ECHAHID HAMMA LAKHDAR UNIVERSITY - EL-OUED
Under the Supervision of the DGRSDT and in collaboration with the CRTI
International Pluridisciplinary PhD Meeting (IPPM'20)
23-26, 2020 1st Edition, February
Theme: Modern Technology and Fineness Life

**IPPM templat**

# ON THE TOPOLOGICAL DEGREE AND ITS APPLICATIONS IN SOME NONLINEAR BOUNDARY VALUE PROBLEMS

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## Abstract

. The aim of this thesis is to study certain essential properties of the topological degree for compact perturbations of identity or the compact perturbation relative to a Fredholm operator of index zero. This theoretical part which will occupy the first part of the thesis will start with reminders on the topological degree of Braouer and of Leray - Schauder and its fundamental properties. The second part will be entirely devoted to the applications of this tool to the resolution of nonlinear boundary value problems associeted with ordinary diferential equations or fractional.

**Keywords :** topological degree-Fredholm operator- Homotopie-Boundary value problems

## 1-Intoduction

the topological degree is a powerful methode for solving non linear boundary value problem. For instance, the fixed point théorems : of Brouwer and of Schauder are prven using degree theory. this paper will present some essential properties of the topological degree of Brouwer and Leray Schauder and its various applications in the resolution of nonlinear BVP.

## 2- Definitions and preliminaries

### 2-1 Construction of Brouwer Degree

**Definition** : Let $\Omega$ be open and bounded subset of $\mathbb{R}^n$ and $f \in C^1(\Omega)$ if $y \notin f(\partial\Omega)$ and $J_f(y) \neq 0$, then we define:

$$\deg(f,\Omega,y) = \sum_{x \in f^{-1}(y)} \operatorname{sgn} J_f(x)$$

Where deg $(f,\Omega,y)$=0 if $f^{-1}(y)=\varnothing$.

**Theorem** : Let $\Omega\subset\mathbb{R}^n$ be an open bounded subset and $f:\Omega\to\mathbb{R}^n$ be a continuous mapping. If $y\notin f(\partial\Omega)$, then there exists an integer deg(f,Ω,y) satisfying the following properties :

**(1) (Normality)** deg(I,Ω,y)=1 if and only if y∈Ω where I denotes the identity mapping

**(2) ( Solvability)** If deg(f,Ω,0)≠0 then f(x)=0 has a solution in Ω.

**3)(Homotopy**) if $f_t(x):[0,1]\times\overline{\Omega}\to\mathbb{R}^n$ is continuous and $y\notin f_t(\partial\Omega)$ then deg( $f_t$,Ω,y) does not depend on t∈ $[0,1]$

**(4) (Additivity)** Suppose that $\Omega_1,\Omega_2$ are two disjoint open subsets of Ω and $y\notin f(\Omega\backslash\Omega_1\cup\Omega_2)$:

$$\deg(f,\Omega,y)=\deg(f,\Omega_1,y)+\deg(f,\Omega_2,y)$$

### Brouwer fixed point theorem

Let $C\subset\mathbb{R}^n$ be a nonempty bounded closed convex subset and f:C→C be a continuous mapping. Then f has a fixed point in C.

## 2-2 Construction of Leray Schauder Degree

### Compact mapping

**Definition**: Let E be a real normed space. A mapping T:D(T)⊂E→E called compact if T maps every bounded subset of D(T) to a relatively compact subset in E. T is said to be completely continuous if T is continuous and compact.

**Lemma** : Let E be a real Banach space, Ω⊂E be an open bounded subset and T:Ω→E be a continuous compact mapping. Then, for any ε≻0, there exist a finite dimensional space F and a continuous mapping $T_\varepsilon:\overline{\Omega}\to F$ such that

$$\|T_\varepsilon(x)-T(x)\| \prec \varepsilon \text{ for all } x\in\overline{\Omega}$$

**Definition** : Let E be a real Banach space, Ω⊂E be an open bounded set and T:Ω→E be an continuous compact mapping now if $y\notin(I-T)(\partial\Omega)$ we define :

$$\deg(I-T,\Omega,y)=\deg(I-T_\varepsilon,\Omega,y)$$

**Theorem :** The Leray Schauder degree has a follouing proprieties :

**(1) (Normality)** deg(I,Ω,y)=1 if and only if y∈Ω where I denotes theidentity mapping

**(2)( Solvability)** If deg(I-T,Ω,0)≠0 then Tx=x has a solution in Ω

**(3)(Homotopy**) If $T_t(x)$ :[0.1]×Ω→E is continuous compact and $T_t(x)\neq x$ then deg(I-T,Ω,0) does not depend on t∈[0.1]

**Theorem ( Schauder fixed point)** : Let C⊂E be a nonempty bounded closed convex subset and T:E→E be a continuous compact mapping. Then T has a fixed point in C.

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