

# On the asymptotic convergence of different boundary problems in a thin layer

Tikialine Belgacem \*, Hadj Ammar Tidjani \*\*, Yassine Letoufa\*\*\*

\* Mathematics departement, LABTHOP: Operator theory laboratory, El-Oued University, Algeria

\*\* Mathematics departement, LABTHOP: Operator theory laboratory, El-Oued University, Algeria

\*\*\* Mathematics departement, LABTHOP: Operator theory laboratory, El-Oued University, Algeria

E-mail: [tikialine44@gmail.com](mailto:tikialine44@gmail.com) , [hat\\_olsz@yahoo.com](mailto:hat_olsz@yahoo.com) , [letoufa54@gmail.com](mailto:letoufa54@gmail.com)

## Abstract

The object of the doctoral thesis is the study of the asymptotic behavior of some boundary problems modeling the behavior of fluids or of different materials in the dynamic and stationary case and this in a bounded domain in dimension three in thin film with the conditions of friction of Tresca.

One of the goals of asymptotic analysis is to obtain an equation in dimension 3 of space (3D) which allows a reasonable description of the phenomenon occurring in a three-dimensional (3D) domain. This process is implemented by moving to the limit towards 0 over the thickness of the supposedly thin (3D) domain.

In the first part of the thesis we will only consider viscoelastic anisotropic materials given by the linear behavior law Kelvin-Voigt, and in the second part it will be a question of seeing to what extent we can generalize the results to the nonlinear case of materials viscoelastic. We are also interested in this doctoral thesis in the asymptotic analysis of thermal flow in a thin film of an incompressible Non-Newtonian fluid whose viscosity depends on temperature. In this case, the mechanical problem leads to a coupled mathematical model, formed from the equation of motion and the equation of energy conservation.

**Keywords:** Boundary conditions, A priori estimates, Viscoelastic bodies, Contact with friction, Reynolds equation, Non-Newtonian fluid.

## Description of the thesis objectives

In this doctoral thesis, we are interested in the asymptotic behavior of certain stationary or dynamic problems for different systems related to elasticity theory or associated with fluid mechanics. More specifically, Kelvin-Voigt model for viscoelastic material, nonlinear viscoelasticity, non-Newtonian fluid, Stokes, Navier Stokes, and that in bounded domains in three dimensions in thin layers with the different nonlinear friction conditions on the edge . We will first consider the isothermal case and then we will try to generalize to the non-isothermal case, in which case the equation of conservation of motion is combined with the equation of conservation of energy deduced from Fourier's law. The objective is to give the variational formulation of the problem considered for which we demonstrate the existence and the uniqueness of the solution. Then we will study the asymptotic analysis of these problems by making a change of scale, in order to bring the study back to a new domain independent of the parameter.

We then look for necessary a priori independent estimates of the parameters of these systems, which will allow us to give the essential convergence theorems allowing to pass to the limit when the thin thickness of the domain (3D) tends towards zero. To close the problem and determine a specified limit model, it must be proven the uniqueness of the limit solution of the limit problem which already obtained. In the same context, we are looking for a weak form of the generalized Reynolds equation of space (2D) associated with .the variational inequality of each problem

We also approach in this thesis a model of transmission problem between two three-dimensional domains in bilateral contact, subject to different laws of behavior, and it is subjected to different volume forces. In this case, we therefore have a difficulty induced by the absence of the symmetry hypotheses that are most often encountered in nature and industry than in the problem of coating. In this framework, we describe the position of the problem and the boundary conditions, concerning the speed field, and the stress field which allow us to make the variational formulation of the starting mechanical problem, then we study the existence and the uniqueness of weak solution of the problem. Then, one seeks to know the asymptotic behavior of this system taking into account the small thickness of the flow. Finally, we formulate a family of Reynolds equations dependent on system data introduced in the boundary conditions, which allows us to .determine the limit field on a two-dimensional space

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