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Descent Propriety of the Convex Combination of FR and PRP Methods

Chenna Nasreddine*, Sellami Badreddine*

* laboratory Informatics and Mathematics (LIM), Mohammed Cherif Messaadia University, Souk ahras, Algeria

E-mail: chennanasreddine206@gmail.com, b.sellami@uni-soukahras.dz

Abstract

In this note, we present a new theory as a modification and alternative to S.Djordjevic's Theorem (2.2) [5], Here we rephrase the text of theory (2.2) by deleting condition (2.16), notations and equation numbers as in S.Djordjevic.

Keywords: unconstrained optimization, conjugate gradient method, global convergence.

1. Introduction

We consider the nonlinear unconstrained optimization problem

$$\min\{f(x) : x \in \mathbb{R}^n\}, \qquad (1.1)$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable function, bounded from below.
There are many different methods for solving the problem (1.1).
We are interested in conjugate gradient methods, which have low memory requirements and strong local and global convergence properties.
For solving the problem (1.1), we consider the conjugate gradient method, which starts from an initial point $x_0 \in R_n$ and generates a sequence $\{x_k\} \subset \mathbb{R}^n$ as follows

$$x_{k+1} = x_k + t_k d_k, \quad (1.2)$$

where $t_k > 0$ is a step size, received from the line search, and the directions d_k are given by

$$d_0 = -g_0,\ d_{k+1} = -1_{k+1} + \beta_k s_k. \quad (1.3)$$

In the relation (1.3) β_k is the conjugate gradient parameter, $s_k = x_{k+1} - x_k$, $g_k = \nabla f(x_k)$.

Let the norm $||.||$ be the Euclidean norm.

The standard Wolfe line search conditions are frequently used in the conjugate gradient methods; these conditions are given by

$$f(x_k + t_k d_k) - f(x_k) \leq \delta t_k g_k d_k, \qquad (1.4)$$

$$g_{k+1} d_k \geq \sigma g_k d_k, \qquad (1.5)$$

where d_k is a descent direction and $0 < \delta \leq \sigma < 1$.
Strong Wolfe conditions consist of (1.4) and the next stronger version of (1.5)

$$|g_{k+1} d_k| \leq -\sigma g_k d_k. \qquad (1.6)$$

In the generalized Wolfe conditions, the absolute value in (1.6) is replaced by a pair of inequalities:

$$\sigma_1 g_k d_k \leq g_{k+1} d_k \leq -\sigma_2 g_k d_k,\quad 0 < \delta < \sigma_1 < 1,\quad \sigma_2 \geq 0. \qquad (1.7)$$

Now, let us denote

$$y_k = 1_{k+1} - 1_k. \qquad (1.8)$$

2.

Theorem 2.2 (S.Djordjevi´c's [5]). Assume that (2.12) and (2.13) hold and let strong Wolfe conditions (1.4)-(1.6) hold with $\sigma < 1/2$. Also, let $\{\| s_k \|\}$ tend to zero, and let there exist some nonnegative constants $\eta 1$, $\eta 2$ such that

$\|g_k\|^2 \geq \eta 1 \|s_k\|^2$; (2.15)

$\|g_{k+1}\|^2 \leq \eta 2 \|s_k\|^2$: (2.16)

Then d_k satisfies the sufficient descent condition for all k.

We suggest a new formula of S.Djordjević's Theorem (2.2).

Theorem 2.2*. Assume that Assumption (2.12) and (2.13) held, let strong Wolfe conditions (1.4)-(1.6) held with $\sigma < 1/2$, and there exists $\eta\ 1 > 0$ such that

$\|g_k\|^2 \geq \eta 1 \|s_k\|^2$, $L \leq \eta 2$.

Then d_k satisfies the sufficient descent condition for all k.

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