2-Orthogonal Polynomials and Darboux Transforms

Chadia Faghmous[1] , Mohamed Cherif Bouras[2] and Karima Ali Khelil[3]

[1]chfaghmous21@gmail.com [2]bourascdz@yahoo.fr

[3]kalikhelil@gmail.com

[1,2,3]LANOS , Badji Mokhtar University of Annaba, P. O. Box 12
23000 Annaba, Algeria

Abstract

In this work we present a new interpretation of Darboux transforms in the context of 2-orthogonal polynomials and find conditions in order for any Darboux transform to yield a new set of 2- orthogonal polynomials. We also introduce the LU and UL factorizations of the monic Jacobi matrix associated with a quasi-definite linear functional defined on the linear space of polynomials with real coefficients.

In 2004, M. I. Bueno, F. Marcellán [2], introduced the LU and UL factorizations of a tridiagonal matrix J, as well as the transformation of Darboux and the Darboux transformation without parameters. They also show how to find the tridiagonal matrix J_1^n associated with the linear functional $\Gamma_1 = x\ \Gamma$ in terms of the matrix J by the application of the Darboux transformation without parameters.

The main purpose of this work is to present a new interpretation of Darboux transforms in the context of $2-$ orthogonal polynomials.

Keywords: 2 orthogonal polynomials, linear functional, Darboux transformation

References

[1] S. Bochner, "Uber Sturm-Liouvillesche Polynomsysteme", *Math. Z. vol. 29, pp. 730–736,1929.*

[2] M. I. Bueno, F. Marcellán, "Darboux transformation and perturbation of linear functionals", *Linear Algebra and its Applications, Vol. 384, pp. 215-242,1 June 2004 .*

[3] K. H. Kwon, L. L. Littlejohn, and B. H. Yoo, Characterizations of orthogonal polynomials satisfying differential equations, SIAM J. Math. Anal. 25 (1994), no. 3, 976–990.

[4] K. H. Kwon, L. L. Littlejohn, and G. J. Yoon, Orthogonal Polynomial Solutions of Spectral Type Differential Equations: Magnus' Conjecture, J. Approximation Theory 112 (2001), no. 2, 189–215.