbf{N} are the number of transmit and receive antennas, respectively. While $\mathbf{M} \geq \mathbf{N}$ when using linear precoding alone, we have $\mathbf{N} \geq \mathbf{M}$ or $\mathbf{M} \geq \mathbf{N}$ when using precoding together with receive the equalizer is designed for the precoded channel:

$$Y = WHTx + Wn \quad (2.2)$$

Where \mathbf{T} is the precoder matrix, \mathbf{W} is the receiver side equalizer The vectors \mathbf{x} and \mathbf{n} are assumed independent.[38]

2.2.3 Linear precoding techniques

Linear pre-coding techniques, sometimes also called beam, have been widely developed over the past decades. He is seen as an advantageous way to exploit the full diversity of the chain. For MU-MIMO systems, assuming a flat fading channel, techniques pre-coding are used to form a beam signal via the target user while reducing interference from other users. In general, in addition to the methods of singular value decomposition (SVD) with capacity optimization, the methods of linear pre-coding can be classified according to two approaches: the approach by forcing zero (ZF) such as ZF and the regularized ZF (RZF) method / root mean square error minimum (MMSE).[26]

2.2.4 SVD

SVD-based pre-coding techniques are considered to be the optimal methods to optimize system performance. In fact, the channel MIMO can consist of a set of independent and parallel links, the maximum number of links being equal to the minimum number of transmit antennas and reception. With perfect CSI, parallel channels are established by applying SVD to the channel matrix:

$$H_P = U_P D_P V_P^H \quad (2.3)$$

Or: $U_P \in C^{M \times M}$ is used by the sender as a pre-coding matrix

$V_P \in C^{K \times K}$ is multiplied by the receiver signal as a post-coding matrix. Of this way, the signals are transmitted through independent beams, which are in fact the eigenvectors of the channel correlation matrix $H_P^H H_P$

The Beam power charges are the singular values squared D_P^2

The disadvantage of the SVD method is the high computational complexity. Of Moreover, the post-coding matrix must be communicated to the users to decode the signal received.[26]

2.2.5 MR

With the maximum ratio (MR), the base station wishes to maximize the received SNR of each stream ignoring the effect of multiuser interference received signal because there is no has no active interference mitigation. Then the pre-coding matrix is given by:

$$F_{MR} = \alpha MR * H^H \quad (2.4)$$

Where αMR is a controlling normalization scalar, the transmit power and the ratio signal to noise MR maximizes the matrix gain of the transmission, but since it neglects the effect of multiuser interference, it performs poorly in scenarios with great interference.[26]

2.2.6 ZF

A linear precoding scheme that removes all interference, interference between symbols and between users, is called forcing to zero (ZF). The matrix of precoding of ZF is given by the pseudo-inverse of the channel:

$$F_{ZF} = \alpha ZF * H^H (H H^H)^{-1} \quad (2.5)$$

Where α_{ZF} is a normalizing scalar.

The main difference between ZF and MR is matrix inversion, which provides the desired interference suppression. This reverse calculation can lead to a significant increase in complexity. But, the properties of MIMO channels massive allow us to significantly reduce the complexity of the calculations by compared to the realization of general matrix inverses.[27]

2.2.6.1 RZF/MMSE

In addition to the MR and ZF pre-encoders, it is possible to use a regularized form of ZF pre-coding (RZF). It is a linear pre-encoder located between MR and ZF, sharing properties with both. The RZF pre-coding matrix can be written as:

$$\alpha_{RZF} * H^H (HH^H + \beta_{reg} I_{KN})^{-1} \quad (2.6)$$

Where the regularization constant β_{reg} can be used to trade between gain of matrix and interference suppression. If β_{reg} is selected to minimize error.

Mean square (MSE) $E \left\| \mu - \frac{1}{\sqrt{\rho}} \hat{u} \right\|^2$ where ρ is a Scaling constant μ is the symbols transmitted to the users, \hat{u} the signals received by the users we get the minimal MSE pre-coder, (MMSE).[27]

2.3 No-linear Precoding:

The precoding techniques can be classified depends upon the amount of the MUI they allow (as zero or non - zero MUI techniques) and their linearity (as linear and nonlinear techniques). The advantage of linear precoding technique is that they require no overhead to provide the demodulation information but they are less computationally expensive and less sensitive to the channel estimation errors at the transmitter than nonlinear counter parts. Since, linear precoding techniques typically suffer from more noise. Than nonlinear precoding and hence are not used in most wireless applications. Nonlinear precoding involves additional transmit signal processing to improve error rate performance. The precoders are designed jointly based on CSI of all the users, based on the equalization techniques such as Decision Feedback Equalization (DFE).[28]

2.3.1 Dirty Paper Coding

DPC is a nonlinear precoding technique, which can be used when the transmitter side knows the channel state information. Here the interference already known to the transmitter is

eliminated and an interference free output can be obtained. For an n th user, the interference caused by the $(n-1)$ users is eliminated. The concept of DPC is that if the interference is known, then the capacity of a system is same as the capacity of a system when there is no interference [29] QR decomposition is used here. Let P be the unitary precoding matrix and R be a lower triangular matrix, then the channel matrix is given by $H=R*P$. DPC along with a modulo operation is the vector precoding technique. To make the transmit power minimum the desired signal d is offset by the integer values of L so that the input to channel after precoding is $x = H^{-1} (d + L)$. For an n th user the received signal is given as $y = d_n + I_n + w_n$. At the receiver a modulo operation is further performed to remove the effect of integer values of L . [30]

2.3.2 Design Consideration

DPC and DPC-like techniques require knowledge of the interference state in a non-causal manner, such as channel state information of all users and other user data. Hence, the design of a DPC-based system should include a procedure to feed side information to the transmitters. [31]

2.3.3 Application

In 2003, Caire and Shamai applied DPC to the multi-antenna multi-user downlink, which is referred to as the 'broadcast channel' by information theorists. Since then, there has been widespread use of DPC in wireless networks and into an interference aware coding technique for dynamic wireless networks. Recently, DPC has also been used for "informed digital watermarking" and is the modulation mechanism used by 10GBASE-T. [31]

2.4 Precoding and Beamforming

The concept of pre-encoding consists of performing digital processing of the signals in the transmission using data coding techniques and distributing it to both Antennas by exploiting prior knowledge on the transmission channel Through beam forming techniques which is a traffic light system for cellular base stations that Determines the most efficient data transmission path for the user Private and reduces interference for nearby users. In stations, MIMO-based mega cues paint the best-Broadcast route over the air for each user. They can then sending individual data packets in many different directions perfectly bring them back from buildings and other things. [32]

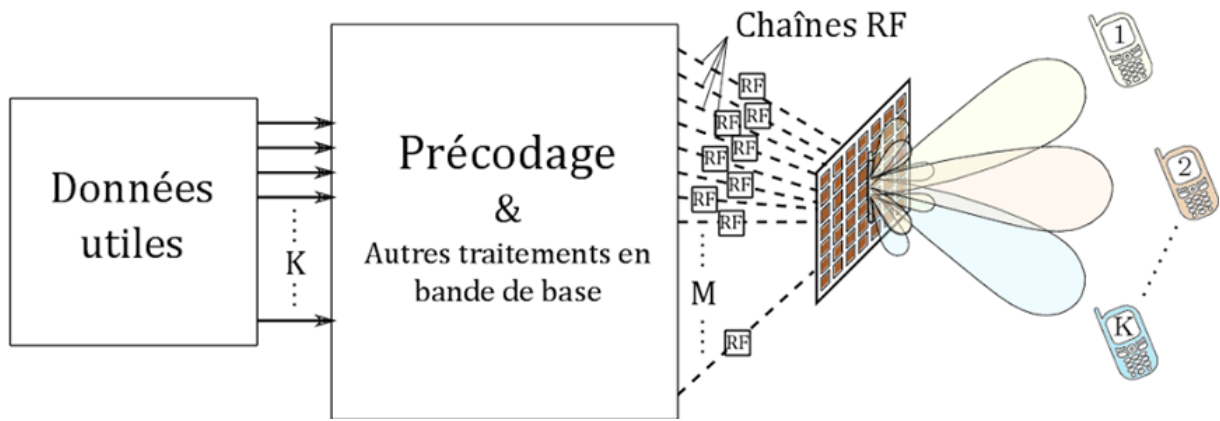


Figure 2.3: Massive MIMO system in multi-user transmission.[32]

2.4.1 The Different Linear Precoders

In this part, we are interested in multi-user systems of the MU-MIMO type where only the spatial dimension is exploited by the precoding to focus the energy towards the different users. **Fig 2.4** shows the structure of a multi-user system with M transmit antennas MIMO Massive Base Station (Mass-BS) and K users in the cell.

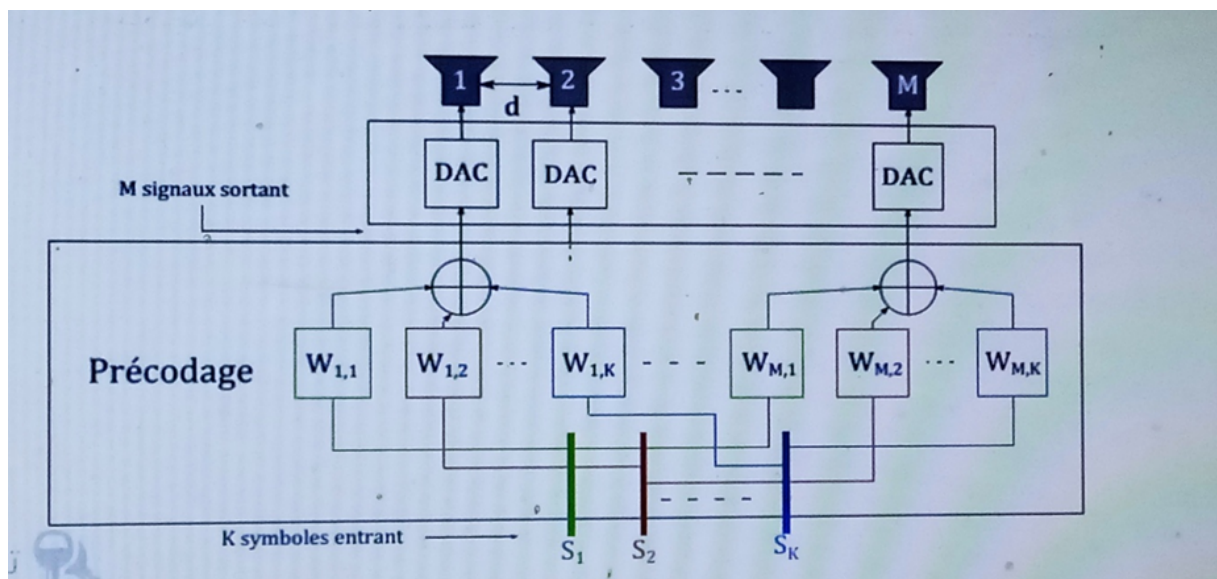


Figure 2.4: Structure of an antenna array with precoding

A multi-user system with M transmit antennas MIMO Massive Base Station (Mass-BS) and K users is described by the following equation:

$$X = \sqrt{\eta} W s \quad (2.7)$$

Or $s = [S_1, S_2, \dots, S_K]^T$ is the vector of the transmitted symbols, $x = [X_1, X_2, \dots, X_M]^T$ vector signals emitted by the M antennas, W the precoding matrix making it possible to distribute the

data on the different transmitting antennas and $\sqrt{\eta}$ is a normalization factor which ensures that the transmitted power remains unchanged by the precoding:

$$\eta = \frac{1}{E [T_r(w^H W)]} \quad (2.8)$$

The antenna \mathbf{M} transmits a linear combination of the K transmitted symbols:

$$X_m = \sqrt{\eta} \sum_k^k = 1 \quad W_{m,k} S_K \quad (2.9)$$

$r = [R1, R2, \dots RK]^T$ the vector of signals received by the different users and \mathbf{n} the vector of their respective white Gaussian noises. The received signal is written:

$$r = Gx + n \quad (2.10)$$

[32]

2.4.2 Signal Detection for Spatially Multiplexed MIMO Systems

Spatial Multiplexed MIMO (SM-MIMO) systems can transmit data faster than MIMO systems using other techniques such as spatiotemporal coding. However, spatial de multiplexing where signal detection at the receiver level is a daunting task for SM MIMO systems the detection and precoding matrices are calculated from the estimation of the H channel there are a number of methods to calculate these matrices. In this work, we will focus on linear precoding/detection methods namely: the maximum ratio (MR), zero forcing (ZF), and minimum mean squared error (MMSE). Derivations full mathematics of these techniques is not detailed, but the criteria for which they are optimized, as well as the advantages and disadvantages of each method are discussed.[33]

2.4.3 Conjugate Beamforming (CB)

Conjugate Beam forming aims to maximize SNR. This precoder will offer a gain of Maximum beam forming to each user but will generate a certain amount.[32]

The Conjugate Beam forming (CB) precoding matrix is given by:

$$W^{(CB)} H^H \quad (2.11)$$

It is about $W_{m,k}^{(CB)} = H_{K,k}$ Between the user \mathbf{k} and the antenna \mathbf{m}

2.4.4 Equal Gain Transmission(EGT)

The EGT, or constant gain transmission, will only compensate for the phase shift caused by the channel on each subcarrier (in the case of a waveform such as OFDM).[32]

$$W_{k,m}^{(EGT)} = e^{-i\phi_{k,m}} \quad (2.12)$$

$\phi^{k,m}$ is the phase shift introduced by the channel between the user \mathbf{k} and the antenna \mathbf{m}

2.4.5 Zero forcing (ZF)

2.4.5.1 Zero Forcing(ZF) Precoding

Zero-forcing (or null-steering) precoding is a method of spatial signal processing by which the multiple antenna transmitter can null multiuser interference signals in wireless communications. Regularized zero-forcing precoding is enhanced processing to consider the impact on a background noise and unknown user interference, where the background noise and the unknown user interference can be emphasized in the result of (known) interference signal nulling.

In particular, null-steering is a method of beamforming for narrowband signals where we want to have a simple way of compensating delays of receiving signals from a specific source at different elements of the antenna array. In general, to make better use of the antenna arrays, we sum and average the signals coming to different elements, but this is only possible when delays are equal. Otherwise, we first need to compensate the delays and then sum them up. To reach this goal, we may only add the weighted version of the signals with appropriate weight values. We do this in such a way that the frequency domain output of this weighted sum produces a zero result. This method is called null steering. The generated weights are of course related to each other and this relation is a function of delay and central working frequency of the source.

If the transmitter knows the downlink channel state information (CSI) perfectly, ZF-precoding can achieve almost the system capacity when the number of users is large. On the other hand, with limited channel state information at the transmitter (CSIT) the performance of ZF-precoding decreases depending on the accuracy of CSIT. ZF-precoding requires the significant feedback overhead with respect to signal-to-noise-ratio (SNR) to achieve the full multiplexing gain. Inaccurate CSIT results in the significant throughput loss because of residual multiuser interferences. Multiuser interferences remain since they cannot be nulled with beams generated by imperfect

CSIT[34].

2.4.5.2 Mathematical description

In a multiple antenna downlink system which comprises N_t transmit antenna access points and K single receive antenna users, such that $K \leq N_t$, The received signal of user k is described as:

$$y_k = h_k^T x + n_k \quad k = 1, 2, \dots, K \quad (2.13)$$

Where $x = \sum_{i=1}^K \sqrt{P_i} s_i w_i$ is the $N_t \times 1$ channel vector w_i is some $N_t \times 1$ linear precoding vector, Here $(.)^T$ is the message signal zero mean and variance $E(|s_i|^2) = 1$.

$$y = H^T W D s + n \quad (2.14)$$

Where \mathbf{y} is the $K \times 1$ received signal vector, $H = [h_1, \dots, h_k]$ is $N_t \times K$ channel matrix, $W = [w_1, \dots, w_k]$ is the $N_t \times K$ precoding matrix, $D = \text{diag}(\sqrt{P_1}, \dots, \sqrt{P_k})$ is a $K \times K$ diagonal power matrix, and $s = [s_1, \dots, s_k]^T$ is a $K \times 1$ transmit signal.

A zero-forcing precoder is defined as a precoder where w_i intended for user i is orthogonal to every channel vector h_j associated with users j where $j \neq i$.

That is $w_i \perp h_j$ **if** $i \neq j$.

Thus, the interference caused by the signal meant for one user is effectively nullified for rest of the users via zero-forcing precoder.

From the fact that each beam generated by zero-forcing precoder is orthogonal to all the other user channel vectors, one can rewrite the received signal as:

$$y_k = h_k^T \sum_{i=1}^K \sqrt{P_i} s_i w_i + n_k = h_k^T w_k \sqrt{P_k} s_k + n_k \quad k = 1, 2, \dots, K \quad (2.15)$$

The orthogonality condition can be expressed in matrix form as:

$$H^T W = Q \quad (2.16)$$

Where Q is some $K \times K$ diagonal matrix? Typically, Q is selected to be an identity matrix. This makes W the right Moore-Penrose pseudo-inverse of H^T give by:

$$W = (H^T)^+ = H(H^T H)^{-1} \quad (2.17)$$

Given this zero-forcing precoder design, the received signal at each user is decoupled from each other as:

$$y_k = \sqrt{P_k}s_k + n_k \quad k = 1, 2, \dots, K \quad [35] \quad (2.18)$$

2.4.5.3 Performance

If the transmitter knows the downlink channel state information (CSI) perfectly, ZF-precoding can achieve almost the system capacity when the number of users is large. On the other hand, with limited channel state information at the transmitter (CSIT) the performance of ZF-precoding decreases depending on the accuracy of CSIT. ZF-precoding requires the significant feedback overhead with respect to signal-to-noise-ratio (SNR) to achieve the full multiplexing gain. Inaccurate CSIT results in the significant throughput loss because of residual multiuser interferences. Multiuser interferences remain since they cannot be nulled with beams generated by imperfect CSIT.[35]

2.4.5.4 Zero forcing (ZF) Beamforming

Zero-forcing Beamforming (ZF-BF) is a spatial signal processing in multiple antenna wireless devices. For downlink, the ZF-BF algorithm allows a transmitter to send data to desired users together with nulling out the directions to undesired users and for uplink, ZF-BF receives from the desired users together with nulling out the directions from the interference users. The concept of interference users in the receive mode is information theoretically dual to undesired users in the transmit mode.[36]

2.4.5.5 Literature Review

This category summarizes techniques of zero-forcing and regularized zero-forcing precoding. If the transmitter knows the downlink channel status information perfectly, ZF-based precoding can achieve close to the optimal capacity especially when the number of users is sufficient. With limited channel status information at the transmitter, ZF-BF requires the amount of feedback overhead proportional to the average signal-to-noise-ratio (SNR) to achieve the full multiplexing gain. Hence, inaccurate channel state information at the transmitter may suffer the significant performance loss of the system throughput because of the interference among transmit streams is remained.[36]

2.4.5.6 Beamforming vectors

ZFBF lets each beamforming vector $w_i^0 \in C^{M \times 1}$ for arbitrary user i be orthogonal to other users' accurate channel state information vectors, i.e., $h_j, j \neq i$.

The beamforming vector obtained using the perfect CSIT is denoted as w_n^0 .

If the perfect channel state information at the transmitter (CSIT) is assumed, the beamforming vectors are chosen to be the normalized rows of the inverse of the channel matrix (h_1, h_2, \dots, h_m) .

Notice that if the channel state information is not perfect, the beamforming vectors cannot be orthogonal to the real channel vectors.

In order to consider more general cases, we discuss the beamforming vector obtained using the limited feedback channel information as w_m . [36]

2.4.5.7 Throughput Analysis

The SNR of transmit stream k in the zero-forcing beam forming systems is given by:

$$\gamma_k = \frac{1}{[(I + \rho H_k^H H_k)^{-1}]_{k,k}} - 1 \quad (2.19)$$

Assuming H_k is $M \times N$ which art matrix with $M < N$, the distribution of γ_k is given by:

$$F_{\gamma_k}(z) = 1 - \frac{e^{-\frac{z}{\rho}}}{(1+z)^{M-1}} L(z) \quad (2.20)$$

Where:

$$L(z) = \sum_{n=1}^N \frac{\sum_{i=0}^{N-n} (M-1, i) z^i}{(n-1)!} \left(\frac{z}{\rho}\right)^{n-1} \quad [36] \quad (2.21)$$

2.4.6 Minimum Mean Square Error (MMSE) detector

The MMSE detector is a linear detector whose transformation matrix is this matrix that minimizes the mean square error between the emission vector and the estimated vector MMSE alleviates the problem of noise enhancement by taking into account the power of noise during the construction of the filtering matrix using the criterion based on the MMSE performance. [32]

2.4.6.1 The Equation Mean Square Error (MSE)

If \hat{d} is an estimator of the unknown parameter d , the mean square error can be defined as:

$$\begin{aligned}
MSE(\hat{o}) &= E(\hat{o} - o)^2 & (2.22) \\
&= E(\hat{o}^2 - 2\hat{o}o + o^2) \\
&= E(\hat{o}) - 2oE(\hat{o}) + o^2 \\
&= E(\hat{o})^2 - [E(\hat{o})]^2 + [E(\hat{o})]^2 - 2oE(\hat{o}) + o^2 \\
&\quad \mathbf{Variance}(\hat{o}) + (E(\hat{o}) - o)^2 \\
&\quad \mathbf{Variance}(\hat{o}) + [biased(o)]^2
\end{aligned}$$

Note: if \hat{o} is an unbiased estimator of o biased $(o) = 0$ $MSE(\hat{o}) = Variance(\hat{o})$ [32]

2.5 Conclusion

The full gain from MIMO precoding is achieved with full CSI at the transmitter since this allows the transmitted signal to be shaped based on the Eigen-structure of the channel. Feedback of CSI to the transmitter can thus enable the transmitter to better exploit channel conditions to improve the MIMO performance. However, the amount of channel information fed back to the transmitter (i.e., the size of the codebook) is limited by the often severely limited feedback control channel bandwidth, thereby preventing the transmitter from obtaining full channel information in order to fully exploit MIMO precoding gain.

compared to the conventional closed-loop precoding scheme, the transceivers have more precoding matrices for optimization, improving the closed-loop precoding performance. The effective number of codebooks that the transceivers can traverse within the channel coherence time determines the actual gain of the rotating codebook over the conventional closed-loop precoding.

CHAPTER 3

SIMULATION AND RESULTS

3.1 Introduction

In this chapter, we present the graphical results of the simulation obtained with Matlab software. These interpretations present simulations of the effect of different parameters on the spectral efficiency of the MIMO system.

In this work, we will focus on the case where we have two transmit and two receive. We will study it in the Rayleigh multi-path channel and the modulation is BPSK.

3.2 Linear Precoding

In a 2×2 MIMO channel, probable usage of the available two transmit antennas can be as follows:

- We have a dispatch sequence, for example $x_1 + x_2 + x_3 + \dots + x_n$
- In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.
- However, as we now have, two transmit antennas; we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second antenna, send x_5 and x_6 in the third time slot and so on.

- Notice that as we are grouping two symbols and sending them in one time slot, we need only $\frac{n}{2}$ time slots to complete the transmission – data rate is doubled.
- These forms (**Figure 3.1**) the simple explanation of a probable MIMO transmission scheme with two transmit antennas and two receive antennas.[39] [40]

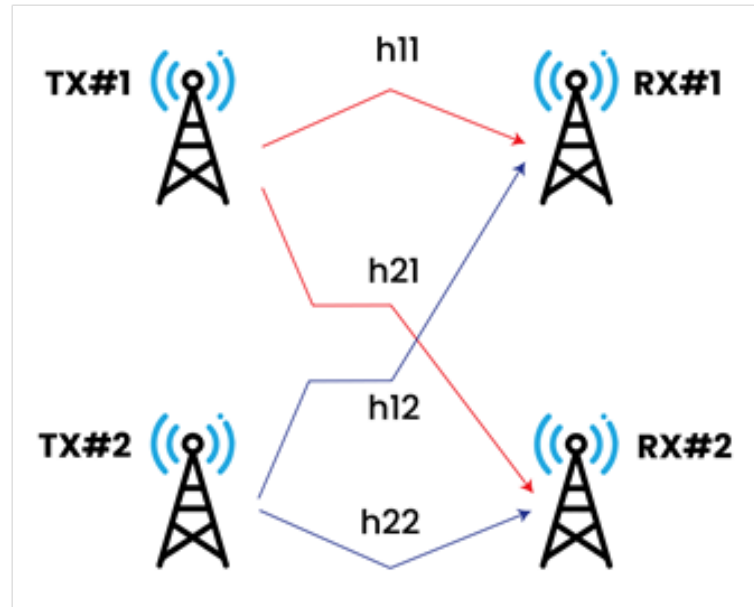


Figure 3.1: Transmit 2 Receive (2×2) MIMO channel

3.3 Transmit Diversity vs Receive Diversity

Using diversity reception is a well-known technique to mitigate the effects of fading over a communications link. However, it has mostly been relegated to the receiver end. Alamouti proposes a transmit diversity scheme that offers similar diversity gains, using multiple antennas at the transmitter. This was conceived to be more practical as, for example, it would only require multiple antennas at the base station in comparison to multiple antennas for every mobile in a cellular communications system.

This section highlights this comparison of transmit vs. receive diversity by simulating coherent binary phase-shift keying (BPSK) modulation over flat-fading Rayleigh channels. For transmit diversity, we use two transmit antennas and one receive antenna (2×1 notationally), while for receive diversity we employ one transmit antenna and two receive antennas (1×2 notationally). The simulation covers an end-to-end system showing the encoded and/or transmitted signal, channel model, and reception and demodulation of the received signal. It also provides the no-diversity link (single transmit- receive antenna case) and theoretical performance of second-order diversity link for comparison. It is assumed here that the channel is known perfectly at the

receiver for all systems. We run the simulation over a range of E_b/N_0 points to generate BER results that allow us to compare the different systems.

We start by defining some common simulation parameters:

Table 3.1: Simulation Parameters for Transmit Diversity vs Receive Diversity

| | |
|-------------------------------|------------------|
| Frame Length | 100 |
| Number of Packets | 1000 |
| E_b/N_0 | varying to 20 dB |
| Maximum Number of Tx antennas | TWO antennas |
| Maximum Number of Rx antennas | TWO antennas |

3.4 Simulation with Results and Discussion

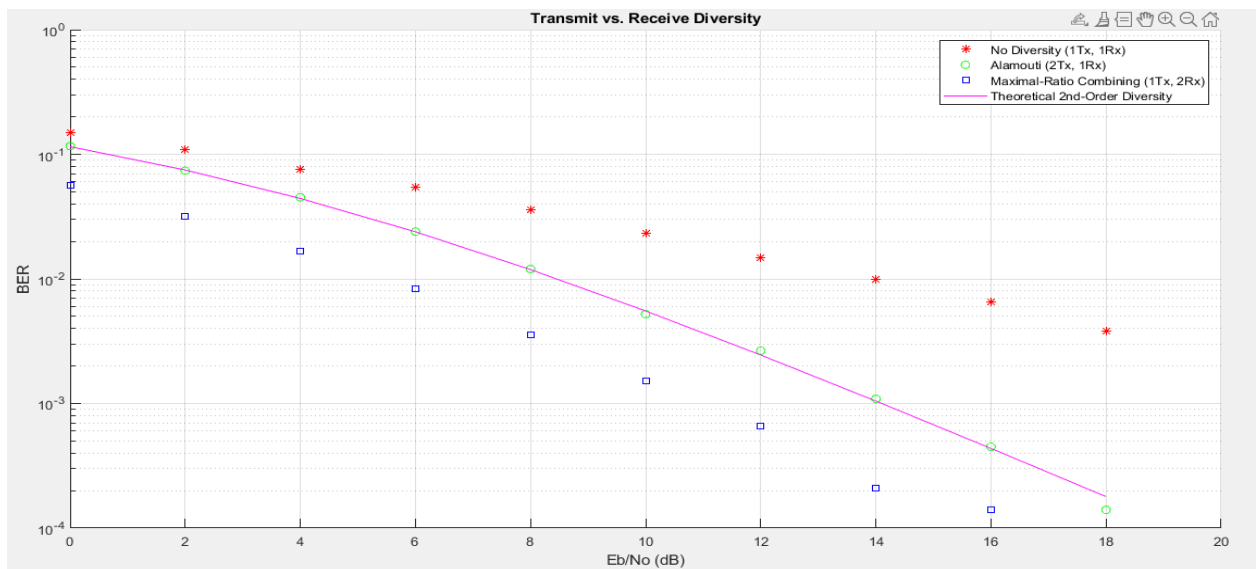


Figure 3.2: BER plot for (1x2) and (2x1) MIMO system with Transmit vs Receive Diversity (BPSK modulation in Rayleigh channel)

- The transmit diversity system has a computation complexity very similar to that of the receive diversity system.
- The resulting simulation in **Figure 3.2** results show that using two transmit antennas and one receive antenna provides the same diversity order as the maximal-ratio combined (MRC) system of one transmit antenna and two receive antennas.

- Also observe that transmit diversity has a 3 dB disadvantage when compared to MRC receive diversity. This is because we modeled the total transmitted power to be the same in both cases. If we calibrate the transmitted power such that the received power for these two cases is the same, then the performance would be identical. The theoretical performance of second-order diversity link matches the transmit diversity system as it normalizes the total power across all the diversity branches.

3.5 Zero Forcing (ZF) Equalizer

The zero-forcing equalizer removes all ISI, and is ideal when the channel is noiseless. However, when the channel is noisy, the zero forcing equalizer will amplify the noise greatly at frequencies f where the channel response $H(j2f)$ has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely. Zero-forcing equalizers ignore the additive noise and may significantly. Amplify noise for channels with spectral nulls.[39][41]

3.6 Zero Forcing (ZF) Algorithm

First, let understand the math for extracting the two symbols, which interfered with each other. In the first time slot, the received signal on the first receive antenna is, the received signal on the first receive antenna is:

$$Y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (3.1)$$

- The received signal on the second receive antenna is:

$$Y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (3.2)$$

Y_1, Y_2 are the received symbol on the first and second antenna respectively. $h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna. $h_{1,2}$ is the channel from transmit antenna to 1st receive antenna, $h_{2,1}$ is the channel from 1st transmit antenna to 1nd receive antenna, $h_{2,2}$ is the channel from 1nd transmit antenna to 1nd receive antenna, x_1, x_2 are the transmitted symbols. n_1, n_2 is the noise on 1st, 2nd receive antennas. We assume that the receiver knows $h_{1,1}, h_{1,2}, h_{2,1}$ and $h_{2,2}$. The receiver also knows Y_1 and Y_2 . The unknowns are x_1 and x_2 . Two equations and two

unknowns. Can we solve it? the Answer is YES. The above equation (y_1, y_2) can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (3.3)$$

Equivalently,

$$y = Hx + n \quad (3.4)$$

To solve for \mathbf{x} , we know that we need to find a matrix \mathbf{W} which satisfies $\mathbf{WH}=\mathbf{I}$ The Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \quad (3.5)$$

This matrix is also known as the pseudo inverse for a general $m \times n$ matrix. The term

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad (3.6)$$

[39]

3.7 BER with ZF equalizer with 2×2 MIMO

Note that the off diagonal terms in the matrix $H^H H$ not zero. Because the off diagonal terms are not zero, the zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e when solving for x_1 the interference from x_2 is tried to be nulled and vice versa. While doing so, there can be amplification of noise. Hence, Zero Forcing equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement. For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as:

$$P_b = \frac{1}{2} \left(1 - \sqrt{\frac{(E_b/N_0)}{(E_b/N_0) + 1}} \right) \quad (3.7)$$

Where P_b presents the error rate.[39][40]

3.8 Simulation with Results and Discussion

We start by defining some simulation parameters:

Table 3.2: Simulation Parameters for ZF

| | |
|--------------------------------|--------------|
| Number of Bits or Symbols | 10^6 |
| Multiple Eb/N0 Values | [0:40] dB |
| Number of Transmitter Antennas | TWO antennas |
| Number of Receiver Antennas | TWO antennas |

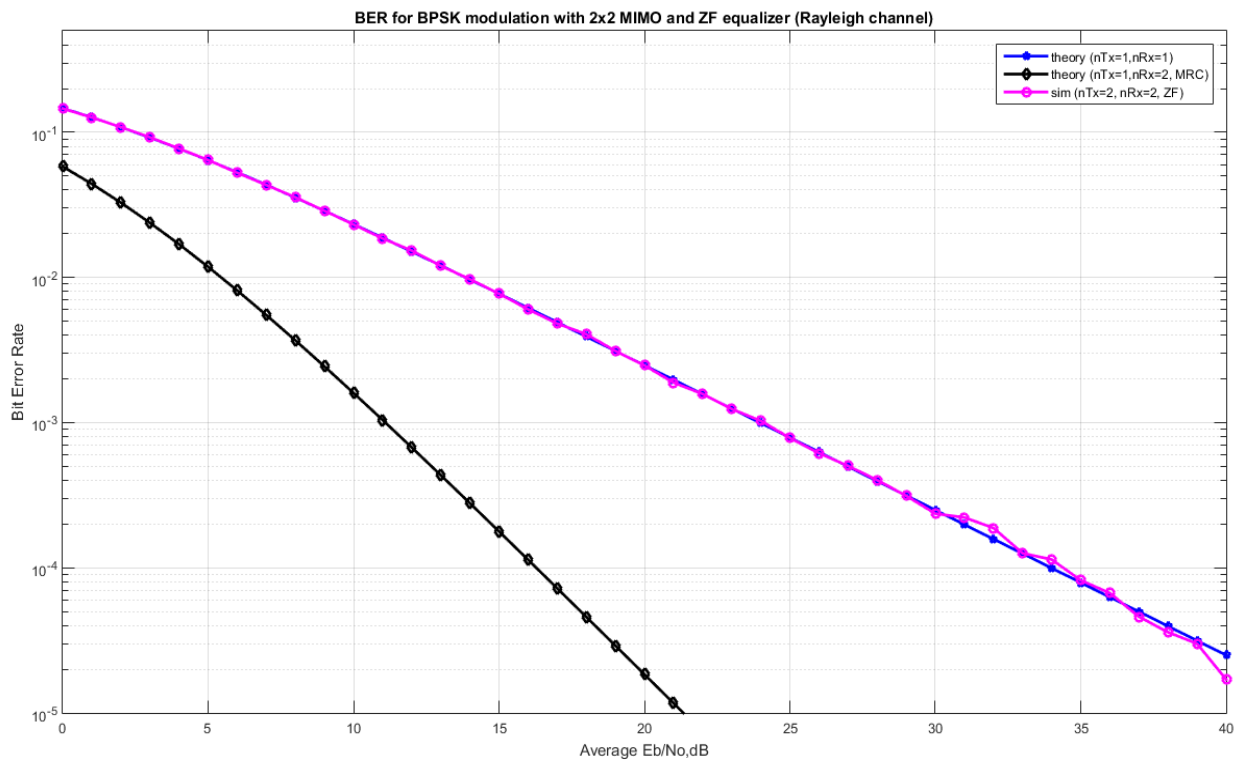


Figure 3.3: Bit rate Error plot for 2×2 MIMO channel with zero forcing equalizer (Binary Phase-Shift Keying modulation in Rayleigh channel)

Figure 3.3 displays the simulation results for a two-transmitter and two-receiver antenna MIMO system with a zero forcing equalization. The findings revealed that the ZF equalization and 1x1 system for BPSK modulation in the Rayleigh channel are identical.

3.9 Minimum mean square equalizer (MMSE)

The Minimum Mean-Square Error Linear Equalizer (MMSE-LE) balances a reduction in ISI with noise enhancement. The MMSE-LE always performs better than the ZFE and is of the

same complexity of implementation. Nevertheless, it is slightly more complicated to describe and analyze than is the ZFE. The MMSE-LE uses a linear time-invariant filter \mathbf{w}_k for R , but the choice of filter impulse response \mathbf{w}_k is different than the ZFE. The MMSE-LE is a linear filter \mathbf{w}_k that acts on \mathbf{y}_k (output signal) to form an output sequence \mathbf{z}_k that is the best MMSE estimate of \mathbf{x}_k (input signal). That is, the filter \mathbf{w}_k minimizes the Mean Square Error (MSE).[42]

3.10 Minimum mean square equalizer (MMSE) algorithm

First, we will use the same equation [(3.1):(3.4)] for the two receivers antenna The Minimum Mean Square Error (MMSE) approach tries to find a coefficient \mathbf{W} , which minimizes the criterion,

$$E \{[\mathbf{V}y - x][\mathbf{V}y - x]^{H-1}\} \quad (3.8)$$

Solving:

$$W = [H^H H + N_0 I]^{-1} H \quad (3.9)$$

When comparing to the equation in Zero Forcing equalizer, apart from the $N_0 I$ term both the equations are comparable. In fact, when the noise term is zero, the **MMSE** equalizer reduces to Zero Forcing equalizer.[39][40]

3.11 Simulation with Results and Discussion

We Use the same simulation parameters in Table3.2

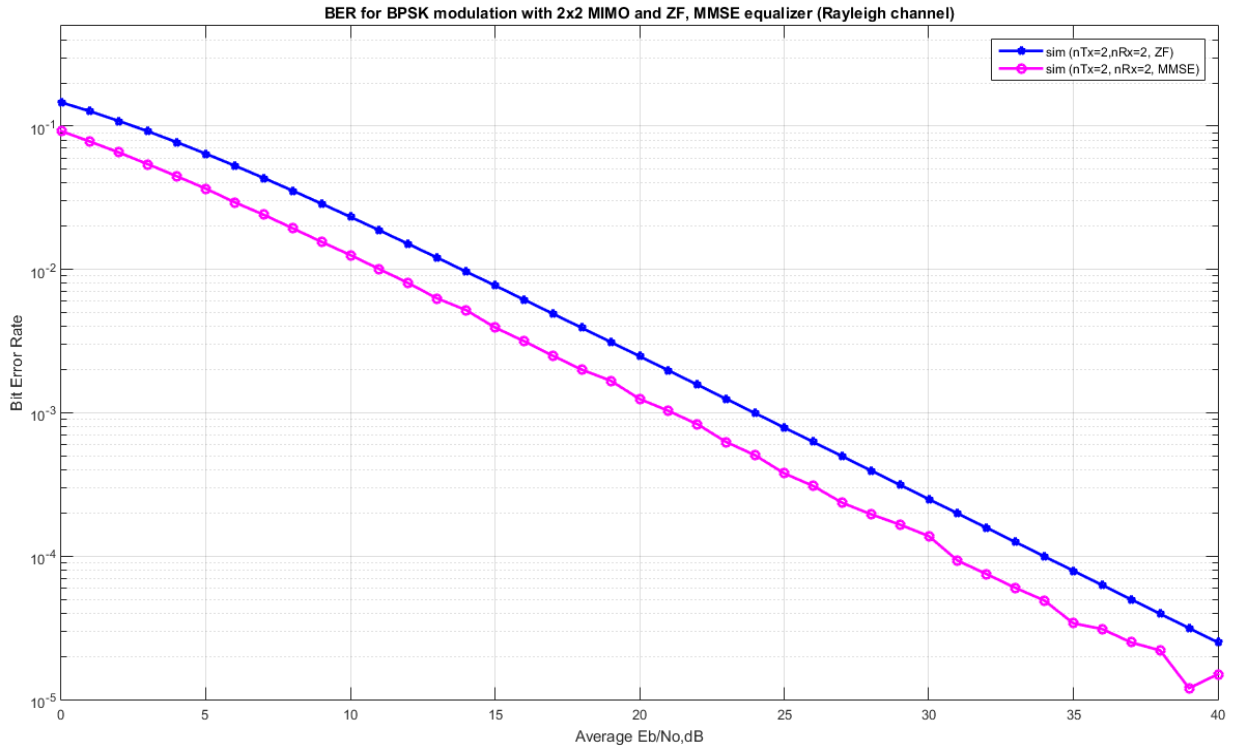


Figure 3.4: BER plot for two receivers antenna and to transmitter antenna MIMO with ZF receiver (Blue) and MMSE (Pink) for BPSK modulation in Rayleigh channel.

The figure 3.4 shows the simulation results:

- The zero forcing equalization (ZFE) exhibits a signal-to-noise ratio of 24 dB for 10^3 bit rate inaccuracy.
- The Minimum Mean-Square Error (MMSE) exhibits a signal-to-noise ratio of 21 dB for the same bit rate error.

Hence, we conclude that there is a good improvement difference between ZF and MMSE in SNR for 2x2 MIMO-OFDM transmission. When compared to the ZF receiver, the MMSE receiver has a 3 dB improvement SNR. This is especially true when spectral efficiencies are low, allowing the MMSE equalizer to reach full spatial variety. The simulation findings show that MMSE receivers outperform ZF receivers in terms of performance.

3.12 Conclusion

It can be concluded from the results presented:

1. Simulated results of 2×2 MIMO system using two transmit antennas and one receive antenna provides the same diversity order as the maximal-ratio combined (MRC) system of one transmit antenna and two receive antennas.
2. Simulated results of 2×2 MIMO using Rayleigh channel BPSK modulation for ZF always show identical results in a 1×1 system of Rayleigh channel BPSK modulation.
3. The Zero Forcing equalizer is not the greatest option for equalizing the received symbol. The zero forcing equalizer assists us in achieving data rate gain but does not allow benefiting from diversity gain (as we have 2 receive antennas).
4. The MMSE detectors perform better in digital transmission for 2×2 MIMO systems with OFDM multiplexing methods and BPSK modulation.
5. For bit error rate values of 10^{-3} , the SNR for MMSE detectors is 21 dB and for ZF detectors is 24 dB, indicating that the MMSE receiver performs better than ZF equalizer receivers.
6. For the same BER values, the MMSE receiver system has a 3 dB improvement SNR than the ZF receiver system.

GENERAL CONCLUSION

In this paper, we have studied the huge technique of "MIMO" systems. Where we explained the most important technique that was adopted by experts and researchers in the design of these multiple input and multiple output systems. and its importance in developing the field of wireless communication and increasing the amount of flow and improved precision. A thorough probabilistic analysis of Rayleigh fading MIMO channels is needed to understand the performance of quantized beamforming systems. Another point of future interest is the derivation of exact expressions for the average probability of error for MIMO equal gain systems. Many papers have derived closed-form probability of error expressions for the SIMO equal gain case but there has been little work on deriving the exact probability of error expressions for MIMO EGT systems.

In general, we present in the first chapter an introductory overview of the MIMO system technology and its current projects. Where MIMO systems create a more stable connection and less congestion.

Then, in the second chapter, a particular focus on the Precoding technique MIMO was presented. Review the benefits and challenges of this technology. MIMO technical precoding represents that an exploits channel-state information at the transmitter (CSIT) to match the transmission to the instantaneous channel conditions Linear and non-linear precoding.

Chapter 3 presents the analysis of the results of the spectral efficiency simulation according to the types of parameters of large MIMO systems we discovered that when the number of antennas approaches infinity, using precoding technology in huge MIMO systems eliminates/cancels the impacts of interference and fading while also enhancing throughput and capacity. Linear, non-linear, and PAPR precoding are the most common types of precoding methods.

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