

# MHD naturel convection flow of a couple stress fluid in a porous vertical channel

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**Abstract**— The effects of the couple-stress parameter only or combined with Darcy/Forchheimer numbers are studied for natural convection of mass and heat transfer, of a couple stress fluid through a porous channel in the presence of a magnetic field. To obtain easy ordinary system, similarity transformations are used for the nonlinear partial differential equations. Then the resulting equations are numerically solved. Graphical illustrations containing non-dimensional velocity, temperature, and concentration are presented mainly for different values of the couple-stress parameter via the large values of the Darcy and Forchheimer numbers.

**Keywords**— *Natural Convection, heat and mass transfer, MHD, porous medium, Couple stress parameter, vertical channel.*

## I. INTRODUCTION

Many scientists have shown great interest, nowadays, in non-Newtonian fluids, since they play an essential role in industrial production in the scientific and technical fields, such as petroleum, biomedical, chemical engineering. Their constitutive equations vary considerably in complexity and are of higher order than the Navier-Stokes equations.

This fluid which has attracted the attention of researchers in fluid mechanics, over the last four decades, is ‘the couple-stress theory’, initiated by Stokes [1]. In this theory, the field of rotation is defined in terms of the velocity field itself, and the rotation vector is equivalent to half the loop of the velocity vector. Kaladhar et al. [2] studied the effects of radiation and thermal diffusion (Soret effect) and the couple-stress parameter on a convective flow of a couple-stress fluid through a vertical channel. Srinivasacharya et al. [3] investigated, for their part, on the Soret and Dufour effects and the couple-stress parameter on a natural convective flow of a couple-stress fluid in a vertical channel with chemical reaction. Tasnim et al. [4] have focused their research on the effects of MHD and couple-stress parameter on the rate of entropy generation through an isothermal porous two-dimensional channel. As for Muthuraj et al. [5], they carried out a numerical study on the combined effects of heat and mass transfer on a stress-couple fluid controlled by a MHD through a porous horizontal channel in presence of viscous dissipation.

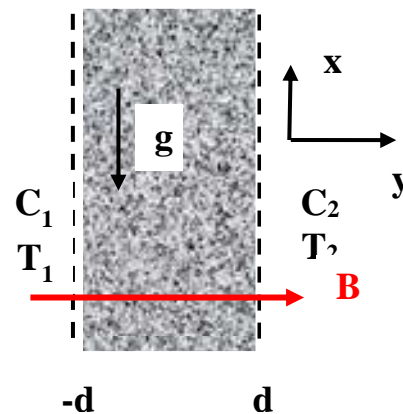
In this work, we study the effects of the couple stress parameter only or combined with the Darcy/Forchheimer numbers on MHD naturel convection flow through a vertical porous channel of a couple stress fluid with chemical reaction. The Dufour and Soret effect are taking into account. The relative technique of boundary values (BVP) is used to solve the nonlinear problem obtained with the

transformations similarity of the partial differential equations.

## II. MATHEMATICAL FORMULATION

Between two vertical parallel plates, at a distance of  $2d$  apart; an incompressible couple stress fluid flow is conducted. The coordinate system is chosen by taking the  $x$ -axis along the vertical upward direction through the central line of the channel and  $y$  is perpendicular to  $x$ .

The plates of the channel are at  $y = \pm d$ . The temperature  $T$  and concentration  $C$  are uniform in the two plates ( $y = \pm d$ ) where the wall 1 is concerning  $T$  and  $C$  of the  $y = +d$  plate and the wall 2 is about the  $y = -d$  plate. The fluid properties and the Soret and Dufour effects are considered constant in the chemical reaction, where the fluid velocity vector  $v = (u, v)$  have to be parallel to the  $x$ -axis, the transpiration cross-flow velocity  $v_0$  remained constant also,  $v_0 < 0$  means the suction and  $v_0 > 0$  is the injection velocity. The details of the application are shown on the fig. 1. A magnetic field is applied in the direction of  $y$ .



**Fig. 1:** Geometrical Configuration

According to the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the steady flow of an incompressible couple stress fluid are:

### Continuity Equation

$$v = v_0 = \text{constant}(1)$$

### Momentum Equations

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} + \rho g \beta_T (T - T_1) + \rho g \beta_C (C - C_1) - \frac{\mu}{K} u - \frac{\rho c_f}{\sqrt{K}} u^2 - \sigma B_0^2 u \quad (2)$$

### Energy Equation

$$v_0 \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + 2 \frac{v}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\eta}{\rho c_p} \left( \frac{\partial^2 u}{\partial y^2} \right)^2 + \frac{D\lambda}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p} u^2 \quad (3)$$

### Concentration Equation

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D\lambda}{T_m} \frac{\partial^2 T}{\partial y^2} - k_1 (C - C_1) \quad (4)$$

$u$  is the velocity component along  $x$  direction,  $\rho$  means the density,  $g$  denotes the acceleration due to gravity,  $p$  notifies the pressure,  $\mu$  signifies the coefficient of viscosity,  $\beta_T$  is the coefficient of thermal expansion,  $\beta_C$  indicates the coefficient of solutal expansion,  $\alpha$  is the thermal diffusivity,  $D$  indicates the mass diffusivity,  $C_p$  is the specific heat capacity,  $c_s$  means the concentration susceptibility,  $\lambda$  is the thermal diffusion ratio,  $T_m$  notifies the mean fluid temperature,  $\lambda$  is the thermal diffusion ratio,  $\eta_1$  is the additional viscosity coefficient that specifies the character of the fluid couple stress,  $k_1$  means the rate of chemical reaction,  $K$  designates permeability of porous medium (hydraulic conductivity),  $c_f$  means Forchheimer constant,  $\sigma$  indicates the electrical conductivity of the fluid and  $B_0$  magnetic field.

The boundary conditions are given by:

$$u = 0 \text{ à } y = +d, u = 0 \text{ à } y = -d \quad (5a)$$

$$u_{,yy} = 0 \text{ à } y = +d, u_{,yy} = 0 \text{ à } y = -d \quad (5b)$$

$$T = T_1, C = C_1 \text{ à } y = -d \quad (5c)$$

$$T = T_2, C = C_2 \text{ à } y = +d \quad (5d)$$

The boundary condition (5a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (5b) implies that the couple stresses are zero at the plate surfaces and introducing the following similarity transformations:

$$y = \eta d, u = \frac{vGr}{d} f, T - T_1 = (T_2 - T_1)\theta, C - C_1 = (C_2 - C_1)\varphi \quad (6)$$

We get from (2) and (4), the following nonlinear system of differential equations:

$$S^2 f^{(IV)} - f'' + Re f' - \theta - N\varphi + \frac{1}{Da} f + FoGr f^2 + Ha^2 f = 0 \quad (7)$$

$$\theta'' - RePr\theta' + 2BrGr^2(f')^2 + S^2 BrGr^2 f'' + D_f Pr \varphi'' + Ha^2 Gr^2 Br f^2 = 0 \quad (8)$$

$$\varphi'' - ReSc\varphi' + S_r Sc\theta'' - K Sc \varphi = 0 \quad (9)$$

Where primes denote differentiation with respect to  $\eta$  alone,  $Re$  represents the Reynolds number,  $Gr$  denotes the Grashof number,  $Pr$  is the Prandtl number,  $Sc$  means the Schmidt number,  $Br$  signifies the Brinkman number,  $K$  notifies the chemical reaction parameter,  $Sr$  is the Soret number,  $D_f$  means the Dufour number,  $N$  denotes the buoyancy ration,  $S$  is the couple stress parameter, the effects of couple-stress are significant for large values of  $S$ ,  $Ha$  means the Hartmann number,  $Fo$  designates the Forchheimer number and finally  $Da$  is the Darcy number.

$$Re = \frac{v_0 d}{\nu}, Gr = \frac{g\beta_T(T_2 - T_1)d^3}{\nu^2}, Pr = \frac{\mu c_p}{\lambda},$$

$$Br = \frac{\mu v^2}{\lambda(T_2 - T_1)d^2}, k = \frac{k_1 d^2}{\nu}, S_r = \frac{D\lambda(T_2 - T_1)}{\nu T_m(C_2 - C_1)},$$

$$D_f = \frac{D\lambda(C_2 - C_1)}{\nu c_s c_p(T_2 - T_1)}, N = \frac{\beta_C(C_2 - C_1)}{\beta_T(T_2 - T_1)}, S = \frac{l}{d} \sqrt{\frac{\eta_1}{\mu}},$$

$$Ha = B_0 d \sqrt{\frac{\sigma}{\mu}}, Fo = \frac{c_f d}{\sqrt{K}}, Da = \frac{K}{d^2}, Sc = \frac{\nu}{D}$$

Boundary conditions (5a-5d) in terms of  $f$ ,  $\theta$  and  $\varphi$  become:

$$\begin{aligned} f = 0, f'' = 0, \theta = 0, \varphi = 0 \text{ à } \eta = -1 \\ f = 0, f'' = 0, \theta = 1, \varphi = 1 \text{ à } \eta = +1 \end{aligned} \quad (10)$$

### III. RESULTS AND DISCUSSION

A numerical calculation was made with the following values:  $Pr = 0,71$ ,  $Sc = 0,22$ ,  $Br = 0,5$ ,  $Re = 2$ ,  $Gr = 0,5$ ,  $N = 2$ ,  $K=1$ ,  $Df=2$ ,  $Sr = 0.03$ ,  $Ha = 0.5$ .

These values are used throughout the calculations unless otherwise stated. Without the presence of the magnetic field and with the couple stress fluid in a not porous medium, a validation (with [2]) of the program written on the basis of the equations (6-10) is obtained via the Lobatto technique to insure more accurate data.

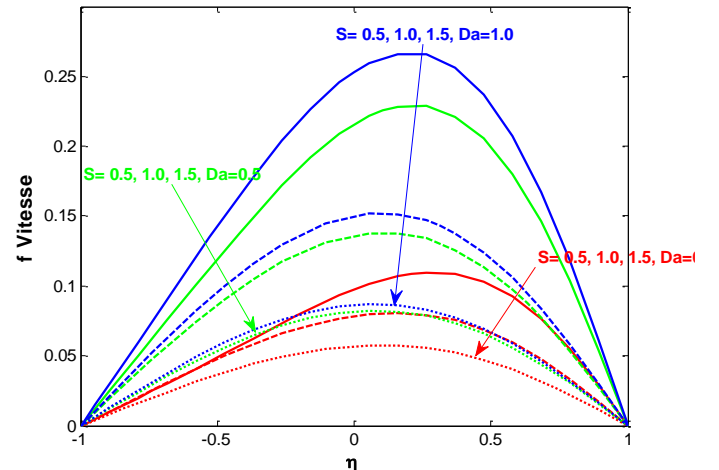
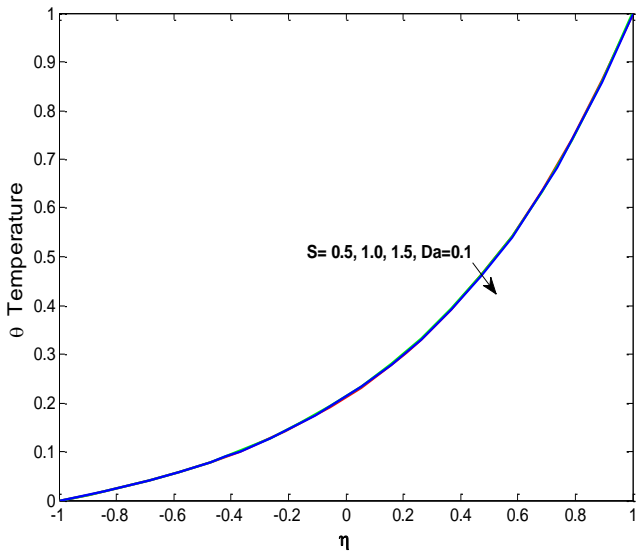


Fig. 2. Velocity profile for different values of  $S$  ( $Da = 0.1, 0.5, 1.0$  and  $Fo = 1.0$ )



**Fig. 3.** Temperature profile for different values of  $S$  ( $Da=1.0$  and  $Fo=1.0$ )

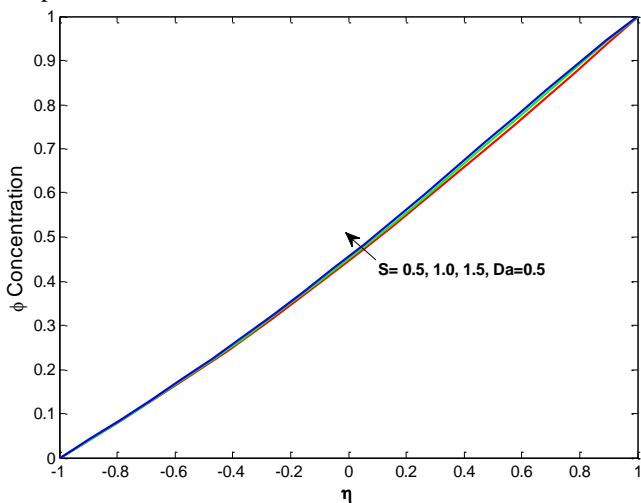
Figs. 2, 3, and 4, show the non-dimensional velocity, temperature and concentration profiles for different values of the couple-stress parameter ‘ $S$ ’ only while the Darcy and Forchheimer are maintained constant.

It should be noted from these figures that the temperature and the concentration profiles remain the same and thus the parameter of the couple stress fluid act only on the velocity profile. The movement of the fluid is substantially affected by the increase of couple-stress parameter.

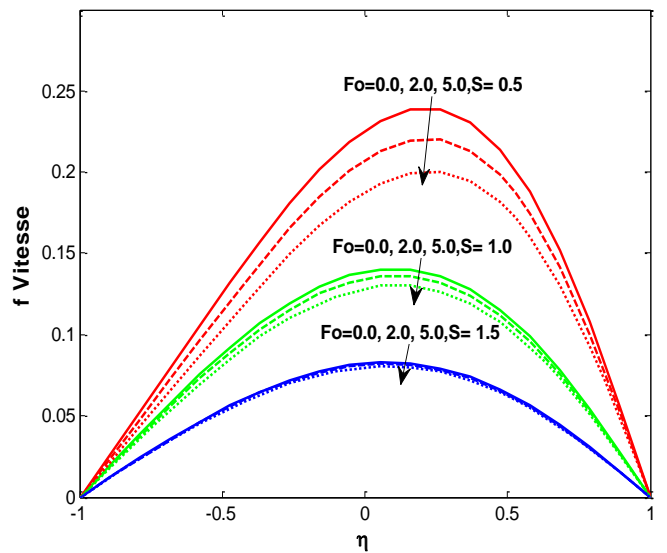
These changes are more pronounced in the region near the centerline of the channel. This is explained by the free rotation and more important of the particles of the fluid as they are moved other than those near the walls.

Also, in the fig. 2, three curves are plotted to highlight the variation of the Darcy number with the variation of  $S$ . It is noted that more is  $Da$ , more is the increase of the velocity.

This is true if we recall that  $Da$  expressed the permeability of the porous medium. This effect can be interpreted also by the short dimensions of the channel. In all cases, the velocity is retracted when the parameter of the couple stress increases.



**Fig. 4.** Concentration profile for different values of  $S$  ( $Da=0.5$  and  $Fo=1.0$ )



**Fig. 5.** Velocity profile for different values of  $S$  and  $Fo$  ( $Da=0.5$ )

On the other hand, at a fixed value of  $Da$ , the effects of the couple-stress parameter ‘ $S$ ’ on the velocity profile with the variations of the Forchheimer number are presented in the Fig.5. Here the curves are near for ones considered with fixed  $Fo$ . Thus, the effect of  $Fo$  is less important than  $Da$ . We can see, unlike the effect of  $Da$ , that the increase of  $Fo$  leads to the decrease of the velocity, in the same tendency with the increase of  $S$ .

#### IV. CONCLUSIONS

In this article, we studied the effect of the couple stress parameter on MHD natural convection flow through a vertical porous channel saturated by a couple stress fluid with chemical reaction. Magnetic field is taken into account with Soret and Dufour effects. This analysis is conducted also with combination between  $S$  and Darcy or Forchheimer numbers.

The conclusions are as follows:

- The Increase of couple-stress parameter causes an important decrease of the velocity and negligible effects are observed for the temperature and concentration profiles. It should be noted that when  $S$  tends to zero, the fluid becomes Newtonian.
- Increasing the number of Darcy increases the velocity with the decrease of the velocity with the couple stress parameter
- The augmentation of Forchheimer number combined with the couple stress parameter causes the reduction of the velocity.
- This trend causes a delicate choice between these numbers to resolve a particular problem in a channel with a couple stress fluid in a porous medium.

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## REFERENCES

- [1] V.K. Stokes, Couple stresses in fluids, *Phys. Fluids*, 9(9):1709–1715. (1966).
- [2] K. Kaladhar et al, Mixed Convection Flow of Couple Stress Fluid in a Vertical Channel with Radiation and Soret Effects, *Journal of Applied Fluid Mechanics*, 9(1):43-50. 2016.
- [3] D. Srinivasacharya, and K. KALADHAR. Soret and Dufour effects on free convection flow of a couple stress fluid in a vertical channel with chemical reaction. *Chem. Ind. Chem. Eng Q.* 19(1):45–55. 2013.
- [4] S.M. Tasnim, S. Mahmud, M.A.H. Mamun. Entropy generation in a porous channel with hydromagnetic effect. *Int. J. Exergy* 3:300–8. 2002.
- [5] R.S. Muthuraj, and al. Heat and mass transfer effects on MHD flow of a couple-stress fluid in a horizontal wavy channel with viscous dissipation and porous medium. *Heat Transfer-Asian Res.*; 42(5):403–21. (2013).