

Natural Frequency Analysis of Rectangular Laminated Sandwichs Plates Using 2D Layerwise FE Model

1st Mohamed-Ouejdi Belarbi, 2nd Abdelouahab Tati, 3rd Abdelhak Khechai

1st Laboratoire de Génie Energétique et Matériaux, LGEM, Université de Biskra, B.P. 145, R.P. 07000, Biskra, Algeria,
3rd Laboratoire de Génie Energétique et Matériaux, LGEM, Université de Biskra, B.P. 145, R.P. 07000, Biskra, Algeria,
2nd Laboratoire de Recherche en Génie Civil, LRG. Université de Biskra, B.P. 145, R.P. 07000, Biskra, Algeria
Corresponding Author email address: Belarbi.m.w@gmail.com

Abstract— The natural frequency analysis of rectangular laminated sandwichs plates has been investigated, using a new higher-order layerwise finite element formulation. A first-order displacement field is assumed for the face sheets, whereas a higher-order displacement field is assumed for the core. Unlike the conventional layerwise models, the number of variables in the present model is fixed and does not increase when increasing the number of lamina layers. This helps to reduce the computational cost of the analysis. The performance and reliability of the proposed formulation are demonstrated by comparing the author's results with those obtained using the three-dimensional elasticity theory, analytical solutions and other advanced finite element models. From the obtained results, it was shown that the proposed finite element model provided a better prediction on the natural frequency a rectangular laminated sandwichs plates.

Keywords— natural frequency, finite element, layerwise, laminated sandwichs.

I. INTRODUCTION

Sandwich structures are widely used in various engineering applications such as civil constructions, marine industry, automobile and aerospace applications, due to their high strength-to-weight ratio, good ability in sound and energy absorbing. To use these structures efficiently, an excellent understanding of their dynamical behavior is needed. There are three major theories that are presented to study the free free vibration behavior of sandwich structures, which are: namely: (1) the classical laminated plate theory (CLPT); (2) the first order shear deformation theory (FSDT); and (3) the higher order shear deformation theory (HSDT). Regarding the approaches used to model the behavior of composite structures, we distinguish the equivalent single layers (ESL) approach where all the laminate layers are referred to the same variables [1]. The main advantages of ESL models are their inherent simplicity and their low computational cost, due to the small number of dependent variables. However, ESL approach fails to capture precisely the local behavior of sandwich structures. This drawback in ESL was circumvented by the Zig-Zag (ZZ) and Layerwise (LW) approaches in which the variables are linked to specific layers [2-5].

In the recent years, many researchers have investigated the dynamic response of laminated composite and sandwich plates using finite element models. According to the literature review on the sandwich models, we found that many authors used finite element models having large number of nodes and/or DOF, especially those based on the LW approach [3, 6-10]. Therefore, the present work aims to propose a new C^0 LW model competitor to the majority of aforementioned finite element models, having a reduced number of nodes and DOF. This new model is used for the calculation of natural frequencies of laminated sandwich plates. Thanks for enforcing the continuity of the interlaminar displacement, the number of variables is independent of the number of layers. The numerical results obtained by developed model are compared favorably with those obtained via analytical solution and numerical results obtained by other models.

II. MATHEMATICAL MODEL

Sandwich plate is a structure composed of three principal layers as shown in figure 1, two face sheets (top-bottom) of thicknesses (h_t), (h_b) respectively, and a central layer named core of thickness (h_c) which is thicker than the previous ones. Total thickness (h) of the plate is the sum of these thicknesses. The plane (x, y) coordinate system coincides with mid-plane plate.

In the present model, the core layer is modeled using the HSDT. Hence, the displacement field is written as a third-order Taylor series expansion of the in-plane displacements

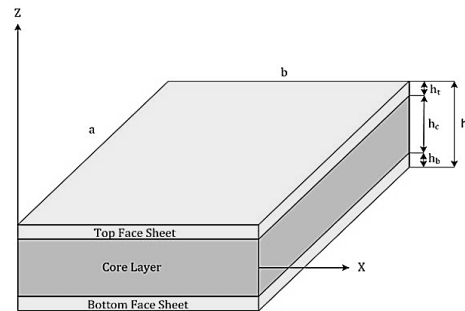


Fig. 1. Geometry and notations of a sandwich plate.

in the thickness coordinate, and as a constant one for the transverse displacement:

$$\begin{aligned} u_c &= u_0 + Z\psi_x^c + Z^2\eta_x^c + Z^3\zeta_x^c \\ v_c &= v_0 + Z\psi_y^c + Z^2\eta_y^c + Z^3\zeta_y^c \\ w_c &= w_0 \end{aligned} \quad (1)$$

where u_0 , v_0 and w_0 are respectively, in-plane and transverse displacement components at the mid-plane of the sandwich plate. ψ_x^c , ψ_y^c represent normal rotations about the x and y axis respectively. The parameters η_x^c , η_y^c , ζ_x^c and ζ_y^c are higher order terms.

The strain-displacement relations for the core layer are given as follows:

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_x^{(0)} + z\chi_x^{(1)} + z^2\chi_x^{(2)} + z^3\chi_x^{(3)} \\ \varepsilon_{yy} &= \varepsilon_y^{(0)} + z\chi_y^{(1)} + z^2\chi_y^{(2)} + z^3\chi_y^{(3)} \\ \gamma_{xy} &= \gamma_{xy}^{(0)} + z\chi_{xy}^{(1)} + z^2\chi_{xy}^{(2)} + z^3\chi_{xy}^{(3)} \\ \gamma_{yz} &= \gamma_{yz}^{(0)} + z\chi_{yz}^{(1)} + z^2\chi_{yz}^{(2)} \\ \gamma_{xz} &= \gamma_{xz}^{(0)} + z\chi_{xz}^{(1)} + z^2\chi_{xz}^{(2)} \end{aligned} \quad (2)$$

For the two face sheets, the FSDT is adopted. The compatibility conditions as well as the displacement continuity at the interface (top face sheet-core- bottom face sheet), leads to the following improved displacement fields:

Top face sheet:

$$\begin{aligned} u_t &= u_0 + \left(\frac{h_c}{2}\right)\psi_x^c + \left(\frac{h_c^2}{4}\right)\eta_x^c + \left(\frac{h_c^3}{8}\right)\zeta_x^c + \left(z - \frac{h_c}{2}\right)\psi_x^t \\ v_t &= v_0 + \left(\frac{h_c}{2}\right)\psi_y^c + \left(\frac{h_c^2}{4}\right)\eta_y^c + \left(\frac{h_c^3}{8}\right)\zeta_y^c + \left(z - \frac{h_c}{2}\right)\psi_y^t \\ w_t &= w_0 \end{aligned} \quad (3)$$

Bottom face sheet:

$$\begin{aligned} u_b &= u_0 - \left(\frac{h_c}{2}\right)\psi_x^c + \left(\frac{h_c^2}{4}\right)\eta_x^c - \left(\frac{h_c^3}{8}\right)\zeta_x^c + \left(z + \frac{h_c}{2}\right)\psi_x^b \\ v_b &= v_0 - \left(\frac{h_c}{2}\right)\psi_y^c + \left(\frac{h_c^2}{4}\right)\eta_y^c - \left(\frac{h_c^3}{8}\right)\zeta_y^c + \left(z + \frac{h_c}{2}\right)\psi_y^b \\ w_b &= w_0 \end{aligned} \quad (4)$$

The constitutive relations for the three layers sandwich plate for k^{th} layer in the global coordinate system can be written as:

$$\{\sigma_x \quad \sigma_y \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz}\}_k^T = \{\bar{Q}\}_k \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}_k^T \quad (5)$$

III. FINITE ELEMENT FORMULATION

In the present study, a C^0 four-node isoparametric element, named QSFT52, with thirteen DOF per node has been developed (figure. 2). Each node contains: two rotational DOF for each face sheet, six rotational DOF for the core, while the three translations DOF are common for

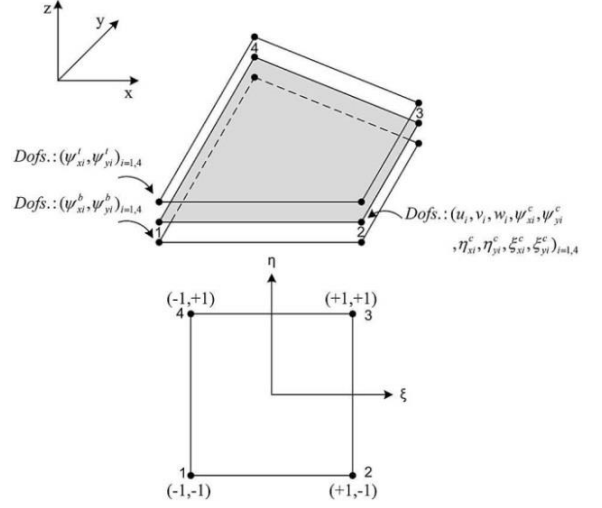


Fig. 2. Geometry and corresponding DOFs of the present QSFT52 element.

sandwich layers. For more details on the formulation of this element, the reader may refer to [5].

Hamilton's principle is used in order to formulate governing static and free vibration problems considered in this work, which is given as:

$$\delta\Pi = \delta\int_{t_1}^{t_2} (\delta U - \delta T) dt = 0 \quad (6)$$

The first variation of the potential energy of the sandwich plate is the summation of contribution from the two face sheets and from the core as:

$$\begin{aligned} \delta U &= \int_{A_t} \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} (\sigma_{xx}^c \delta\varepsilon_{xx}^c + \sigma_{yy}^c \delta\varepsilon_{yy}^c + \sigma_{xy}^c \delta\varepsilon_{xy}^c + \sigma_{xz}^c \delta\varepsilon_{xz}^c + \sigma_{yz}^c \delta\varepsilon_{yz}^c) dV_c \\ &+ \int_{A_t} \int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_s} (\sigma_{xx}^t \delta\varepsilon_{xx}^t + \sigma_{yy}^t \delta\varepsilon_{yy}^t + \sigma_{xy}^t \delta\varepsilon_{xy}^t + \sigma_{xz}^t \delta\varepsilon_{xz}^t + \sigma_{yz}^t \delta\varepsilon_{yz}^t) dV_t \\ &+ \int_{A_b} \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}-h_s} (\sigma_{xx}^b \delta\varepsilon_{xx}^b + \sigma_{yy}^b \delta\varepsilon_{yy}^b + \sigma_{xy}^b \delta\varepsilon_{xy}^b + \sigma_{xz}^b \delta\varepsilon_{xz}^b + \sigma_{yz}^b \delta\varepsilon_{yz}^b) dV_b \end{aligned} \quad (7)$$

The first variation of kinetic energy of the three layers sandwich plate can be expressed as:

$$\begin{aligned} \delta T &= \int_{V_t} \rho_t (\ddot{u}_t \delta u_t + \ddot{v}_t \delta v_t + \ddot{w}_t \delta w_t) dV_t \\ &+ \int_{V_c} \rho_c (\ddot{u}_c \delta u_c + \ddot{v}_c \delta v_c + \ddot{w}_c \delta w_c) dV_c \\ &+ \int_{V_b} \rho_b (\ddot{u}_b \delta u_b + \ddot{v}_b \delta v_b + \ddot{w}_b \delta w_b) dV_b \end{aligned} \quad (8)$$

where u_i , v_i and w_i are the displacement in x , y and z directions, respectively, of the three-layered sandwich ($i = t, c, b$), ρ_i and V_i are the density of the material and volume, of each component, respectively, and $(\ddot{\cdot})$ is a second derivative with respect to time.

In the present analysis, the work done by external forces and the damping are neglected. Hence, Eq. (6) leads to the following dynamic equilibrium equation of a system:

$$[M_e]\{\ddot{\delta}\} + [K_e]\{\delta\} = 0 \quad (9)$$

The total element stiffness matrix is the summation of contribution from the two face sheets and from the core as:

$$[K_e] = \iint \left(\begin{array}{c} [B^t]^T [D^t] [B^t] + [B^c]^T [D^c] [B^c] \\ + [B^b]^T [D^b] [B^b] \end{array} \right) dA \quad (10)$$

The total element mass matrix, for the three-layer sandwich plate, can be written as:

$$[M_e] = \iint \left(\begin{array}{c} [N]^T [m^{(t)}] [N] + [N]^T [m^{(c)}] [N] \\ + [N]^T [m^{(b)}] [N] \end{array} \right) dA \quad (11)$$

Now, after evaluating the stiffness and mass matrices for all elements, the governing equations for free vibration analysis can be stated in the form of generalized eigenvalue problem.

$$[K]\{\chi\} - \omega^2 [M]\{\chi\} = 0 \quad (12)$$

IV. RESULTS AND DISCUSSIONS

1. Rectangular sandwich plate having two-ply laminated stiff sheets at the faces (0/90/C/0/90)

In this problem, a simply supported rectangular sandwich plate having two laminated stiff layers is investigated. The mechanical properties MM1 and MM2 of table 1 are adopted, respectively, for laminated face sheets and core. In the present analysis, different aspect ratios (a/b), keeping the same reports ($a/h = 10$ and $h_c/h_f = 10$), are considered. The non-dimensional natural frequencies are presented in table 2 using a mesh size of 12×12 . The obtained results are compared with the 3D-elasticity solution [11], the analytical results based on HSDT [12] and those obtained with the FEM-Q8 solution based on global-local higher order shear deformation theory (GLHSDT) [4]. It is clear, from the table 2, that the results of developed element are in excellent agreement with numerical results found in the literature.

2. Influence of the geometric properties on the natural frequencies

In order to study the influence of the plate dimensions on the natural frequencies, a seven-layer simply supported square sandwich plate (0/90/0/core/0/90/0) is considered. The core is made of HEREX-C70.130 PVC foam (MM3) and the face sheets are made of glass polyester resin (MM4). The ratio of the thickness of the core to the total thickness (h_c/h) is assumed to be equal to 0.88. The effect of the thickness ratios (a/h) and aspect ratio (a/b) on the natural frequencies for different aspect ratios is examined in figure 3. It is found that the non-dimensional natural frequencies decrease with decreasing a/h ratio. This can be explained by the fact that the FRP sandwich plates are not assumed to be infinitely stiff through the thickness.

TABLE I. MATERIAL MODELS (MM) CONSIDERED FOR DIFFERENT LAMINATED AND SANDWICH PLATE

Elastic properties	Unit	MM1	MM2	MM3	MM4
E_{11}	GPa	131.0	0.00690	24.51	0.1036
E_{22}	GPa	10.34	0.00690	7.77	0.1036
G_{12}	GPa	6.9	0.00344	3.34	0.05
G_{13}	GPa	6.2	0.00344	3.34	0.05
G_{23}	GPa	6.9	0.00345	1.34	0.05
ν_{12}	-	0.22	10^{-5}	0.078	0.32
ρ	Kg/m ³	1627	97	1800	130

TABLE II. NON-DIMENSIONAL FUNDAMENTAL FREQUENCIES WITH DIFFERENT ASPECT RATIO (A/B) FOR SIMPLY SUPPORTED SANDWICH PLATE WITH LAMINATED FACE SHEETS (0/90/C/0/90).

$\frac{a}{b}$	Present element QSFT52	Rao et al. [13] 3D-Elast.	Zhen et al. [4] FEM-Q8-GLHSDT	Kant and Swaminathan [12] HSDT-Anal
0.5	5.7288	5.7326	6.1069	15.0316
1.0	1.8481	1.8480	1.9712	4.8519
1.5	1.0919	1.0900	1.1644	2.8188
2.0	0.8071	0.8048	0.8584	2.4560
2.5	0.6654	0.6627	0.7045	1.5719
3.0	0.5834	0.5804	0.6145	1.3040
5.0	0.4524	0.4494	0.4676	0.8187

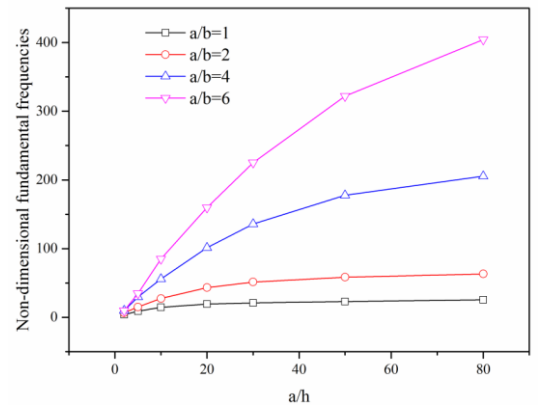


Fig. 3. Effect of the plate aspect ratio (a/b) and the length/thickness ratio (a/h) on the non-dimensional fundamental frequencies of a simply supported sandwich plate having laminated face sheets.

V. CONCLUSIONS

In this paper, a new layerwise finite element model was proposed for the natural frequency analysis of rectangular laminated sandwich plates. The developed model is based on a proper combination of higher-order and first-order, shear deformation theories. These combined theories satisfy interlaminar displacement continuity. The performance and the efficiency of the newly developed FE model are demonstrated. The results obtained by our model were compared with those obtained by the analytical results and other finite element models found in literature. The comparison showed that the element has an excellent accuracy and a broad range of applicability.

REFERENCES

- [1] Kant, T. and K. Swaminathan, Free vibration of isotropic, orthotropic, and multilayer plates based on higher order refined theories. *Journal of sound and vibration*, 2001. 241(2): p. 319-327.
- [2] Chalak, H.D., et al., Free vibration analysis of laminated soft core sandwich plates. *Journal of Vibration and Acoustics*, 2013. 135(1): p. 011013.
- [3] Pandey, S. and S. Pradyumna, A new C^0 higher-order layerwise finite element formulation for the analysis of laminated and sandwich plates. *Composite Structures*, 2015. 131: p. 1-16.
- [4] Zhen, W., C. Wanji, and R. Xiaohui, An accurate higher-order theory and C^0 finite element for free vibration analysis of laminated composite and sandwich plates. *Composite Structures*, 2010. 92(6): p. 1299-1307.
- [5] Belarbi, M.-O., et al., Development of a 2D isoparametric finite element model based on the layerwise approach for the bending analysis of sandwich plates. *Structural Engineering and Mechanics*, 2016. 57(3): p. 473-506.
- [6] Elmalich, D. and O. Rabinovitch, A high-order finite element for dynamic analysis of soft-core sandwich plates. *Journal of Sandwich Structures and Materials*, 2012. 14(5): p. 525-555.
- [7] Marjanović, M. and D. Vuksanović, Layerwise solution of free vibrations and buckling of laminated composite and sandwich plates with embedded delaminations. *Composite Structures*, 2014. 108: p. 9-20.
- [8] Nabarrete, A., S.F. M. De Almeida, and J.S. Hansen, Sandwich-plate vibration analysis: three-layer quasi-three-dimensional finite element model. *AIAA Journal*, 2003. 41(8): p. 1547-1555.
- [9] Ramtekkar, G., Y. Desai, and A. Shah, Mixed finite-element model for thick composite laminated plates. *Mechanics of Advanced Materials and Structures*, 2002. 9(2): p. 133-156.
- [10] Araújo, A., C.M. Soares, and C.M. Soares, Finite element model for hybrid active-passive damping analysis of anisotropic laminated sandwich structures. *Journal of Sandwich Structures and Materials*, 2010. 12(4): p. 397-419.
- [11] Kulkarni, S.D. and S. Kapuria, Free vibration analysis of composite and sandwich plates using an improved discrete Kirchhoff quadrilateral element based on third-order zigzag theory. *Computational Mechanics*, 2008. 42(6): p. 803-824.
- [12] Kant, T. and K. Swaminathan, Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher-order refined theory. *Composite Structures*, 2001. 53(1): p. 73-85.
- [13] Rao, M., et al., Natural Vibrations of Laminated and Sandwich Plates. *Journal of Engineering Mechanics*, 2004. 130(11): p. 1268-1278.