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EXISTENCE RESULT FOR A CLASS OF HYPERBOLIC QUASIVARIATIONAL INEQUALITIES

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Abstract

This work proves the existence of solution for a quasivariational inequalities of hyberbolic type by using fixed-point theorem. We illustrate the abstract results by an application to dynamic contact frictional problem for viscoelastic materials.

Keywords: Quasivariational inequalities, hyperbolic, fixed-point technique, frictional, contact problem.

1. Introduction

In this paper we establish the existence of solution to hyperbolic quasivariational inequalities with the viscosity term arising in dynamic viscoelastic frictional contact problem, where the friction contact is modeled by the Coulomb law of dry friction, where we use a fixed point theorem [5] to find a displacement field $u : [0, T] \to V$ such that for almost every $t \in [0, T]$

Here, V is a Banach space of admissible displacements, and we introduce A and B are operators related to the viscoelastic constitutive law, φ is a convex functional related to contact boundary conditions, and . The function f represents the given body forces and surface traction, and u_0, u_1 represents the initial displacement and velocity, respectively. Moreover, the derivative of displacement u' is with respect to the time variable t, the interval of interest is $[0, T]$.

2. Existence of solution

In this section , we consider an evolution of triple spaces $V \subset H \subset V^*$, where V is a strictly convex, reflexive and separable Banach space, H is a separable Hilbert space. For $T > 0$ and $2 \leq p < +\infty$ and q denote its conjugate exponent. Define a vector-valued space $\mathcal{W} = \{w \in L^p(0, t; V) | w' \in L^q(0, t; V^*)\}$, where $v' = \partial v/\partial t$ is the time derivative in the sense of vector-valued distributions.

We make the following assumptions:

H(A): The operator $A : [0, T] \times V \to 2^{V^*}$ is such that

A1 $A(., v) : [0, T] \times V \to 2^{V^*}$ is measurable for all $v \in V$;

A2 $A(t,v)$ is maximal monotone for $a.e.t \in [0,T]$; there exist $a_1 \in L^q(0,T)$ and a constant $c_1 > 0$ such that for all $v \in V$

$$\|\xi(t)\|_{V^*} \leq a_1(t) + c_1\|\xi(t)\|_V^{p-1}, \ \ \forall \ \xi(t) \in A(t,v) \ \text{ and } a.e \ t \in [0,T]$$

A3 there exist $a_2 \in L^1_+(0,T)$ and $c_2 > 0$ such that for all$v \in V$

$$\langle \xi(t), v\rangle \geq c_2\|v\|_V^p - a_2(t), \ \ \forall \ \xi(t) \in A(t,v) \ \text{ and } a.e \ t \in [0,T]$$

H(B): $B : V \to V^*$ is linear, bounded, symmetric and monotone, i.e.,
B1 $B \in \mathcal{V}, \mathcal{V}^*$ and $\forall \ v \in V, \ \|B(v)\|_{V^*} \leq c_3\|v\|_V$ with $c_3 > 0$;
B2 $\langle B(v), v\rangle = \langle B(v), v\rangle \geq 0$, $\forall u, v \in V$

H(φ): $\varphi : M \times M \to \mathbb{R}$ is such that
(φ1) $\varphi(., u)$ is measurable for all $u \in M$ and $\varphi(t, .)$ is proper, convex and lower semicontinuous for a.e. $t \in [0,T]$
(φ2) there exist a function $a_3 \in L^q(0,T)$ and $c_4 > 0$ such that

$$\|\eta\|_{M^*} \leq a_3(t) + c_4\|v\|_M^{p-1}, \ \forall v \in M, \ \eta \in \partial\varphi(t,v) \ \ a.e.t \in [0,T]$$

H(f): $f \in L^q(0,T;V^*)$ and $u_0 \in V$.

We have the following theorem of existence.

Theorem 2.1. *Assume that assumptions H(A),H(B)and H(f) holds. Then the hyberbolic quasivariational inequality has a solution.*

The proof of Theorem 2.1 is based on three basic steps.

(1) We consider an auxiliary problem with parameter ω of finding $u \in W^{1,p}(0;T;V)$ which represent a class of evolutionary variational inequalities which solved in [1].
(2) We define an operator S which we prove it has a fixed point by using Kakutani fixed point theorem.
(3) then we conclude that the hyperbolic quasivariational inequality has a solution.

3. Dynamic contact frictional problem

In this section, we consider a dynamic frictional problem, and we prove the existence and uniqueness of weak solution by using the abstract result in Section 2, then we introduce frictional dynamic contact models for viscoelastic materials.

4. Conclusion

As conclusion, it is evident that this study has shown that the hyperbolic quasivariational inequality a solution in the space $W^{1,p}(0;T;V)$ Further study of the issue would be of interest when the viscosity term is vanished in Problem 1 in order to obtain an existence result for an elastodynamic Signorini problem with Coulomb friction law.

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